Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 7 Version 1.0 June 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

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GJSFR-F Classification MSC 2010: 34L30.

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A Comparison of Adomian's Decomposition Method and Picard Iterations Method in Solving Nonlinear Differential Equations

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Abstract - In this paper,nonlinear differential equations are solved through Adomian decomposition method(ADM) and the results are compared with those of Picard iterations method. It is noted that ADM takes the form of a convergent series with easily computable components. The ADM method is able to solve large class of nonlinear equations effectively, more easily and accurately; and thus the method has been widely applicable to solve any class of equations in sciences and engineering.

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I. Introduction

Large classes of linear differential equations can be solved by the Adomian decomposition method $[1,2,3,5]$ and Picard iteration method $[9]$. The ADM was first compared with the Picard method by Rach $[4]$ and Bellomo and Sarafyan $[6]$ on a number of examples.Golberg$[7]$ showed that the Adomian approach to llinear differential equations was equivalent to the classical method of successive approximations (Picard method). However,we shall show that the equivalence does not hold for nonlinear differential equations. We consider the generalized first order nonlinear differential equations in the form:

$$y' = f(t,y), \quad y \in R^d, \quad f : R\,\mathrm{x}R^d \to R^d, \qquad 1.1$$

With initial condition $y(0) = y_0 \in R^d$. We shall proceed to discuss the basic concepts of the two methods.

II. Derivation of The Methods

In reviewing the basic methodology, we consider an abstract system of differential equation (1.1) and assume that $f(t,y)$ is nonlinear and analytic near $y = y_0, t = 0$. It is equivalent to solve the initial value problem (1.1) and the Voltera integral equation of first kind,

$$y(t) = y_0 = \int_0^t f(s, y(s))\,ds \qquad 1.2$$

To set up the Adomian method, Rach [4] write (1.1) in an operator form:

$$Ly = f(t,y), \qquad 1.3$$

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Ref.

1. Ibijola E.A and Adegboyegun B.J on AdomianDecompositoon Method (ADM) for Numerical Solution of Ordinary Differential Equation. Advances in Natural Applied Sciences, 2(3): 165 – 169, 2008 U.S

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Where $L=\frac{d}{dt}$.He applied L^{-1} (a one fold integral) on (1.3) and imposed the initial condition to obtain:

$$y(t)=y_0+L^{-1}f(t,y)$$

Adomian considered the solution $y(t)$ in the series form:

$$y(t)=y_0+\sum_{n=1}^{\infty}y_n \tag{1.4}$$

And write the nonlinear $f(t,y)$ as the series of function

$$f(t,y)=\sum_{n=0}^{\infty}A_n(t,y_0,y_1,\cdots y_n) \tag{1.5}$$

The dependence of A_n on t and y_0 may be non-polynomial. Formally, A_n is obtained by

$$A_n=\frac{1}{n!}\frac{d^n}{d\ell^n}f(t,\sum_{i=0}^{\infty}\ell^i y_i)|_{\ell=0}, \quad n=0,1,2\cdots \tag{1.6}$$

Where ℓ is a formal parameter. Functions A_n are polynomials in $(y_1,y_2,\cdots y_n)$, which are referred to as the Adomian polynomials. The first few Adomian for $d=1$ are listed by Zhu and Chang [8] as follows:

$$A_0=f(t,y_0)$$

$$A_1=y_1f^{'}(t,y_0)$$

$$A_2=y_2f^{'}(t,y_0)+\frac{1}{2}y_1^2f^{''}(t,y_0)$$

$$A_3=y_3f^{'}(t,y_0)+y_1y_2f^{''}(t,y_0)+\frac{1}{6}y_1^3f^{'''}(t,y_0)$$

where primes denote the partial derivatives with respect to y.

It was shown by Cherruault and Abbaoui [3] that the Adomian polynomials A_n are defined by the explicit formulae:

$$A_n=\sum_{k=1}^{n}\frac{1}{k!}f^k(t,y_0)\left(\sum_{p_1+\cdots+p_k=n}y_{p_1}\cdots y_{p_k}\right), \quad n\geq 1 \tag{1.7}$$

Khelifa and Cherruault [8] proved a bound for Adomian polynomials by,

$$|A_n|\leq\frac{(n+1)^n}{(n+1)!}M^{n+1} \tag{1.8}$$

Where $\sup_{t\in J}|f^{(k)}(t,y_0)|\leq M$ for a given time interval $J\subset R$.

By substituting (1.4) and (1.5) into (1.2) gives a recursive equation for y_{n+1} in terms of $(y_0,y_1,y_2,\cdots y_n)$:

$$y_{n+1}=\int_0^t A_n(s,y_0(s),y_1(s),\cdots y_n(s))ds, \quad n=0,1,2,\cdots \tag{1.9}$$

The Picard's method is used for the proof of existence and uniqueness of solutions of a system of differential equations. The method starts with analysis of Volterra's integral equation (1.2). Assume that $f(t,y)$ satisfies a local Lipschitz condition in a ball around

Ref.

8. Y. Zhu, Q. Chang, and S. Wu, "A new algorithm for calculating Adomian Polynomials", Appl. Math. Comput., 169, 402-416 (2005).
3. N. Himoun, K. Abbaoui, and Y. Cherruault, "Short new results on Adomian method",Kybernetes, 32, 523-539 (2003).

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$t=0$ and $y=y_0$:

$$\forall|t|\le t_0,\ \forall|\tilde{y}-y_0|\le\delta_0:\quad |f(t,y)-f(t,\tilde{y})|\le K|y-\tilde{y}|, \qquad 1.10$$

Where K is lipshitz constant and $|y|$ is any norm in R^d

Let $y^{(0)}=y_0$ and define a recurrence relation

$$y^{(n+1)}(t)=y_0+\int_0^t f(s,y^n(s))\,ds,\qquad n=0,1,2,\cdots \qquad 1.11$$

If t_0 is small enough, the new approximation $y^{(n+1)}(t)$ belongs to the same ball $|y-y_0|\le\delta_0$ for all $|t|\le t_0$ and (1.11) is a contraction in the sense that

$$|\int_0^t[f(s,y(s))-f(s,\tilde{y}(s))]d\,s|\le Q\sup_{|t|\le t_0}|y(t)-\tilde{y}(t)|, \qquad 1.12$$

Where $Q=Kt_0<1,$ so that $t_0<\frac{1}{k}$.

By the Banach fixed point theorem, there exists a unique solution $y(t)$ in $C([-t_0,t_0],B_{\delta_0}(y_0))$ where $B_{\delta_0}(y_0)$ is an open ball in R^d centered at y_0 with radius δ_0. The computations of Picard iterative algorithm were reported recently by Youssef [9].

III. Applications and Results

Since the problems given in the article at reference [9] on the computations and analysis of Picard's iterative scheme are relevant for our aim, we reuse some of the problems.

Problem 1

Consider a scalar first order ODE,

$$\frac{dy}{dt}=y^p,\quad y(0)=1 \qquad 1.13$$

where $p\ge1$. This differential equation has exact solution;

$$y(t)=\frac{1}{(1-(p-1)t)^{\frac{1}{p-1}}} \qquad 1.14$$

Following the Adomian method, we write (1.13) as $y(t)=1+\int_0^t y^p(s)\,ds$ and compute the Adomian polynomials from $f(t,y)=y^p$ in the form:

$$A_0=y_0^p,$$

$$A_1=py_0^{p-1}y_1,$$

$$A_2=\frac{p(p-1)}{2}y_0^{p-2}y_1^2+py_0^{p-1}y_2,$$

$$A_3=\frac{p(p-1)(p-2)}{6}y_0^{p-3}y_1^3+p(p-1)y_0^{p-2}y_1y_2+py_0^{p-1}y_3,$$

Ref.

9. I. K. Youssef, "Picard iteration algorithm combined with Gauss-Seidel technique for initial value problems", Applied mathematics and computation, 190, 345-355 (2007).

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Using (1.9), we determine few terms of the Adomian series:

$y_0(t) = 1$

$y_1(t) = t$

$y_2(t) = \dfrac{pt^2}{2}$

$y_3(t) = \dfrac{p(2p-1)t^3}{3!}$

$y_4(t) = \dfrac{p(6p^2-7p+2)t^4}{4!}$

From (1.4)

$$y(t) = y_0 + y_1 + y_2 + y_3 + y_4 + \cdots = 1 + t + \frac{pt^2}{2} + \frac{p(2p-1)t^3}{3!} + \frac{p(6p^2-7p+2)t^4}{4!} + \cdots$$

Expanding (1.13) in a power series of t, we can see that theAdomian method recovers the power series solution $y(t) = \dfrac{1}{(p-(p-1)t)^{\frac{1}{p-1}}}$

$$= 1 + t + \frac{pt^2}{2} + \frac{p(2p-1)t^3}{3!} + \frac{p(6p^2-7p+2)t^4}{4!} + 0(t^5)$$

Using the Picard iterations method,

$$y^{(n+1)} = 1 + \int_0^t (y^{(n)}(s))^p \, ds,$$

We obtain successive approximations in the form:

$y^{(0)} = 1$

$y^{(1)} = 1 + t$

$y^{(2)} = \dfrac{p}{1+p} + \dfrac{(1+t)^{1+p}}{1+p}$

$y^{(3)} = 1 + \displaystyle\int_0^t (y^{(2)})^p ds$

It is noted from $y^{(2)}$ that Picard iteration mix up powers of t which make $y^{(n)}$ being different from the nth partial sum of the power series.If $p = 2$, then

$y^{(0)} = 1 = y_0$,

$y^{(1)} = 1 + t = y_0 + y_1$,

$y^{(2)} = 1 + t + t^2 + \dfrac{t^3}{3} = y_0 + y_1 + y_2 + \dfrac{t^3}{3}$,

Notes

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Since $\sum_{i=0}^{n} y_i(t)$ is a partial sum of the power series (1.13), we conclude that the Adomian method better approximates the exact power series solution compared to the Picard method.

Problem 2

Notes

Consider the nonlinear differential equation,

$$\frac{dy}{dt} = 2y - y^2, \quad y(0) = 1 \qquad 1.15$$

with the exact solution $y(t) = 1 + \tanh(t)$

By the ADM, we write (1.15) in the integral form $y(t) = 1 + \int_0^t \left(2y(s) - y^2(s)\right) ds$ and compute the Adomian polynomials for $f(y) = 2y - y^2$ in the form;

$A_0 = 2y_0 - y_0^2$,

$A_1 = 2y_1 - 2y_0 y_1$,

$A_2 = 2y_2 - 2y_0 y_2 - y_1^2$,

$A_3 = 2y_3 - 2(y_0 y_3 + y_1 y_2)$,

$A_4 = 2y_4 - 2(y_0 y_4 + y_1 y_3) - y_2^2$.

Using (1.9) we determine few terms of the series;

$$y_0 = 1; \quad y_1 = t; \quad y_2 = 0; \quad y_3 = \frac{-t^3}{3}; \quad y_4 = 0; \quad y_5 = \frac{2t^5}{15}.$$

Thus, $y(t) = 1 + t - \frac{t^3}{3} + \frac{2t^5}{15} - \cdots = 1 + \tanh(t)$

By using Picard method, we write (1.15) in the integral form, $y^{n+1} = 1 + \int_0^t \left(2y^{(n)}(s) - (y^{(n)}(s))^2\right) d$ and we obtain the successive approximations in the form;

$y^{(0)} = 1 = y_0$,

$y^{(1)} = 1 + t = y_0 + y_1$,

$y^{(2)} = 1 + t - \frac{t^3}{3} = y_0 + y_1 + y_2 - \frac{t^3}{3}$,

$y^{(3)} = 1 + t - \frac{t^3}{3} + \frac{2t^5}{15} - \frac{t^7}{63} = y_0 + y_1 + y_2 + y_3 + \frac{2t^5}{15} - \frac{t^7}{63}$

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We can equally see from $y^{(2)}$ and $y^{(3)}$ that Picard method mix up powers of t which also make $y^{(n)}$ being different from the nth partial sum of the series.

IV. Conclusion and Discussion

In this paper, ADM has been successfully applied to finding the solutions of nonlinear ODE. The obtained results are compared with those of Picard iterations method. It is noted that the Picard method mixes up powers of the partial sum for the exact solutions, while the Adomian series is equivalent to the power series in time and Adomian method requires analyticity of $f(t, y)$,which is more restrictive than the Lipschitz condition required for the Picard method.The results show that ADM is a powerful mathematical tool for solving nonlinear differential equations, and therefore, can be widely applied in the field of science and engineering.

Notes

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