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An Integral Associated With H-Function of Several Complex Variables

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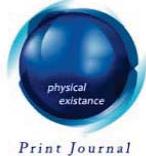
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An Integral Associated With H-Function of Several Complex Variables

Rakeshwar Purohit^a, Ashok Singh Shekhawat^a & Jyoti Shaktawat^b

Abstract - The object of present paper is to derive an integral pertaining to a product of Fox's H-function [1], generalized polynomials Srivastava [7], general class of multivariable polynomials Srivastava and Garg [8] and H-function of multivariables given by Srivastava and Panda [9] with general arguments of quadratic nature. This paper is capable of yielding numerous result involving classical orthogonal polynomials hitherto scattered in the literature.

Keywords : fox's H-function, general polynomials, general class of multivariable polynomials, multivariable H-function, generalized Lauricella function, G-function.

I. INTRODUCTION

The H-function of multivariable is defined by Srivastava and Panda [9] as:

$$H[x_1, \dots, x_r] = H_{A,C:[B';D];\dots;[B^{(r)},D^{(r)}]}^{0,\lambda:(u',v);\dots;(u^{(r)},v^{(r)})} \left[\begin{matrix} [(p):\theta;\dots;\theta^{(r)}]:[(q):\Delta];\dots;[(q^{(r)}):\Delta^{(r)}]; \\ (s):\psi';\dots;\psi^{(r)}]:[(t):\delta];\dots;[(t^{(r)}):\delta^{(r)}]; \end{matrix}; x_1, \dots, x_r \right] \quad (1.1)$$

The Fox's H-function [1]:

$$H_{P,Q}^{L,R} \left[x \begin{pmatrix} (m_P, M_P) \\ (n_Q, N_Q) \end{pmatrix} \right] = \sum_{G=0}^{\infty} \sum_{g=1}^L \frac{(-1)^G}{G! N_g} \phi_{(\eta_G)} x^{\eta_G}, \quad (1.2)$$

where

$$\phi_{(\eta_G)} = \frac{\prod_{j=1}^L \Gamma(n_j - N_j \eta_G) \prod_{j=1}^R \Gamma(1 - m_j + M_j \eta_G)}{\prod_{j=L+1}^Q \Gamma(1 - n_j + N_j \eta_G) \prod_{j=R+1}^P \Gamma(m_j - M_j \eta_G)}$$

and

$$\eta_G = \frac{(\eta_g + G)}{\eta_g}$$

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The H-function of multivariable in (1.1) converges absolutely if

$$\left| \arg(x_i) \right| < \frac{1}{2} \pi T_i, \quad (1.3)$$

where

$$T_i = - \sum_{j=1+\lambda}^A \theta_j^{(i)} + \sum_{j=1}^{v^{(i)}} \Delta_j^{(i)} - \sum_{j=1+v^{(i)}}^{B^{(i)}} \Delta_j^{(i)} - \sum_{j=1}^C \Psi_j^{(i)} + \sum_{j=1}^{u^{(i)}} \delta_j^{(i)} - \sum_{j=1+u^{(i)}}^{D^{(i)}} \delta_j^{(i)} > 0, \quad \forall i \in (1, \dots, r) \quad (1.4)$$

Srivastava and Garg introduced and defined a general class of multivariable polynomials [8] as follows

$$S_E^{F_1, \dots, F_s}[z_1, \dots, z_s] = \sum_{k_1, \dots, k_s=0}^{F_1 k_1 + \dots + F_s k_s \leq E} (-E)_{F_1 k_1 + \dots + F_s k_s} A(E; k_1, \dots, k_s) \frac{(z_1)^{k_1}}{k_1!} \cdots \frac{(z_s)^{k_s}}{k_s!} \quad (1.5)$$

where F_1, \dots, F_s are imaginary positive integer and the coefficients $A(E; k_1, \dots, k_s)$, $(E; k_i \geq 0, i = 1, \dots, s)$ are arbitrary constants, real or complex.

Srivastava has defined and introduced the general polynomials ([7], p.185, eq.(7))

$$S_{N_1, \dots, N_s}^{M_1, \dots, M_s}[z_1, \dots, z_r] = \sum_{\beta_1=0}^{[N_1/M_1]} \cdots \sum_{\beta_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1 \beta_1}}{\beta_1!} \cdots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \\ \cdot B[N_1, \beta_1; \dots; N_s, \beta_s] z_1^{\beta_1} \cdots z_s^{\beta_s}, \quad (1.6)$$

where $N_i = 0, 1, 2, \dots$, $\forall i = (1, \dots, s)$; M_1, \dots, M_s are arbitrary positive integers and the coefficients $B[N_1, \beta_1; \dots; N_s, \beta_s]$ are arbitrary constants, real or complex.

II. THE MAIN RESULT

We shall establish the following result:

$$\int_0^\infty y^{1-\beta} (p + qy + sy^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{y}{p + qy + sy^2} \right)^\sigma \middle| (m_p, M_p) \right. \\ \left. \cdot S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right] \right]$$

Ref.

8. H.M. Srivastava and M. Garg, Some integrals involving a general class of polynomials and the multivariable H-function, Rev. Roumaine Phys., 32 (1987), 685-692.

Notes

$$\cdot S_E^{F_1, \dots, F_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right]$$

$$\cdot H \left[x_1 \left(\frac{y}{p + qy + sy^2} \right)^{\sigma_1}, \dots, x_r \left(\frac{y}{p + qy + sy^2} \right)^{\sigma_r} \right] dy$$

$$= \sqrt{\frac{\pi}{c}} \sum_{G=0}^{\infty} \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{k_1, \dots, k_s=0}^{F_1 K_1 + \dots + F_s K_s \leq E} \frac{(-1)^G}{G! F_g} \frac{(-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \Phi(\eta_G)$$

$$(-E)_{F_1 k_1 + \dots + F_s k_s} B[N_1, \beta_1; \dots; N_s \beta_s] A[E; k_1, \dots, k_s] \frac{z_1^{(\beta_1+k_1)}}{k_1!} \dots \frac{(z_s)^{(\beta_s+k_s)}}{k_s!}$$

$$(q + 2\sqrt{sp})^{\beta - \sigma \eta_G - \sum_{i=1}^s n_i(\beta_i + k_i) - 1}$$

$$\cdot H^{0, \lambda+1: (u', v'); \dots; (u^{(r)}, v^{(r)})}_{A+1, C+1: [B', D']; \dots; [B^{(r)}, D^{(r)}]} \left[\begin{array}{l} x_1 (q + 2\sqrt{sp})^{-\sigma_1} \\ \vdots \\ x_r (q + 2\sqrt{sp})^{-\sigma_r} \end{array} \right] \left[\begin{array}{l} [\beta - \sigma \eta_G - \sum_{i=1}^s n_i(\beta_i + k_i); \sigma_1; \dots; \sigma_r], \\ [(s); \Psi'; \dots; \Psi^{(r)}], \end{array} \right]$$

$$\left. \left[\begin{array}{l} [(p); \theta'; \dots; \theta^{(r)}] \\ : [(q'); \Delta'] ; \dots ; [(q^{(r)}); \Delta^{(r)}] \end{array} \right] \right]_{[\beta - \sigma \eta_G - \sum_{i=1}^s n_i(\beta_i + k_i) - \frac{1}{2}; \sigma_1; \dots; \sigma_r]; [(t'); \delta'] ; \dots ; [(t^{(r)}); \delta^{(r)}]} \quad (2.1)$$

provided that $\operatorname{Re}(p) > 0$, $\operatorname{Re}(q) > 0$, $s > 0$ and

$$\sigma \min \left[\operatorname{Re} \left(\frac{n_j}{N_j} \right) \right] + \sum_{i'=1}^r \sigma'_{i'} \min \left[\operatorname{Re} \left(\frac{t^{(i')}}{\delta_{j'}^{(i')}} \right) \right] > \beta - 2, \quad j = 1, \dots, M \text{ and } j' = 1, \dots, u^{(i')}.$$

Proof:

In order to prove (2.1) first we express the Fox H-function and a general polynomials in form of series and the H-function of multivariable in terms of Mellin-Barnes contour integrals. Now interchanging the order of summations and integration which is permissible under the stated condition, we obtain

$$\sum_{G=0}^{\infty} \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{k_1, \dots, k_s=0}^{F_1 K_1 + \dots + F_s K_s \leq E} \frac{(-1)^G (-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G)$$

$$\cdot (-E)_{F_1 k_1 + \dots + F_s k_s} B[N_1, \beta_1; \dots; N_s \beta_s] A[E; k_1, \dots, k_s] \frac{(z_1)^{(\beta_1+k_1)}}{k_1!} \dots \frac{(z_s)^{(\beta_s+k_s)}}{k_s!}$$

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$$\cdot \frac{1}{(2\pi i)^r} \int_{I_1} \dots \int_{I_r} \psi(\xi \gamma_1, \dots, \gamma_r) \Delta_1(v_1) \dots \Delta_r(\gamma_r) x_1^{\gamma_1} \dots x_r^{\gamma_r}$$

$$\left. \cdot \left\{ \int_0^\infty y^{1 - [\beta - \sigma \eta_G - \sum_{i=1}^s n_i(\beta_i + k_i) - \sigma_1 \gamma_1 - \dots - \sigma_r \gamma_r]} \right. \right. \\ \left. \left. \cdot (p + qy + sy^2)^{(\beta - \sigma \eta_G - \sum_{i=1}^s n_i(\beta_i + k_i) - \sigma_1 \xi \gamma_1 - \dots - \sigma_r \gamma_r) - \frac{3}{2}} dy \right\} d\gamma_1 \dots d\gamma_r, \right. \quad (2.2)$$

On solving above y-integral with the help of known theorem (Saxena [6]) and reinterpreting the result obtained in terms of H-function of r variable, we reached at the desired result.

III. SPECIAL CASES

(a) If $\lambda = A$, $u^{(i)} = 1$, $v^{(i)} = B^{(i)}$ and $D^{(i)} = D^{(i)} + 1$, $\forall i \in (1, \dots, r)$ the result in (2.1) reduces to the following integral transformation:

$$\int_0^\infty y^{1-\beta} (p + qy + sy^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{y}{p + qy + sy^2} \right)^\sigma \middle| (m_P, M_P) \right. \\ \left. (n_Q, N_Q) \right]$$

$$\cdot S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right]$$

$$\cdot S_E^{F_1, \dots, F_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right]$$

6. R.K.Saxena, An Integral Involving G-Function, Proc. Nat. Inst. Sci. India, 26A(1960), 661-664.

Ref.

$$\cdot E_{C:D';...;D^{(r)}}^{A:B';...;B^{(r)}} \left[-x_1 \left(\frac{y}{p + qy + sy^2} \right)^{\sigma_1}, \dots, -x_r \left(\frac{y}{p + qy + sy^2} \right)^{\sigma_s} \middle| \begin{matrix} [1-(p):\theta';...;\theta^{(r)}] \\ [1-(s):\Psi';...;\Psi^{(r)}] \end{matrix} \right]$$

$$\left. :[1-(q'):\Delta'];...;[1-(b^{(r)}):\Delta^{(r)}] \atop [1-(t'):\delta'];...;[1-(d^{(r)}):\delta^{(r)}] \right] dy$$

$$= \sqrt{\frac{\pi}{c}} \sum_{G=0}^{\infty} \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{k_1,...,k_s=0}^{F_1 K_1 + \dots + F_s K_s \leq E} \frac{(-1)^G}{G! F_g} \frac{(-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G)$$

$$\cdot (E)_{F_1 k_1 + \dots + F_s k_s} B(N_1, \beta_1; \dots; N_s \beta_s)$$

$$\cdot A[E; k_1, \dots, k_s] \frac{z_1^{(\beta_1+k_1)}}{k_1!} \dots \frac{(z_s)^{(\beta_s+k_s)}}{k_s!} (q + 2\sqrt{sp})^{\beta - \sigma \eta_G - \sum_{i=1}^s n_i(\beta_i+k_i)-1}$$

$$\frac{\Gamma(1-\beta + \sigma \eta_G + \sum_{i=1}^s n_i(\beta_i+k_i))}{\Gamma(\frac{3}{2}-\beta + \sigma \eta_G + \sum_{i=1}^s n_i(\beta_i+k_i))} E_{C+1:D';...;D^{(r)}}^{A+1:B';...;B^{(r)}}$$

$$\cdot \left[-x_1 (q + 2\sqrt{sp})^{-\sigma_1}, \dots, -x_r (q + 2\sqrt{sp})^{-\sigma_r} \right] \left. \begin{matrix} [1-\beta + \sigma \eta_G + \sum_{i=1}^s n_i(\beta_i+k_i):\sigma_1;...;\sigma_r], \\ [1-(s):\Psi';...;\Psi^{(r)}], \end{matrix} \right]$$

$$\left. \begin{matrix} [1-(p):\theta';...;\theta^{(r)}]:[1-(q'):\Delta'];...;[1-(q^{(r)}):\Delta^{(r)}] \\ [\frac{3}{2}-\beta + \sigma \eta_G + \sum_{i=1}^s n_i(\beta_i+k_i):\sigma_1;...;\sigma_r]:[1-(t'):\delta'];...;[1-(t^{(r)}):\delta^{(r)}] \end{matrix} \right] \quad (3.1)$$

provided that $\operatorname{Re}(p) > 0$, $\operatorname{Re}(q) > 0$, $s > 0$, the series on the right side exists.

(b) If $\theta', \dots, \theta^{(r)} = \Delta', \dots, \Delta^{(r)} = \psi', \dots, \psi^{(r)} = \delta', \dots, \delta^{(r)} = \sigma_1, \dots, \sigma_r = \beta', \dots, \beta^{(r)}$ in (2.1), we get the following integral transformation:

$$\int_0^\infty y^{1-\beta} (p + qy + sy^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{y}{p + qy + sy^2} \right)^\sigma \middle| \begin{matrix} (m_P, M_P) \\ (n_Q, N_Q) \end{matrix} \right]$$

Notes

$$\cdot S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right]$$

$$\cdot S_E^{F_1, \dots, F_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right]$$

$$\cdot F_{A, C; [B', D'] \dots; [B^{(r)}, D^{(r)}]}^{0, \lambda: (u', v'); \dots; u^{(r)}, v^{(r)}} \left[x_1^{1/\beta'} \left(\frac{y}{p + qy + sy^2} \right), \dots, x_r^{1/\beta^r} \left(\frac{y}{p + qy + sy^2} \right), \begin{matrix} (p); (q); \dots; (q^{(r)}) \\ (s); (t); \dots; (t^{(r)}) \end{matrix} \right] dy$$

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$$= \sqrt{\frac{\pi}{c}} \sum_{G=0}^{\infty} \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\beta_s=0}^{[N_s/M_s]} \sum_{k_1, \dots, k_s=0}^{F_1 K_1 + \dots + F_s K_s \leq E} \frac{(-1)^G}{G! F_g} \frac{(-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G)$$

$$\cdot (-E)_{F_1 k_1 + \dots + F_s k_s} B(N_1, \beta_1; \dots; N_s \beta_s)$$

$$\cdot A[E; k_1, \dots, k_s] \frac{z_1^{(\beta_1+k_1)}}{k_1!} \dots \frac{(z_s)^{(\beta_s+k_s)}}{k_s!} (q + 2\sqrt{sp})^{\beta - \sigma \eta_G - \sum_{i=1}^s n_i (\beta_i + k_i) - 1}$$

$$\frac{\Gamma(1 - \beta + \sigma \eta_G + \sum_{i=1}^s n_i \alpha_i)(\beta_i + k_i)}{\Gamma(\frac{3}{2} - \beta + \sigma \eta_G + \sum_{i=1}^s n_i (\beta_i + k_i))} E_{A+1; B'; \dots; B^{(r)}}^{C+1; D'; \dots; D^{(r)}}$$

$$\cdot F_{A+1, C+1; [B', D'] \dots; [B^{(r)}, D^{(r)}]}^{0, \lambda+1: (u', v'); \dots; (u^{(r)}, v^{(r)})} \left[x_1^{1/\beta'} (q + 2\sqrt{sp})^{-1}, \dots, x_r^{1/\beta^r} (q + 2\sqrt{sp})^{-1} \right]$$

$$\left[\begin{array}{l} \left[\beta - \sigma \eta_G - \sum_{i=1}^s n_i (\beta_i + k_i) \right], (p); (q); \dots; (q^{(r)}) \\ (s), \left[\beta - \sigma \eta_G - \sum_{i=1}^s n_i (\beta_i + k_i) - \frac{1}{2} \right]; (t'); \dots; (t^{(r)}) \end{array} \right] \quad (3.2)$$

provided that $\operatorname{Re}(p) > 0$, $\operatorname{Re}(q) > 0$, $s > 0$; $\beta^{(i)} > 0$ ($i = 1, \dots, r$), $2(u^{(i)} + v^{(i)}) > (A + C + B^{(i)} + D^{(i)})$

$$|\arg(z_i)| < \left[u^{(i)} + v^{(i)} - \frac{A}{2} - \frac{C}{2} - \frac{B^{(i)}}{2} - \frac{D^{(i)}}{2} \right] \pi \text{ and}$$

Notes

$$\sigma \left\{ \min_{1 \leq j \leq M} [\operatorname{Re}(n_j / N_j)] \right\} + \sum_{i=1}^r \left\{ \min_{1 \leq j \leq u^{(i)}} [\operatorname{Re}(t_j^{(i)})] \right\} > \beta - 2.$$

(c) When we put $\lambda = A = C = 0$ in (2.1), we get the following transformation

$$\text{Notes} \quad \int_0^\infty y^{1-\beta} (p + qy + sy^2)^{\beta-3/2} H_{P,Q}^{L,R} \left[\left(\frac{y}{p + qy + sy^2} \right)^\sigma \middle| (m_P, M_P) \middle| (n_Q, N_Q) \right]$$

$$S_{N_1, \dots, N_s}^{M_1, \dots, M_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right]$$

$$\cdot S_E^{F_1, \dots, F_s} \left[z_1 \left(\frac{y}{p + qy + sy^2} \right)^{n_1}, \dots, z_s \left(\frac{y}{p + qy + sy^2} \right)^{n_s} \right]$$

$$\cdot \prod_{i=1}^r H_{B^{(i)}, D^{(i)}}^{u^{(i)}, v^{(i)}} \left[x_i \left(\frac{y}{p + qy + sy^2} \right)^{\sigma_i} \middle| [(b^{(i)}) : \phi^{(i)}] \middle| [d^{(i)} : \delta^{(i)}] \right] dy$$

$$= \sqrt{\frac{\pi}{c}} \sum_{G=0}^{\infty} \sum_{g=1}^L \sum_{\beta_1=0}^{[N_1/M_1]} \dots \sum_{\alpha_s=0}^{[N_s/M_s]} \sum_{k_1, \dots, k_s=0}^{F_1 K_1 + \dots + F_s K_s \leq E} \frac{(-1)^G}{G! F_g} \frac{(-N_1)_{M_1 \beta_1}}{\beta_1!} \dots \frac{(-N_s)_{M_s \beta_s}}{\beta_s!} \phi(\eta_G)$$

$$\cdot (-E)_{F_1 k_1 + \dots + F_s k_s} B[N_1, \beta_1; \dots; N_s, \beta_s] A[E; k_1, \dots, k_s] \frac{z_1^{(\beta_1 + k_1)}}{k_1!} \dots \frac{(z_s)^{(\beta_s + k_s)}}{k_s!}$$

$$(q + 2\sqrt{sp})^{\beta - \sigma \eta_G - \sum_{i=1}^s n_i (\beta_i + k_i) - 1} H_{1,1; [B', D']; \dots; [B^{(r)}, D^{(r)}]}^{0,1; (u', v'); \dots; (u^{(r)}, v^{(r)})}$$

$$\begin{bmatrix} x_1 (q + 2\sqrt{sp})^{-\sigma_1} & [\beta - \sigma \eta_G - \sum_{i=1}^s n_i (\beta_i + k_i) : \sigma_1; \dots; \sigma_r] : [(q') : \Delta'] ; \dots ; [(q^{(r)}) : \Delta^{(r)}] \\ \vdots \\ x_r (q + 2\sqrt{sp})^{-\sigma_r} & [\beta - \sigma \eta_G - \sum_{i=1}^s n_i (\beta_i + k_i) - \frac{1}{2} : \sigma_1; \dots; \sigma_r] : [(t') : \delta'] ; \dots ; [(t^{(r)}) : \delta^{(r)}] \end{bmatrix} \quad (3.3)$$

valid and the same condition which is obtained from (2.1).

(d) Taking $N_i \rightarrow 0$, ($i = 1, \dots, s$), $E \rightarrow 0$, $p = 0$, $s = 1$, the result in (2.1) reduces to the known result after a slight simplification obtained by Goyal and Mathur [4].

(e) If $r = 1$ and $M_i, N_i \rightarrow 0$ ($i = 2, \dots, s$), $E \rightarrow 0$ the result in (2.1) reduces to the known result with a slight modification recently derived by Gupta and Jain [5].

(f) Taking $E \rightarrow 0$, the result in (2.1) reduces to the known result given in [3], after a little simplification.

IV. ACKNOWLEDGEMENT

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Notes