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An Integral Involving Extended Jacobi Polynomial

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Abstract - An attempt has been made to establish an integral concerning the product of two H-function of several complex variables (Srivastava and Panda [8] with extended Jacobi polynomial [5]). Mainly we are using the series representation of H-function given by Olkha and Chaurasia [6,7]. By assigning suitable values to the parameters, the results can be reduced to many new, known and unknown results.

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Ref.

6. Olkha, G.S. and Chaurasia, V.B.L. – Some integral transform involving the H-function of several complex variables, Kyungpook Math. J., 22 (1982), 309-315.

An Integral Involving Extended Jacobi Polynomial

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Abstract - An attempt has been made to establish an integral concerning the product of two H-function of several complex variables (Srivastava and Panda [8] with extended Jacobi polynomial [5]). Mainly we are using the series representation of H-function given by Olkha and Chaurasia [6,7]. By assigning suitable values to the parameters, the results can be reduced to many new, known and unknown results.

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I. Introduction

The series representation of the H-function of several complex variable studied by Olkha and Chaurasia [6,7] is given as follows:

$$H[z_1,\ldots,z_r] = H^{0,\lambda':(u',v');\ldots;(u^{(r)},v^{(r)})}_{A',C':[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left[\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]: & [b':\phi'];\ldots;[b^{(r)}:\phi^{(r)}];\\ [(c):\psi',\ldots,\psi^{(r)}]: & [d':\delta'];\ldots;[d^{(r)}:\delta^{(r)}];\end{matrix}\ z_1,\ldots,z_r\right]$$

$$= \sum_{m_i=1}^{u^{(i)}} \sum_{n_i=0}^{\infty} \Phi_1 \Phi_2 \frac{\prod_{i=1}^{r} (z_i)^{U_i} (-1)^{\sum_{i=1}^{r}(n_i)}}{\prod_{i=1}^{r} \left(\delta^{(i)}_{(m_i)}\, n_i!\right)}, \qquad (1.1)$$

where

$$\Phi_1 = \frac{\prod_{j=1}^{\lambda'} \Gamma\left[1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}U_i\right]}{\prod_{j=\lambda'+1}^{A'} \Gamma\left[a_j-\sum_{i=1}^{r}\theta_j^{(i)}U_i\right]\prod_{j=1}^{C} \Gamma\left[1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}U_i\right]}, \qquad (1.2)$$

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$$\Phi_2 = \frac{\prod_{\substack{j=1\\ j\neq m_j}}^{u^{(i)}} \Gamma(d_j^{(i)} - \delta_j^{(i)} U_i) \prod_{j=1}^{v^{(i)}} \Gamma(1 - b_j^{(i)} + \phi_j^{(i)} U_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)} U_i) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma(b_j^{(i)} - \phi_j^{(i)} U_i)}, \tag{1.3}$$

$$U_i = \frac{d_{m_i}^{(i)} + n_i}{\delta_{m_i}^{(i)}}, \; i = 1, \ldots, r \tag{1.4}$$

which is valid under the following conditions

$$\delta_{m_i}^{(i)} [d_j^{(i)} + p_i] \neq \delta_j^{(i)} [d_{m_i}^{(i)} + n_i] \tag{1.5}$$

for $j \neq m_i, m_i = 1, \ldots, u^{(i)}; p_i, n_i = 0, 1, 2, \ldots; z \neq 0$

$$\nabla_i = \sum_{j=1}^{\lambda'} \theta_j^{(i)} - \sum_{j=1}^{C'} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} < 0 \quad \forall i = 1, \ldots, r \tag{1.6}$$

Srivastava and Panda [8] have introduced the multivariable H-function

$$H[y_1, \ldots, y_R] = H^{0,\lambda : (\alpha', \beta') ; \ldots ; (\alpha^{(R)}, \beta^{(R)})}_{A, C : [M', N'] ; \ldots ; [M^{(R)}, N^{(R)}]} \left[\begin{matrix} [(g) : \gamma', \ldots, \gamma^{(R)}] : [q' : n'] ; \ldots ; (\alpha^{(R)}, \beta^{(R)}) : \\ [(f) L \xi', \ldots, \xi^{(R)}] : [p' : \in'] ; \ldots ; [p^{(R)}, \in^{(R)}] : \end{matrix} \; y_1, \ldots, y_R \right] \tag{1.7}$$

For the sake of brevity

$$T_i = \sum_{j=1}^{\lambda} \gamma_j^{(i)} - \sum_{i=1}^{C} \xi^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \in_j^{(i)} \leq 0, \tag{1.8}$$

$$\Omega_i = -\sum_{j=\lambda+1}^{A} \gamma_j^{(i)} - \sum_{i=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{\beta^{(i)}} \eta_j^{(i)} - \sum_{j=\beta^{(i)}+1}^{M^{(i)}} \eta_j^{(i)} + \sum_{j=1}^{\alpha^{(i)}} \in_j^{(i)} - \sum_{j=\alpha^{(i)}+1}^{N^{(i)}} \in_j^{(i)} > 0, \tag{1.9}$$

$$|\arg(y_i)| < \frac{1}{2} T_i \pi, \; \forall \; i = 1, \ldots, R.$$

II. Main Theorem

The transformation valid under the following conditions:

(i) $\mathrm{Re}(\rho) > -1$,

Ref.

8. Srivastava, H.M. and Panda, R. – Expansion theorems for the H-function of several complex variables, J. Reine Angew. Math., 288 (1976), 129-145.

(ii) $\quad \operatorname{Re}\left(\sigma+\sum_{i=1}^{R} h_i \frac{p_j^{(i)}}{\epsilon_j^{(i)}}+\sum_{i'=1}^{r} h'_{i'} \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > -1,$

(iii) $\quad h_i, h'_{i'}, T_i, k>0,\ t=k\,(q-p), i=1,\dots,R,\ i'=1,\dots,r,$

$|\arg(y_i)| < \frac{1}{2}\Omega_i, T_i, \Omega_i$ are given in (1.8) and (1.9).

(iv) $\quad F_n(\rho,\omega;x)$ is Fujiwara's polynomial.

Thus, the following transformation holds

$$\int_p^q (x-p)^{\rho}(q-x)^{\sigma} F_n(\rho,\omega;x)\, H[z_1(q-x)^{h'},\dots,z_r(q-x)^{h_r}]$$

$$.\ H[y_1(q-x)^{h},\dots,y_R(q-x)^{h_R}]dx$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{n_i=0}^{\infty}\frac{(-1)^{\sum_{i=1}^{r}(n_i)+n}\prod_{i=1}^{r}(z_i)^{U_i}\, t^n}{\prod_{i=1}^{r}((\delta_{m_i}^{(i)} n_i!)\, n!}(q-p)^{\rho+\sigma+1+\sum_{i=1}^{r} h'_i U_i}\,\Gamma(1+\rho+n)\,\Phi_1\Phi_2$$

$$.\ H^{0,\lambda+2\ :(\alpha',\beta')\ ;\dots;(\alpha^{(R)},\beta^{(R)})}_{A+2,C+2:[M',N'];\dots;[M^{(R)},N^{(R)}]}\left[\begin{matrix}[\omega-\rho-\sum_{i=1}^{r} h'_iU_i:h_1,\dots,h_R],[-\sigma-\sum_{i=1}^{r} h'_iU_i:h_1,\dots,h_R],\\ [\omega+n-\sigma-\sum_{i=1}^{r} h'_iU_i:h_1,\dots,h_R],[-1-\rho-n-\sigma\sum_{i=1}^{r} h'_iU_i:h_1,\dots,h_R],\end{matrix}\right.$$

$$\left.\begin{matrix}[(g):\gamma',\dots,\gamma^{(R)}]:[q',\eta'];\dots;[q^{(R)},\eta^{(R)}];\\ [(f):\xi',\dots,\xi^{(R)}]:[p',\epsilon'];\dots;[p^{(R)},\epsilon^{(R)}];\end{matrix}\ y_1(q-p)^{h_1},\dots,y_R(q-p)^{h_R}\right]. \qquad (2.1)$$

III. Proof

To prove (2.1), we express the multivariable H-function in series form with the help of (1.1) and then changing the order of integration and summations which is valid under the conditions stated and evaluating the remaining integral with the help of a known result of Chaurasia and Sharma ([2], p.269, eqn. (2.1)), we get the required result.

IV. Special Cases

(a) Giving suitable values to the parameters and making use of a transformation formula given by Srivastava and Panda ([8], p.139, eqn. 4.11), after a little simplification, we have the following result

Ref.

2. Chaurasia, V.B.L. and Sharma, S.C. – An integral involving extended Jacobi polynomial and H-function of several complex variables, Vij. Pari. Anu. Pat., 27(3) (1984), 267-272.

Theorem (A)

The transformation valid under the following conditions

(i) $\operatorname{Re}(\rho)>-1, \operatorname{Re}(\sigma)>-1$

(ii) $h_i>0, h^{i}_{i'}>0, \Delta_{i'}\geq 0, i=1,\ldots,R, i'=1,\ldots,r, t=k(q-p), k>0$

where

$$\Delta_j = 1+\sum_{i=1}^{C}\xi^{(j)}+\sum_{i=1}^{B^{(j)}}\epsilon_i^{(j)}-\sum_{i=1}^{\lambda}\gamma_i^{(j)}-\sum_{i=1}^{\alpha^{(j)}}\eta_i^{(j)}\ (j=1,\ldots,R)$$

(iii) The equality holds when $|y_j|<L_j$, j = 1,...,R, with the L_j defined by equation (5.3), p.157 in [10].

Thus, the following transformation holds

$$\int_p^q (x-p)^{\rho}(q-x)^{\sigma}F_n(\rho,\omega;x)\,F^{A:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}}[z_1(q-x)^{h'},\ldots,z_r(q-x)^{h'_r}]$$

$$.\ F^{\lambda:\alpha';\ldots;\alpha^{(R)}}_{\mu:\beta';\ldots;\beta^{(R)}}[y_1(q-x)^{h},\ldots,y_R(q-x)^{h_R}]dx$$

$$=\sum_{m_1,\ldots,m_r=0}^{\infty}\frac{\prod_{i=1}^{A}(a_i)_{m_1\theta_1+\ldots+m_r\theta_i^{(r)}}\prod_{i=1}^{B'}(b'_i)_{m_1\phi'_i}\cdots\prod_{i=1}^{B^{(r)}}(b_i^{(r)})_{m_r\phi_i^{(r)}}}{\prod_{i=1}^{C}(c_i)_{m_1\psi_1+\ldots+m_r\psi_i^{(r)}m_r}\prod_{i=1}^{D'}(d'_{(i)})_{m_1\delta'_i}\cdots\prod_{i=1}^{D^{(r)}}(d_i^{(r)})_{m_r\delta_i^{(r)}}}\frac{z_1^{m_1}}{m_1!}\cdots\frac{z_r^{m_r}}{m_r!}\frac{t^n(-1)^n}{n!}$$

$$.(q-p)^{\rho+\sigma+1+\sum_{i=1}^{r}h'm_i}\frac{\Gamma(1+\rho+n)\,\Gamma(1+\sigma+\sum_{i=1}^{r}h'm_i)\Gamma(1+\sigma-\omega+\sum_{i=1}^{r}h'm_i)}{\Gamma(1+\sigma-\omega-n+\sum_{i=1}^{r}h'm_i)\Gamma(2+\omega+n+\sigma+\sum_{i=1}^{r}h'm_i)}$$

$$.\ F^{\lambda+2:\alpha';\ldots;\alpha^{(R)}}_{\mu+2:\beta';\ldots;\beta^{(R)}}\left[\begin{matrix}[1+\sigma+\sum_{i=1}^{r}h'm_i:h_1,\ldots,h_R],[1+\sigma-\omega+\sum_{i=1}^{r}h'm_i:h_1,\ldots,h_R],\\ [1+\sigma-\omega-n+\sum_{i=1}^{r}h'm_i:h_1,\ldots,h_R],[2+\omega+n+\sigma+\sum_{i=1}^{r}h'm_i:h_1,\ldots,h_R],\end{matrix}\right.$$

$$\left.\begin{matrix}[(g):\gamma',\ldots,\gamma^{(R)}]:[(q'):\eta'];\ldots;[(q^{(R)}):\eta^{(R)}];\\ [(f):\xi',\ldots,\xi^{(R)}]:[(p'):\epsilon'];\ldots;[(p^{(R)}):\epsilon^{(R)}];\end{matrix}\ y_1(q-p)^{h_1},\ldots,y_R(q-p)^{h_R}\right]. \tag{4.1}$$

Ref.

10. Srivastava, H.M. and Daoust, M.C. – A note on convergence of Kampé de Fériets double hypergeometric series, Math. Nachr. 53 (1972), 151-159.

(b) For r = 1 =R in (2.1), we obtain the following result

Theorem (B)

The transformation valid under the following conditions

(i) $\mathrm{Re}(\rho) > -1, h > 0, h' > 0, T > 0, |\arg y| < \frac{1}{2}T\pi, t = k(q-p), k > 0$

(ii) $\mathrm{Re}(\sigma + h'\frac{p_j}{\in_j} + h\frac{d_{j'}}{\delta_{j'}} + 1 > 0, j = 1,\ldots,u, j' = 1,\ldots,\alpha.$

Notes

Thus, the following transformation holds

$$\int_p^q (x-p)^{\rho}(q-x)^{\sigma} F_n(\rho,\omega;x) H_{B,D}^{u,v}\left[\begin{matrix}[b:\phi]\\ [d:\delta]\end{matrix}\middle| z(q-x)^{h'}\right] H_{M,N}^{\alpha,\beta}\left[\begin{matrix}(q,\eta)\\ [p,\in]\end{matrix}\middle| y(q-x)^{h}\right]dx$$

$$= \sum_{m_1=1}^{u}\sum_{n_1=0}^{\infty} \frac{(-1)^{n_1} z^U t^n (q-p)^{\rho+\sigma+1+h'U}}{n!\ \ n_1!\ \ \delta n_1}\Gamma(1+\rho+n)$$

$$.\ H_{M+2,N+2}^{\alpha,\beta+2}\left[\begin{matrix}[\omega-\rho-h'U:h],[-\sigma-hU:h],[b:\phi]\\ [d:\delta],[\omega+n-\sigma-h'U:h],[-1-\rho-n-\sigma-h'U:h]\end{matrix}\right]. \tag{4.2}$$

(c) Taking $p = 1 = q, \lambda = 1, h_{i'}' = 1, i' = 1,\ldots,r,$ we get a known result due to Srivastava and Panda [8].

(d) Letting $h_i' = 1, i = 1,\ldots,r$ in (2.1), we have a result due to Chaurasia and Sharma [2].

(e) Putting $h_i' = 1, i = 1,\ldots,r$ the result in (4.1) reduces to a known result obtained by Chaurasia and Sharma in [2].

(f) The result already established by the equation (3.2) and (3.3) in [2] can be deduced from our results.

A great number of interesting integral formulae as particular cases our main result can be deduced, but we omit them here for lack of space.

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Notes