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Common Fixed Point Theorem for Weakly Compatible Maps Satisfying E.A Property in Intuitionistic Fuzzy Metric Spaces using Implicit Relation

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COMMON FIXED POINT THEOREM FOR WEAKLY COMPATIBLE MAPS SATISFYING E.A PROPERTY IN INTUITIONISTIC FUZZY METRIC SPACES USING IMPLICIT RELATION

Strictly as per the compliance and regulations of :

Ref.

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Common Fixed Point Theorem for Weakly Compatible Maps Satisfying E.A Property in Intuitionistic Fuzzy Metric Spaces using Implicit Relation

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Abstract - In this paper, we use the notion of E.A. property in intuitionistic fuzzy metric space to prove a common fixed point theorem which generalizes Theorem-2 of Turkoglu, Alaca,Cho and Yildiz [12].

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I. Introduction

In 1986, Jungck[6] introduced the notion of compatible maps for a pair of self mappings. Several papers have come up involving compatible maps in proved the existence of common fixed points in the classical and fuzzy metric spaces. Aamri and El. Moutawakil[1] generalized the concept of non compatibility by defining the notion of property E.A. and proved common fixed point theorems under strict contractive conditions. Atanassove [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets and later there has been much progress in the study of intuitionistic fuzzy sets by many authors [4,5]. In 2004, Park [9] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [5]. Fixed point theory has important applications in diverse disciplines of mathematics, statistics, engineering, and economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc. Several authors [5,8] proved some fixed point theorems for various generalizations of contraction mappings in probabilistic and fuzzy metric space. Turkoglu [12] gave a generalization of Jungck's common fixed point theorem [6] to intuitionistic fuzzy metric spaces. In this paper, we use the notion of E.A property in intuitionistic fuzzy metric space to prove a common fixed point theorem for a pair of self mappings in intuitionistic fuzzy metric space. Our result generalizes Theorem-2 of Turkoglu, Alaca,Cho and Yildiz [12].

II. Preliminaries

The concepts of triangular norms (t–norm) and triangular conorms (t-conorm) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [8] in study of statistical metric spaces.

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Definition 2.1. A binary operation $*$: $[0, 1]\times[0, 1] \to [0, 1]$ is continuous t-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;

(ii) $*$ is continuous;

(iii) $a * 1 = a$ for all $a \in [0, 1]$;

(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.

Definition 2.2. A binary operation $\diamond$: $[0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\diamond$ satisfies the following conditions:

(i) $\diamond$ is commutative and associative;

(ii) $\diamond$ is continuous;

(iii) $a \diamond 0 = a$ for all $a \in [0, 1]$;

(iv) $a \diamond b \le c \diamond d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.

Alaca et al. [2] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous tconorms as a generalization of fuzzy metric space due to Kramosil and Michalek [7] as :

Definition 2.3 [2]: A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous tconorm and M, N are fuzzy sets on X2×$[0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \le 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(vi) for all $x, y \in X$, $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous;

(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;

(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(xi) $N(x, y, t) \diamond N(y, z, s) \ge N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii) for all $x, y \in X$, $N(x, y, .) : [0, \infty)\to[0, 1]$ is right continuous;

(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X:

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M (x, y,t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Ref.

2. C. Alaca, D. Turkoglu, and C. Yildiz, Fixed points in Intuitionistic fuzzy metric spaces, Chaos, Solitons & Fractals 29, 1073-1078 (2006).

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Notes

Remark 2.1 [2]: Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *, ◊) such that t-norm * and t-conorm ◊ are associated as x ◊ y = 1-((1-x) * (1-y)) for all x, y ∈ X.

Remark 2.2 [2]: In intuitionistic fuzzy metric space (X, M, N, *, ◊), M(x, y, *) is nondecreasing and N(x, y, ◊) is non-increasing for all x, y ∈ X.

Proof : Suppose that M(x, y, *) is non-increasing, therefore for t ≤ s, we have

$$M(x, y, t) \geq M(x, y, s).$$

For all x, y, z ∈ X, we have $M(x, z, t + s) \geq M(x, y, t) * M(y, z, s)$.
In particular for z = y, we have $M(x, y, t + s) \geq M(x, y, t) * M(y, y, s)$.
$M(x, y, t + s) \geq M(x, y, t) * 1 = M(x, y, t)$, a contradiction, hence M(x, y, *) is non-decreasing.

Again, suppose N(x, y, ◊) is non-decreasing, therefore for t ≤ s,
we have $N(x, y, s) \geq N(x, y, t)$.

For all x, y, z ∈ X, we have $N(x, z, t + s) \leq N(x, y, t) \diamond N(y, z, s)$.

In particular for z = y, we have $N(x, y, t + s) \leq N(x, y, t) \diamond N(y, y, s)$

$N(x, y, t + s) \leq N(x, y, t) \diamond 0 = N(x, y, t)$, a contradiction, hence N(x, y, ◊) is nonincreasing.
Hence the result.
Alaca, Turkoglu and Yildiz [2] introduced the following notions:

Definition 2.4 [2] : Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then
(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point x ∈ X if, for all t > 0,

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \text{ and } \lim_{n\to\infty} N(x_n, x, t) = 0.$$

Definition 2.5 [2] : An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Example 2.1 [2] : Let $X = \{1/n : n \in N\} \cup \{0\}$ and let * be the continuous t-norm and ◊ be the continuous t-conorm defined by a * b = ab and a ◊ b = min{1, a + b} respectively, for all $a, b \in [0,1]$. For each $t \in (0, \infty)$ and $x, y \in X$, define (M, N) by

$$M(x, y, t) = \begin{cases} \dfrac{t}{t+|x-y|}, & t > 0, \\ 0 & t = 0 \end{cases} \quad \text{and } N(x, y, t) = \begin{cases} \dfrac{|x-y|}{t+|x-y|}, & t > 0, \\ 1 & t = 0 \end{cases}$$

Clearly, (X, M, N, *, ◊) is complete intuitionistic fuzzy metric space.

Definition 2.6 [2] : A pair of self mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be commuting if

$$M(fgx, gfx, t) = 1 \text{ and } N(fgx, gfx, t) = 0 \text{ for all } x \in X.$$

Global Journal of Science Frontier Research (F) Volume XII Issue VI Version I June 2012

Definition 2.7 [1] : A pair of self mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to satisfy the E.A property if there exist a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} M(fx_n, gx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(fx_n, gx_n, t) = 0.$$

Example 2.2 [1] : Let X =[0, ∞). Consider (X, M, N, *, ◊) be an intuitionistic fuzzy metric space as in Example 2.1. Define f, g : X →X by $fx = \frac{x}{5}$ and $gx = \frac{2x}{5}$ for all x ∈ X. Then for sequence $\{x_n\} = \left\{\frac{1}{n}\right\}$,

$$\lim_{n\to\infty} M(Sx_n, Tx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(Sx_n, Tx_n, t) = 0.$$

Then f and g satisfies E.A property.

Definition 2.8 [12] : A pair of self mappings (f, g) of a intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be weakly compatible if they commute at coincidence points i.e. if fu = gu for some u in X, then fgu = gfu.

It is easy to see that two compatible maps are weakly compatible.

Lemma 1[2] : Let (X, M, N, *, ◊) be intuitionistic fuzzy metric space and for all x, y in X, t > 0 and if for a number k ∈ (0, 1),

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t)$$

Then x = y.

III. Main Results

Turkoglu et al.[12] proved the following Theorem:

Theorem 3.1 : Let (X, M, N, *, ◊) be a complete intuitionistic fuzzy metric space. Let f and g be self mappings of X satisfying the following conditions:

(a) g(X) ⊆ f(X)

(b) there exist 0 < k < 1 such that $M(gx, gy, kt) \geq M(fx, fy, t)$ and $N(gx, gy, kt) \leq N(fx, fy, t)$,

(c) f is continuous

Then f and g have a unique common fixed point provided f and g commute.

Now, we prove a common fixed point theorem using E.A property in intuitionistic fuzzy metric space, which is a generalization of Theorem-3.1 in the following way.

I. to relax the continuity requirement of maps completely,

II. E.A property buys containment of ranges.

Theorem 3.2 : Let (X, M, N, *, ◊) be a intuitionistic fuzzy metric space with continuous t-norm and continuous t-conorm defined by a * a ≥ a and (1 – a) ◊ (1 – a) ≤ (1 – a) where a, b in [0, 1]. Let S and T be two weakly compatible self mappings of X satisfying the following conditions:

(i) T and S satisfy the E.A property,

(ii) for each x, y ∈ X, t >0, there exist 0 < k < 1 such that

$$M(Tx, Ty, kt) \geq M(Sx, Sy, t) \text{ and}$$
$$N(Tx, Ty, kt) \leq N(Sx, Sy, t),$$

Ref.
12. D. Turkoglu, C. Alaca, Y. J. Cho, and C. Yildiz, Common fixed point theorems in Intuitionistic fuzzy metric spaces, J. Appl. Math. & Computing 22, 411-424 (2006).

Notes

(iii) S(X) or T(X) is complete subspace of X.

Then S and T have a unique common fixed point.

Proof : In view of (i), there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n, = x_0$ for some x_0 in X. Suppose that S(X) is complete subspace of X, therefore, every convergent sequence of points of S(X) has a limit point in S(X) implies $\lim_{n\to\infty} Sx_n = Sa = u = \lim_{n\to\infty} Tx_n$, for some $a \in X$, which implies that $u = S\,a \in S\,(X)$.

Now, we prove that Ta = S a.

From (ii) take $x = x_n$, $y = a$, we get

$M(Tx_n, Ta, kt) \geq M(Sx_n, Sa, t)$ and $N(Tx_n, Ta, kt) \leq N(Sx_n, Sa, t)$.

Taking limit $n\to\infty$ on both sides,

We get, $M(Sa, Ta, kt) \geq M(Sa, Sa, t)$ and $N(Sa, Ta, kt) \leq N(Sa, Sa, t)$

This implies by using definition of IFS, Sa = Ta.

Therefore, u = S a = T a.

This shows that 'a' is coincident point of T and S.

As T and S are weakly compatible, therefore, TS(a) = ST(a) = SS(a) = TT(a).

Now, we show that Ta is the common fixed point of T and S.

From (ii) take x = a, y = Ta,

$$M(Ta, TTa, kt) \geq M(Sa, STa, t) \text{ and } N(Ta, TTa, kt) \leq N(Sa, STa, t),$$

$$M(Ta, TTa, kt) \geq M(Ta, TTa, t) \text{ and } N(Ta, TTa, kt) \leq N(Ta, TTa, t),$$

This implies by Lemma 1, TTa = Ta = STa.

This proves that Ta is the common fixed point of T and S.

Now, we prove the uniqueness of common fixed point of T and S. If possible, let x_0 and y_0 be two common fixed points of S and T. Then by condition (ii),

$$M(x_0, y_0, kt) = M(Tx_0, Ty_0, kt) \geq M(Sx_0, Sy_0, t) = M(x_0, y_0, t)$$

$$N(x_0, y_0, kt) = N(Tx_0, Ty_0, kt) \leq N(Sx_0, Sy_0, t) = N(x_0, y_0, t),$$

Then by Lemma 1, we have $x_0 = y_0$.

Therefore, the mappings S and T have a unique common fixed point.

This completes the proof.

Example 3.1: Let $X = \left\{\frac{1}{n} : n = 1, 2, 3, \ldots\right\} \cup \{0\}$ with the usual metric d defined by d (x, y) = $|x-y|$ for all $x, y \in X$ and $t>0$, define

$$M(x, y, t) = \begin{cases} \dfrac{t}{(t+|x-y|)}, & t > 0 \\ 0, & t = 0 \end{cases}$$

and

Global Journal of Science Frontier Research (F) Volume XII Issue VI Version I June 2012

$$N(x, y, t) = \begin{cases} \dfrac{|x-y|}{(kt+|x-y|)}, & t > 0, k > 0 \\ 1 & , t=0 \end{cases}$$

Clearly, (X, M, N, *, ◊) is intuitionistic fuzzy metric space, where * and ◊ are defined by a * b = ab and a ◊ b = min –1, a + b″ respectively. Define $T(x) = \frac{x}{12}$, $S(x) = \frac{x}{4}$ for all x ∈ X. Clearly, S and T are weakly compatible mappings on X,

(1) S and T satisfy the E.A property for the sequence $\{x_n\}=\left\{\frac{1}{n}\right\}$,

(2) Also for $k = \frac{1}{3}$, the condition (ii) of above theorem is satisfied ,

(3) S(X) is complete subspace of X and

Thus all the conditions of theorem 3.2 are satisfied and so S and T have the common fixed point x = 0.

Theorem 3.3 : Let (X, M, N, *, ◊) be a intuitionistic fuzzy metric space with continuous t-norm and continuous t-conorm defined by a * a ≥ a and (1 – a) ◊ (1 – a) ≤ (1 – a), where a, b in [0, 1]. Let f and g be two weakly compatible self mappings of X satisfying the following conditions:

(i) f and g satisfy the E.A property ,

(ii) for each x, y ∈ X, t >0, there exist 0 < k < 1 such that

$$M(fx,fy,kt) \geq \phi\ (M(gx,gy,t), M(fx,gx,t), M(fy,gy,t), M(fx,gy,t), M(fy,gx,t)) \text{ and}$$

$$N(fx,fy,kt) \leq \psi\ (N(gx,gy,t), N(fx,gx,t), N(fy,gy,t), N(fx,gy,t), N(fy,gx,t))$$

where ϕ, ψ is a mapping from [0,1] to [0,1], which is upper semi-continuous, nondecreasing in each coordinate variable and such that

$\phi\ (1,1,t,1,\ t) \geq t$, $\phi\ (t,1,1,t,\ t) \geq t$ and $y\ (1,1,t,1,\ t) \leq t$, $\psi\ (t,1,1,t,\ t) \leq t$ where t in [0,1]

(iii) the range of g is a closed subspace of X.

Then f and g have a unique common fixed point.

Proof. In view of (i), there exist a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n, = p$ for some p in X. As g(X) is a closed subspace of X, there is u in X such that p = gu .

Therefore, $\lim_{n\to\infty} fx_n = p = gu = \lim_{n\to\infty} gx_n$.

Now, we prove that fu = gu.

From (ii) take $x = x_n$, y = u,

$M(fx_n, fu, kt) \geq \phi\ (M(gx_n, gu, t), M(fx_n, gx_n, t), M(fu, gu, t), M(fx_n, gu, t), M(fu, gx_n, t))$

$N(fx_n, fu, kt) \leq \psi\ (N(gx_n, gu, t), N(fx_n, gx_n, t), N(fu, gu, t), N(fx_n, gu, t), N(fu, gx_n, t))$

As n→∞, we get

$$M(gu,fu,kt) \geq \phi\ (M(gu,gu,t),M(gu,gu,t),M(fu,gu,t),M(gu,gu,t),M(fu,gu,t))$$

$$= \phi\ (1,1,M(gu,fu,t),1,M(gu,fu,t)) \geq M(gu,fu,t). \text{ (By using (ii))}$$

Notes

$$N(gu,fu,kt)) \leq \psi\ (N(gu,gu,t),N(gu,gu,t),N(fu,gu,t),N(gu,gu,t),N(fu,gu,t))$$
$$= \psi\ (1,1,M(gu,fu,t),1,M(gu,fu,t)) \leq N(gu,fu,t). \text{ (By using (ii))}$$

By using lemma 1, we deduce that fu = gu. Denote fu by z.

Therefore, fu = gu = z.

This shows that 'u' is coincident point of f and g.

From weak compatibility of the mappings f and g it follows that fg(u) = gf(u)

This implies, fz = gz.

Now, we show that z is the common fixed point of f and g.

From (ii) take x = z, y = u,

$$M(fz,z,t) = M(fz,fu,t)$$
$$\geq \phi\ (M(gz,gu,t),M(fz,gz,t),M(fu,gu,t),M(fz,gu,t),M(fu,gz,t)),$$

that is,

$$M(fz,z,t) \geq \phi\ (M(fz,z,t),1,1,M(fz,z,t),M(z,fz,t)) \geq M(z,fz,t).$$

And

$$N(fz,z,t) = N(fz,fu,t)$$
$$\leq \psi\ (N(gz,gu,t),N(fz,gz,t),N(fu,gu,t),N(fz,gu,t),N(fu,gz,t)),$$

that is,

$$N(fz,z,t) = N(fz,fu,t)$$
$$\leq \psi\ (\ N(fz,z,t),\ 1\ ,\ 1\ ,\ N(fz,z,t),N(z,fz,t))\ \Theta\ N(fz,z,t),$$

By using lemma 1, we deduce that, fz = z = gz and thus we obtain that z is a common fixed point of f and g.

Now, we prove the uniqueness of common fixed point of f and g. If possible, let 'a' and 'b' be two common fixed points of f and g. Then by condition (ii) take x = a, y = b we get,

$$M(fa,fb,kt) \geq \phi\ (M(ga,gb,t),M(fa,ga,t),M(fb,gb,t),M(fa,gb,t),M(fb,ga,t))$$

$$M(a,b,kt) \geq \phi\ (M(a,b,t),M(a,a,t),M(b,b,t),M(a,b,t),M(b,a,t))$$

$$M(a,b,kt) \geq \phi\ (M(a,b,t),\ 1\ ,1\ ,M(a,b,t),M(a,b,t)) \geq M(a,b,t)$$

and

$$N(fa,fb,kt) \leq \psi\ (N(ga,gb,t),N(fa,ga,t),N(fy,gb,t),N(fa,gb,t),N(fb,ga,t))$$

$$N(a,b,kt) \leq \psi\ (N(a,b,t),N(a,a,t),N(b,b,t),N(a,b,t),N(b,a,t))$$

$$N(a,b,kt) \leq \psi\ (N(a,b,t)\ 1,\ 1,N(a,b,t),\ N(a,\ b\ ,t)) \leq N(a,b,t)$$

Then by Lemma 1, we have a = b.

Therefore, the mappings f and g have a unique common fixed point.

This completes the proof.

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Notes