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Modelling of Incompressible Elastic Thin Plastic Plate

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MODELLING OF INCOMPRESSIBLE ELASTIC THIN PLASTIC PLATE

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A. Ait Moussa[α] & M. Verid Ould M. Moulaye[σ]

Abstract - The aim of this work to study the asymptotic behavior of an elasticity problem, of containing structure, in incompressible elastic thin oscillating layer of thickness and stiffness depending of small parameter. We use the epi-convergence method to approximate the limit problem modeling.

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I. Introduction

The study of the inclusion between two elastic adherents bodies involves introducing a very thin third body, filled by adhesive with an oscillating boundary, between them. In general, the computation of solution using numerical methods is very difficult. In one hind, this is because the thickness of the adhesive requires a fine mesh, which in turn implies an increase of the degrees of exible than the adherents, and this produces numerical instabilities in the stiffness matrix. To overcame this difficulties, thanks to Goland and Reissner [4] find a limit problem in which the adhesive is treated on this theoretical approach, see for example A. Ait Moussa and J. Messaho [1], A. Ait Moussa and L. Zla ji [3], Licht and Michail [2], and Acerbi, Buttazzo and Perceivable [6].

This work is specially intereted in approximating a minimization problem $(\mathbb{P}_\epsilon)$ where ϵ is a small parameter linked to the thickness and the stiffness of the adhesive. In particular, we associate to each component of gradient an independent of stiffness parameter. We use epi-convergent method introduced in a paper by De Giorgi and Franzoni in [9],to proof a weak limit of a $(\mathbb{P}_\epsilon)$- minimizing sequence with is a solution of $(\mathbb{P})$.

This paper is organized in the following way. In section 2, we express the problem to study, and we give some notation and we define functional spaces for this study in the section 3. In the section 4, we study the problem (4.0). The section 5 is reserved to the determination of the limits problems and our main result.

II. Notation and Preliminaries

we consider a structure, occupying a bonded domain $\Omega \in \mathbb{R}^3$ with Lipschitzian boundary $\partial\Omega$. It is constituted of two elastic bodies joined together by an incompressible rigid thin layer with oscillating boundary, the latter obeys to nonlinear elastic low of power type. More precisely, the stress field is related to the displacement's field by

$$\sigma^\varepsilon = \lambda\, |e(u^\varepsilon)|^{-1}\, e(u^\varepsilon), \qquad \lambda > 0.$$

The structure occupies the regular domain $\Omega = B_\varepsilon \cup \Omega_\varepsilon$, where B_ε is given by $B_\varepsilon = \{x = (x', x_3)\ /\ |x_3| < \frac{\varepsilon}{2}\}$, and $\Omega_\varepsilon = \Omega \setminus B_\varepsilon$ represent the regions occupied by the thin plate and the two elastic bodies, ε being a positive parameter intended to approach 0, and $\Sigma = \{x = (x', x_3)\ /\ |x_3| = 0\}$.

The structure is subjected to a density of forces of volume $f,\ f : \Omega \to \mathbb{R}^3$, and it is fixed on the boundary $\partial\Omega$. Equations which relate the stress field σ^ε, $\sigma^\varepsilon : \Omega \to \mathbb{R}^9_S$, and the field of displacement u^ε, $u^\varepsilon : \Omega \to \mathbb{R}^3$, are

$$\begin{cases} div(\sigma^\varepsilon) + f = 0 \text{ in } \Omega \\ \sigma^\varepsilon_{ij} = a_{ijkh} e_{kh}(u^\varepsilon) \text{ in } \Omega_\varepsilon \\ \sigma^\varepsilon = \lambda |e(u^\varepsilon)|^{-1} e(u^\varepsilon) \text{ in } B_\varepsilon \qquad (\mathbb{P}_\epsilon) \\ div(u^\varepsilon) = 0 \text{ in } B_\varepsilon \\ u^\varepsilon = 0 \text{ on } \partial\Omega \end{cases}$$

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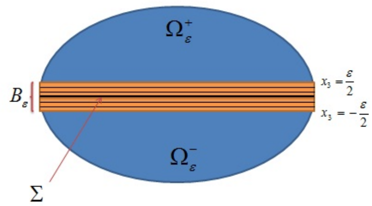
Where a_{ijkh} are the elasticity coefficients and $\mathbb{R}^9_S$ the vector space of the square symmetrical matrices of order three, $e_{ij}(u)$ are the components of the linearized tensor of deformation $e(u)$. In the sequel,we assume that the elasticity coefficients a_{ijkh} satisfy to the following hypotheses :

$$a_{ijkh} \in L^{\infty}(\Omega) \qquad (2.1)$$

$$a_{ijkh} = a_{jikh} = a_{khij}, \qquad (2.2)$$

$$a_{ijkh}\tau_{ij}\tau_{kh} \geq C\tau_{ij}\tau_{ij}, \ \forall\tau \in \mathbb{R}^9_S \qquad (2.3)$$



III. Notation and Function Setting

a) Notations

We begin by introducing some notation which is used throughout the paper.

$x = (x', x_3)$, Where $x' = (x_1, x_2)$,
$\tau \otimes \zeta = (\tau_i\zeta_i)_{1\leq i,j\leq 3}$ And

$$\tau \otimes_s \zeta = \frac{\tau \otimes \zeta + \zeta \otimes \tau}{2}, \ \forall\tau, \zeta \in \mathbb{R}^3$$

In the following C will denote any constant with respect to ε, $[v]$ is the jump of displacement field v through Σ, and ν, H_2 respectively the Lebesgue Hausdorff measures. Also, we use the convention $0.(+\infty)$.

To describe a global subadditive theorem, we consider $\mathcal{B}_b(\mathbb{R}^d)$ the family of Borel bounded subsets of $\mathbb{R}^d$ and δ. Euclidean distance in $\mathbb{R}^d$, for every $A \in \mathcal{B}_b(\mathbb{R}^d)$, $\rho(A) = \sup\{r \geq 0 : \exists\overline{B}_r(x) \subset A\}$, where $\overline{B}_r(x) = \{y \in \mathbb{R}^d : \delta(x,y) \leq r\}$. A sequence $(B_n)_{n\in\mathbb{N}} \subset \mathcal{B}_b(\mathbb{R}^d)$ is called regular if there exist an increasing sequence of intervals $(I_n)_n \subset \mathbb{Z}^d$ and a constant C independent of n such that $B_n \subset I_n$ and meas $(I_n) \leq C$ meas(B_n),$\forall n$. The global subadditive theorem is essentially based on subadditive $\mathbb{Z}^d$-invariant functions.

A function $S : A \in \mathcal{B}_b(\mathbb{R}^d) \rightarrow S_A \in \mathbb{R}$ is called subadditive $\mathbb{Z}^d$-periodic if it satisfy the following conditions:

(i) For all $A, B \in \mathcal{B}_b(\mathbb{R}^d)$ such that $A \cap B = \emptyset$, $S_{A\cup B} \leq S_A + S_B$.

(ii) For all $A \in \mathcal{B}_b(\mathbb{R}^d)$, all $z \in \mathbb{Z}^d$, $S_{A+z} = S_A$.

Now, we shall see the global subadditive theorem, firstly used in the setting of the calculus of variation by Licht and Michaille [2]

Theorem 3.1: [2; page24] Let S be a subadditive $\mathbb{Z}^d$- invariant function such that

$$\gamma(S) = \inf\{\frac{S_I}{\text{meas}\, I} : I = [a, b[, a, b \in \mathbb{Z}^d \ and \ a_i < b_i \forall 1 \leq i \leq d\}$$

$$\gamma(S) > -\infty$$

In addition, we suppose that S satisfies the dominant property: There exists C(S), for every Borel convex subset $A \subset [0,1[^d$, $|S_A| \leq C(S)$. Let $(A_n)_n$ be a regular sequence of Borel convex subsets of $\mathcal{B}_b(\mathbb{R}^d)$ with $\lim_{n\rightarrow+\infty} \rho(A_n) = +\infty$. Then $\lim_{n\rightarrow+\infty} \frac{S_{A_n}}{\text{meas}\, A_n}$ exists and is equal to

$$\lim_{n\rightarrow+\infty} \frac{S_{A_n}}{\text{meas}\, A_n} = \inf_{m\in\mathbb{N}^*}\{\frac{S_{[0,m[^d}}{m^d}\} = \gamma(S)$$

We have the following stability result for epiconvergence.

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Proposition 3.2 : [7; p:40] Suppose that F^ε epi-convergence to F in $(\mathbb{X},\tau)$ and that $G:\mathbb{X}\to\mathbb{R}\cup\{+\infty\}$, is $\tau-$ continues. Then $F^\varepsilon+G$ epi-converges to $F+G$ in $(\mathbb{X},\tau)$

This epi-convergence is a special case of the Γ- convergence introduced by De Giorgi (1979) [9]. It is well suited to the asymptotic analysis of sequences of minimization problems since one has the following fundamental result.

Theorem 3.3 : [7; Theorem1:10]. Suppose that

(1) F^ε admits a minimizer on $\mathbb{X}$;

(2) the sequence (u^ε) is τ relatively compact

(3) The sequence F^ε epi-converges to F in this topology τ. on $\mathbb{X}$

Then every cluster point u^* of the sequence (u^ε) minimizer F on $\mathbb{X}$ and $\lim\limits_{\varepsilon'\to 0} F^{\varepsilon'}(u^{\varepsilon'})=F(u^*)$

where $(u^{\varepsilon'})_{\varepsilon'}$ denotes any subsequence of $(u^\varepsilon)_\varepsilon$ which converges to u^*

b) Function setting

First, we introduce the space

$$V^\varepsilon=\left\{\begin{array}{c} u\in W_0^{1,2}(\Omega_\varepsilon,\mathbb{R}_s^3)\times W^{1,1}(B_\varepsilon,\mathbb{R}_s^3), \\ u=0 \text{ on } \partial\Omega \text{ and } div(u)=0 \text{ in } B_\varepsilon \end{array}\right\}$$

we easily show that V^ε is a Banach space with respect to the norm

$$u\to\|e(u)\|_{L^2(\Omega_\varepsilon,\mathbb{R}_s^9)}+\|e(u)\|_{L^1(B_\varepsilon,\mathbb{R}_s^9)}.$$

IV. Study of Problem

Problem $\mathbb{P}_\varepsilon$ is equivalent of the minimization problem

$$inf_{v\in V^\varepsilon}\left\{\begin{array}{c}\frac{1}{2}\int_{\Omega_\varepsilon} aijhke_{ij}(v)e_{hk}(v)dx+ \\ +\lambda\int_{B_\varepsilon}|e(v)|-\int_\Omega fvdx\end{array}\right\}$$

To study problem $\mathbb{P}_\varepsilon$, we will study the minimization problem (4.0). The existence and uniqueness of solutions to (4.0) is given in the following proposition.

Proposition 4.1 : Under the hypotheses (2:1), (2:2), (2:3) and for $f\in L^\infty$, problem (4:0) admits an unique solution.

Proof. From (2:1) and (2:3), we show easily that the energy functional in (4:0) is weakly lower semicontinuous, strictly convex and coercive over V^ε, Since V^ε is not reexive, so we may not apply directly result given in Dacorogna [17; p:48], but we can follow our proof by using the compact imbedding to the $W^{1,1}(\Omega)$ Sobolev space, in the reflexivity space $L^q(\Omega)$, or $q=\frac{2}{3}$ for more information see you Adams [14; p:95].

On the other hand, let u_n be a minimizing sequence for (4:0), to simplify the writing let

$$F^\varepsilon(v)=\frac{1}{2}\int_{\Omega_\varepsilon} aijhke_{ij}(v)e_{hk}(v)dx+\lambda\int_{B_\varepsilon}|e(v)|$$
$$-\int_\Omega fvdx$$

so, we have $F^\varepsilon(u_n)\to\inf_{v\in V^\varepsilon}F(v)$. Using the coercivity of F^ε, we may then deduce that there exists a constant $C>0$, independent of n, such that

$$\|u_n\|_{V^\varepsilon}\leq C,$$

then u_n bounded in L^q, therefore a subsequence of u_n , still denoted by u_n , there exists $u_0\in V^\varepsilon$ such that $u_n \rightharpoonup u_0$ in V^ε. The weak lower semicontinuity and the strict convexity of F^ε imply then the result.

Lemma 4.2. Assuming that for any sequence $(u^\varepsilon)_\varepsilon\subset V^\varepsilon$ there exists a constant $C>0$ such that $F^\varepsilon(u^\varepsilon)\leq C$, under (2:1), (2:3) and for $f\in L^\infty(\Omega,\mathbb{R}^3)$, $(u^\varepsilon)_{\varepsilon>0}$ satisfies

$$\|e(u^\varepsilon)\|^2_{L^2(\Omega_\varepsilon,\mathbb{R}^9_s)} \le C \qquad (4.1)$$

$$\int_{B_\varepsilon} |e(u^\varepsilon)| \le C. \qquad (4.2)$$

moreover u^ε is bounded in $W_0^{1,1}(\Omega,\mathbb{R}^3)$.

Proof. Science $F^\varepsilon(u^\varepsilon) \le C$, we have

$$\frac{1}{2}\int_{\Omega_\varepsilon} a_{ijhk}e_{ij}(u^\varepsilon)e_{hk}(u^\varepsilon)dx + \lambda\int_{B_\varepsilon}|e(u^\varepsilon)| - \int_\Omega fu^\varepsilon dx \le C$$

Then, taking advantage of the fact that u^ε vanishes on $\partial\Omega$:

$$\int_\Omega fu^\varepsilon dx \le |f|_{L^\infty(\Omega)}|u^\varepsilon|_{L^1(\Omega)} \le C|e(u^\varepsilon)|_{L^1(\Omega)}.$$

by Hölder and Young the inequalities, we obtain

$$|e(u^\varepsilon)|_{L^1(\Omega)} \le C + C\|e(u^\varepsilon)\|_{L^2(\Omega_\varepsilon)} + |e(u^\varepsilon)|_{L^1(B_\varepsilon)}$$

According to (2:3) and (4:3), we have

$$\|e(u^\varepsilon)\|^2_{L^2(\Omega_\varepsilon,\mathbb{R}^9_s)} + \lambda\int_{B_\varepsilon}|e(u^\varepsilon)| \le C + C\int_\Omega fu^\varepsilon dx$$

$$\le C + C\|e(u^\varepsilon)\|_{L^2(\Omega_\varepsilon,\mathbb{R}^9_s)} + \int_{B_\varepsilon}|e(u^\varepsilon)|$$

To facilitate writing, we denote by

$X_1 = \|e(u^\varepsilon)\|_{L^2(\Omega_\varepsilon,\mathbb{R}^9_s)}$, and $X_2 = \int_{B_\varepsilon}|e(u^\varepsilon)|$ We have

$$X_1^2 + X_2 \le C(X_1 + X_2).$$

Then

$$(X_1 - \frac{C}{2})^2 + (1-C)X_2 \le \frac{C^2}{4}$$

according to the values of *C*,

- If $(1-C) \ge 0$: we see easily the result.
- If $(1-C) < 0$:

$$(X_1 - \frac{C}{4})^2 \le \frac{C^2}{4} - (1-C)X_2,$$

this inequality means that, the tensor of deformation form a straight line on the ground in B_ε,below parable about Ω_ε, which is contradicted to the situation of problem. Therefore, we will have (4:2) and (4:3). In another problem we can supposed that $(1-C) > 0$ and completed the proof.

According to (4:2) and (4:3), and for a small enough ε the sequence (u^ε) is bounded in $W_0^{1,1}(\Omega,\mathbb{R}^3)$.

Remark 4.3. The solution u^ε of the problem (4:0) satisfy to the Lemma 4:2.

To apply the epi-convergence method, we need to characterize the topological spaces containing any cluster point of the solution of the problem (4:0) with respect to the used topology, therefore the weak topology to use is insured by the lemma 4.1. So the topological spaces characterization is given in the following proposition.

Proposition 4.4. The solution u^ε of the problem (4.0) possess a cluster point u^* in $BD(\Omega) \cap L^1(\Omega)$.

Proof. According to the Remark 4.3 and Lemma 4.2, for a small enough ε, the solution u^ε is bounded in $BD(\Omega)$, since there is a compact embedding of $BD(\Omega)$ in $L^1(\varepsilon)$. Hence the result of Temam [13; p:152], there exists
.

$$u^* \in L^1(\Omega), \text{ such that } u^\varepsilon \rightharpoonup u^* \text{ in } L^1(\Omega).$$

Remark 4.5. Proposition 4.4 remains valid for any weak cluster point of a sequence u^ε in V^ε, that satisfies (4:2) (4:3)

VI. Limit Behavior

Let

$$F_\varepsilon(v)=\begin{cases}\frac{1}{2}\int_{\Omega_\varepsilon} a_{ijhk}e_{ij}(v)e_{hk}(v)dx+\lambda\int_{B_\varepsilon}|e(v)| & \text{if } v\in V^\varepsilon\\ +\infty & \text{if } v\notin V^\varepsilon\end{cases} \quad (5.1)$$

$$G(v)=-\int_\Omega fvdx,\quad \forall v\in V^\varepsilon$$

We design by τ_f the weak topology on the space. In the sequel, we shall characterize, the epi-limit of the energy functional given by (5:1) in the following theorem.

Theorem 5.1. Under (2:2), (2:3), (2:4) and for $f\in L^\infty(\Omega,\mathbb{R}^3)$, there exists a functional F: $W^{1,1}(\Omega)\to\mathbb{R}\cup\{+\infty\}$ such that

$$\tau_f-\lim_\varepsilon F^\varepsilon=F\quad in\ W_0^{1,1}(\Omega)$$

where F is given by

$$F(u)=\begin{cases}\frac{1}{2}\int_\Omega a_{ijhk}e_{ij}(u)e_{hk}(u)dx+\lambda\int_\Sigma|[u]\otimes_s e_3|\\ \quad if\ u\in W_0^{1,1}(\Omega)\\ +\infty\quad if\ u\notin W_0^{1,1}(\Omega)\end{cases}.$$

Before launching our proof of this theorem we need the following lemma

Lemma 5.2.

Let Ω be a Lipschitzian in $\mathbb{R}^3$ and $p>1$:

(i) – If $\pounds\in W^{-1,s'}(\Omega,\mathbb{R}^3)$ and $<\pounds,\Phi>=0,\forall\Phi\in W_0^{1,p}(\Omega,\mathbb{R}^3)$, with $\boldsymbol{div}(\Phi)=0$, then tere exists $q\in L^{q'}$ such that $\pounds=\boldsymbol{grad}(q)$.

(ii) – If $q\in L^q$ and $\int_\Omega q=0$, then there exists $v\in W_0^{1,p}(\Omega,\mathbb{R}^3)$, such that $\boldsymbol{div}(v)=q$.

Proof. this result is classical if $p=2$, is less well known if $p=1$, thus the domain Ω is connected lipschitizian boundary. We can be shown (i) and (ii) such as in Tartar [10; p:29 - 30].

Proof. [theorem 5.1]

- — (a) We are now in position to describe the lower epi-limit.

Let $u\in W_0^{1,1}(\Omega)$ and $(u^\varepsilon)\in V^\varepsilon$ such that $u^\varepsilon\rightharpoonup u$ in $W_0^{1,1}(\Omega)$. If $\liminf_{\varepsilon\to0}F^\varepsilon(u^\varepsilon)=+\infty$, there is nothing to prove, because

$$\frac{1}{2}\int_\Omega a_{ijhk}e_{ij}(u)e_{hk}(u)dx+\lambda\int_\Sigma|[u]\otimes_s e_3|\leq+\infty.$$

Otherwise, $\liminf_{\varepsilon\to0}F^\varepsilon(u^\varepsilon)<+\infty$, there exists a subsequence of

$F^\varepsilon(u^\varepsilon)\leq C$, which implies that

$$\|e(u^\varepsilon)\|^2_{L^2(\Omega_\varepsilon,\mathbb{R}^9_s)}\leq C,$$

$$\int_{B_\varepsilon}|e(u^\varepsilon)|\leq C,$$

then $\chi_{\Omega_\varepsilon}e(u^\varepsilon)$ is bounded in $L^2(\Omega,\mathbb{R}^9_s)$, so for a subsequence of $\chi_{\Omega_\varepsilon}e(u^\varepsilon)$, still denoted by $\chi_{\Omega_\varepsilon}e(u^\varepsilon)$, we have

$$\chi_{\Omega_\varepsilon}e(u^\varepsilon)\rightharpoonup e(u)\quad \text{in } L^2(\Omega,\mathbb{R}^9_s).$$

Form the semirespectability's inequality of $u\to\frac{1}{2}\int_{\Omega_\varepsilon}a_{ijhk}e_{ij}(u)e_{hk}(u)dx$, and passing to the lower limit, we obtain

$$\liminf_{\varepsilon\to 0}\frac{1}{2}\int_{\Omega_\varepsilon} a_{ijhk}e_{ij}(u^\varepsilon)e_{hk}(u^\varepsilon)dx$$
$$\geq \frac{1}{2}\int_{\Omega} a_{ijhk}e_{ij}(u)e_{hk}(u)dx. \qquad (5.2)$$

To describe the lower limit in the domain B^ε, denote by $C_\rho(x_0)$ the cylinder $S_\rho(x_0)$ is the open ball in $\mathbb{R}^2$ with radios ρ in centered at x_0 on Σ, it suffices to establish for H_2 which represents the Hausdorff measure, almost all x_0 on Σ

$$\lim_{\rho\to 0}\frac{\nu(C_\rho(x_0))}{H_2(S_\rho(x_0))}\geq [u]\otimes_s e_3,$$

where ν represent the Lebesgue measure in $L^1(\Omega)$, then

$$\frac{\nu(C_\rho(x_0))}{H_2(S_\rho(x_0))}=\lim_{\varepsilon\to 0}\frac{\nu_\varepsilon(C_\rho(x_0))}{H_2(S_\rho(x_0))}$$
$$=\lim_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\int_{S_\rho(x_0)\times]-\varepsilon,\varepsilon[} e(u^\varepsilon)dx. \qquad (5.3)$$

Other way, let u in $W_0^{1,1}(\Omega)$ there exists a sequence u^n in $C_0^\infty(\Omega)$ such that $u^n \rightharpoonup u$ in $W_0^{1,1}(\Omega)$ when $n\to +\infty$ We will use the smoothing operator R_ε, define by

$$R_\varepsilon u=\begin{cases}\frac{[u]^-(x_3)}{2}\Psi_\varepsilon(x)+\frac{[u]^+(x_3)}{2} & \text{if } |x_3|<\frac{\varepsilon}{2}\\ u(x) & \text{if } |x_3|>\frac{\varepsilon}{2}\end{cases}$$

where

$$[u]^+(x_3)=\frac{u(x',|x_3|)+u(x',-|x_3|)}{2},$$
$$[u]^-(x_3)=\frac{u(x',|x_3|)-u(x',-|x_3|)}{2},$$

And

$$\Psi_\varepsilon(x)=sign(x_3)\min(\frac{|x_3|}{\varepsilon},1),$$

We denote by $u^{\varepsilon,n}=R_\varepsilon u^n$, We easily show that

$$e(u^n)=\frac{1}{\varepsilon}[u](x_0)\otimes_s e_3+e(u^n-R_\varepsilon(u^n)). \qquad (5.4)$$

Otherwise, let

$$q^{\varepsilon,n}=\begin{cases}\mathbf{div}u^{\varepsilon,n} & \text{in } B_\varepsilon\\ \frac{1}{\Omega_\varepsilon}\int_{B_\varepsilon}\mathbf{div}u^{\varepsilon,n} & \text{in } \Omega_\varepsilon\end{cases}$$

We have $q^{\varepsilon,n}\in L^1(\Omega)$ and

$$\int_\Omega q^{\varepsilon,n}=0,$$

according to the Lemma 5.2, there exists $v^{\varepsilon,n}\in W_0^{1,1}(\Omega,\mathbb{R}^3)$ such that $q^{\varepsilon,n}=\mathbf{div}v^\varepsilon$ and $v^{\varepsilon,n}$ depending linearly and continuously of $q^{\varepsilon,n}$ so there exists a constant $C>0$ such that

$$\|v^{\varepsilon,n}\|_{W_0^{1,1}}\leq C\|q^{\varepsilon,n}\|_{L^1}$$

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Using that $\int_\Omega q^{\varepsilon,n}=0$, implies $q^{\varepsilon,n}=0\ \ p.p$ for this result see you Roudin [15], so we have $v^{\varepsilon,n}\to 0$ in $W_0^{1,1}(\Omega,\mathbb{R}^3)$.

Let

$$w^{\varepsilon,n}=u^{\varepsilon,n}-v^{\varepsilon,n}.$$

Since $\mathbf{div} v^{\varepsilon,n}=\mathbf{div} u^{\varepsilon,n}$ in B_ε, so $\mathbf{div} w^{\varepsilon,n}=0$ in B_ε, it follows that $w^{\varepsilon,n}\in V^\varepsilon$, and as $u^{\varepsilon,n}\rightharpoonup u^n$ and $v^{\varepsilon,n}\to 0$ in $W_0^{1,1}(\Omega,\mathbb{R}^3)$.

According to the (5:3) and (5:4), we have

$$\lim_{\rho\to 0}\lim_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\int_{S_\rho(x_0)\times]-\varepsilon,\varepsilon[}|e(u^{\varepsilon,n})|dx \tag{5.5}$$

$$=\lim_{\rho\to 0}\lim_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\int_{S_\rho(x_0)\times]-\varepsilon,\varepsilon[}|e(R_\varepsilon(u))-e(u^{\varepsilon,n}-R_\varepsilon(u))|dx$$

$$=\lim_{\rho\to 0}\lim_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\{$$

$$\int_{S_\rho(x_0)\times]-\varepsilon,\varepsilon[}|\frac{1}{\varepsilon}[u]^-(x_0)\otimes_s e_3+e(u^{\varepsilon,n}-R_\varepsilon(u))|dx\}.$$

We can modify $u^{\varepsilon,n}-R_\varepsilon(u)$ in the boundary of $S_\rho(x_0)\times]-\varepsilon,\varepsilon[$ by a function $\varphi_\varepsilon\in W_0^{1,1}(S_\rho(x_0)\times]-t(\varepsilon),t(\varepsilon)[,\mathbb{R}^3)$ where $\lim_{\varepsilon\to 0}\frac{t(\varepsilon)}{\varepsilon}=1$, for more information see you Licht and Michaille [16], so that

$$\lim_{\rho\to 0}\lim_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\{$$

$$\int_{S_\rho(x_0)\times]-\varepsilon,\varepsilon[}|\frac{1}{\varepsilon}[u]^-(x_0)\otimes_s e_3+e(u^{\varepsilon,n}-R_\varepsilon u)|dx\}$$

$$\geq$$

$$\limsup_{\rho\to 0}\limsup_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\{$$

$$\int_{S_\rho(x_0)\times]-t(\varepsilon),t(\varepsilon)[}|\frac{1}{\varepsilon}[u]^-(x_0)\otimes_s e_3+e(\varphi_\varepsilon)|dx\}.$$

Recalling (5:5), then

$$\lim_{\rho\to 0}\lim_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\int_{S_\rho(x_0)\times]-\varepsilon,\varepsilon[}|e(u^{\varepsilon,n})|dx$$

$$\geq$$

$$\limsup_{\rho\to 0}\limsup_{\varepsilon\to 0}\frac{1}{|A_\varepsilon|}\inf\{$$

$$\int_{A_\varepsilon}[u]^-(x_0)\otimes_s e_3+e(\varphi)|dx:\varphi\in W_0^{1,1}(A_s,\mathbb{R}^3)\}$$

where $A_\varepsilon=\frac{1}{\lambda}S_\rho(x_0)\times]-t(\varepsilon),t(\varepsilon)[$, the subadditive process

$$A\to S_A=$$

$$\inf\{\int_A\{[u]^-(x_0)\otimes_s e_3+e(\varphi)|dx:\varphi\in W_0^{1,1}(A,\mathbb{R}^3)\}$$

satisfies all the condition of the global theorem thus finally obtain

$$\lim_{\varepsilon\to 0}\frac{\lambda}{H_2(S_\rho(x_0))}\{$$

$$\int_{S_\rho(x_0)\times]-\varepsilon,\varepsilon[}e(u^\varepsilon)dx\geq[u]^-(x_0)\otimes_s e_3\}$$

For $u \in W_0^{1,1}(\Omega)$ and $u^\varepsilon \in V^\varepsilon$, such that $u^\varepsilon \rightharpoonup u$ in $W_0^{1,1}(\Omega)$, Assume that

$$\liminf_{\varepsilon\to 0} F^\varepsilon(u^\varepsilon) < +\infty.$$

So there exists a constant $C > 0$ and a subsequence of $F^\varepsilon(u^\varepsilon)$, still de noted by $F^\varepsilon(u^\varepsilon)$, such that

$$F^\varepsilon(u^\varepsilon) < C.$$

So u^ε verifies the following evaluation (4.2) and (4.3), as $u^\varepsilon \rightharpoonup u$ in $W_0^{1,1}(\Omega)$ thanks to the Remark 4.5 we have $u \in W^{1,1}(\Omega)$ what contracdicts the fact that $u \in W^{1,1}(\Omega) \setminus W_0^{1,1}(\Omega)$, consequently we have

$$\liminf_{\varepsilon\to 0} F^\varepsilon(u^\varepsilon) = +\infty$$

- — (b) We are now in position to determine the upper epi-limit, we have

$$F^\varepsilon(w^{\varepsilon,n}) = \frac{1}{2}\int_{\Omega_\varepsilon} a_{ijhk} e_{ij}(w^{\varepsilon,n}) e_{hk}(w^{\varepsilon,n}) dx + \lambda \int_{B_\varepsilon} |e(w^{\varepsilon,n})|$$

which implies that

$$F^\varepsilon(w^{\varepsilon,n}) = \frac{1}{2}\int_{\Omega_\varepsilon} a_{ijhk} e_{ij}(w^n) e_{hk}(w^n) dx + \lambda \int_{B_\varepsilon} |e(w^{\varepsilon,n})|$$

$$= S_1 + S_2$$

so that

$$\lim_{\varepsilon\to 0} S_1 = \lim_{\varepsilon\to 0} \frac{1}{2}\int_{\Omega} \chi_{\Omega_\varepsilon} a_{ijhk} e_{ij}(w^n) e_{hk}(w^n) dx$$

$$= \int_{\Omega} a_{ijhk} e_{ij}(w^n) e_{hk}(w^n) dx$$

we have

$$S_2 = \lambda \int_{B_\varepsilon} |e(w^{\varepsilon,n})|$$

as in [8] we chow that

$$\lim_{\varepsilon\to 0} |e(w^{\varepsilon,n}) - \frac{1}{\varepsilon}[w^n] \otimes_s e_3|$$

Consequently,

$$\limsup_{\varepsilon\to 0} F^\varepsilon(w^{\varepsilon,n}) =$$

$$= \frac{1}{2}\int_{\Omega} a_{ijhk} e_{ij}(w^n) e_{hk}(w^n) dx + \lambda \int_{\Sigma} |[w^n] \otimes_s e_3|.$$

Science $w^n \to u$ in $W_0^{1,1}(\Omega)$ because $v^{\varepsilon,n} \to 0$, there fore according to the classic result, digitalization's Lemma, see [7, p.32], there exists a real function $n(\varepsilon) : \mathbb{R}^+ \to \mathbb{N}$ increasing to $+\infty$, such that $w^\varepsilon, n(\varepsilon) \rightharpoonup u$ in $W_0^{1,1}(\Omega)$ when $\varepsilon \to 0$.

Consequently, we have

$$\limsup_{\varepsilon\to 0} F^\varepsilon(w^{\varepsilon,n(\varepsilon)}) \le \limsup_{\varepsilon\to 0} \limsup_{n\to+\infty} F^\varepsilon(w^{\varepsilon,n})$$

$$\le \frac{1}{2}\int_{\Omega} a_{ijhk} e_{ij}(w^n) e_{hk}(w^n) dx + \lambda \int_{\Sigma} |[w^n] \otimes_s e_3|.$$

For $u \in W_0^{1,1}(\Omega)$, so for any sequence $u^\varepsilon \rightharpoonup u$ in $L^1(\Omega)$, we obtain

$$\limsup_{\varepsilon\to 0} F^\varepsilon(u^\varepsilon) \leq +\infty$$

Hence the proof is complete.

In the sequel, we determine the limit problem linked to (4:0), when ε approaches to zero. Thanks to the epi-convergence results, see section 2 [Theorem 3.3, and Proposition 3.2] and the theorem 5.1, according to the τ_f -continuity of the functional G in $W_0^{1,1}(\Omega)$, we have $F^\varepsilon + G$ τ_f -epiconverges to $F+G$ in $W_0^{1,1}(\Omega)$

Proposition 5.3. For any $f \in L^1(\Omega,\mathbb{R}^3)$, there exists $u^* \in W_0^{1,1}(\Omega,\mathbb{R}^3)$ satisfies

$$u^\varepsilon \rightharpoonup u^* \quad in\ W_0^{1,1}(\Omega,\mathbb{R}^3),$$

$$F(u^*) + G(u^*) = \inf_{u\in W_0^{1,1}(\Omega)} \{F(u)+G(u)\}.$$

Proof. Thanks to Lemma 4.2, the family $(u^\varepsilon)_\varepsilon$ is bounded in $L^1(\Omega)$, therefore it passess a τ_f – cluster point u^* in $L^1(\Omega)$. And thanks to a classical epiconvergence method, theorem 3.3, it follows that u^* is a solution of the problem : Find

$$\inf_{u\in W_0^{1,1}(\Omega)} \{F(u)+G(u)\} \tag{5.2}$$

Since $F = +\infty$ on $W^{1,1}(\Omega) \setminus W_0^{1,1}(\Omega)$, so (5.6) became

$$\inf_{u\in W^{1,1}(\Omega)} \{F(u)+G(u)\}.$$

According to the uniqueness of solutions of problem (5:6), so u^ε admits an unique τ_f -cluster point u^*, and therefore $u^\varepsilon \rightharpoonup u^*$ in $W_0^{1,1}(\Omega)$.

VI. Conclusion

Using the epi-convergence method we showed that the structure, constituted of two elastic bodies joined together by a incompressible elastic thin oscillating layer of thickness, rigidity, and periodicity parameter depending on (ϵ), obeying to a nonlinear elastic law, whose parameters depend on the negative powers of e, behaves at the limit like an elastic body embedded on the boundary and subjected to a density of forces of volume f, according to the powers of ε , the layer behaves like a rather rigid nonlinear elastic material surface with membrane effect, too rigid inextensible material surface, a material surface with effect of infection or the structure is embedded on the interface Σ , We found the same result of A. Ait Moussa and J. Messaho [1].

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