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New Theorems Involving the Generalized Mellin-Barnes Type of Contour Integrals and General Class of Polynomials

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Abstract - In the present investigation, First we establish three new theorems, which involves generalized Mellin-Barnes type of contour integrals and general class of polynomials. Next, we obtain certain new integrals and expansion formulas by the application of our theorems. By giving suitable values to the parameters, main integral reduces to Fox's H-function and generalized wright hypergeometric function, etc. Our Main findings provide interesting unification and extensions of a number of new results.

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New Theorems Involving the Generalized Mellin-Barnes Type of Contour Integrals and General Class of Polynomials

Praveen Agarwal [α] & Mehar Chand [σ]

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Keywords : $\overline{H}$ *-function, general class of polynomials, generalized wright hypergeometric function.*

I. Introduction

In 1987, Inayat-Hussain [1, 2] introduced generalization form of Fox's H-function, which is popularly known as $\overline{\mathrm{H}}$-function. Now $\overline{\mathrm{H}}$-function stands on fairly firm footing through the research contributions of various authors [1, 2, 3, 9, 10, 14, 15, and 16]. $\overline{\mathrm{H}}$-function is defined and represented in the following manner [10].

$$\overline{\mathrm{H}}_{p,q}^{m,n}[z]=\overline{\mathrm{H}}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L z^{\xi}\overline{\phi}(\xi)d\xi \quad (z\neq 0) \tag{1.1}$$

where

$$\overline{\phi}(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

It may be noted that the $\overline{\phi}(\xi)$ contains fractional powers of some of the gamma function and m,n,p,q are integers such that $1\le m\le q, 1\le n\le p$ $(\alpha_j)_{1,p},(\beta_j)_{1,q}$ are positive real numbers and $(A_j)_{1,n},(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(\alpha_j)_{1,p}$ and $(\beta_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\overline{\mathrm{H}}$-function can be seen in the papers [9, 10]

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Ref.

3. A.K. Rathie, A new generalization of generalized hypergeometric functions, Le Mathematic he Fasc. II 52 (1997), 297-310.
9. K.C. Gupta and R.C. Soni, On a basic integral formula involving the product of the H-function and Fox H-function, J.Raj.Acad.Phy. Sci., 4 (3) (2006), 157-164.

The behavior of the $\bar{H}$-function for small values of $|z|$ follows easily from a result given by Rathie [3]:

$$\bar{H}^{m,n}_{p,q}[z] = o\left(|z|^{\alpha}\right);\ \text{Where}$$

$$\alpha = \min_{1\le j\le m} \operatorname{Re}\left(\frac{b_j}{\alpha_j}\right), |z| \to 0 \tag{1.3}$$

$$\Omega = \sum_{j=1}^{m} |B_j| + \sum_{j=m+1}^{q} |b_j B_j| - \sum_{j=1}^{n} |a_j A_j| - \sum_{j=n+1}^{q} |A_j| > 0,\ 0 < |z| < \infty \tag{1.4}$$

The following function which follows as special cases of the $\bar{H}$-function will be required in the sequel [10]

$${}_p\bar{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};z\right] = \bar{H}^{1,p}_{p,q+1}\left[-z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\ (0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right] \tag{1.5}$$

The general class of polynomials $S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}[x]$ will be defined and represented as follows [6, p.185, eqn. (7)]:

$$S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}[x] = \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} x^{l_i} \tag{1.6}$$

where $n_1,\ldots,n_r = 0,1,2,\ldots; m_1,\ldots m_r$ are arbitrary positive integers, the coefficients $A_{n_i,l_i}(n_i,l_i \ge 0)$ are arbitrary constants, real or complex. $S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}[x]$ yields a number of known polynomials as its special cases. These includes, among other, the Jacobi polynomials, the Bessel Polynomials, the Lagurre Polynomials, the Brafman Polynomials and several others [8, p. 158-161].

The following formulas [12, p.77, Ens. (3.1), (3.2) & (3.3)] will be required in our investigation.

$$\int_0^{\infty}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx = \frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad (a>0; b\ge 0; c+4ab>0; \operatorname{Re}(p)+1/2>0) \tag{1.7}$$

$$\int_0^{\infty}\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx = \frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad (a\ge 0; b>0; c+4ab>0; \operatorname{Re}(p)+1/2>0) \tag{1.8}$$

$$\int_0^{\infty}\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx = \frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad (a>0; b>0; c+4ab>0; \operatorname{Re}(p)+1/2>0) \tag{1.9}$$

II. Main Theorems

In our investigation following result [11, p. 75] is also required.

$$\text{If } (1-y)^{\alpha+\beta-\gamma}\ {}_2F_1(2\alpha,2\beta;2\gamma;y) = \sum_{r=0}^{\infty} a_r y^r \tag{2.1}$$

Ref.

6. H.M.Srivastava, A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials, Paci c J.Math.117,(1985), 183-191.
10. K.C. Gupta, R. Jain and R. Agarwal, On existence conditions for a generalized Mellin-Barnes type integral Natl Acad Sci Lett. 30(5-6) (2007), 169-172.

then

$$ {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)=\sum_{r=0}^{\infty}\frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r}a_r X^r \tag{2.2}$$

Let X stands for $\left(ax+\frac{b}{x}\right)^2+c$

Notes

First Theorem:

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_i X^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\middle|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right] \tag{2.3}$$

The above result will be converge under the following conditions

i. $a>0;b\ge 0;c+4ab>0$ and $\mu_i>0,\delta\ge 0$.

ii. $\mathrm{Re}\left[\lambda+\delta\min_{1\le j\le m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z|<\frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

iv. $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Second Theorem:

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_i X^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\middle|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right] \tag{2.4}$$

The above result will be converge under the following conditions

i. $a\ge 0;b>0;c+4ab>0$ and $\mu_i>0,\delta\ge 0$.

ii. $\mathrm{Re}\left[\lambda+\delta\min_{1\le j\le m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z| < \frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

iv. $-\frac{1}{2} < (\alpha - \beta - \gamma) < \frac{1}{2}$

Third Theorem:

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] \overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^\infty \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i}(y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^\delta}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (2.5)$$

The above result will be converge under the following conditions

i. $a>0; b>0; c+4ab>0$ and $\mu_i>0, \delta\geq 0$.

ii. $\mathrm{Re}\left[\lambda+\delta\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z| < \frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

iv. $-\frac{1}{2} < (\alpha - \beta - \gamma) < \frac{1}{2}$

Proof :

To prove the first theorem, using the result given by equation (2.2) and express $\overline{\mathrm{H}}$ -function occurring on the L.H.S. of equation (2.3) in terms of Mellin-Barnes type of contour integral given by equation (1.1) and the general class of polynomials $S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]$ in series form with the help of equation (1.6) and then interchanging the order of integration and summation we get:

$$\sum_{r=0}^\infty \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i}(y_i)^{l_i} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \frac{1}{2\pi i}\int_L \overline{\phi}(\xi) z^\xi \left[\int_0^\infty \left[\left(ax+\frac{b}{x}\right)+c\right]^{-\lambda+r-\sum_{i=1}^k \mu_i l_i-\delta\xi-1} dx\right] d\xi \quad (2.6)$$

Further using the result (1.7) the above integral becomes

$$\sum_{r=0}^\infty \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i}(y_i)^{l_i} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$$\frac{1}{2\pi i}\int_L \overline{\phi}(\xi) z^\xi \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+r+\sum_{i=1}^k \mu_i l_i+\delta\xi+1/2}} \frac{\Gamma\left(\lambda-r+\sum_{i=1}^k \mu_i l_i+\delta\xi+1/2\right)}{\Gamma\left(\lambda-r+\sum_{i=1}^k \mu_i l_i+\delta\xi+1\right)} d\xi \quad (2.7)$$

Notes

Then interpreting with the help of (1.1) and (2.7) provides first integral.

Proceeding on the same parallel lines, theorems second and third given by (2.4) and (2.5) can be obtained by using the results (1.8) and (1.9) respectively.

Special Cases :

(3.1) If we put $A_j = B_j = 1$, $\overline{\mathrm{H}}$-function reduces to Fox's H-function [7, p. 10, Eqn. (2.1.1)], then the equation (2.3), (2.4) and (2.5) takes the following form.

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i\,!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),\left(a_j,\alpha_j\right)_{1,p}\\ \left(b_j,\beta_j\right)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \qquad (3.1.1)$$

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i\,!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),\left(a_j,\alpha_j\right)_{1,p}\\ \left(b_j,\beta_j\right)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \qquad (3.1.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i\,!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),\left(a_j,\alpha_j\right)_{1,p}\\ \left(b_j,\beta_j\right)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \qquad (3.1.3)$$

The Conditions of validity of (3.1.1), (3.1.2) and (3.1.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.2) By applying the our results given in (2.3), (2.4) and (2.5) to the case of Hermite polynomials [4, 5] by setting $S_n^2(x)\to x^{n/2}H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $m_1,\ldots,m_k=2;n_1,\ldots,n_k=n;k=1;v_i=v,y_i=y,A_{n_i,l_i}=(-1)^l$, we have the following interesting results.

Ref.

7. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publishers, New Dehli, Madras (1982).

Global Journal of Science Frontier Research (F) Volume XII Issue III Version I March 2012

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2}H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^\delta}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.2.1}$$

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2}H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^\delta}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.2.2}$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2}H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^\delta}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.2.3}$$

The Conditions of validity of (3.2.1), (3.2.2) and (3.2.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.3) By applying the our results given in (2.3), (2.4) and (2.5) to the case of Lagurre polynomials [4, 5] by setting $S_n^2(x)\rightarrow L_n^{(\alpha')}[x]$ in which $m_1,\ldots,m_k=1;n_1,\ldots,n_k=n;k=1,v_i=v,y_i=y,A_{n_i,l_i}=\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}$, we have the following interesting results.

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

Ref.

4. C. Szego, Orthogonal polynomials, Amer. Math. Soc. Colloq. Publ. 23 Fourth edition, Amer. Math. Soc. Providence, Rhode Island (1975).
5. E.M. Wright, The asymptotic expansion of the generalized Bessel Function. Proc. London Math. Soc. (Ser.2), 38(1935), 257-260.

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.3.1}$$

$$\int_0^{\infty}\frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.3.2}$$

$$\int_0^{\infty}\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.3.3}$$

The Conditions of validity of (3.3.1), (3.3.2) and (3.3.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.4) If we put $n=p,m=1,q=q+1,b_1=0,\beta_1=1,a_j=1-a_j,b_j=1-b_j$, then the $\overline{\mathrm{H}}$-function reduces to generalized wright hypergeometric function [17] i.e. $\overline{H}^{1,p}_{p,q+1}\left[z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right]={}_p\overline{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\right]$, the equations (2.3), (2.4) and (2.5) takes the following form.

$$\int_0^{\infty}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\ldots,m_k}_{n_1,\ldots,n_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\overline{\psi}_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$${}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \tag{3.4.1}$$

$$\int_0^{\infty}\frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\ldots,m_k}_{n_1,\ldots,n_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\overline{\psi}_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

Ref.

17. Wright, E.M., The asymptotic expansion of the generalized hypergeometric function. J. London Math. Soc. 10 (1935a), 286-293.

$$
{}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i;\delta,1\right),\left(a_j,\alpha_j;A_j\right)_{1,p}\\ \left(b_j,\beta_j;B_j\right)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \tag{3.4.2}
$$

$$
\int_0^\infty\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\overline{\psi}_q\left[zX^{-\delta}\right]dx
$$

$$
=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times
$$

$$
{}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),\left(a_j,\alpha_j;A_j\right)_{1,p}\\ \left(b_j,\beta_j;B_j\right)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \tag{3.4.3}
$$

The Conditions of validity of (3.4.1), (3.4.2) and (3.4.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.5) If we put $\alpha=\gamma$, in the main theorem, the value of a_r in (2.1) comes out to be equal to $\frac{\beta_r}{r!}$ and the result (2.3), (2.4) and (2.5) gives the following interesting integral.

$$
\int_0^\infty X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx
$$

$$
=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times
$$

$$
\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.5.1}
$$

$$
\int_0^\infty\frac{1}{X^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx
$$

$$
=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times
$$

$$
\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.5.2}
$$

Notes

Notes

$$\int_0^\infty \left(a+\frac{b}{X^2}\right) X^{-\lambda-1}\, {}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] \overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.5.3)$$

The conditions of validity of (3.5.1), (3.5.2) and (3.5.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.6) If we put $\beta=\alpha+\frac{1}{2}$ and $\alpha=-f$ (f is non-negative integer) in (3.5.1), (3.5.2) and (3.5.3), we have:

$$\int_0^\infty X^{-\lambda-1}(1-X)^f S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{f}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.6.1)$$

$$\int_0^\infty \frac{1}{X^2}X^{-\lambda-1}(1-X)^f S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.6.2)$$

$$\int_0^\infty \left(a+\frac{b}{X^2}\right)X^{-\lambda-1}(1-X)^f S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.6.3)$$

The conditions of validity of (3.6.1), (3.6.2) and (3.6.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

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Notes