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Relation Between Weakly Prime Elements and Weakly Prime Sub Modules

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Abstract - In this paper we give the defition of weakly prime element of a module. Therefore we give a new definition of factorization in a module, which is called weakly factorization. So we call a module weakly unique factorization which is unique. We give the relation between weakly prime elements and weakly prime sub modules Then we characterize such weakly unique factorization modules.

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Relation Between Weakly Prime Elements and Weakly Prime Sub Modules

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Abstract - In this paper we give the defition of weakly prime element of a module. Therefore we give a new definition of factorization in a module, which is called weakly factorization. So we call a module weakly unique factorization which is unique. We give the relation between weakly prime elements and weakly prime sub modules Then we characterize such weakly unique factorization modules.

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I. Introduction

The study of factorization in tortion free modules was begun0 in Nicolas (5). She defined the module M to be factorial if (1) every non zero element of M has a irreducible factorization,(2) every irreducible element of R is prime, and (3)every irreducible element of M is primitive. She showed that if M is factorial then R is a UFD. After this she showed that M is a un0ique factorization module (UFM)if and only if (1)every element of M 0has an irreducible factorization, and(2)if x= $a_1a_2\ldots\ldots a_km = b_1b_2\ldots\ldots b_tm'$ are two factorization of x € M then k = $a_i \sim b_i$ for all i € { 1,2,……..k } and m ~ m'. Later, Lu(3) gives some characterizations of UFM and relations between prime submodules and primitive elements such modules. Further she investigates polynomial modules. There is an another work about factorization of modules, by Anderson and Valdes-Leon(1).They generalize factorization of any modules over a ring with zero divisor, which have non zero tortion elements. They showed that their defition and definition of Nicolas are concides if M is tortion free module and R is and integral domain.

We give a new definition of factorization for modules, named weakly factorization, and give relations between ewakly prime elements and weakly prime submodules. After this investigate the direct sum of modules, the direct product of modules, fractions of modules and polynomial of modules.

Throughout this paper all rings, R are commutative ring with identity 1 and all modules, M are non zero torsion free module which are unitary.

We will give some definitions:

Definition 1. Let M be a tortion free R-module and m be anon zero element of M.

(1) m is irreducible in M if m=am' implies that a € U(R) for every a € R and m' €M

(2) M primitive in M if m | am' implies m | m' for all 0 ≠ a € R and m'€M.

(3) An irreducible element p of R is called prime to the module M if p | am implies p|a in R or p | m in M.

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Definition 2. Let M be an R- module. Then a submodule N of M is called pure sub module if for all a € R we have aM∩ N =An

Definition 3. Anonzero element m of M is called weakly prime (w-prime) if for a,b € R and m' € M , m | abm' implies m | am' or m | bm'.

Definition 4. A submodule N of an R-module M is called weakly prime if abk € N implies ak € N or bk € N for all k € M and a,b € R

Definition 5. Atortion- free module M over a commutative ring with identity R is called a weakly unique factorization module (w-UFM) or w-factorial module if the following two conditions are satisfied:

(w-ufm 1) Each nonzero element x€ M has a w-factorization, $x=a_1a_2\ldots\ldots a_km$, where a_i's are irreducible elements in R (possibly with k=0) and m is a w-prime element in M.

(w-ufm2) if $x=a_1a_{2\ldots\ldots a}k_m = b_1b_2\ldots\ldots b_tm'$ are two factorization of x, then k= t , $a_i \sim b_i$ and $m \sim m'$ for all i € { 1,2,……,k}.

Definition 6. Let Mbe an R-module and a € R, m € M

(1) An element d € R is called greatest common divisor (gcd) of a and m if the following two condition hold

i) d | a in R and d | m in M , and

ii) if there is an element c € R such that c|a in R and c | m in M then c is a divisor of d.

(2) An element m' € M is called least common multiple (lcm) of ab and m if the following two condition hold

i) a|m' and m| m' in M respetively, and

ii) if there is an element n € M such that a | n and m | n in M then m' is a factor of n

The following propositions are given by (3) without their proof, we will give now their proof

Proposition 1. Let Mbe an R- module then every w-primitive element of M is an irreducible element.

Proof, suppose that m is aprimitive element of M and let m =am' for some a € R, m' € M. Then m | am' and since m is primitive we get m | m'. Since m = am´ implies m´ | m. Therefore m ~ m´, hence m is a irreducible element.

Proposition 2. Let M be an R module, then every primitive element of M is w-prime.

Proof, Assume that m | abm´ for some a,b € R and m´ € M .Then since is primitive. We get m | m´ .Hence m | am´ and m | bm´.

Example 1. Let R be a commutative ring with an identity and M = R| ẍ|, the polynomial ring over Ris an R- module then x € M is w-prime (primitive, irreducible) element.

Example 2. Let R = Z and M = Z | x|. Then the element 2x is a w-prime element but is neither primitive nor irreducible.

Theorem1. Let M be a torsion- free R-module. Then M is a UFM if and only if M is w-UFM.

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Proof, The follows from theorem "Let M be a module over a UFD R which satisfie (w-ufm 1) Then M is a w-UFM if and only if every weakly prime element of M is primitive.

With this there m we get that in a w-UFD M, every weakly prime element of M is irreducible element of M. And this gives u that weakly factorial modules and factorial module concides. From this note we obtain the following corollaries.

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Corllary 1, Let M be an R –module then two primitive elements m and m^{J} of M are non associates if and only if $Rm' \cap Rm = 0$

Corollary 2. Every vector space is w-UFM.

Theorem 2. Let { M_i / i € I } be a set of modules over a UFD R. Then the following statements are equivalent.

i) $\prod M_i$ is a w-UFM over R,

ii) O M_i is a w-UFM over R ,

iii) Each M_i is a w-UFM over R

Proof. (i) => (ii) => (iii) it is clear

(iii) implies(i) now assume that each M_i is a w-UFM over R for i €

I .Let $M = \prod_i M_i$ and $m = (m_i)_{i \in I}$ € M where $m_i = a_i m_i'$ for some a_i€ R and a w-prime element m_i' of M_i. First we will show that $m = (m_i)_{i \in I}$ € M is a w-prime element in M iff $\{ a_i \}_{i \in I}$ has no gcd. In R. Let m € M be a w- prime element. Assume that d = g.c.d $\{a_i\}$ And set $a_i = db_i$ for b_i € R. Then m =dm´ where m' = $(b_i m_i)_{i \in} t'$. Then m | m =dm´ but m | m´ gives us a contradiction. For the converse assume that $\{ a_i \}_{i \in I}$ has no g.cd. in R. Let m | cbn for c,b € R and $n = (n_i)_{i \in I}$ € M then there exist r € R uch that rm =cbn. So this gives us that for all i € I $rm_i = cbn_i$. Thus for all i € I , $Ra_i m_i' = cbn_i$ Since m_i' is w-prime and $\{ a_i \}_{Ii \in I}$ has no g.c.d. then for all i € I we get $a_i m_i' | cn_i$ or $a_i m_i' | bn_i$. Hence m | cn or m | bn, so m is w-prime. Now we will show that M is a w-UFM. Let $m = (m = (m_i)_{i \in I}$ € M, since each M_i is a w-UFM over R we have a w-factorization for m_i € M and i € I such that m_i € M_i i € I such that $m_i = a_i m_i'$ where m_i' is w- prime in M_i .If we let d =g.c.d. { a_i } then for $a_i = db_i$ we obtain the equation m = dm´ where m´ = $(b_i m_i')$. Now by Theorem "Let M be a w-factorization module over a UFD R such that $Pm \neq M$ for every non unit element p € R .Then the following statements are equivalent:

i) p is prime to M

ii) pM is a weaky prime submodule of M with (pM:M) =(p) "m´ is w-prime in M and since R is UFD M satisfies w-UFMI. Now, let p € R be a w-irreducible element such that p | abm in M for some a,b € R and m = $(m_i)_{i \in I}$ € M. Then for all i € I , p $|abm_i$ in M_i is w-UFM if p| ab then p | m_i for all i € I. Consequently p | m and therefore M is w-UFM.

Corollary 3, Every free module over a UFD is w-UFM

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