Global Journal of Science Frontier Research
Physics & Space Science
Volume 12 Issue 1 Version 1.0 January 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Deformation due to various sources in micropolar elastic solid with voids under inviscid liquid half space

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GJSFR-A Classification : FOR Code: 010599

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Deformation due to various sources in micropolar elastic solid with voids under inviscid liquid half space

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Abstract - The present investigation deals with the deformation of a micropolar elastic solid with void overlying a semi-infinite inviscid fluid subjected at the plane interface due to various sources . Laplace and Fourier transform techniques have been used to solve the problem. The expressions of the displacement components, stress, couple stress and change in volume fraction field are obtained in the transformed domain. As an application of the approach (i) concentrated force (ii) uniformly distributed force (iii) linearly distributed force (iv) moving couple have been taken to illustrate the utility of the approach. A particular case of interest have been deduced from the investigation.

I. Introduction

A non-linear theory concerning solid elastic materials consisting of various pores (voids) distributed throughout the body has been formulated by Nunziato and Cowin (1979). Later, Cowin and Nunziato (1983) developed a theory of linear elastic materials with voids, for the mathematical study of the mechanical behavior of porous solids. They introduced the presence of pores in the classical continuum model by assigning an additional degree of freedom to each material particle, namely fraction of elementary volume which results void of matter; consequently, the bulk mass density of such materials is given by the product of two fields, the void volume fraction and the mass density of matrix material.

Classical mechanics deals with the basic assumption that the effect of the microstructure of a material is not essential for describing mechanical behavior. Such an approximation has been shown in many well-known cases. Often, however, discrepancies between the classical theory and experiments are observed, indicating that the microstructures might be important. For example, discrepancies have been found in the stress concentrations in the areas of holes, notches and cracks; elastic vibrations characterized by a high frequency and small wavelengths, particularly in granular composites consisting of stiff inclusions embedded in a weaker matrix, fibers or grains; and the mechanical behavior of complex fluids such as liquid crystals, polymeric suspensions, and animal blood. In general, granular composites, for example, porous materials are widely used in the area of passive noise control as sound absorbers and the aspect of acoustical waves characterized by high frequencies and small wavelengths become significant.

To explain the fundamental departure of microcontinuum theories from classical continuum theories, the formal is continuum model embedded with microstructures to describe the microscopic motion or a non local model to describe the long range material interaction. This extends the application of the continuum model to microscopic space and short time scales. Micromorphic Theory (Eringen and Suhubi, 1964; Eringen, 1999) treats a material body as continuous collection of a large number of deformable particles, with each particle possessing finite size and inner structure. Using assumptions such as infinitesimal deformation and slow motion, micromorphic theory can be reduced to Mindlin's Microstructure Theory (1964) . When the microstructure of the material is considered rigid, it becomes the micropolar theory (Eringen, 1966) . Eringen's micropolar theory is more appropriate for geological materials like rocks, soils, since their theory takes into account the intrinsic rotation and predicts the behavior of the material with inner structure.

Different researchers has discussed different type of problems in micropolar elasticity with voids. Scarpetta (1990), Marin (1996-a, 1996-b) , discussed some problems in micropolar theory of elastic solids with voids. Passarella (1996) derived the constitutive relations and field equations for anisotropic micropolar porous theomoelastic materials and also derived some basic results.

Mondal and Acharya (2006) studied the effect of voids on the propagation of surface waves in a homogeneous micropolar elastic solids medium which contains distribution of vacuous pores, Kumar, Deswal and Tomar (2002) , discussed the surface wave propagation in micropolar liquid saturated porous layer over a micropolar liquid saturated porous half space of different elastic properties. Kumar and Deswal (2000) studied the propagation of surface waves in a micropolar liquid saturated porous solids line under a uniform layer of liquid.

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Kumar and Ailawalia (2007) studied the interaction in a micropolar thermoelastic medium with voids due to distributed loads. Kumar and Ailawalia (2009) studied the influence of various sources in micropolar thermoelastic medium with voids. Kumar and Kumar (2009) studied the analysis of waves motion in transversely isotropic medium with voids under a inviscid liquid layer.

Ghiba (2008) studied the asymptotic partition of energy in the micropolar mixture theory of porous media. Huang and Zhao (2009) developed a mixture theory for multicomponent micropolar porous media with a combination of the hybrid mixture theory and the microcontinuum theory. Madeo and Gavrilyuk (2010) studied the propagation of acoustic waves in porous media and their reflection and transmission at a pure-fluid/porous-medium permeable interface. Tomar and Khurana (2011) investigated the reflection and transmission phenomena of plane longitudinal wave from a plane interface between two distinct micropolar porous elastic solid half space in welded contact. Kumar and Kumar (2011) studied the wave propagation in transversely isotropic generalized thermoelastic half space with voids under initial stress and discusses the reflection characteristics of these waves under consideration of stress free, thermally insulated or isothermal boundary conditions. Kumar and Kumar (2011) studied the wave propagation in orthotropic generalized thermoelastic half space with voids under initial stress.

In this paper the deformation due to various sources in micropolar elastic with void overlying a semi-infinite invicsid fluid is studied. The expressions for components of displacement , stress and acoustic pressure are obtained by using Laplace and Fourier Transforms due to concentrated force, uniformly distributed force, linearly distributed force and moving couple .

II. BASIC EQUATIONS

Following Eringen (1968) and Iesan (1985) the equations of motion and constitutive relations in a homogenous isotropic micropolar material with voids are given by:

$$(\lambda+2\mu+K)\nabla\left(\nabla.\vec{u}\right)-(\mu+K)\nabla\times\nabla\times\vec{u}+K\left(\nabla X\vec{\phi}\right)+\xi\nabla\psi=\rho\frac{\partial^2\vec{u}}{\partial t^2}\ , \qquad (1)$$

$$(\alpha+\beta+\gamma)\nabla\left(\nabla.\vec{\phi}\right)-\gamma\nabla\times\left(\nabla\times\vec{\phi}\right)+K\left(\nabla\times\vec{u}\right)-2K\vec{\phi}=\rho j\frac{\partial^2\vec{\phi}}{\partial t^2}\ , \qquad (2)$$

$$d\nabla^2\psi-\xi\nabla.\vec{u}-\omega_1^*\frac{\partial\vec{\psi}}{\partial t}-a\psi=\rho\chi\frac{\partial^2\psi}{\partial t^2}\ , \qquad (3)$$

$$t_{ij}=\lambda\delta_{ij}u_{r,r}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\in_{ijk}\phi_k\right)+\xi\psi\delta_{ij}\ , \qquad (4)$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\phi_{i,j}+\gamma\phi_{j,i}+\varsigma\psi\delta_{ij}\ , \qquad (5)$$

where the list of symbols is given at the end of the paper.

III. PROBLEM FORMULATION

We consider a homogeneous isotropic micropolar elastic half space with void (medium M1) and underlying a uniform homogeneous inviscid liquid(medium M2). We take the rectangular coordinate system $(Ox_1x_2x_3)$ with origin at the interface of M1and M2 and X_3 - axis is pointing normally into the medium M1.

We restrict our analysis to the plane deformation parallel to x_1x_3 plane with displacement vector $\vec{u}$ and microrotation vector $\vec{\phi}$. For two-dimensional problem, we take

$$\vec{u}=(u_{1,}0,u_3)\ ,\quad \vec{\phi}=(0,\phi_2,0) \qquad (6)$$

We introduce the non-dimensional quantities defined by the expressions

$$x_1' = \frac{\omega^*}{c_1} x_1, \quad x_3' = \frac{\omega^*}{c_1} x_3, \quad u_1' = \frac{\omega^*}{c_1} u_1, \quad u_3' = \frac{\omega^*}{c_1} u_3, \quad \phi'_2 = \left(\frac{\rho c_1^2}{k}\right)\phi_2$$

$$\psi' = \left(\frac{\rho c_1^2}{k}\right)\psi, \quad \phi'^f = \frac{\omega^*}{c_1^2}\phi^f, \quad t'_{31} = \frac{t_{31}}{\mu}, \quad t'_{33} = \frac{t_{33}}{\mu}, \quad m'_{32} = \frac{m_{32}}{\mu}\frac{\omega^*}{c_1}$$

$$t' = \omega^* t, \quad \omega^{*2} = \frac{k}{\rho j}, \quad \bar{u}^{f'} = \frac{\omega^* \bar{u}^f}{c_1}, \quad p' = \frac{p}{\lambda^f} \tag{7}$$

Equations (1) - (3) with the aid of (6) and (7) after suppressing the primes, yield

$$a_1 \frac{\partial e}{\partial x_1} + a_2 \nabla^2 u_1 - a_3 \frac{\partial \phi_2}{\partial x_3} + a_4 \frac{\partial \psi}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}, \tag{8}$$

$$a_1 \frac{\partial e}{\partial x_3} + a_2 \nabla^2 u_3 + a_3 \frac{\partial \phi_2}{\partial x_1} + a_4 \frac{\partial \psi}{\partial x_3} = \frac{\partial^2 u_3}{\partial t^2}, \tag{9}$$

$$a_5 \nabla^2 \phi_2 + a_6 \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) - 2a_7 \phi_2 = \frac{\partial^2 \phi_2}{\partial t^2}, \tag{10}$$

$$a_8 \left(\frac{\partial^2 \psi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_3^2}\right) - a_9 e - a_{10} \frac{\partial \psi}{\partial t} - a_{11} \psi = \frac{\partial^2 \psi}{\partial t^2}, \tag{11}$$

where

$$a_1 = \frac{\lambda + \mu}{\rho c_1^2}, a_2 = \frac{k + \mu}{\rho c_1^2}, a_3 = \frac{k^2}{\rho^2 c_1^4}, a_4 = \frac{\xi k}{\rho^2 c_1^4}, a_5 = \frac{\gamma}{\rho j c_1^2}, a_6 = \frac{c_1^2}{j \omega^{\bullet 2}},$$

$$a_7 = \frac{k}{\rho j \omega^{\bullet 2}}, a_8 = \frac{d}{\chi \rho c_1^2}, a_9 = \frac{\xi c_1^2}{\chi k \omega^{\bullet 2}}, a_{10} = \frac{\omega_1^*}{\chi \rho \omega^{\bullet}}, a_{11} = \frac{a}{\chi \rho \omega^{\bullet 2}}, \tag{12}$$

The displacement components u_1 and u_3 are related to the potential functions as,

$$u_1 = \frac{\partial \Phi}{\partial x_1} - \frac{\partial \Psi}{\partial x_3}, \quad u_3 = \frac{\partial \Phi}{\partial x_3} + \frac{\partial \Psi}{\partial x_1}, \tag{13}$$

Substituting the values of u_1 and u_3 from equation (13) in the equations (8)-(11), we obtain,

$$\nabla^2 \Phi + a_4 \psi - \frac{\partial^2 \Phi}{\partial t^2} = 0, \tag{14}$$

$$a_2 \nabla^2 \Psi + a_3 \phi_2 - \frac{\partial^2 \Psi}{\partial t^2} = 0, \tag{15}$$

$$a_5\nabla^2\phi_2 - a_6\nabla^2\Psi - 2a_7\phi_2 - \frac{\partial^2\phi_2}{\partial t^2} = 0 , \tag{16}$$

$$a_8\nabla^2\psi - a_9\nabla^2\Phi - a_{10}\frac{\partial\psi}{dt} - a_{11}\psi - \frac{\partial^2\psi}{dt^2} = 0 . \tag{17}$$

We define Laplace Transforms as

$$\bar{f}(x_1,x_3,s) = \int_0^\infty f(x_1,x_3,t)e^{-st}dt \tag{18}$$

and its inverse as

$$f(x_1,x_3,t) = \frac{1}{2\pi}\int_{x-i\infty}^{x+i\infty} f(x_1,x_{3,}s)e^{st}ds \tag{19}$$

Fourier Transforms as

$$\hat{\bar{f}}(\xi,x_3,s) = \int_{-\infty}^{\infty}\bar{f}(x_1,x_3,s)e^{-i\xi x_1}dx \tag{20}$$

and its inverse as

$$\bar{f}(x_1,x_3,s) = \frac{1}{2\pi i}\int_{-\infty}^{\infty}\hat{\bar{f}}(\xi,x_3,s)e^{i\xi x_1}d\xi \tag{21}$$

Applying Laplace and Fourier Transform defined by (18) and (20) on (14) - (17) and after simplification, we obtain,

$$\left(\frac{d^4}{dz^4} + A\frac{d^2}{dz^2} + B\right)\left(\hat{\bar{\Phi}},\hat{\bar{\psi}}\right) = 0 , \tag{22}$$

$$\left(\frac{d^4}{dz^4} + C\frac{d^2}{dz^2} + D\right)\left(\hat{\bar{\phi}}_2,\hat{\bar{\Psi}}\right) = 0 , \tag{23}$$

where

$$A = -\left[1 + b_7 s + b_8 + b_6 s^2 + (\xi^2 + s^2) + b_9\right]$$

$$B = (\xi^2 + s^2)\left[b_7 s + b_8 + b_6 s^2 + \xi^2\right] - b_9\xi^2$$

$$C = b_1 b_3 - 2(b_4 + \xi^2) - s^2(b_5 + b_2)$$

$$D = (b_2 s^2 + \xi^2)(b_5 s^2 + 2b_4 + \xi^2) - b_1 b_3 \xi^2$$

$$\hat{\bar{\psi}} = \left(\frac{s^2}{b_{14}\nabla^2 + a_4 - b_{15}s - b_{16} - s^2}\right)\hat{\bar{\Phi}}$$

$$\hat{\bar{\phi}}_2 = \left(\frac{s^2}{b_{10}\nabla^2 + a_3 - 2b_{11} - b_{12}s^2}\right)\hat{\bar{\Psi}} \tag{24}$$

The solution of equations (20) and (21) satisfying radiation conditions that $\hat{\Phi}, \hat{\psi}, \hat{\phi}_2, \hat{\Psi} \to 0$ as $x_3 \to \infty$ are

$$\hat{\Phi} = A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3} \tag{25}$$

$$\hat{\psi} = A_1 d_1 e^{-m_1 x_3} + A_2 d_2 e^{-m_2 x_3} \tag{26}$$

$$\hat{\Psi} = B_1 e^{-m_3 x_3} + B_2 e^{-m_4 x_3} \tag{27}$$

$$\hat{\phi}_2 = B_1 d_3 e^{-m_3 x_3} + B_2 d_4 e^{-m_4 x_3} \tag{28}$$

where

$$d_1 = \left(\frac{s^2}{b_{14}(m_1^2 - \xi^2) + a_4 - b_{15}s - b_{16} - s^2} \right)$$

$$d_2 = \left(\frac{s^2}{b_{14}(m_2^2 - \xi^2) + a_4 - b_{15}s - b_{16} - s^2} \right)$$

$$d_3 = \left(\frac{s^2}{b_{10}(m_3^2 - \xi^2) + a_3 - 2b_{11} - b_{12}s^2} \right)$$

$$d_4 = \left(\frac{s^2}{b_{10}(m_4^2 - \xi^2) + a_3 - 2b_{11} - b_{12}s^2} \right)$$

Substituting the values of $\hat{\Phi}, \hat{\Psi}$ from (25) and (27) in (13) and with the aid of (18) and (20) yield the displacement components as

$$\hat{\tilde{u}}_1 = -i\xi\left(A_1 e^{-m_1 x_3} + A_2 e^{-m_2 x_3}\right) + \left(m_3 B_1 e^{-m_3 x_3} + m_4 B_2 e^{-m_4 x_3}\right) \tag{29}$$

$$\hat{\tilde{u}}_3 = -\left(m_1 A_1 e^{-m_1 x_3} + m_2 A_2 e^{-m_2 x_3}\right) - i\xi\left(B_1 e^{-m_3 x_3} + B_2 e^{-m_4 x_3}\right) \tag{30}$$

From equations (4)-(7), (13),(18),(20) and with the help of (25)-(28) , we obtain the components of stresses, couple stress as

$$\hat{\tilde{t}}_{33} = \left(f + g{m_1}^2 + hd_1\right)A_1 e^{-m_1 x_3} + \left(f + g{m_2}^2 + hd_2\right)A_2 e^{-m_2 x_3} + \left(fm_3 + i\xi g m_3\right)B_1 e^{-m_3 x_3} + \left(fm_4 + i\xi g m_4\right)B_2 e^{-m_4 x_3} \tag{31}$$

$$\hat{\tilde{t}}_{31} = Km_1 A_1 e^{-m_1 x_3} + Km_2 A_2 e^{-m_2 x_3} - \left(\xi^2 + b_{18}d_3 + b_{17}{m_3}^2\right)B_1 e^{-m_3 x_3} + \left(\xi^2 + b_{18}d_4 + b_{17}{m_4}^2\right)B_2 e^{-m_4 x_3} \tag{32}$$

$$\hat{\tilde{m}}_{32} = -b_{22}\left(m_3 d_3 B_1 e^{-m_3 x_3} + m_4 d_4 B_2 e^{-m_4 x_3}\right) \tag{33}$$

Following Achenbach (1973), the field equations in terms of velocity potential for inviscid fluid are given by

$$p = -\rho^f \frac{\partial \varphi^f}{\partial t} \tag{34}$$

$$\nabla^2 \varphi^f = \frac{1}{\alpha^{f^2}} \frac{\partial^2 \varphi^f}{\partial t^2} \tag{35}$$

$$u_3{}^f = \frac{\partial \varphi^f}{\partial x_3} \tag{36}$$

$$\vec{u}^{\,f} = \nabla \varphi^f \tag{37}$$

where ρ^f is the density, p is the acoustic pressure, $u_3{}^f \left(= \frac{\partial \varphi^f}{\partial x_3} \right)$ is the normal component of velocity $\vec{u}^{\,f}$.

Applying Laplace & Fourier Transform defined by (18) and (20) on (34)-(37) , we obtain

$$\hat{\bar{u}}_3{}^f = -m_5 D e^{-m_5 x_3}, \tag{38}$$

$$\hat{\bar{p}} = -b_{13} s e^{-m_5 x_3}. \tag{39}$$

where

$$m_5 = \sqrt{\xi^2 + \delta_1{}^2 s^2} \tag{40}$$

Boundary Conditions : At the surface $x_3 = 0$ are,

$$\begin{aligned} &1)\quad t_{33} - p = -F_1(x,t)\ , \\ &2)\quad t_{31} = 0\ , \\ &3)\quad m_{32} = 0\ , \\ &4)\quad \frac{d\psi}{dx_3} = 0\ , \\ &5)\quad \dot{u}_3 = u_3^f\ , \end{aligned} \tag{41}$$

Where F(x,t) is the known function. Applying the Laplace and Fourier Transform defined by (18)-(21) on (34) and with the help of (29) to (34), we obtain the system of five non-homogeneous equations and after some calculations, we obtain the stress components and acoustic pressure as

$$\hat{\bar{t}}_{33} = \frac{1}{\Delta}\left(\Delta_{A_1} J e^{-m_1 x_3} + \Delta_{A_2} \mathrm{Re}^{-m_2 x_3} + \Delta_{B_1} V e^{-m_3 x_3} + \Delta_{B_2} U e^{-m_4 x_3}\right)\ , \tag{42}$$

$$\hat{\bar{t}}_{31} = \frac{1}{\Delta}\left(\Delta_{A_1} K m_1 e^{-m_1 x_3} + \Delta_{A_2} K m_2 e^{-m_2 x_3} - \Delta_{B_1} G e^{-m_3 x_3} - \Delta_{B_2} H e^{-m_4 x_3}\right)\ , \tag{43}$$

$$\hat{\bar{m}}_{32} = -\frac{b_{22}}{\Delta}\left(m_3 d_3 \Delta_{B_1} e^{-m_3 x_3} + m_4 d_4 \Delta_{B_2} e^{-m_4 x_3}\right), \tag{44}$$

$$\hat{\bar{p}} = -\frac{b_{13} s}{\Delta}\left(\Delta_D e^{-m_5 x_3}\right), \tag{45}$$

where

$$\Delta = \begin{vmatrix} J & R & V & U & L \\ Km_1 & Km_2 & G & H & 0 \\ 0 & 0 & -b_{22}m_3d_3 & -b_{22}m_4d_4 & 0 \\ m_1d_1 & m_2d_2 & 0 & 0 & 0 \\ m_1 & m_2 & i\xi & i\xi & -m_5 \end{vmatrix}$$

$$\Delta = m_4d_4(2GQ + GTR + YKVm_5 + i\xi Y) - m_3d_3(HQ + THR + YUKm_5) - S(G+H) \quad (46)$$

$$\Delta_{A_1} = \hat{\bar{F}}_1(\xi,s)m_5d_2m_2b_{22}(Gm_4d_4 - Hm_3d_3) \quad (47)$$

$$\Delta_{A_2} = \hat{\bar{F}}_1(\xi,s)m_5d_1m_1b_{22}(Gm_4d_4 - Hm_3d_3) \quad (48)$$

$$\Delta_{B_1} = \hat{\bar{F}}_1(\xi,s)m_5d_4m_4b_{22}Km_1m_2(d_2 - d_1) \quad (49)$$

$$\Delta_{B_2} = -\hat{\bar{F}}_1(\xi,s)m_5d_3m_3b_{22}Km_1m_2(d_2 - d_1) \quad (50)$$

$$\Delta_D = \hat{\bar{F}}_1(\xi,s)[b_{22}m_1m_2(d_1 - d_2)\{d_3m_3(H - Ki\xi) - d_4m_4(G - Ki\xi)\}] \quad (51)$$

Applying the Inverse Laplace and Fourier transform defined by (19) and (21) on (42)-(45) , we obtain the components of stress, normal velocity and pressure in the fluid as

$$t_{33} = \frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left(\Delta_{A_1}Je^{-m_1x_3} + \Delta_{A_2}\mathrm{R}e^{-m_2x_3} + \Delta_{B_1}Ve^{-m_3x_3} + \Delta_{B_2}Ue^{-m_4x_3}\right)e^{i\xi x}e^{st}\,d\xi ds \quad (52)$$

$$t_{31} = \frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left(\Delta_{A_1}Km_1e^{-m_1x_3} + \Delta_{A_2}Km_2e^{-m_2x_3} - \Delta_{B_1}Ge^{-m_3x_3} - \Delta_{B_2}He^{-m_4x_3}\right)e^{i\xi x}e^{st}\,d\xi ds \quad (53)$$

$$m_{32} = -\frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\frac{b_{22}}{\Delta}\left(m_3d_3\Delta_{B_1}e^{-m_3x_3} + m_4d_4\Delta_{B_2}e^{-m_4x_3}\right)e^{i\xi x}e^{st}\,d\xi ds \quad (54)$$

$$p = -\frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\frac{b_{13}s}{\Delta}\left(\Delta_D e^{-m_5x_3}\right)e^{i\xi x}e^{st}\,d\xi ds \quad (55)$$

IV. Particular Case

In the absence of voids effect $\xi, d, \chi, w_1^* \rightarrow 0$, the boundary conditions given by (41) reduce to

1) $t_{33} - p = -F_1(x,t)$
2) $t_{31} = 0$
3) $m_{32} = 0$ (56)
4) $u_3 = u_3^f$

and the components of stress and acoustic pressure reduces to

$$t_{33}=\frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left(\Delta_{A_1}Je^{-m_1x_3}+\Delta_{A_2}\,\mathrm{Re}^{-m_2x_3}+\Delta_{B_1}Ve^{-m_3x_3}+\Delta_{B_2}Ue^{-m_4x_3}\right)e^{i\xi x}e^{st}d\xi ds \tag{57}$$

$$t_{31}=\frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left(\Delta_{A_1}Km_1e^{-m_1x_3}+\Delta_{A_2}Km_2e^{-m_2x_3}-\Delta_{B_1}Ge^{-m_3x_3}-\Delta_{B_2}He^{-m_4x_3}\right)e^{i\xi x}e^{st}d\xi ds \tag{58}$$

$$m_{32}=\frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\left(-\frac{b_{22}}{\Delta}\left(m_3d_3\Delta_{B_1}e^{-m_3x_3}+m_4d_4\Delta_{B_2}e^{-m_4x_3}\right)\right)e^{i\xi x}e^{st}d\xi ds \tag{59}$$

$$p=\frac{1}{4\pi i}\int_{x-i\infty}^{x+i\infty}\int_{-\infty}^{\infty}\left(-\frac{b_{13}s}{\Delta}\left(\Delta_De^{-m_5x_3}\right)\right)e^{i\xi x}e^{st}d\xi ds \tag{60}$$

where

$$\Delta=\begin{vmatrix} N & V & U & L \\ Km_1 & -G & -H & 0 \\ 0 & -b_{22}m_3d_3 & -b_{22}m_4d_4 & 0 \\ 0 & 0 & 0 & 0 \\ m_1 & i\xi & i\xi & -m_5 \end{vmatrix}$$

$$\Delta=-m_3d_3b_{22}[m_5(m_1KU+NH)+b_{13}sm_1(H+Ki\xi)]+ \\ b_{22}m_4d_4[m_5(NG+KVm_1)+b_{13}sm_1(Ki\xi+G)] \tag{61}$$

$$\Delta_{A_1}=\hat{\bar{F}}_1(\xi,s)m_5b_{22}(Gm_4d_4-Hm_3d_3) \tag{62}$$

$$\Delta_{B_1}=\hat{\bar{F}}_1(\xi,s)m_5m_1m_4d_4b_{22}K \tag{63}$$

$$\Delta_{B_2}=-\hat{\bar{F}}_1(\xi,s)m_5m_3m_1d_3b_{22}K \tag{64}$$

$$\Delta_D=\hat{\bar{F}}_1(\xi,s)[d_4m_4b_{22}m_1(Ki\xi+G)-d_3m_3b_{22}m_1(Ki\xi+H)] \tag{65}$$

V. Application

As an application of the approach, we consider the following cases.

Case I : Concentrated force

Here $F(x,t)=F(x)\delta(t)$

To determine stress components and pressure due to concentrated force described by Dirac delta function $F(x)=\delta(x)$ must be used with

$$\hat{\bar{F}}(\xi,s)=F \tag{66}$$

Case II : The solution due to uniformly distributed force over a strip of dimensionless width 2a applied on the liquid half is obtained by setting

$$F(x,t)=[H(x+a)-H(x-a)]\;\delta(t) \tag{67}$$

in equations (50) – (54).

Using (66) and then apply Laplace and Fourier transform defined by (18) and (20) on (67), we obtain

$$\hat{\bar{F}}(\xi,s)=\frac{2\sin(\xi a)}{\xi} \tag{68}$$

Case III : The solution due to linearly distributed force is obtained by using

$$\begin{aligned} F(x,t) &= \left[1-\frac{|x|}{a}\right]\delta(t) \quad ,if\,|x|\le a \\ &= 0 \qquad ,if\,|x|>a \end{aligned} \tag{69}$$

Its Laplace and Fourier transform yield

$$\hat{\bar{F}}(\xi,s)=\frac{2(1-\cos\xi a)}{\xi^2 a} \tag{70}$$

Case IV : Moving couple:

The boundary condition for a couple with its axis parallel to x_2-axis and moving along x_1-axis with constant speed V are given by

$$F_1(x,t)=H(t)\frac{\partial}{\partial x_1}\delta(x_1-Vt) \tag{71}$$

and its Laplace and Fourier transforms yields,

$$\hat{\bar{F}}_1(\xi,s)=\frac{F_0(i\xi)}{(s-i\xi V)} \tag{72}$$

Substituting the values of $\hat{\bar{F}}_1(\xi,s)$ from (66),(68),(70) and (72) in (57) - (60) we obtain the components of stress and acoustic pressure for (i) concentrated force (ii) uniformly distributed force (iii) linearly distributed force (iv) moving couple.

In the absence of void effect the corresponding results reduces to micropolar elastic with void under an inviscid fluid.

VI. NOMENCLATURE

λ,μ – lame's constant

K, α, β, γ – micropolar constant

$a,\xi,\varsigma,d,w_1^*,\chi$ – material constant due to presence of void

ρ – density constant

j – microinertia

$\vec{u}$ – displacement vector

$\vec{\phi}$ – microrotation vector

ψ – change in volume fraction field

t_{ij} – component of stress tensor

m_{ij} – component of couple stress tensor

δ_{ij} – Kronecker delta.

t – time

ϵ_{ijk} – alternative tensor

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