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Invention of a Summation Formula Accumulated with Hypergeometric Function

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INVENTION OF A SUMMATION FORMULA ACCUMULATED WITH HYPERGEOMETRIC FUNCTION

Strictly as per the compliance and regulations of :

Notes

Invention of a Summation Formula Accumulated with Hypergeometric Function

Salahuddin [α] , M. P. Chaudhary [σ] & Vinesh Kumar [ρ]

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Keywords : *Contiguous relation, Recurrence relation, Gauss second summation theorem.*

I. Introduction

Generalized Gaussian Hypergeometric function of one variable :

$$ {}_AF_B\left[\begin{matrix} a_1, a_2, \cdots, a_A & ; \\ & & z \\ b_1, b_2, \cdots, b_B & ; \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relations :

[Andrews p.367(8)]

$$ c\,(1-z)\,{}_2F_1\left[\begin{matrix} a, & b\ ; \\ & c\ \ ; \end{matrix}\ z\right] = c\,{}_2F_1\left[\begin{matrix} a-1, & b & ; \\ & c & ; \end{matrix}\ z\right] - (c-b)\,z\,{}_2F_1\left[\begin{matrix} a, & b & ; \\ & c+1 & ; \end{matrix}\ z\right] \tag{2} $$

[Abramowitz p.558(15.2.19)]

$$ (a-b)\,(1-z)\,{}_2F_1\left[\begin{matrix} a, & b\ ; \\ & c\ \ ; \end{matrix}\ z\right] = (c-b)\,{}_2F_1\left[\begin{matrix} a, & b-1 & ; \\ & c & ; \end{matrix}\ z\right] + (a-c)\,{}_2F_1\left[\begin{matrix} a-1, & b & ; \\ c\ ; & & \end{matrix}\ z\right] \tag{3} $$

Recurrence relation :

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{4} $$

Gauss second summation theorem [Prud.,p. 491(7.3.7.8)]

$$ {}_2F_1\left[\begin{matrix} a, & b & ; \\ & \frac{a+b+1}{2} & ; \end{matrix}\ \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{5} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{6} $$

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Author ρ : J.N.U., New Delhi, India

A new summation formula [Ref.[3], p.337(10)]

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\} + \frac{2\,\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \tag{7} $$

II. Main Summation Formula

For the main formula $a \neq b$

For $a < 1$ and $a > 35$

$$ {}_2F_1\left[\begin{matrix} a, & b & ; & \frac{1}{2} \\ \frac{a+b-35}{2} & & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-35}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-35}{2})}\left\{\frac{(221643095476699771875a)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+\right.\right. $$

$$ +\frac{(-53792893588976422650 0a^2 + 517596339235489288425a^3 - 277705505168550027360a^4)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(95853765344939263692a^5 - 23003823190786749936a^6 + 4025053173951400564a^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(-529465109186351520a^8 + 53416477786629594a^9 - 4184718381424152a^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(256190750871198a^{11} - 12267387269280a^{12} + 457279214236a^{13} - 13119887088a^{14})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(283927812a^{15} - 4479840a^{16} + 48603a^{17} - 324a^{18} + a^{19} - 221643095476699771875b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(794460568958575915725a^2b - 954106896831586263360a^3b + 583473807108908300484a^4b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(-200209016127040580160a^5b + 52111887197927490740a^6b - 8592784683577819200a^7b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(1229578887992586390a^8b - 112651380427012416a^9b + 9758821555276134a^{10}b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(-521382728433600a^{11}b + 28326660226388a^{12}b - 873242129088a^{13}b + 29555243172a^{14}b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

Ref.

3. Arora, Asish, Singh, Rahul, Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems , Journal of Rajasthan Academy of Physical Sciences., 7(2008), 335-342.

$$+\frac{(-483675840a^{15}b+9621813a^{16}b-63936a^{17}b+629a^{18}b+537928935889764226500b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-794460568958575915725ab^{2}+539930822961781218744a^{3}b^{2}-386145326367083741616a^{4}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

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$$+\frac{(167890730348008826916a^{5}b^{2}-38716437757273456320a^{6}b^{2}+7935347639494594536a^{7}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-908895245233633416a^{8}b^{2}+108166740015493698a^{9}b^{2}-6820153444748160a^{10}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(508087027139016a^{11}b^{2}-17810342469360a^{12}b^{2}+849592365636a^{13}b^{2}-15388862400a^{14}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(461771544a^{15}b^{2}-3328668a^{16}b^{2}+58275a^{17}b^{2}-517596339235489288425b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(954106896831586263360ab^{3}-539930822961781218744a^{2}b^{3}+123336195405232240356a^{4}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-56552210538568153920a^{5}b^{3}+18623361536764442808a^{6}b^{3}-2926284502864922496a^{7}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(486797755152510186a^{8}b^{3}-38116127387131200a^{9}b^{3}+3830565140713080a^{10}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-158626558911744a^{11}b^{3}+10223320539684a^{12}b^{3}-212083704000a^{13}b^{3}+8868220680a^{14}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-71736192a^{15}b^{3}+1888887a^{16}b^{3}+277705505168550027360b^{4}-583473807108908300484ab^{4})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(386145326367083741616a^{2}b^{4}-123336195405232240356a^{3}b^{4}+11976026816214506892a^{5}b^{4})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-3621627756895390704a^{6}b^{4}+942055925530878444a^{7}b^{4}-99657364322023200a^{8}b^{4})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(13764795916582260a^9b^4 - 703215442424880a^{10}b^4 + 60448094196756a^{11}b^4)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-1480020897600a^{12}b^4 + 83034151620a^{13}b^4 - 772160400a^{14}b^4 + 28312548a^{15}b^4)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-9585376534493926369 2b^5 + 2002090161270405801 60ab^5 - 1678907303480088269 16a^2b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(5655221053856815392 0a^3b^5 - 1197602681621450689 2a^4b^5 + 555797963695121532a^6b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-110524284078975360a^7b^5 + 23405963382833724a^8b^5 - 1596308168613312a^9b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(186344810270388a^{10}b^5 - 5597348621760a^{11}b^5 + 415953681948a^{12}b^5 - 4559958144a^{13}b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(223926516a^{14}b^5 + 2300382319078674993 6b^6 - 5211188719792749074 0ab^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(3871643775727345632 0a^2b^6 - 1862336153676444280 8a^3b^6 + 3621627756895390704a^4b^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-555797963695121532a^5b^6 + 13009040083811376a^7b^6 - 1638821697792048a^8b^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(289107691712532a^9b^6 - 11485059284160a^{10}b^6 + 1149394449672a^{11}b^6 - 15389858736a^{12}b^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(997490844a^{13}b^6 - 4025053173951400564b^7 + 8592784683577819200ab^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-7935347639494594536a^2b^7 + 2926284502864922496a^3b^7 - 942055925530878444a^4b^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(110524284078975360a^5b^7 - 13009040083811376a^6b^7 + 153898964562420a^8b^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

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$$+\frac{(-11143637208000a^9b^7+1667462215320a^{10}b^7-29314016640a^{11}b^7+2544619500a^{12}b^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(529465109186351520b^8-1229578887992586390ab^8+908895245233633416a^2b^8)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-486797755152510186a^3b^8+99657364322023200a^4b^8-23405963382833724a^5b^8)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1638821697792048a^6b^8-153898964562420a^7b^8+859647308850a^9b^8-27266346360a^{10}b^8)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(3511574910a^{11}b^8-534164777866295946b^9+112651380427012416ab^9)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-108166740015493698a^2b^9+38116127387131200a^3b^9-13764795916582260a^4b^9)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1596308168613312a^5b^9-289107691712532a^6b^9+11143637208000a^7b^9-859647308850a^8b^9)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1767263190a^{10}b^9+41847183814241152b^{10}-9758821555276134ab^{10}+6820153444748160a^2b^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-3830565140713080a^3b^{10}+703215442424880a^4b^{10}-186344810270388a^5b^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(11485059284160a^6b^{10}-1667462215320a^7b^{10}+27266346360a^8b^{10}-1767263190a^9b^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-256190750871198b^{11}+521382728433600ab^{11}-508087027139016a^2b^{11})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(158626558911744a^3b^{11}-60448094196756a^4b^{11}+5597348621760a^5b^{11}-1149394449672a^6b^{11})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(29314016640a^7b^{11}-3511574910a^8b^{11}+12267387269280b^{12}-28326660226388ab^{12})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(17810342469360a^2b^{12}-10223320539684a^3b^{12}+1480020897600a^4b^{12}-415953681948a^5b^{12})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(15389858736a^6b^{12}-2544619500a^7b^{12}-457279214236b^{13}+873242129088ab^{13})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-849592365636a^2b^{13}+212083704000a^3b^{13}-83034151620a^4b^{13}+4559958144a^5b^{13})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-997490844a^6b^{13}+131198870888b^{14}-29555243172ab^{14}+15388862400a^2b^{14})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-8868220680a^3b^{14}+772160400a^4b^{14}-223926516a^5b^{14}-283927812b^{15}+483675840ab^{15})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-461771544a^2b^{15}+71736192a^3b^{15}-28312548a^4b^{15}+4479840b^{16}-9621813ab^{16})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(3328668a^2b^{16}-1888887a^3b^{16}-48603b^{17}+63936ab^{17}-58275a^2b^{17}+324b^{18})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-629ab^{18}-b^{19})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}\Bigg\}+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-34}{2})}\Bigg\{\frac{(-342004140004602190500a+618589042130777249700a^2)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-439776869912495100000a^3+207660990003844961376a^4-55646018917045943664a^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(12692779497598637424a^6-1727329171783934880a^7+227673953705453472a^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-17503765732175448a^9+1437382773094488a^{10}-64498668078240a^{11}+3385607303328a^{12})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-86814694512a^{13}+2872970352a^{14}-38294880a^{15}+745824a^{16}-3876a^{17}+36a^{18})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(342004140004602190500b-565735909026242999520a^2b+605915064265530229056a^3b)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

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$$+\frac{(-2661315155267773877728a^4b+9200504905766856608Oa^5b-16857776376518415520a^6b)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(306366104862810630 4a^7b-293176469928015080a^8b+32203327729514304a^9b)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-1723576078229280a^{10}b+121070246033344a^{11}b-3609181157456a^{12}b+164273746112a^{13}b)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-2510666080a^{14}b+72098496a^{15}b-430236a^{16}b+7104a^{17}b-61858904213077724970 0b^2)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(565735909026242999520ab^2-222917145657651459168a^3b^2+162758440229415575344a^4b^2)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-46807570830991840800a^5b^2+12487467579332170432a^6b^2-1576581661708571232a^7b^2)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(23455052407533498 4a^8b^2-15467084885720160a^9b^2+1437907007343488a^{10}b^2)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-50905841320992a^{11}b^2+3084537853040a^{12}b^2-54812066400a^{13}b^2+2168792000a^{14}b^2)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-14914848a^{15}b^2+369852a^{16}b^2+439776869912495100000b^3-605915064265530229056ab^3)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(222917145657651459168a^2b^3-321884444171374011 20a^4b^3+17333176390267879232a^5b^3)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-3333735232568168736a^6b^3+711712084009024384a^7b^3-60978459051109600a^8b^3)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(7580367458452800a^9b^3-328001164876000a^{10}b^3+26128258084096a^{11}b^3)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-549526101600a^{12}b^3+28918904000a^{13}b^3-231371360a^{14}b^3+7970688a^{15}b^3)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

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$$+\frac{(-2076609900038449613 76b^4+2661315155267773877 28ab^4-1627584402294155753 44a^2b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(321884444171374011 20a^3b^4-2043265785712729424a^5b^4+853645273887046256a^6b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-10903971079783584 0a^7b^4+1913244909089360 0a^8b^4-106308372802632 0a^9b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(112220361169840a^{10}b^4-2866811616480a^{11}b^4+197641864000a^{12}b^4-1867224240a^{13}b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(85795600a^{14}b^4+5564601891704594366 4b^5-9200504905766856608 0ab^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(4680757083099184080 0a^2b^5-1733317639026787923 2a^3b^5+2043265785712729424a^4b^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-619832405087772 80a^6b^5+2081748801903654 4a^7b^5-169696496409316 8a^8b^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(249794015368128a^9b^5-8121009770720a^{10}b^5+737446564288a^{11}b^5-8423256016a^{12}b^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(506662016a^{13}b^5-1269277949759863742 4b^6+1685777637651841552 0ab^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-1248746757933217043 2a^2b^6+3333735232568168736a^3b^6-853645273887046256a^4b^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(61983240508777280a^5b^6-915171126674112a^7b^6+253701418913712a^8b^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-11953460867040a^9b^6+1500360345792a^{10}b^6-21659801184a^{11}b^6+1709984304a^{12}b^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(1727329171783934880b^7-3063661048628106304ab^7+1576581661708571232a^2b^7)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

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$$+\frac{(-7117120840090243 84a^3b^7+1090397107978358 40a^4b^7-2081748801903654 4a^5b^7)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(915171126674112a^6b^7-6204081708000a^8b^7+1448936280000a^9b^7-30032497440a^{10}b^7)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(3257112960a^{11}b^7-22767395370545347 2b^8+29317646992801508 0ab^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-2345505240753349 84a^2b^8+609784590511096 00a^3b^8-191324490908936 00a^4b^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(1696964964093168a^5b^8-253701418913712a^6b^8+6204081708000a^7b^8-15147970200a^9b^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(3029594040a^10b^8+17503765732175448b^9-3220332772951430 4ab^9)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(15467084885720160a^2b^9-7580367458452800a^3b^9+1063083728026320a^4b^9)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-249794015368128a^5b^9+11953460867040a^6b^9-1448936280000a^7b^9+15147970200a^8b^9)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-1437382773094488b^{10}+1723576078229280ab^{10}-1437907007343488a^2b^{10})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(328001164876000a^3b^{10}-112220361169840a^4b^{10}+8121009770720a^5b^{10})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-1500360345792a^6b^{10}+30032497440a^7b^{10}-3029594040a^8b^{10}+64498668078240b^{11})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-121070246033344ab^{11}+50905841320992a^2b^{11}-26128258084096a^3b^{11})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(2866811616480a^4b^{11}-737446564288a^5b^{11}+21659801184a^6b^{11}-3257112960a^7b^{11})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-33856073303328b^{12}+3609181157456ab^{12}-3084537853040a^2b^{12}+549526101600a^3b^{12})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-197641864000a^4b^{12}+8423256016a^5b^{12}-1709984304a^6b^{12}+868146945 12b^{13})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-164273746112ab^{13}+54812066400a^2b^{13}-28918904000a^3b^{13}+1867224240a^4b^{13})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-506662016a^5b^{13}-28729703352b^14+2510666080ab^{14}-2168792000a^2b^{14}+231371360a^3b^{14})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-85795600a^4b^{14}+38294880b^{15}-72098496ab^{15}+14914848a^2b^{15}-7970688a^3b^{15})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-745824b^{16}+430236ab^{16}-369852a^2b^{16}+3876b^{17}-7104ab^{17}-36b^{18})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}\Bigg\}\Bigg] \qquad (9)$$

III. Derivation of Main Summation Formula

Substituting $c=\frac{a+b-35}{2}$ and $z=\frac{1}{2}$ in equation (3), we get

$$(a-b)\,{}_2F_1\left[\begin{matrix}a,\ b & ; & \frac{1}{2}\\ \frac{a+b-35}{2} & ; & \end{matrix}\right]=(a-b-35)\,{}_2F_1\left[\begin{matrix}a,\ b-1 & ; & \frac{1}{2}\\ \frac{a+b-35}{2} & ; & \end{matrix}\right]+(a-b+35)\,{}_2F_1\left[\begin{matrix}a-1,\ b & ; & \frac{1}{2}\\ \frac{a+b-35}{2} & ; & \end{matrix}\right]$$

Now using the same method of Ref[9] the main result is derived.

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Notes