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Past, Present and Future of the Avogadro Number

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I. Introduction

Considering strong gravity, Erasmo Recami says [1]: *A consequence of what stated above is that inside a hadron (i.e., when we want to describe strong interactions among hadron constituents) it must be possible to adopt the same Einstein equations which are used for the description of gravitational interactions inside our cosmos; with the only warning of scaling them down, that is, of suitably scaling, together with space distances and time durations, also the gravitational constant G (or the masses) and the cosmological constant Λ.*

In 3+1 dimensions, experiments and observations reveals that, if strength of strong interaction is unity, with reference to the strong interaction, strength of gravitation is 10^{-39}. If this is true, any model or theory must explain this astounding fact. At least in 10 dimensions also, till today no model including String theory [2-4] or Super gravity [5,6] has succeeded in explaining this fact. Note that in the atomic or nuclear physics, till today no experiment reported or estimated the value of the gravitational constant. Note that G is quite difficult to measure, as gravity is much weaker than the other fundamental forces, and an experimental apparatus cannot be separated from the gravitational influence of other bodies. Furthermore, till today gravity has no established relation to other fundamental forces, so it does not appear possible to calculate it indirectly from other constants that can be measured more accurately, as is done in other areas of physics. It is sure that something is missing in the current understanding of unification. This clearly indicates the need of revision of our existing physics foundations.

So far even in 10 dimensions also, no unified model proposed a methodology for estimating the rest masses of the basic constituents of matter like electron, proton & neutron and the nuclear binding energy. In this sensitive and critical situation, considering Avogadro number as an absolute proportionality ratio in 3+1 dimensions, in this paper an attempt is made to understand the basics of gravitational and non-gravitational interactions in a unified manner. This paper is the simplified form of the authors 15 published papers. Including "low and high energy super symmetry", authors made an attempt to understand the unification with only 4 simple assumptions.

a) Extra dimensions and the strong gravity

In unification, success of any model depends on how the gravitational constant is implemented in atomic, nuclear and particle physics. David Gross [7] says: *But string theory is still in the process of development, and although it has produced many surprises and lessons it still has not broken dramatically with the conceptual framework of relativistic quantum field theory. Many of us believe that ultimately string theory will give rise to a revolution in physics, as important as the two revolutions that took place in the 20th century, relativity and quantum mechanics. These revolutions are associated with two of the three fundamental dimensionful parameters of nature, the velocity of light and Planck's constant. The revolution in string theory presumably has to do with Newton's constant, that defines a length, the Planck length of 10^{-33} cm. String theory, I believe, will ultimately modify in a fundamental way our concepts at distances of order this length.*

In this connection the fundamental questions to be answered are: What is the 'physical base' for extra dimensions and their compactification? What is the physical entity next to length, area and volume? Why the

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assumed 10 dimensional compactification is ending at the observed (3+1) dimensions? During the dimensional compactification: 1) How to confirm that that there is no variation in the magnitude of the observed (3+1 dimensional) physical constant or physical property? 2) If space-time is curled up to the least possible (planck) size, how to interpret or understand the observed (3+1 dimensional) nuclear size and atomic sizes which are very large compared to the tiny planck size?

The concept of 'extra dimension' is very interesting but at the same time one must see its 'real existence' and 'workability' in the real physical world. Kaluza and Klein [8] showed that if one assumed general relativity in five dimensions, where one dimension was curled up, the resulting theory would look like a four-dimensional theory of electromagnetism and gravity. When gravity is existing in 3+1 dimensions, what is the need of assuming it in 5 dimensions? In the reality of (4+1) dimensional laboratory, how to confirm that, (3+1) dimensional gravity will not change in (4+1) dimensions? When gravity and electromagnetism both are existing in 3+1 dimensions, unifying them within 5 dimensions seems to be very interesting but impracticable. **More over to unify 2 interactions if 5 dimensions are required, for unifying 4 interactions 10 dimensions are required. For 3+1 dimensions if there exist 4 (observed) interactions, for 10 dimensions there may exist 10 (observable) interactions. To unify 10 interactions 20 dimensions are required. From this idea it can be suggested that- with 'n' new dimensions 'unification' problem cannot be resolved**.

Erasmo Recami says [1]: *Let us recall that Riemann, as well as Clifford and later Einstein, believed that the fundamental particles of matter were the perceptible evidence of a strong local space curvature. A theory which stresses the role of space (or, rather, space-time) curvature already does exist for our whole cosmos: General Relativity, based on Einstein gravitational field equations; which are probably the most important equations of classical physical theories, together with Maxwell's electromagnetic field equations. Whilst much effort has already been made to generalize Maxwell equations, passing for example from the electromagnetic field to Yang-Mills fields (so that almost all modern gauge theories are modeled on Maxwell equations), on the contrary Einstein equations have never been applied to domains different from the gravitational one. Even if they, as any differential equations, do not contain any inbuilt fundamental length: so that they can be used a priori to describe cosmoses of any size. Our first purpose is now to explore how far it is possible to apply successfully the methods of general relativity (GR), besides to the world of gravitational interactions, also to the domain of the so-called nuclear, or strong, interactions: namely, to the world of the elementary particles called hadrons. A second purpose is linked to the fact that the standard theory (QCD) of strong interactions has not yet fully explained why the hadron constituents (quarks) seem to be permanently confined in the interior of those particles; in the sense that nobody has seen up to now an isolated "free" quark, outside a hadron. So that, to explain that confinement, it has been necessary to invoke phenomenological models, such as the so-called "bag" models, in their MIT and SLAC versions for instance. The "confinement" could be explained, on the contrary, in a natural way and on the basis of a well-grounded theory like GR, if we associated with each hadron (proton, neutron, pion,...) a particular "cosmological model".*

b) Significance of large number ratios in unification

In his large number hypothesis P. A. M. Dirac [9, 10] compared the ratio of characteristic size of the universe and classical radius of electron with the electromagnetic and gravitational force ratio of electron and proton. If the cosmic closure density is, $\rho_0 \cong \frac{3H_0^2}{8\pi G}$, number of nucleons in a Euclidean sphere of radius (c/H_0) is equal to $\frac{c^3}{2Gm_nH_0}$ where H_0 is the Hubble's constant and m_n is the nucleon rest mass. It can be suggested that coincidence of large number ratios reflects an intrinsic property of nature.

It can be supposed that elementary particles construction is much more fundamental than the black hole's construction. If one wishes to unify electroweak, strong and gravitational interactions it is a must to implement the classical gravitational constant G in the sub atomic physics [11-13]. By any reason if one implements the planck scale in elementary particle physics and nuclear physics automatically G comes into subatomic physics. Then a large 'arbitrary number' has to be considered as proportionality constant. With this large arbitrary number it is be possible to understand the mystery of the strong interaction and strength of gravitation. Anyhow, the subject under consideration is very sensitive to human thoughts, experiments and observations.

In this critical situation here let us consider the valuable words of Einstein: *'The successful attempt to derive delicate laws of nature, along a purely mental path, by following a belief in the formal unity of the structure of reality, encourages continuation in this speculative direction, the dangers of which everyone vividly must keep in sight who dares follow it".*

II. About the Avogadro Number

Avogadro's number, N is the fundamental physical constant that links the macroscopic physical

world of objects that we can see and feel with the submicroscopic, invisible world of atoms. In theory, N specifies the exact number of atoms in a palm-sized specimen of a physical element such as carbon or silicon. The name honors the famous Italian mathematical physicist Amedeo Avogadro (1776-1856), who proposed that equal volumes of all gases at the same temperature and pressure contain the same number of molecules. Long after Avogadro's death, the concept of the mole was introduced, and it was experimentally observed that one mole (the molecular weight in grams) of any substance contains the same number of molecules.

Determination of N, and hence k_B, was one of the most difficult problems of chemistry and physics in the second half of the 19th century. The constant N was (and still is) so fundamental that for its verification and precise determination every new idea and theory appeared in physics are at once used. Many eminent scientists devoted definite periods of their research life to the study of this problem: beginning from I. Loschmidt (1866), Van der Vaals (1873), S. J.W. Rayleigh (1871), etc. in the 19th century, and continuing in the 20th century, beginning from Planck (1901), A. Einstein and J. Perrin (1905-1908), Dewer (1908), E. Rutherford and Geiger (1908-1910), I. Curie, Boltwood, Debierne (1911), and many others. The value obtained by Planck on the basis of his famous black body radiation formula was, $N \approx 6.16\times10^{23}\ \text{mol}^{-1}$. More accurate definition of the value of N involves the change of molecular magnitudes and, in particular, the change in value of an elementary charge. The latter is related with N through the so-called "Helmholtz relation" $Ne = F$, where F is the Faraday constant, a fundamental constant equal to 96485.3415(39) C.mol^{-1}.

Today, Avogadro's number is formally defined to be the number of carbon-12 atoms in 12 grams of unbound carbon-12 in its rest-energy electronic state [14-18]. The current state of the art estimates the value of N, not based on experiments using carbon-12, but by using X-ray diffraction in crystal silicon lattices in the shape of a sphere or by a watt-balance method. According to the National Institute of Standards and Technology (NIST), the current accepted value for $N \cong (6.0221415 \pm 0.0000010)\times10^{23}$. The CODATA recommended value is $N \cong 6.02214179(30)\times10^{23}$. **This definition of N and the current experiments to estimate it, however, both rely on the precise definition of "one gram"! Hence most of the scientists consider it as an ad-hoc number. But in reality it is not the case**. Please see the following sections.

a) The Boltzmann constant: Bridge from macroscopic to microscopic physics

In statistical mechanics [19] that makes theoretical predictions about the behavior of macroscopic systems on the basis of statistical laws governing its component particles, the relation of energy and absolute temperature T is usually given by the inverse thermal energy $\frac{1}{k_B T}$. The constant k_B, called the Boltzmann constant is equal to the ratio of the molar gas constant R_U and the Avogadro number N.

$$k_B = \frac{R_U}{N} \cong 1.38065(4)\times10^{-23}\ \text{J/}^0\text{K} \qquad (1)$$

where $R_U \cong 8.314504(70)\ \text{J/mol.}^0\text{K}$ and N is the Avogadro number. k_B has the same units as entropy. k_B plays a crucial role in this equality. It defines, in particular, the relation between absolute temperature and the kinetic energy of molecules of an ideal gas. The product $k_B T$ is used in physics as a scaling factor for energy values in molecular scale (sometimes it is used as a pseudo-unit of energy), as many processes and phenomena depends not on the energy alone, but on the ratio of energy and $k_B T$. Given a thermodynamic system at an absolute temperature T, the thermal energy carried by each microscopic "degree of freedom" in the system is of the order of $(k_B T/2)$.

As Planck wrote in his Nobel Prize lecture in 1920, [20]: *This constant is often referred to as Boltzmann's constant, although, to my knowledge, Boltzmann himself never introduced it - a peculiar state of affairs, which can be explained by the fact that Boltzmann, as appears from his occasional utterances, never gave thought to the possibility of carrying out an exact measurement of the constant.* The Planck's quantum theory of light, thermodynamics of stars, black holes and cosmology totally depend upon the famous Boltzmann constant which in turn depends on the Avogadro number. From this it can be suggested that, Avogadro number is more fundamental and characteristic than the Boltzmann constant and indirectly plays a crucial role in the formulation of the quantum theory of radiation.

b) Current status of the Avogadro number

The situation is very strange and sensitive. Now this is the time to think about the significance of 'Avogadro number' in a unified approach. It couples the gravitational and non-gravitational interactions. It is observed that, either in SI system of units or in CGS system of units, value of the order of magnitude of Avogadro number $\cong N \approx 6\times10^{23}$ but not 6×10^{26}. But the most surprising thing is that, without implementing the gravitational constant in atomic or nuclear physics this fact cannot understood. It is also true that till today no unified model (String theory or Super gravity) successfully implemented the gravitational constant in the atomic or nuclear physics. Really this is a challenge

to the modern nuclear physics and astrophysics. **Please note that, ratio of planck mass and electron mass is very close to** $(N/8\pi)$.

$$m_e c^2 \cong \frac{8\pi}{N}\sqrt{\frac{\hbar c}{G}}\cdot c^2 \quad \cong 0.50952547 \text{ MeV} \tag{2}$$

This is a very strange coincidence[20]. But interpretation seems to be a very big puzzle. Any how it gives a clue for fitting and coupling the electron rest mass with the planck scale.

III. Mystery of the Gram Mole

If $M_P \cong \sqrt{\hbar c/G}$ is the Planck mass and m_e is the rest mass of electron, semi empirically it is observed that,

$$M_g \cong N^{-\frac{1}{3}}\cdot\sqrt{(N\cdot M_P)(N\cdot m_e)} \cong 1.004412\times10^{-3} \text{ Kg} \tag{3}$$

$$M_g \cong N^{\frac{2}{3}}\cdot\sqrt{M_P m_e} \tag{4}$$

Here M_g is just crossing the mass of one gram. If m_p is the rest mass of proton,

$$M_g \div m_p \cong N \cong 6.003258583\times10^{23} \tag{5}$$

$$\frac{\sqrt{M_P m_e}}{m_p} \cong N^{\frac{1}{3}} \tag{6}$$

Thus obtained $N \cong 5.965669601\times10^{23}$. More accurate empirical relation seems to be

$$\frac{\sqrt{M_P m_e}\ c^2}{\frac{m_p c^2 + m_n c^2 - B_a}{2} + m_e c^2} \cong N^{\frac{1}{3}} \tag{7}$$

where m_n is the rest mass of neutron and $B_a \cong 8$ MeV is the mean binding energy of nucleon. Obtained value of $N \cong 6.020215677\times10^{23}$. Here accuracy depends only on the 'mean binding energy per nucleon'. **Qualitatively and quantitatively - from this coincidence it is possible to say that, in atomic and nuclear physics, Avogadro number plays a very interesting role.** The unified atomic mass-energy unit $m_u c^2$ can be expressed as [20]

$$m_u c^2 \cong \left(\frac{m_p c^2 + m_n c^2}{2} - B_a\right) + m_e c^2 \tag{8}$$

$$\cong 931.4296786 \text{ MeV}$$

In this way, in a very simplified manner, Avogadro number can be estimated from the nuclear physics.

IV. The Key Assumptions in Unification

Assumption-1: In atomic and nuclear physics, atomic gravitational constant (G_A) is Avogadro number times the classical gravitational constant (G_C).

$$G_A \cong N G_C \tag{9}$$

Thus it is reasonable to say that - since the atomic gravitational constant is N times the classical gravitational constant, atoms are themselves arranged in a systematic manner and generate the "gram mole". In this paper mostly the subject under presentation is limited to this assumption only.

Assumption-2: The key conceptual link that connects the gravitational and non-gravitational forces is - the classical force limit

$$F_C \cong \left(\frac{c^4}{G_C}\right) \cong 1.21026\times10^{44} \text{ newton} \tag{10}$$

It can be considered as the upper limit of the string tension. In its inverse form it appears in Einstein's theory of gravitation [1] as $\frac{8\pi G_C}{c^4}$. It has multiple applications in Black hole physics and Planck scale physics [21,22]. It has to be measured either from the experiments or from the cosmic and astronomical observations.

Assumption-3: Ratio of 'classical force limit (F_C)' and ' weak force magnitude (F_W)' is N^2 where N is a large number close to the Avogadro number.

$$\frac{F_C}{F_W} \cong N^2 \cong \frac{\text{Upper limit of classical force}}{\text{nuclear weak force magnitude}} \tag{11}$$

Thus the proposed weak force magnitude is $F_W \cong \frac{c^4}{N^2 G_C} \cong 3.33715\times10^{-4}$ newton. Considering this F_W, Higgs fermion and boson masses can be fitted. In this connection please refer our earlier published papers [23,24,25].

Assumption-4: Ratio of fermion and its corresponding boson mass is not unity but a value close to $\Psi \approx 2.2627$. This idea can be applied to quarks, leptons, proton and the Higgs fermion. One can see "super symmetry" in low energies as well as high energies. This is a fact and cannot be ignored. Authors explained these facts in detail [23,24]. For the time

being its value can be fitted with the relation, $\Psi^2 \ln\left(1+\sin^2\theta_W\right) \cong 1$ where $\sin\theta_W$ can be considered as the weak coupling angle. Please see section-5.

V. The Weak Mixing Angle

David Gross [7] says: *After sometime in the late 1920s Einstein became more and more isolated from the mainstream of fundamental physics. To a large extent this was due to his attitude towards quantum mechanics, the field to which he had made so many revolutionary contributions. Einstein, who understood after better than most the implications of the emerging interpretations of quantum mechanics, could never accept it as a final theory of physics. He had no doubt that it worked, that it was a successful interim theory of physics, but he was convinced that it would be eventually replaced by a deeper, deterministic theory. His main hope in this regard seems to have been the hope that by demanding singularity free solutions of the nonlinear equations of general relativity one would get an over determined system of equations that would lead to quantization conditions.* These statements clearly suggest that, at fundamental level there exists some interconnection in between quantum mechanics and gravity. It is noticed that

$$\hbar \approx \frac{N}{2}\sqrt{\left(\frac{e^2}{4\pi\varepsilon_0 c}\right)\cdot\left(\frac{G_C m_e^{\,2}}{c}\right)} \cong 1.135\times10^{-34}\ \text{J.sec} \qquad (12)$$

If it is really true, this may be considered as the beginning of unified quantum mechanics. From accuracy point of view here factor $(1/2)$ can be replaced with the weak mixing angle $\sin\theta_W$. Considering $\sin\theta_W$ as a characteristic number in fundamental physics,

$$\hbar \cong N\sin\theta_W \cdot \sqrt{\left(\frac{e^2}{4\pi\varepsilon_0 c}\right)\cdot\left(\frac{G_C m_e^{\,2}}{c}\right)} \qquad (13)$$

Thus the weak mixing angle can be expressed as

$$\sin\theta_W \cong \left(\frac{\hbar}{m_e c}\right) \div \sqrt{\frac{e^2}{4\pi\varepsilon_0 F_W}} \cong 0.464433353 \qquad (14)$$

Here $(\hbar/m_e c)$ is the Compton wave length of electron and $\sqrt{\frac{e^2}{4\pi\varepsilon_0 F_W}}$ seems to be a characteristic length of weak interaction.

VI. To Fit the Rest Masses of Proton and Neutron

Similar to the planck mass $\sqrt{\hbar c/G_C}$ and with reference to the elementary charge (e), it is possible to construct a mass unit as $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_C}}$. By considering the proposed atomic gravitational constant, it takes the form $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}}$. To a first approximation, guess that, nucleon rest mass is close to the geometric mean mass of m_e and $\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}}$.

$$m_x c^2 \cong k\sqrt{m_e\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}}}\cdot c^2 \qquad (15)$$

where k is a proportionality number. When $k = \alpha\ln\left(\frac{1}{\alpha}\right) \cong 0.035904752$ it is noticed that, $m_x c^2 \cong 940.923$ MeV. Thus

$$\frac{m_x c^2}{m_e c^2} \cong \alpha\ln\left(\frac{1}{\alpha}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_A m_e^2}\right)^{\frac{1}{4}} \qquad (16)$$

Then it is noticed that,

$$m_n c^2 \cong \frac{k}{1+k^2}\cdot\sqrt{m_e\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}}}\cdot c^2 \cong 939.71\ \text{MeV} \qquad (17)$$

$$m_p c^2 \cong \frac{k}{\left(1+k^2\right)^2}\cdot\sqrt{m_e\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_A}}}\cdot c^2 \qquad (18)$$

$\cong 938.50$ MeV. These obtained values can be compared with the experimental values [20]. But here the term $k = \alpha\ln\left(\frac{1}{\alpha}\right)$ seems to be a complicated one and needs a clear explanation. It plays a very interesting role in fitting the nuclear binding energy constants and the maximum mean binding energy per nucleon. With reference to the actual proton rest mass, $N \cong 6.028037223\times10^{23}$. From the above coincidences, it can be expressed as,

$$m_x c^2 - m_n c^2 \approx m_n c^2 - m_p c^2 \approx 1.21\ \text{MeV} \qquad (19)$$

In this way 93.56% of the neutron, proton mass difference can be understood.

a) Nuclear binding energy constants

The semi-empirical mass formula (SEMF) is used to approximate the mass and various other

properties of an atomic nucleus [26,27]. As the name suggests, it is based partly on theory and partly on empirical measurements. The theory is based on the liquid drop model proposed by George Gamow and was first formulated in 1935 by German physicist Carl Friedrich von Weizsäcker. Based on the 'least squares fit', volume energy coefficient is $a_v = 15.78$ MeV, surface energy coefficient is $a_s = 18.34$ MeV, coulombic energy coefficient is $a_c = 0.71$ MeV, asymmetric energy coefficient is $a_a =$ 23.21 MeV and pairing energy coefficient is $a_p = 12$ MeV. The semi empirical mass formula is

$$BE \cong Aa_v - A^{\frac{2}{3}}a_s - \frac{Z(Z-1)}{A^{\frac{1}{3}}}a_c - \frac{(A-2Z)^2}{A}a_a \pm \frac{1}{\sqrt{A}}a_p \tag{20}$$

In a unified approach it is noticed that, the energy coefficients are having strong inter-relation with the above defined number $k = \alpha \ln\left(\frac{1}{\alpha}\right)$. The interesting semi empirical observations can be expressed in the following way.

h) The maximum mean binding per nucleon is

$$(B_A)_{\max} \cong \frac{1}{2}k \cdot \tan\theta_W \cdot m_p c^2 \cong 8.8335 \text{ MeV} \tag{21}$$

i) The coulombic energy coefficient is (a_c)

$$\cong \sqrt{\alpha}(B_A)_{\max} \cong 0.7546 \text{ MeV} \tag{22}$$

j) The volume energy coefficient is (a_v)

$$\cong 2(B_A)_{\max} - 2a_c \cong 16.158 \text{ MeV} \tag{23}$$

k) The surface energy coefficient is (a_s)

$$\cong 2(B_A)_{\max} + 2a_c \cong 19.176 \text{ MeV} \tag{24}$$

l) The pairing energy coefficient (a_p)

$$\cong \frac{4}{3}(B_A)_{\max} \cong 11.778 \text{ MeV} \tag{25}$$

m) The asymmetry energy coefficient (a_a)

$$\cong 2a_p \cong \frac{8}{3}(B_A)_{\max} \cong 23.556 \text{ MeV} \tag{26}$$

n) $a_a + a_p \cong a_v + a_s \cong 2k \cdot \tan\theta_W \cdot m_p c^2 \cong 4(B_A)_{\max}$

$$\cong 35.334 \text{ MeV} \tag{27}$$

In table-1 within the range of $(Z = 26; A = 56)$ to $(Z = 92; A = 238)$ nuclear binding energy is calculated and compared with the measured binding energy [28]. Column-3 represents the calculated binding energy and column-4 represents the measured binding energy.

Table 1 : SEMF binding energy with the proposed energy coefficients

Z	A	$(BE)_{cal}$ in MeV	$(BE)_{meas}$ in MeV
26	56	490.8	492.254
28	62	543.62	545.259
34	84	725.65	727.341
50	118	1004.79	1004.950
60	142	1181.17	1185.145
79	197	1552.89	1559.40
82	208	1623.33	1636.44
92	238	1801.89	1801.693

Qualitatively and quantitatively - from these coincidences it is possible to say that, in atomic and nuclear physics, the operating gravitational constant is Avogadro number times the Newton's gravitational constant.

b) Proton-nucleon stability relation

It is noticed that

$$\frac{A_s}{2Z} \cong 1 + 2Z\left(\frac{a_c}{a_s}\right)^2 \tag{28}$$

where A_s is the stable mass number of Z. This is a direct relation. Assuming the proton number Z, in general, for all atoms, lower stability can be fitted directly with the following relation [26].

$$A_s \cong 2Z\left[1 + 2Z\left(\frac{a_c}{a_s}\right)^2\right] \cong 2Z + Z^2 * 0.0062 \tag{29}$$

if $Z = 21$, $A_s \cong 44.73$; if $Z = 29$, $A_s \cong 63.21$; if $Z = 47$, $A_s \cong 107.69$; if $Z = 53$, $A_s \cong 123.42$ if $Z = 60$, $A_s \cong 142.32$; if $Z = 79$, $A_s \cong 196.69$; if $Z = 83$, $A_s \cong 208.71$; if $Z = 92$, $A_s \cong 236.48$;

Stable super heavy elements can be predicted with this relation. In between $Z = 30$ to $Z = 60$ obtained A_s is lower compared to the actual A_s. It is noticed that, upper stability in light and medium atoms up to $Z \approx 56$ can be fitted with the following relation.

$$A_s \cong 2Z\left[1 + 2Z\left(\left(\frac{a_c}{a_s}\right)^2 + \left(\frac{a_c}{4(B_A)_{\max}}\right)^2\right)\right] \tag{30}$$

$$\cong 2Z + Z^2 * 0.008$$

From this relation for $Z = 56,$ obtained upper $A_s \cong 137.09.$ Note that, for $Z = 56,$ actual stable $A_s \cong 137 \cong \frac{1}{\alpha}$ where α is the fine structure ratio. This seems to be a nice and interesting coincidence. In between 0.0062 and 0.008, for light and medium atoms up to $Z \approx 56$ or $A_s \approx 137,$ mean stability can be fitted with the following relation.

$$A_s \cong 2Z + Z^2 * 0.00711 \tag{31}$$

Surprisingly it is noticed that, in this relation, $0.0071 \approx \alpha.$ Thus up to $Z \cong 56$ or $A_s \approx 137,$ mean stability can be expressed as

$$A_s \approx 2Z + \left(Z^2\alpha\right) \tag{32}$$

VII. To Fit the Rms Radius of Proton

Let R_p be the rms radius of proton. Define two radii R_1 and R_2 as follows.

$$R_1 \cong \left(\frac{\hbar c}{G_A m_p^2}\right)^2 \frac{2G_C m_p}{c^2} \cong 1.9637 \times 10^{-25} \text{ m} \tag{33}$$

$$R_2 \cong \left(\frac{\hbar c}{G_A m_p^2}\right)^3 \frac{2G_C m_p}{c^2} \cong 5.521 \times 10^{-11} \text{ m} \tag{34}$$

It is noticed that,

$$R_p \cong \left(R_1 R_2^2\right)^{\frac{1}{3}} \cong 8.4278 \times 10^{-16} \text{ m} \tag{35}$$

Thus,

$$R_p \cong \left(\frac{\hbar c}{G_A m_p^2}\right)^{8/3} \frac{2G_C m_p}{c^2} \tag{36}$$

This can be compared with the 2010 CODATA recommended rms radius of proton $0.8775(51)$ fm. Recent work on the spectrum of muonic hydrogen (an exotic atom consisting of a proton and a negative muon) indicates a significantly lower value for the proton charge radius, $R_p \cong 0.84184(67)$ fm and the reason for this discrepancy is not clear. This is 10 times more precise than all the previous determinations [29,30]. **Qualitatively and quantitatively - from this coincidence it is possible to say that, in atomic and nuclear physics, the operating gravitational constant is Avogadro number times the Newton's gravitational constant**. Thus from proton rest mass and rms radius,

$$G_A \cong \left(\frac{2G_C m_p}{R_p c^2}\right)^{3/8} \left(\frac{\hbar c}{m_p^2}\right) \tag{37}$$

$$N \cong \left(\frac{2G_C m_p}{R_p c^2}\right)^{3/8} \left(\frac{\hbar c}{G_C m_p^2}\right) \tag{38}$$

Here the most interesting thing is that, R_2 is very close to the Bohr radius of Hydrogen atom. It is very interesting to note that, with R_2 ionic radii of atoms can be fitted very easily as

$$(R)_A \cong A^{1/3} \cdot \left(\frac{R_2}{\sqrt{2}}\right) \cong A^{1/3} \cdot 3.904 \times 10^{-1\,1} \text{m} \tag{39}$$

where $(R)_A$ is the ionic radius of mass number $A.$ If $A = 7, (R)_A \cong 0.0747$ nm, if $A = 23, (R)_A \cong 0.111$ nm and if $A = 39, (R)_A \cong 0.132$ nm. Their corresponding recommended radii are 0.076 nm, 0.102 nm and 0.138 nm respectively [31,32].

a) Scattering distance between electron and the nucleus

If $R_0 \cong 1.21$ to 1.22 fm is the minimum scattering distance between electron and nucleus [32] it is noticed that,

$$R_0 \cong \left(\frac{\hbar c}{G_A m_e{}^2}\right)^2 \cdot \frac{2G_C m_e}{c^2} \cong 1.21565 \text{ fm} \tag{40}$$

Qualitatively and quantitatively - from this coincidence also it is possible to say that, in atomic and nuclear physics, the operating gravitational constant is Avogadro number times the Newton's gravitational constant.

$$N \cong \sqrt{\frac{2\hbar^2}{G_C m_e^3 R_0}} \tag{40}$$

$$G_C \cong \frac{2\hbar^2}{N^2 m_e{}^3 R_0} \tag{41}$$

b) Vibrations of the basic charged leptonic string in 3+1 dimensions

Muon and tau rest masses can be fitted in the following way [33]. The key relation seems to be

$$\left(\frac{\hbar c}{G_A m_e^2}\right)^2 \cong \frac{R_0 c^2}{2G_C m_e} \tag{42}$$

Considering the ratio of the volumes $\frac{4\pi}{3}R_0^3$ and $\frac{4\pi}{3}\left(\frac{2G_C m_e}{c^2}\right)^3$, let

$$\ln\left(\frac{R_0 c^2}{2G_C m_e}\right)^3 \cong \gamma \cong 289.805 \qquad (43)$$

Now muon and tau masses can be fitted with the following relation [23,24].

$$\left(m_l c^2\right)_x \cong \left[\gamma^3 + \left(x^2\gamma\right)^x \sqrt{N}\right]^{\frac{1}{3}} \cdot \frac{m_e c^2}{\gamma} \qquad (44)$$

where x = 0,1 and 2. At x = 0, $\left(m_l c^2\right)_0 \cong m_e c^2$. This relation can be considered as the representation of the basic charged leptonic string in 3+1 dimensions. At x = 1, $\left(m_l c^2\right)_1 \cong 107.23$ MeV and can be compared with the rest mass of muon (105.66 MeV). At x = 2, $\left(m_l c^2\right)_2 \cong 1788.07$ MeV and can be compared with the rest mass of tau (1777.0 MeV). x = 0,1 and 2 can be considered as the 3 characteristic vibrating modes. Best fit can be obtained at, $\gamma \cong 295.0606338$. Please refer [23,24]. **Qualitatively and quantitatively - from these coincidences also it is possible to say that, in atomic and nuclear physics, the operating gravitational constant is Avogadro number times the Newton's gravitational constant**.

VIII. Magnetic Moments of Nucleons

In the earlier published papers [23-25] authors suggested that, magnetic moment of electron is due to weak force magnitude and similarly nucleon's magnetic moment is due to the strong force magnitude or strong interaction range. Based on the proposed concepts and representing $\hbar$ in terms of Avogadro number and $\sin\theta_W$, magnetic moment of electron [33,34] takes the following form.

$$\mu_e \cong \frac{1}{2}\sin\theta_W \cdot ec \cdot \sqrt{\frac{e^2}{4\pi\varepsilon_0 F_W}} \cong 9.274\times10^{-24}\ \text{J/tesla} \qquad (45)$$

where F_W is the proposed weak force magnitude. Similarly the magnetic moment of proton can be expressed as

$$\mu_p \cong \frac{1}{2}\sin\theta_W \cdot ec \cdot R_0 \cong 1.356\times10^{-26}\ \text{J/tesla} \qquad (46)$$

where $R_0 \cong 1.21565\times10^{-15}$ m. If proton and neutron are the two quantum states of the nucleon, by considering the "rms" radius of proton as the radius of neutron, magnetic moment of neutron can be fitted as

$$\mu_n \cong \frac{1}{2}\sin\theta_W \cdot ec \cdot R_P \cong 9.59\times10^{-27}\ \text{J/tesla} \qquad (47)$$

where $R_P \cong 0.86\times10^{-15}$ m is the radius of proton. This seems to be a very nice and interesting fitting.

IX. To Fit the Characteristic Potential Radius of Nucleus

It is noticed that, gram mole is a black hole where the operating gravitational constant is (G_A) but not (G_C). That means for the simplest case of Hydrogen gram mole, there exist N number of protons and N number of electrons. Let it follows the concept of Schwarzschild radius. It can be expressed in the following way.

$$R_N \cong \frac{2G_A\left[N\left(m_p^2 m_e\right)^{1/3}\right]}{c^2} \qquad (48)$$

Here the only change is that, instead of the proton mass or instead of the electron mass, $\left(m_p^2 m_e\right)^{\frac{1}{3}}$ is considered for fitting the experimental radius of 1.4 fm. Volume of R_N is

$$V_N \cong \frac{4\pi}{3}R_N^3 \qquad (49)$$

The characteristic mean distance can be obtained as

$$\lambda_0 \cong \left(\frac{V_N}{N}\right)^{\frac{1}{3}} \cong 1.404\times10^{-15}\ \text{meter} \qquad (50)$$

This can be compared with the characteristic alpha scattering experimental radius [31] of nucleus ≈ 1.4 fm. Based on the Yukawa's Pion exchange model nuclear interaction range is 1.4 fm [33,35,36]. Thus if $m_\pi^\pm$ is the charged pion rest mass,

$$N \cong \left(\frac{3}{32\pi}\right)^{\frac{1}{5}}\left(\frac{\hbar c}{G_C\left(m_p^2 m_e\right)^{1/3} m_\pi^\pm}\right)^{3/5} \qquad (51)$$

X. To Fit the Rest Mass of Proton or Electron

Semi empirically it is also noticed that

$$\ln\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_C m_p^2}} \cong \sqrt{\frac{m_p}{m_e} - \ln\left(N^2\right)} \qquad (52)$$

where m_p is the proton rest mass and m_e is the electron rest mass. Considering this as a characteristic relation, and by considering the electron rest mass as a fundamental input, proton rest mass can be fitted accurately in the following way.

$$\left(e^{\sqrt{\frac{m_p}{m_e} - \ln\left(N^2\right)}}\right)^2 m_p^2 \cong \frac{e^2}{4\pi\varepsilon_0 G_C}. \qquad (53)$$

Thus by trial-error method, proton rest mass can be estimated from this relation. Here interpretation seems to be a big puzzle. **Alternatively by considering the proton rest mass as a fundamental input, without considering the electron rest mass, the proton-electron mass ratio can be estimated from this relation. It comes out to be 1836.1 and is a very nice fitting. Thus the electron rest mass can be fitted!** Here the important question is: What is the role of squared Avogadro number in grand unified physics? Authors are working in this new direction.

The accuracy of the measured value of *G* has increased only modestly since the original Cavendish experiment. The 2007 recommended value of $G = 6.6742867\times10^{-11}\ m^3Kg^{-1}sec^{-2}$. Based on the newly developed "interferometry techniques" [9], measured value of $G = 6.693\times10^{-11}\ m^3Kg^{-1}sec^{-2}$. Fitting the gravitational constant with the atomic and nuclear physical constants is a challenging task. From equ. (52)

$$G_C \cong \left(e^{\sqrt{\frac{m_p}{m_e} - \ln\left(N^2\right)}}\right)^{-2} \cdot \frac{e^2}{4\pi\varepsilon_0 m_p^2} \qquad (54)$$

$$\cong 6.666270179\times10^{-11}\ m^3Kg^{-1}sec^{-2}.$$

Avogadro number can be expressed as

$$N \cong \sqrt{exp\left[\frac{m_p}{m_e} - \left(\ln\sqrt{\frac{e^2}{4\pi\varepsilon_0 G_C m_p^2}}\right)^2\right]} \qquad (55)$$

$$\cong 6.174407621\times10^{23}.$$

Qualitatively and quantitatively - from this coincidence it is possible to say that, in atomic and nuclear physics, Avogadro number plays a very interesting role.

XI. Conclusion

In this paper authors mostly discussed the first assumption and it is the base for the other assumptions and applications. **For any theory, its success depends on its mathematical formulation as well as its workability in the observed physical phenomena**. Initially string theory was originated in an attempt to describe the strong interactions. It is having many attractive features. Then it must explain the ratio of (3+1) dimensional strong interaction strength and the gravitational interaction strength. Till date no single hint is available in this direction. This clearly indicates the basic drawback of the current state of the art string theory. Proposed relations clearly show the applications in different ways.

Now this is the time to decide, whether Avogadro number is an arbitrary number or a characteristic unified physical number. Developing a true unified theory at 'one go' is not an easy task. Qualitatively and quantitatively proposed new concepts and semi empirical relations can be given a chance in understanding and developing the unified concepts. If one is able to fine tune the "String theory" or "Super gravity" with the proposed weak and strong force magnitudes (within the observed 3+1 dimensions), automatically planck scale, nuclear scale and atomic scales can be interlinked into a theory of "strong gravity" [37-50]. But this requires further observations, analysis, discussions and encouragement. Authors request the science community to kindly look into this new approach.

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