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MATHEMATICAL MODELING AND THIN LAYER DRYING KINETICS OF CARROT SLICES

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Abstract - The thin-layer drying characteristics of carrot slices were investigated under four microwave powers; 200, 300, 400 and 500 W and slice thickness of 2.5 mm. Data were analyzed to obtain diffusivity values from the period of falling drying rate. Four mathematical models for describing the thin-layer drying behavior of carrot were investigated. The results show that the Midilli et al. is the most appropriate model for drying behaviour of thin layer carrot slices. An analysis of variance (ANOVA) revealed that microwave power significantly affected the drying time, drying rate, effective diffusivity and specific energy consumption. The effective diffusivity varied from 1.90×10^{-8} to 3.99×10^{-8} m^2/s, and the activation energy was determined to be 36.40. Specific energy consumption values ranged 8.58 to 12.46 MJ/kg and the optimized specific energy consumption was obtained 540 W microwave levels.

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I. INTRODUCTION

Drying is the process of removing the moisture in the product up to certain threshold value by evaporation. In this way, the product can be stored for a long period, since the activities of the microorganisms, enzymes or ferments in the material are suppressed via drying [1]. In particular, convective hot-air drying is extensively employed as a preservation technique. The major draw-back of convective hot-air drying method, from an energy point of view, is the longer drying period, higher drying temperature and therefore high energy consumption, which may be as high as 6000 kJ/kg of water evaporated [2]. In general, energy efficiency in drying is closely related to drying times.

Microwave is an electromagnetic wave in the frequency range of 300–30000 MHz. The conversion of microwave energy into heat in the food is because of the presence of water. The quick absorption of energy by water molecules causes rapid evaporation of water, resulting in high drying rates of the food.

Compared to hot air drying, microwave or hybrid/combined microwave drying techniques can greatly reduce the drying time (up to 50%) of biological materials without quality degradation, therefore microwave method offers significant energy savings [3, 4]. The most relevant aspects of drying technology are the mathematical modeling of the process and the experimental setup [5]. The modeling is basically based on the design of a set of equations to describe the system as accurately as possible. Drying characteristics of the particular products being dried and simulation models are needed in the design, construction and operation of drying systems [6].

The aim of this research was (i) to determine the influence of microwave power on the energy consumption and drying kinetics during microwave dehydration of carrot slices and (ii) to fit the experimental moisture data to four mathematical models.

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3. Soysal A., Oztekin S. and Eren O; Microwave drying of parsley: modelling, kinetics, and energy aspects. Biosys Eng, 93 (4) (2006), 403–413.
4. Beaudry C., Raghavan G. S. V. and Rennie T. J; Microwave finish drying of osmotically dehydrated cranberries. Dry Tech, 21(9) (2003), 1797–1810.

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II. MATERIALS AND METHODS

Carrot samples were procured from local vegetable market in Tehran, Iran. The samples were stored at 4ffi0.5 flC before they were used in experiments. Carrots were washed under running water to remove the adhering impurities, and thinly sliced in thicknesses of 2.5 using a sharp stainless steel knife. The average initial moisture content of the samples were found to be 83.8% wet basis, as determined by using convective oven at 105 °C for 24h.

A domestic microwave oven (M945, Samsung Electronics Ins) with maximum output of 1000 W at 2450MHz was used for the drying experiments. The dimensions of the microwave cavity were 327×370×207 mm. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying chamber was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. Experiments were performed at four microwave powers of 200, 300, 400 and 500 W. The moisture losses of samples were recorded at 30s intervals during the drying process by a digital balance (GF-600, A & D, Japan) and an accuracy of ffi0.01 g. For measuring the weight of the sample during experimentation, the tray with sample was taken out of the drying chamber, weighed on the digital top pan balance and placed back into the chamber. Drying was carried out until the final moisture content reaches to a level less than 5% (w.b.).

The experimental drying data were used to calculate the moisture ratio (MR) and drying rate (DR) using the following equations:

$$MR = \frac{M_t - M_e}{M_0 - M_e} \quad (1)$$

$$DR = \frac{M_{t+dt} - M_t}{dt} \quad (2)$$

where M_t, M_0 and M_e are the moisture content at any time of drying; initial moisture content and equilibrium moisture content (kg water/kg dm), respectively, DR is the drying rate (kg waret/kg dm.min), Mt+dt is the moisture content at t+dt (kg water/kg dm), and t is drying time (min). Since the values of me are very small compared to mt or m_0, Eq. (1) can be simplified to M_t/M_0 [7].

Four well-known thin-layer drying models in Table 1 were tested to select the best model for describing the drying curve of the carrot slices. The terms used to evaluate goodness of fit of the tested models to the experimental data were the coefficient of determination (R^2); root mean square error (RMSE) and the reduced chi-square (χ^2) between the experimental and predicted moisture ratio values. Statistical values are defined as follows:

$$\chi^2 = \frac{\sum_{i=1}^{N}\left(MR_{\exp,i} - MR_{pre,i}\right)^2}{N - z} \quad (3)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(MR_{\exp,i} - MR_{pre,i}\right)^2}{N}} \quad (4)$$

In these equations, N is the number of observations, z is the number of constants, MR_{exp} and MR_{pre} are the experimental and predicted moisture ratios, respectively.

Table 1 : Thin-layer drying models

Model name	Model	References
Page	$MR=\exp(-kt^{n})$	[7]
Wang and Singh	$MR = 1 + bt + at^{2}$	[3]
Logarithmic	$MR=a \exp(-kt) + b$	[5]
Midilli et al.	$MR=a \exp(-kt^{n})+ bt$	[10]

**where, k is the drying constant and a, b, n are equation constants*

During the drying process, diffusivity is assumed to be the only physical mechanism for the transfer of water to material surface and can be defined by Fick's second law of diffusion for a slab as follows:

$$\frac{\partial M}{\partial t} = D_{eff} \frac{\partial^{2} M}{\partial x^{2}} \tag{5}$$

By using appropriate initial and boundary conditions, Crank [8] gave the analytical solutions for various geometries and the solution for spherical object with constant diffusivity is given as:

$$MR = \frac{8}{\pi^{2}} \sum \frac{1}{(2n+1)^{2}} \exp\left(- \frac{(2n+1)\pi^{2} D_{eff}}{4L^{2}} t \right) \tag{6}$$

where D_{eff} is the effective diffusivity (m^2/s), and L is the half thickness of slab (m). For long drying times, only the first term (n=0) in the series expansion of the above equation can give good estimate of the solution, which is expressed in logarithmic forms as follows:

$$\ln(MR) = \ln\left(\frac{8}{\pi^{2}}\right) - \left(\frac{\pi^{2} D_{eff}}{4L^{2}}\right) t \tag{7}$$

The diffusion coefficients are typically determined by plotting experimental drying data in terms of ln(MR) versus drying time (t), because the plot gives a straight line with a slope as $\pi^2 D_{eff}/4L^2$.

Specific energy consumption (E_s) of the drying process was expressed in MJ/kg water evaporated. Therefore, the Es could be determined as follows [9]:

$$E_s = \frac{Pt}{m_w} \tag{8}$$

where P is the microwave power input (W); and mw is the mass of water evaporated (kg).

Inasmuch as temperature is not precisely measurable inside the microwave drier, the activation energy is found as modified from the revised Arehnious equation. In this method it is assumed as related to effective diffusion coefficient and the ratio of microwave output power to sample weight (m/p) instead of to air temperature. Then Equation (9) can be effectively used [10] as follows:

$$D_{eff} = D_0 \exp\left(- \frac{E_a . m}{P} \right) \tag{9}$$

Ref.

8. Crank J; The Mathematics of Diffusion. 2nd ed. Oxford (UK): Clarendon Press; 1975.
10. Ozbek B. and Dadali G; Thin-layer drying characteristics and modelling of mint leaves undergoing microwave treatment. J Food Eng, 83(2007), 541-549.

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where E_a is the activation energy (W/g), m is the mass of raw sample (g), and D_0 is the pre-exponential factor (m^2/s).

All measurements were carried out in triplicate. ANOVA test was performed in order to examine the effect of microwave power on drying kinetics and energy consumption. The SPSS version 17.0 was used for statistical investigations. For all statistical analysis, the level of significance is fixed at 95%. Each factor having a P value $\leq$ 0.05 was considered significant.

III. RESULTS AND DISCUSSION

Notes

The moisture ratios versus drying time for the carrot slices at the selected powers are shown in Fig. 1. The total drying times to reach the final moisture content for the carrot sample were 19.5, 15.5, 10.5 and 7 min at 200, 300, 400, and 500 W, respectively. Obviously, within a certain microwave power range (200-500W in this study), increasing output power speeds up the drying process, thus shortening the drying time (up to 61%).

As seen in Figs. 2 and 3, all curves have two stages. The drying rate rapidly increases and then slowly decreases as drying progresses. The moisture content of the material was very high during the initial phase of the drying which resulted in a higher absorption of microwave power and higher drying rates due to the higher moisture diffusion. As the drying progressed, the loss of moisture in the product caused a decrease in the absorption of microwave power and resulted in a fall in the drying rate. Similar findings were reported in previous studies [3,11,12].The drying rate by the microwave method can be described by Eq. (10):

$$DR = At^u + \frac{ht}{1+\exp\left(t^d\right)} \tag{10}$$

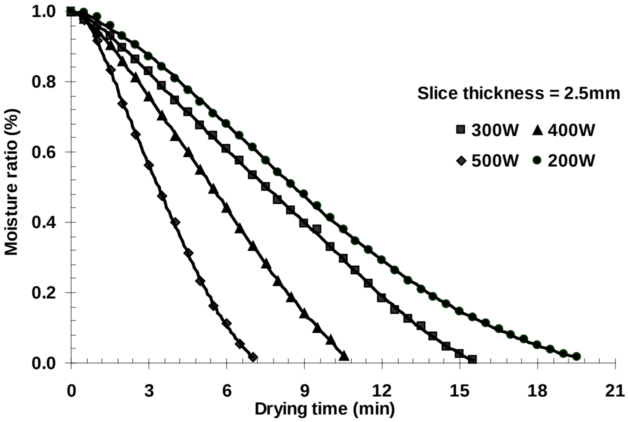

Fig.1 : Variation of moisture ratio with drying time for the carrot slice

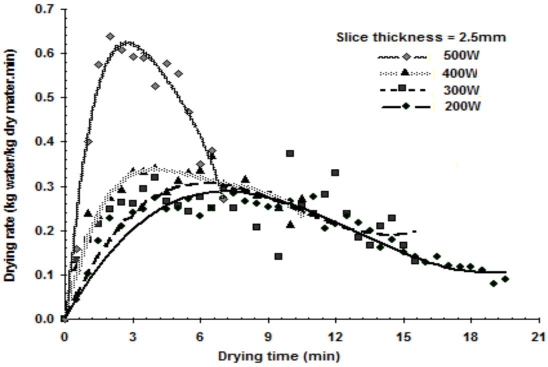

Fig.2 : Variation of drying rate with drying time for the carrot slice

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Parameters a, b, c and d of that function are given in Table 2. In order to take into account the effect of microwave power on the constant and coefficients of the Eq. (10), namely, A, u, h and d, the regression analysis was used to set up the relations between these parameters and the microwave power. Thus, the regression equations of these parameters against microwave power and the drying rate model are as follows:

$h = 0.0074P - 2.1312 \qquad R^2 = 0.971$
$u = 64.431\ P^{-0.84} \qquad R^2 = 0.983$
$d = -6\times10^{-7}P^3 + 7\times10^{-4}P^2 - 0.237P + 24.667 \qquad R^2 = 0.999$
$A = 6\times10^{-8}P^3 - 6\times10^{-5}P^2 + 0.0192P - 1.549 \qquad R^2 = 0.999$

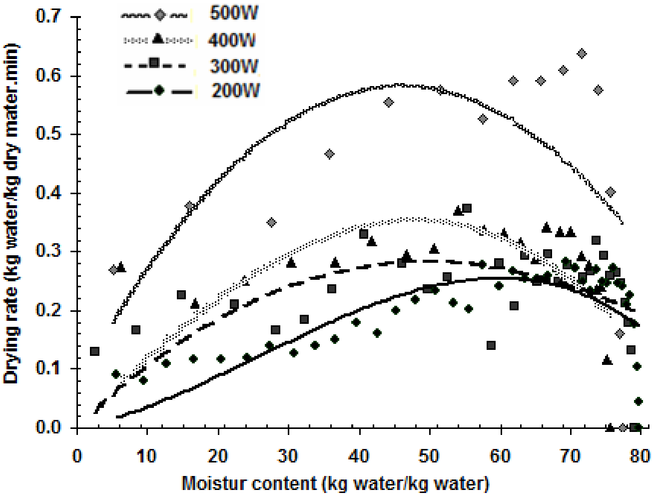

Fig.3 : Variation of drying rate with moisture content for the carrot slice

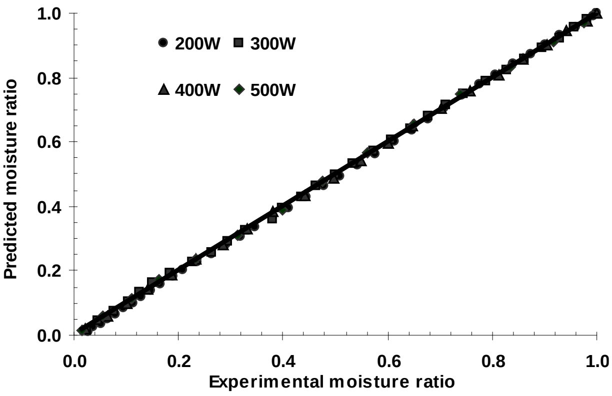

Fig.4 : Comparison of moisture ratios determined by experimentation and prediction using the Midilli et al. model for the carrot slices

The statistical results from models are summarized in Table 3. The best model describing the thin-layer drying characteristics of carrot slices was chosen as the one with the highest R^2 values and the lowest χ^2 and RMSE values. The statistical parameter estimations showed that R^2, χ^2 and RMSE values were ranged from 0.9900 to 0.9999, 0.00001 to 0.10269, and 0.00072 to 0.29833, respectively. Of all the models tested, the Midilli et al. model gives the highest value of R^2 and the lowest values of χ^2 and RMSE. It was determined that the value of the drying rate constant (k) increased with the increase in microwave powers. This implies that with increase in microwave power drying curve becomes steeper indicating increase in drying rate.

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Fig. 4 compares experimental data with those predicted with the Midilli et al. model for carrot slice samples at the different microwave powers. The prediction using the model showed MR values banded along the straight line, which showed the suitability of these models in describing drying characteristics of carrot slices.

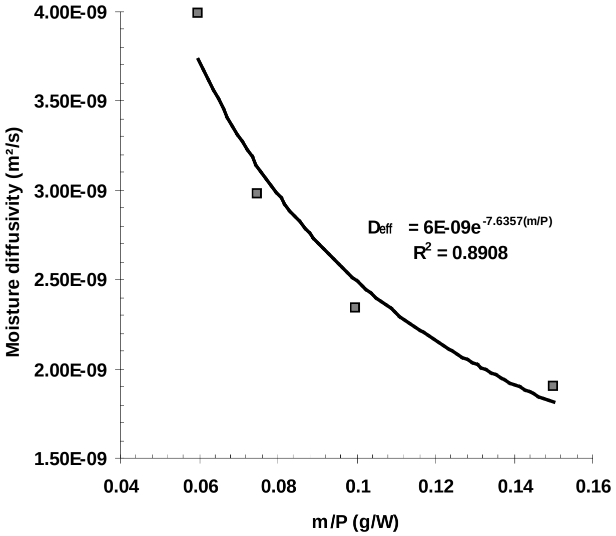

Fig.5 : The relationship between the values of D_{eff} versus sample amount/power

The effective moisture diffusivities of carrot slices for different microwave powers are presented in Table 4. The values ranged from 1.90×10^{-9} to 3.99×10^{-9} m^2/s. In Table 4, it was noted that D_{eff} increased progressively with the increase of drying microwave power. This might be explained by the increased heating energy, which would increase the activity of the water molecules leading to higher moisture diffusivity when samples were dried at higher microwave power. The values for D_{eff} obtained from this study lie within the general range 10^{-11}–10^{-8} m^2/s for drying of food materials [13, 14]. The values of D_{eff} are comparable with the reported values of 1.0465×10^{-8} to 9.1537×10^{-8} m^2/s mentioned for apple pomace microwave drying [11], 1.14×10^{-6} to 6.09×10^{-6} m^2/s for tomato pomace microwave drying at 160-800W [2], 0.55×10^{-7} to 3.5×10^{-7} m^2/s for Gundelia tournefortii microwave drying at 90-800W [15].

The values of effective diffusivity versus m/P shown in Fig. 5 accurately fit to Eq. (9) with coefficient of determination (R^2) of 0.8908. Then, D_0 and E_a values were estimated as 6×10^{-9} m^2/s and 7.636W/g.

Table 4 : Values of effective diffusivity obtained for carrot slice at different microwave powers

P(W)	Effective diffusivity (m^2/s)
200	1.90×10^{-9}
300	2.34×10^{-9}
400	2.98×10^{-9}
500	3.99×10^{-9}

Fig.6 shows the microwave specific energy consumption values at different amounts of microwave powers for the drying of carrot slices. Statistical analyses showed that microwave power was significant on the specific energy consumption values of carrot slices (α = 0.05). The values ranged from 8.58 to 12.46 MJ/kg water evaporated.

Ref.

11. Wang Z., Sun J., Chen F., Liao X. and Hu X; Mathematical modelling on thin layer microwave drying of apple pomace with and without hot air pre-drying. J Food Eng, 80 (3007), 536–544.

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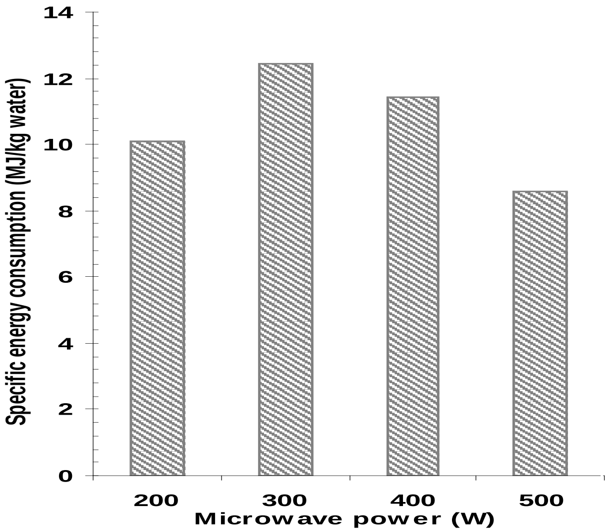

Fig.6 : Specific energy consumption for microwave drying of carrot slices

Table 2 : Parameters A, u, h and d of the functions describing the drying rate of carrot slices

P(W)	Drying kinetic parameters				
	A	u	h	d	R^2
500	0.001	0.336	1.694	0.642	0.904
400	0.001	0.446	0.736	9.521	0.917
300	0.213	0.522	-0.096	-0.958	0.630
200	0.270	0.751	-0.496	0.026	0.931

IV. CONCLUSION

Thin layer drying experiments were conducted to determine the thin layer drying characteristics and energy consumption of carrot slices in a microwave dryer. Four thin layer drying models were evaluated for their suitability. The Midilli et al. model was found to be the most suitable model for describing the thin layer drying of carrot slices. The effective moisture diffusivity was obtained based on Fick's second law. The values ranged from 1.90×10^{-9} to 3.99×10^{-9} m^2/s. The activation energy required to detach and move the water out from melon slices during the drying process was found to be 7.636 W/g. Increasing the microwave power was caused to increase the drying rate and decrease the energy consumption. The values ranged from 8.58 to 12.46 MJ/kg water evaporated.

Table 3 : Results of statistical analysis on the modeling of moisture content and drying time for the microwave dried carrot slices

Model	P (W)	Model constants	R^2	χ^2	RMSE
Page	200	k=0.017, n=1.749	0.9980	0.00029	0.01673
	300	k=0.023,n=1.722	0.9900	0.00101	0.03084
	400	k=0.034, n=1.824	0.9940	0.00070	0.02532
	500	k=0.075, n=1.859	0.9970	0.00044	0.01947
Wang and Singh	200	a= 0.0004, b= - 0.0610	0.9930	0.00125	0.03450
	300	a= -5E-05, b= - 0.0655	0.9975	0.00026	0.01580
	400	a= -0.0012, b= - 0.0840	0.9960	0.0165	0.12246
	500	a= 0.0142, b= - 0.2489	0.9971	0.10269	0.29833

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Logarithmic	200	k= 0.032, a=2.334, b= -1.266	0.9950	0.00064	0.02441
	300	k= 0.015, a=5.019, b= -3.982	0.9990	0.00018	0.01278
	400	k= 0.018, a= 5.867, b= -4.813	0.9970	0.00063	0.02334
	500	k= 0.037, a= 4.656, b= -3.601	0.9950	0.00082	0.02557
Midilli et al.	200	k= 0.020, a= 0.999, b= -0.005, n=1.605	0.9999	0.00014	0.01110
	300	k= 0.027, a= 1.003, b= -0.023, n=1.342	0.9999	0.00006	0.00072
	400	k= 0.034, a= 1.001, b= -0.024, n=1.550	0.9999	0.00001	0.00380
	500	k= 0.080, a=1.007, b= -0.019, n=1.634	0.9990	0.00004	0.00537

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