Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 11 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Some Remarks on Product Summability of Sequences

By Suyash Narayan Mishra
Amity University, India

Abstract - In [4], the definition of product summability method $(\boldsymbol{DD},\boldsymbol{kk})$(C,l) for functions was given and some of its properties were investigated. In [2], $(\boldsymbol{D},\boldsymbol{k})$ $(\boldsymbol{C},\boldsymbol{\alpha},\boldsymbol{\beta})$ $(k>0, \alpha>0, \beta>-1)$ summability for functions are defined and some of its properties are investigated. In this paper $(\boldsymbol{D},\boldsymbol{k})(\boldsymbol{C},\boldsymbol{\alpha},\boldsymbol{\beta})$ $(k>0, \alpha>0, \beta>-1)$ summability for sequences are defined and some of its properties investigated.

GJSFR-F Classification : MSC 2010: 28A20

SOME REMARKS ON PRODUCT SUMMABILITY OF SEQUENCES

Strictly as per the compliance and regulations of :

Some Remarks on Product Summability of Sequences

Suyash Narayan Mishra

Abstract - In [4], the definition of product summability method $(\boldsymbol{D},\boldsymbol{k})$(C,l) for functions was given and some of its properties were investigated . In [2], $(\boldsymbol{D},\boldsymbol{k})(\boldsymbol{C},\boldsymbol{\alpha},\boldsymbol{\beta})$ $(k>0,\alpha>0,\beta>-1)$ summability for functions are defined and some of its properties are investigated. In this paper $(\boldsymbol{D},\boldsymbol{k})(\boldsymbol{C},\boldsymbol{\alpha},\boldsymbol{\beta})$ $(k>0,\alpha>0,\beta>-1)$ summability for sequences are defined and some of its properties investigated.

I. Introduction

Kuttner [1], introduced the summability method for functions and investigated some of its properties. Pathak [4], defined the summability method for functions and investigated some of its properties. Mishra and Srivastava [3], introduced the summability method for functions by generalizing summability method. Mishra and Mishra [2], introduced the summability method for functions and investigated some of its properties. In this paper we define summability method for sequences and investigate some of its properties.

II. Some Relations and Definitions

Let $f(x)$ be any function which is Lebesgue-measurable, and that $f:[0,+\infty)\to R$, and integrable in $(0,\ x)$, for any finite x and which is bounded in some right hand neighbourhood of origin. Integrals of the form $\int_0^\infty$ are throughout to be taken as $\lim_{x\to\infty}\int_0^x$, $\int_0^x$ being a Lebesgue integral. For any n¿o, we write $a_n(x)$ for the n^{th} integral,

$$a_n(x) = \frac{1}{\Gamma(n)} \int_0^x (x-y)^{n-1}\, a(y)dy\,,$$

$$\boldsymbol{a}\_(\mathbf{0})(\boldsymbol{x}) = \boldsymbol{a}(\boldsymbol{x})$$

The (C,α,β) transform of $a(t)$, which we denote by $\partial_{\alpha,\beta}(t)$ is given by

$$a(t) \qquad (\alpha=0)$$

Author : *Amity University, Uttar Pradesh (Lucknow Campus) Viraj Khand-5,Gomti Nagar Lucknow U.P. E-mail : drsnm2010@gmail.com*

Ref.

4. **S.N.Pathak,** Some investigations of summability of functions, Ph.D. Thesis, Gorakhpur University, Gorakhpur (1986).

$$\frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha)\Gamma(\beta+1)}\frac{1}{t^{\alpha+\beta}}\int_0^x (t-u)^{\alpha-1}u^{\beta}a(y)dy\,,\ (\alpha>0,\beta>-1)\,, \tag{2.1}$$

If, for t > 0, the integral defining $\partial_{\alpha,\beta}(t)$ exists and if $\partial_{\alpha,\beta}(t) \to s$ as $t \to \infty$, we say that a (x) is summable (C,α,β) to s, and we write $\boldsymbol{a(x)} \to$ s (C, α, β) . We write

$$g(t) = g^{(k)}(t) = kt\int_0^\infty \frac{x^{k-1}}{(x+t)^{k+1}}a(x)dx\ ,(\ k>0\ ) \qquad (2.2)$$ if this exists ,We also write

$$U_{k,\alpha,\beta}(t) = kt\int_0^\infty \frac{x^{k-1}}{(x+t)^{k+1}}\partial_{\alpha,\beta}(x)dx\,, \qquad (2.3)$$ if this exists.

Notes

With the usual terminology, we say that the sequence a_n is summable,

(I) (D, k) to the sum s, if g (t) tends to a limit s as t→ ∞,

(II) (D, k) (C,α,β) to s, if $U_{k,\alpha,\beta}(t)$ tends to s as t→ ∞. When $\beta = 0$,$(D,k)(C,\alpha,\beta)$ and $(D,k)(C,\alpha)$ denote the same method . The case $\beta = 0$ is due to Pathak [5]. We know that for any fixed t > 0, k > 0, it is necessary and sufficient for the convergence of (2.3) that

$$\int_1^\infty \frac{\partial_{\alpha,\beta}(x)}{x^2}\,dx \text{ should converge.} \tag{2.4}$$

If (2.4) converges, write for x > 0, $F_{\alpha,\beta}(x) = \int_x^\infty \frac{\partial_{\alpha,\beta}(t)}{t^2}\,dt$.

We note that $F_{\alpha,\beta}(x) = o(1)$ as x→ ∞. Further, (since f(x) is bounded in some right hand neighbourhood of the origin) we have,

$$F_{\alpha,\beta}(x) = o\left(\frac{1}{x}\right) \text{ as } x\to 0+.$$

III. Main Results

In this section, we have following theorems for sequences analogous to [2].

Theorem 3.1: If $\alpha > \gamma \geq 1,\ k > 0$ then $a\,(x) \to s\,(D,k)(C,\alpha-1,\beta)$, whenever $a(x) \to s(D,k)(C,\gamma-1,\beta)$.

Theorem 3.2: Let $\alpha > \gamma \geq 0,\ \beta > -1$, and suppose that a(x) is summable (C,γ,β) to s and that $\int_1^\infty \frac{\partial_{\gamma,\beta}(x)}{x^2}\,dx$ converges. Then a(x) is summable $(D,k)(C,\alpha,\beta)$ to s.

References Références Referencias

1. **B. Kuttner,** On translated quasi –Cesaro summability, Cambridge Philos.Soc.,62 (1966), 705-712.
2. **B. P. Mishra and S.N. Mishra,** Some remarks on product summability methods.Prog.of Maths.Vol 35 (nos.1 & 2) 2001.
3. **B. P. Mishra and A.P.Srivastava,** Some remarks on absolute summability of functions based on (C,α,β) summability methods. To appear in Jour.Nat.Acad.Math.
4. **S.N.Pathak,** Some investigations of summability of functions, Ph.D. Thesis,Gorakhpur University,Gorakhpur (1986).