Global Journal of Science Frontier Research
Mathematics & Decision Sciences
Volume 12 Issue 3 Version 1.0 March 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Accelerating and Decelerating Hypersurface-Homogeneous Cosmological Models in Barber's Second Self-Creation Theory

By S. D. Katore, R. S. Rane, K. S. Wankhade & S.A.Bhaskar
SGB Amravati University

Abstract - We study the hypersurface-homogeneous cosmological model in presence of perfect fluid within the framework of Barber's [1982, GRG, 14, 117] second self-creation theory of gravitation. We have shown that the field equations are solvable for any arbitrary cosmic scale function and then obtained exact solutions for two values of a specific parameter. While doing so, we have used the general equation of state $p = m\rho$ where m $(0 \le m \le 1)$ is a constant. We also discussed the physical aspects of the models of the universe.

Keywords : *Hypersurface-homogeneous. Perfect fluid. Barber's self-creation theory.*

GJSFR-F Classication : *MSC 2010: 32S25*

ACCELERATING AND DECELERATING HYPERSURFACE-HOMOGENEOUS COSMOLOGICAL MODELS IN BARBERS SECOND SELF-CREATION THEORY

Strictly as per the compliance and regulations of :

Open Association of Research Society, USA
Research | Diversity | Ethics

Notes

Accelerating and Decelerating Hypersurface-Homogeneous Cosmological Models in Barber's Second Self-Creation Theory

S. D. Katore[α], R. S. Rane[σ], K. S. Wankhade[σ] & S.A.Bhaskar[α]

Abstract - We study the hypersurface-homogeneous cosmological model in presence of perfect fluid within the framework of Barber's [1982, GRG, 14, 117] second self-creation theory of gravitation. We have shown that the field equations are solvable for any arbitrary cosmic scale function and then obtained exact solutions for two values of a specific parameter. While doing so, we have used the general equation of state $p = m\rho$ where $m\,(0 \le m \le 1)$ is a constant. We also discussed the physical aspects of the models of the universe.

Keywords : *Hypersurface-homogeneous. Perfect fluid. Barber's self-creation theory.*

I. Introduction

In recent years, there has been a considerable interest in alternative theories of gravitation. Brans-Dicke (1961) theory develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution of matter in motion. Brans-Dicke theory is a scalar-tensor theory of gravitation in which the tensor field is identified with the space-time of Riemannian geometry and scalar field is alien to geometry. This theory does not allow the scalar field to interact with fundamental principles and photons. However, Barber (1982) has modified scalar-tensor Brans-Dicke theory to develop a continuous creation of matter in the large scale structure of the universe. As a result, two self-creation theories are proposed by Barber (1982) in which the mass of the universe is seem to be created out of self-contained gravitational, scalar and matter field. Brans (1987) has pointed out that Barber's first theory is in disagreement with experiment as well as inconsistent, in general. Hence Barber's first theory is not accepted because this theory violets the equivalence principle.

The second theory retains the attractive features of the first theory and overcomes previous objections. These modified theories create the universe out of self-contained gravitational and matter fields. In Barber's second self-creation theory, the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space-time manifold. Barber's second theory is a modification of general relativity to include continuous creation and is within observational limits, thus it modifies general relativity to become a variable G-theory. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, with the scalar acting as a reciprocal gravitational constant. The scalar field is postulated to couple to the trace of the energy-momentum

Author α : PG Department of Mathematics, SGB Amravati University, Amravati. E-mail: katoresd@rediffmail.com

Author σ : Department of Mathematics, Y. C. Science and Arts college, Mangrulpir. E-mails : rsrane53@rediffmail.com, wankhade.kishor@rediffmail.com

tensor. Moreover, the most significant feature of self-creation is that it is as consistent with cosmological constraints in the distant supernovae data, the Cosmic microwave Background anisotropies and primordial nucleo-synthesis, as the standard paradigm. Unlike that model, however, it does not require the addition of the undiscovered physics of Inflation, dark non-baryonic matter or dark energy. Nevertheless, it does demand an exotic equation of state, which requires the presence of false vacuum energy at a moderate density determined by the Einstein's field equations. The consistency of Barber's second theory motivates us to study cosmological model in this theory.

Astronomical observations of the large-scale distribution of galaxies in the universe show that the distribution of matter can be satisfactorily described by a perfect fluid. Many authors have studied the Barber's self-creation theory of gravitation to produce mass creation in presence of perfect fluid satisfying the equation of state in the context of different space times. Pimentel (1985) and Soleng (1987) have discussed FRW models by using a power law relation between the expansion factor of the universe and the scalar field. Singh (1984), Reddy (1987) and Reddy *et al.* (1987) have studied Bianchi type space-times solutions in Barber's second theory of gravitation while Reddy and Venkateswarlu (1989) presented Bianchi type $-\mathrm{VI}_0$ cosmological model in Barber's second self-creation theory of gravitation. Shanti and Rao (1991) studied Bianchi type II and III space-times in this theory, both in vacuum as well as in presence of stiff fluid. Ram and Singh (1998) have discussed the spatially homogeneous and isotropic Robertson-Walker and Bianchi type-II models of the universe in Barber's self-creation theory in presence of perfect fluid by using gamma law equation of state. Pradhan and Pandey (2002), Pradhan and Vishwakarma (2002), Panigrahi and Sahu (2004), Vishwakarma and Narlikar (2005), Sahu and Mohanty (2006), Singh and Kumar (2007), Singh *et al.* (2008) , Venkateswarlu *et al.* (2008), Reddy and Naidu (2008) and Katore *et al.* (2010), are some of the authors who have studied various aspects of different cosmological models in Barber's second self-creation theory. In recent years, Verma and Shri Ram (2010) studied Hypersurface-homogeneous bulk viscous fluid space-times with time-dependent cosmological term and Shri Ram and Verma (2010) have discussed bulk viscous fluid hypersurface-homogeneous cosmological models with time varying G and Λ.

Motivated by these works, in this paper, we have investigated hypersurface-homogeneous cosmological model in presence of perfect fluid within the framework of Barber's second self-creation theory of gravitation. We first show that the field equations are solvable for any arbitrary cosmic scale function and then we obtain exact solutions for two values of a specific parameter. While doing so, we have used the general equation of state $p = m\rho$ where m $(0 \le m \le 1)$ is a constant. We also discuss the physical aspects of the models of the universe. This paper is organized as follows: The metric and field equations are considered in Sect. 2. In Section 3, solutions of Barber's field equations are obtained while the models are considered in Sect. 3.1 and 3.2. Some concluding remarks are given in Sect. 4.

II. The Metric and Field Equations

Stewart and Ellis (1968) have obtained general solutions of Einstein's field equations for a perfect fluid distribution satisfying a barotropic equation of state for the Hypersurface-homogeneous space time given by the metric

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + f_K^2(y)dz^2\right], \tag{1}$$

Notes

where $f_K(y) = \sin y,\ y,\ \sinh y$ respectively when $K = 1,\ 0,\ -1$.

Hajj-Boutros (1985) developed a method to build exact solutions of field equations in case of the metric (1) in presence of perfect fluid and obtained exact solutions of the field equations which add to the rare solutions not satisfying the barotropic equation of state. Recently Verma and Shri Ram (2010b) obtained some hypersurface-homogeneous bulk viscous fluid cosmological models with time-dependent cosmological term. Very recently Shri Ram and Verma (2010) presented bulk viscous fluid hypersurface-homogeneous cosmological models with time varying G and Λ term. The energy-momentum tensor in presence of perfect fluid has the form

Notes

$$T_{ij} = (\rho + p)u_i u_j - p\, g_{ij} \tag{2}$$

together with the relation

$$g_{ij} u^i u^j = 1 \tag{3}$$

and perfect fluid obeys the equation of state

$$p = m\rho \tag{4}$$

where m ($0 \le m \le 1$) is a constant. Here p is the pressure in the fluid and ρ is the energy density of the fluid and u^i is the four velocity vector defined by $u^i = \delta^i_4$, where $i = 1, 2, 3, 4$. We use the co-ordinate to be co-moving so that $u^i = (0,0,0,1)$. For a universe field with perfect fluid, from (2) one finds

$$T_1^1 = T_2^2 = T_3^3 = -p, \quad T_4^4 = \rho \quad \text{and} \quad T = \rho - 3p. \tag{5}$$

The field equation in Barber's second self-creation theory of gravitation are

$$R_{ij} - \frac{1}{2} R g_{ij} = -8\pi\varphi^{-1} T_{ij} \tag{6}$$

and the scalar field equation is defined by

$$\varphi = \frac{8\pi}{3}\lambda T, \tag{7}$$

where R_{ij} is the Ricci tensor and R is the scalar curvature. $\varphi = \varphi_{;k}^{\;k}$ is the invariant d'Alembertian and T is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field theory. Barber scalar field φ is a function of t due to the nature of space-time which plays the role analogous to the reciprocal of Newtonian gravitational constant i. e. $\varphi = \frac{1}{G}$. λ is a coupling constant to be determined from the experiment $|\lambda|$ ¡ 10^{-1}. In the limit as $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect.

In a commoving coordinate system the Barber's field equations (6) and (7) for the metric (1) with the help of (5) take the form

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = -8\pi\varphi^{-1} p, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\varphi^{-1} p \tag{9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB} = -8\pi\varphi^{-1}p \tag{9}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{K}{B^2} = 8\pi\varphi^{-1}\rho \tag{10}$$

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) = \frac{8\pi\lambda}{3}(\rho-3p) \tag{11}$$

where λ is a coupling constant to be determined from experiments ($|\lambda|\le 10^{-1}$, $\lambda=0$) and $G_N = \varphi^{-1}$. Equation (11) is the scalar field cosmological equation. Here overhead dots (.) indicate the differentiation with respect to t The energy conservation equation of general relativity $T^{ij}_{;j}=0$ takes the form

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=0 \tag{12}$$

For the line element (1), we define the following physical and geometrical parameters, to be used in solving the Barber's field equations. The average scale factor (S), Volume scale factor (V), expansion scalar (θ) and shear scalar (σ) are

$$S=(AB^2)^{\frac{1}{3}} \tag{13}$$

$$V=S^3=AB^2 \tag{14}$$

$$\theta=v^i_{;i}=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{15}$$

and

$$\sigma^2=\frac{1}{3}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2 \tag{16}$$

The generalized mean Hubble parameter H is given by

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{17}$$

where $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}=H_3$.

An important observational quantity is the deceleration parameter q which is defined as

$$q=\frac{-V\ddot{V}}{\dot{V}^2} \tag{18}$$

Notes

The sign of q indicates whether the model inflates or not. The positive sign corresponds to the standard decelerating model whereas the negative sign indicates inflation.

III. Solution of the Field Equations

Recently, Shri Ram and Verma (2010) have investigated the hypersurface-homogeneous cosmological models with time varying G and Λ term in the presence of bulk viscous fluid. They showed that the field equations are solvable for any arbitrary cosmic scale function. We follow the same approach to find exact solutions of the field equations.

Notes

From (8) and (9), we obtain

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{B}^2}{B^2}-\frac{\dot{A}\dot{B}}{AB}+\frac{K}{B^2}=0 \tag{19}$$

which on integration, yields

$$-B^2\dot{A}+AB\dot{B}=-K\int A dt+c_1 \tag{20}$$

where c_1 is an arbitrary constant.

We can write (20) in the form

$$\frac{d}{dt}(B^2)-2\frac{\dot{A}}{A}B^2=F(t), \tag{21}$$

where $F(t)=-2\frac{K}{A}\int A dt+c_1$ (22)

The linear differential equation (22) has the general solution given by

$$B^2=A^2\left[\int\frac{F(t)}{A^2}dt+c_2\right], \tag{23}$$

where c_2 is an integration constant. It is clear that the solution of Barber's field equations reduces to integration of (23) if $A(t)$ is known as a explicit function of time. We now obtain a particular solution of the field equations for a simple choice of the function $A(t)$. we choose

$$A=t^n \tag{24}$$

where n is a real number.

Integrating (23), we obtain

$$B^2=\frac{Kt^2}{n^2-1}+c_1t^{1+2n}+c_2t^{2n} \tag{25}$$

Without loss of generality, we take $c_1=c_2=0$. The solution (25) becomes

$$B^2=\frac{Kt^2}{n^2-1}\quad,\quad n\neq\pm1 \tag{26}$$

Hence the geometry of the universe in Barber's second self-creation theory for the hypersurface-homogeneous space-time corresponding to the solution (24) and (26) takes the form

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{Kt^2}{n^2-1}\left[dy^2 + f_K{}^2(y)dz^2\right] \tag{27}$$

We also consider the usual barotropic equation of state relating the perfect fluid pressure p to energy density ρ i.e. Equation (4).

Using (24), (26) and (4) in (12), we obtain the explicit form of the physical quantities p and ρ as

$$p = m\,c_3 t^{-(m+1)(n+2)} \quad . \tag{29}$$

and

$$\rho = c_3 t^{-(m+1)(n+2)} \tag{28}$$

From (11) and (4), we obtain the solution for scalar field $\varphi(t)$ in Barber's second self-creation theory is given by

$$\varphi(t) = c_4 t^{2-(m+2)(n+2)} \quad . \tag{30}$$

3.1 Model I:

For $K = 1$, the hypersurface-homogeneous cosmological model in (27) reduces to

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{n^2-1}\left[dy^2 + \sin^2 y\, dz^2\right] \tag{31}$$

This model is well defined for $n^2 - 1 > 0$.

For the model (31), the physical and geometrical parameters are given by

$$\theta = \frac{(n+2)}{t} \tag{32}$$

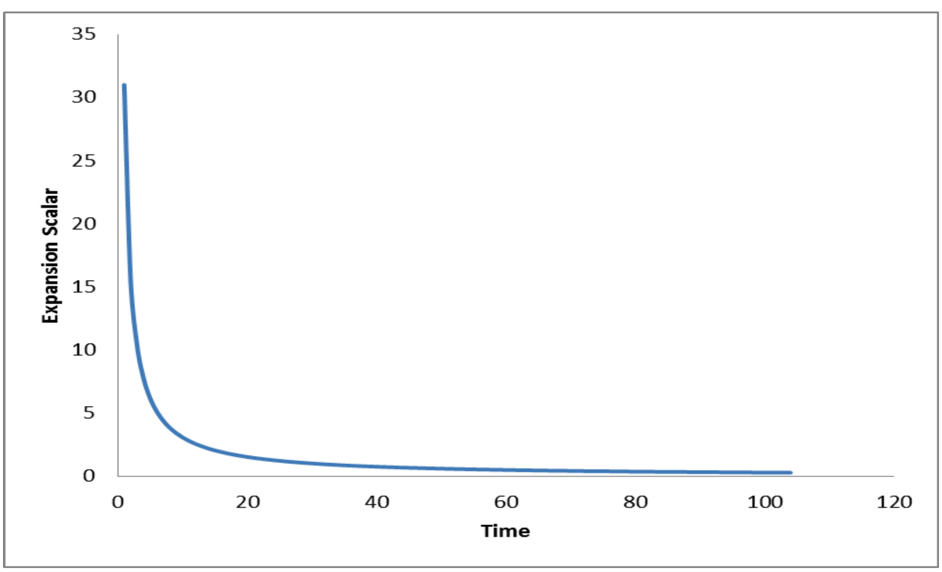

Figure 1 : Expansion Scalar Vs Time

Notes

$$\sigma^2 = \frac{(n-1)^2}{3t^2} \tag{33}$$

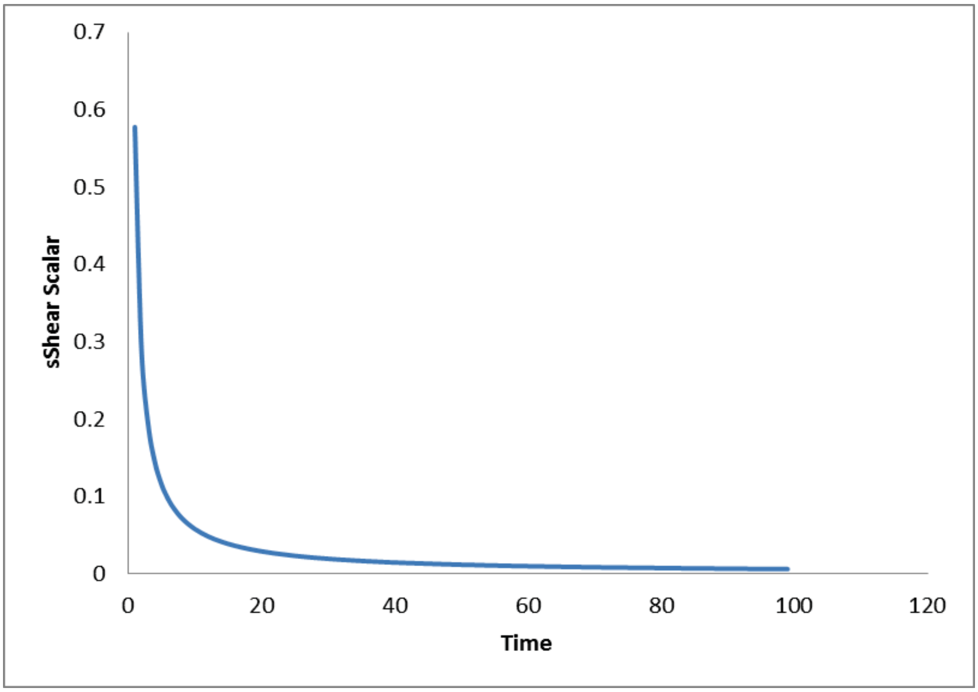

Figure 2 : Shear Scalar Vs Time .

$$S = \left(\frac{t^{n+2}}{n^2-1}\right)^{1/3} \tag{34}$$

$$V^3 = \frac{t^{n+2}}{n^2-1} \tag{35}$$

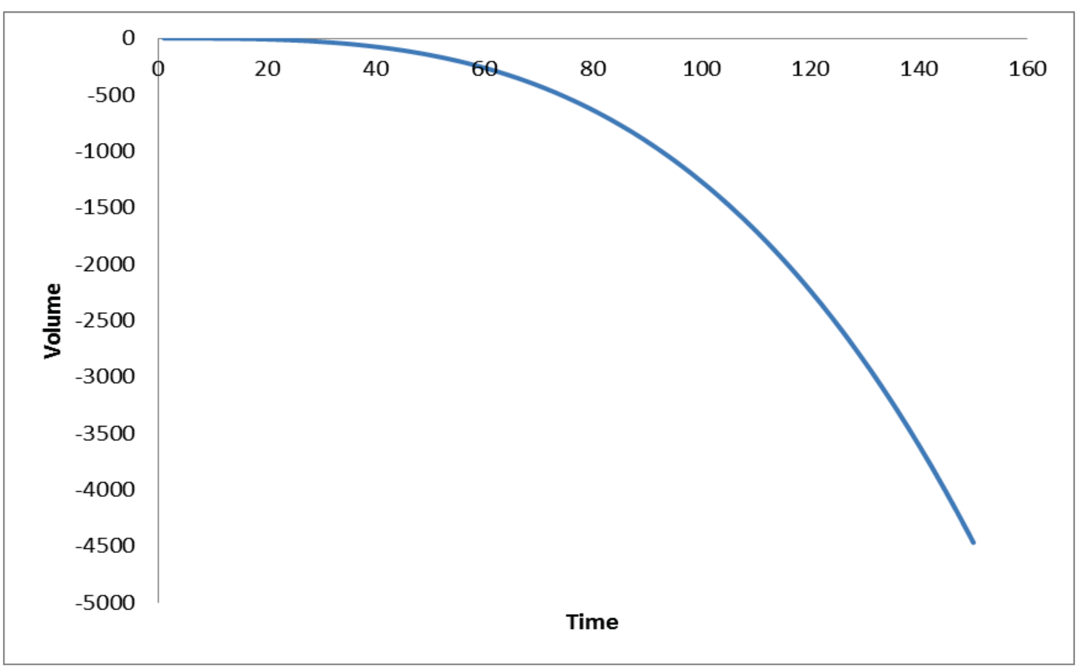

Figure 3 : Volume Vs Time.

Notes

$$H = \frac{n+2}{3t} \quad (36)$$

The deceleration parameter q is given by

$$q = \frac{1-n}{2+n} \quad (37)$$

For model (31), we observed that the spatial volume increases with time when $(n+2) > 0$ and it becomes infinite for large value of t. At t = 0, all the physical parameters ρ, σ, θ are infinite and the spatial volume is zero. Therefore, the cosmological model starts evolving with a big-bang at $t = 0$. Also the physical parameters decreases as time increases and tend to zero for large time. Since $\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\left(\frac{n-1}{n+2}\right)$ the anisotropy in the universe is maintained throughout. The deceleration parameter q is positive for $-2 < n < -1$. In this case, the model (31) represents a decelerating universe. When n ¿1 , the value of deceleration parameter q is negative and thus (31) corresponds to an inflationary model of the universe.

3.2 Model II

For K = -1, the metric (27) of our solution can be written in the form

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{1-n^2}\left[dy^2 + \sinh^2 y\, dz^2\right] \quad (38)$$

For the model (38), the expansion scalar θ, shear scalar σ and the generalized mean Hubble's parameter have the expressions given by (32), (33) and (36) respectively. The spatial volume V and the deceleration parameter q are given by the following expressions

$$V^3 = \frac{t^{n+2}}{1-n^2} \quad (39)$$

$$q = \frac{1-n}{2+n}, \quad (40)$$

where n is not less than -2. The deceleration parameter q is positive since $-1 < n < 1$. The model decelerates because of the fact that the deceleration parameter is positive constant. The cosmological model (38) starts with a big-bang at t = 0. The physical behaviors of this model are same as of the model (31).

IV. Conclusion

In this paper, we have obtained hypersurface-homogeneous cosmological model in presence of perfect fluid within the frame work of Barber's second self-creation theory. The cosmic fluid satisfies the barotropic equation of state. It is shown that Barber's field equations for hypersurface-homogeneous cosmological model are solvable for any arbitrary

Notes

cosmic scale function. Two classes of exact solutions of Barber s field equations are presented for $K = 1$ and $K =$ -1 which represent expanding, shearing, non-rotating, decelerating / accelerating models of the universe. In present models of the universe the anisotropy is maintained throughout. It is also observed that the Barber scalar field φ increases when t increases.

References Références Referencias

Notes

1. Barber, G.A.: Gen. Relativ. Gravit. **14**, 117 (1982).
2. Brans, C.: Gen. Relativ. Gravit. **19**, 949 (1987).
3. Brans, C., Dicke, R.H.: Phys. Rev. **124**, 925 (1961).
4. Pimentel, L.O.: Astrophys. Space Sci. **116**, 935 (1985).
5. Panigrahi, U.K., Sahu,R.C.: Theor. Appl. Mech. **30**,163 (2003).
6. Pradhan, A., Pandey, H.R.: arXiv: gr-qc/0207027 vl 4 Jul 2002.
7. Pradhan, A.,Vishwakarma, A.K.: Int.J.Mod.Phys. D **11**, 1195 (2002).
8. Reddy, D.R.K.: Astrophys. Space Sci. **133**, 389 (1987).
9. Reddy, D.R.K., Avadhanulu,M.B., Venkateswarlu,R.: Astrophys. Space Sci. **134**, 201 (1987).
10. Reddy, D.R.K., Venkateswarlu,R.: Astrophys. Space Sci. **155**, 135 (1989).
11. Reddy, D.R.K., Naidu, R.L.: Int. J. Theor. Phys. DOI10.1007/s100773-008-9774-2.
12. Sahu, R.C., Mohanty, G..: Astrophys. Space Sci. **306**, 179 (2006).
13. Singh, C.P., Kumar, S.: Astrophys. Space Sci. **310**, 31 (2007).
14. Singh, J.P., Tiwari, R.K., Kumar, S.: Astrophys. Space Sci. **314,** 145 (2008).
15. Singh, T.: Astrophys. Space Sci. **102**, 67 (1984).
16. Soleng, H.H..: Astrophys. Space Sci. **139**, 13 1987).
17. Shanti, K, Rao, V.U.M..: Astrophys. Space Sci. **179**, 147 (1991).
18. Venkateswarlu,K., Rao,V.U.M., Kumar,K.P.: Int.J.Theor.Phys .**47**, 640 (2008).
19. Vishwakarma, R.G., Narlikar, J.V..: Int. J. Mod. Phys. D **14**, 345 (2005).
20. Shri Ram, Singh, C.P.: Astrophysics Space sci. **257**, 287 (1998).
21. Katore, S.D., Rane, R.S., Wankhade, K.S.: Int. J. Theor. Phys. **49**, 187 (2010).
22. Verma, M.K., Shri Ram : Astrophys. Space Sci. **326**, 299 (2010b).
23. Shri Ram, Verma, M.K.: Astrophys. Space Sci. **330**, 151 (2010).
24. Stewart, J.M., Ellis, G.F.R. : J. Math. Phys. **9**, 1072 (1968).
25. Hajj-Boutros, J. : J. Math. Phys. **26**, 2297 (1985).

Notes

This page is intentionally left blank