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Computational Algorithm for Gravity Turn Maneuver

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GJSFR-F Classification : *MSC 2010: 83C27*

Computational Algorithm for Gravity Turn Maneuver

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I. Introduction

It is known that (Thomson 1986) the tangent of the optimum thrust attitude φ for placing space vehicle into an orbit is always linear function of time Likewise, the optimum thrust attitude for maximum range can be shown to be $\varphi =$ constant. These conditions may be satisfactory for a rocket traveling in vacuum but, owing to the large angle of attack α (see Fig.1) which results from such trajectories, they are not feasible through the atmosphere. Thus for flight through the atmosphere, a trajectory known as gravity turn or zero –lift turn is generally used.

A gravity turn maneuver is used in launching a spacecraft into, or descending from, an orbit around a celestial body such as a planet or a moon (ShangKristian et al 2011, Mehedi et al 2011). It is a trajectory optimization that uses gravity to steer the vehicle onto its desired trajectory. It offers two main advantages over a trajectory controlled solely through vehicle's own thrust. Firstly, the thrust doesn't need to be used to change the ship's direction so more of it can be used to accelerate the vehicle into orbit. Secondly, and more importantly, during the initial ascent phase the vehicle can maintain low or even zero angle of attack. This minimizes transverse aerodynamic stress on the launch vehicle, allowing for a lighter launch vehicle (Samuel 1965). The term gravity turn can also refer to the use of a planet's gravity to change a spacecraft's direction in other situations than entering or leaving the orbit (Roger 1964).

In a gravity turn, the thrust vector is kept parallel to the velocity vector at all times (see Fig 2) starting with some nonvertical initial velocity vector $\mathbf{v}_0$.

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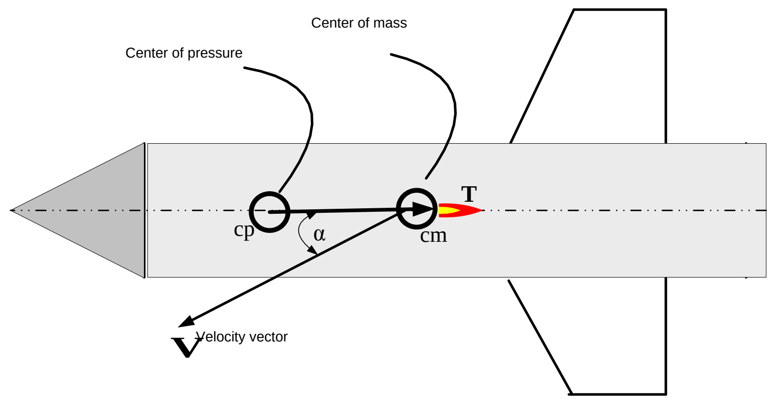

Fig1 : The angle of attack

In this paper, computational algorithm for gravity turn maneuver is established for variable thrust-to-weight ratio. The application of the algorithm was illustrated graphically.

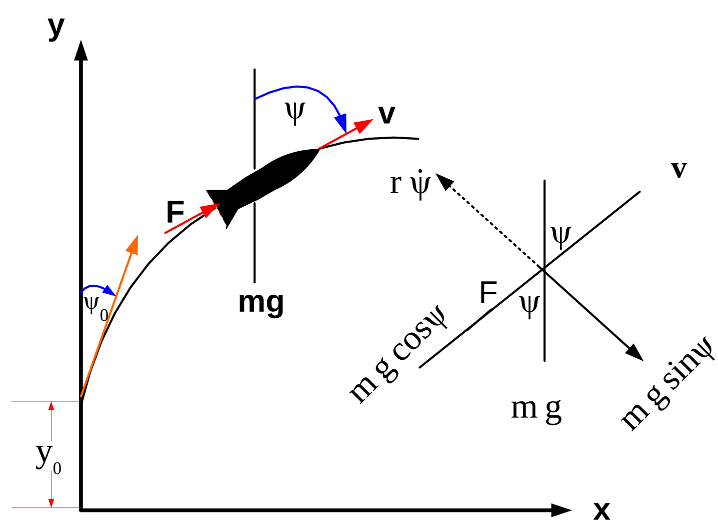

Fig. 2 : Gravity turn trajectory

II. FORCES EQUATIONS

It is convenient here to measure the angle made by the velocity vector from vertical, as shown in Fig.2. Assuming zero aerodynamic drag and constant gravity field g, we can write the force equations as:

$$\frac{1}{g}\frac{dv}{dt} = \frac{F}{m\,g} - \cos\psi\,, \tag{1}$$

$$\frac{v}{g}\frac{d\psi}{dt} = \sin\psi\ , \tag{2}$$

where F is the magnitude of thrust vector and m is the instantaneous vehicle mass.

Notes

These equations are nonlinear and no analytical solution is known when F/mg varies with time.

III. Numerical Solution for Varying F/mg

When F/mg is constant, Equations (1) and (2) can be solved analytically. For F/mg to be constant, the thrust F must decrease with time, this is because, the mass m decreases with the time t, consequently F should decreases with t so as to keep the ratio constant.

Notes

Let $F/mg = n$ over short increment of the flight path. It could be shown that(Thomson 1986) the solution for gravity turn trajectory when n is constant is represented by the following three equations

$$v = C z^{n-1} (1+z^2) \ . \tag{3}$$

The constant C can be evaluated from the initial conditions that at $z = z_0$, $v = v_0$ to get:

$$C = \frac{v_0}{z_0^{n-1} (1+z_0^2)} . \tag{4}$$

$$\Delta t = \frac{C}{g} \left\{ z^{n-1} \left(\frac{1}{n-1} + \frac{z^2}{n+1} \right) - z_0^{n-1} \left(\frac{1}{n-1} + \frac{z_0^2}{n+1} \right) \right\} . \tag{5}$$

To apply Equations (3) (4) and (5) for a varying F/mg, the following algorithm is devoted

a) *Computational algorithm*

▲ Purpose: To compute the coordinates(x,y) and the tangential velocity v of space vehicle along gravity turn path with varying F/mg ratio.

▲ Input: $t_0, \psi_0, v_0, x_0, y_0, n$

▲ Computational steps:

1 - $\Delta\psi_0 = \psi_0/100$

2 - $z_0 = \tan\frac{1}{2}\psi_0$

3 - $C = \dfrac{v_0}{z_0^{n-1}(1+z_0)^2}$

4 - $\psi = \psi_0 + \Delta\psi_0$

5 - $z = \tan\frac{1}{2}\psi$

6 - $v = Cz^{n-1}(1+z^2)$

7 - $$\Delta t = \frac{C}{g}\left\{ z^{n-1}\left(\frac{1}{n-1}+\frac{z^2}{n+1}\right) - z_0^{n-1}\left(\frac{1}{n-1}+\frac{z_0^2}{n+1}\right)\right\}$$

8 - $$\Delta x = \frac{1}{2}(v_0 \sin\psi_0 + v\sin\psi)\,\Delta t \quad ; \quad \Delta y = \frac{1}{2}(v_0\cos\psi_0 + v\cos\psi)\,\Delta t$$

9 - $$x = x_0 + \Delta x \; ; \quad y = y_0 + \Delta y$$

10 - $$x_0 = x \; ; \quad y_0 = y ; \; \psi_0 = \psi \; ; \quad t_0 = t + \Delta t$$

12 - Go to step 2

The procedure can be repeated up to any time

b) Graphical illustrations

The above algorithm was applied with the initial conditions

$$t_0 = 0 \quad ; \quad \psi_0 = 10^{o} \; ; \quad v_0 = 500\text{ft/sec} \; ; \; x_0 = 0 \quad ; \; y_0 = 3000\text{ft}$$

with n variable according to the formula: $n(t) = 3e^{-5t}$. Note that, the initial and the computed coordinates referred to the geocentric coordinate system. The output are illustrated graphically in the following figures.

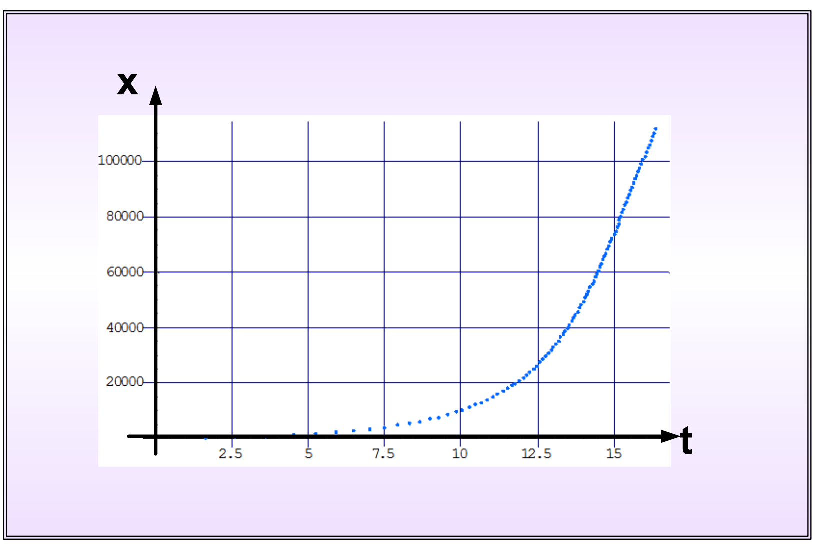

Fig. 3 : The variation of the x coordinate with time along gravity turn path with:

$$t_0 = 0 \quad ; \quad \psi_0 = 10^{o} \; ; \quad v_0 = 500\text{ft/sec} \; ; \; x_0 = 0 \quad ; \; y_0 = 3000\text{ft} \quad ; \; n(t) = 3e^{-5t}$$

Notes

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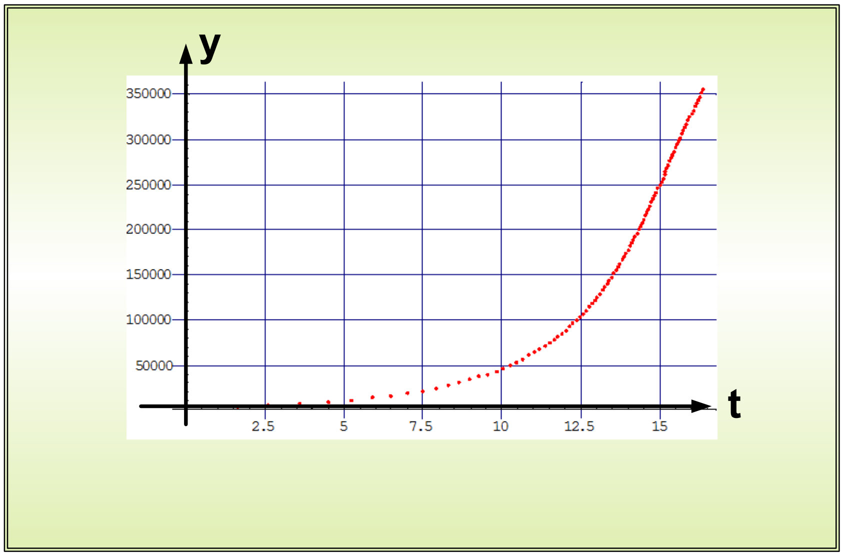

Fig. 4 : The variation of the y coordinate with time along gravity turn path with:

$t_0 = 0$; $\psi_0 = 10^o$; $v_0 = 500 ft/sec$; $x_0 = 0$; $y_0 = 3000 ft$; $n(t) = 3e^{-5t}$

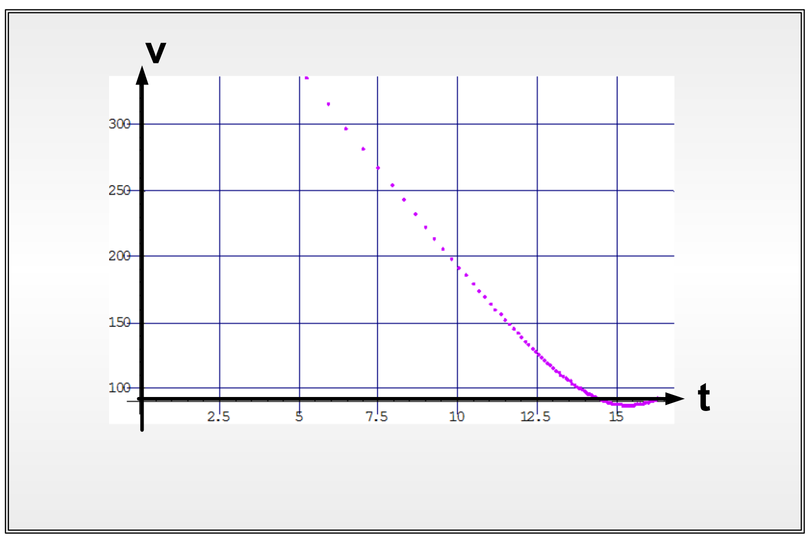

Fig. 5 : The variation of the velocity with time along gravity turn path with:

$t_0 = 0$; $\psi_0 = 10^o$; $v_0 = 500 ft/sec$; $x_0 = 0$; $y_0 = 3000 ft$; $n(t) = 3e^{-5t}$

In concluded the present paper, computational algorithm for gravity turn maneuver is established for variable thrust-to-weight ratio. The applications of the algorithm was illustrated graphically.

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