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Orbit - Orbit Resonance of Pluto and Neptune

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Orbit - Orbit Resonance of Pluto and Neptune

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Abstract - In the present paper, an algorithm for the planar restricted circular threebody problem in rotating symbolic system is developed to determine orbit-orbit resonance of Pluto and Neptune.

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I. Introduction

Because of Pluto is locked into a 3:2 resonance with Neptune, Pluto completes 2 orbits every 3 orbits of the Sun completed by Neptune. Although, the ratio is not exactly 3:2, sometimes Pluto's period is slightly faster than its average value or slower. It is the possibility to indefinitely close approach if the perihelion and nod of Pluto are unrestricted, where, the radius of perihelion of Pluto is less than the radius of Neptune's orbit. This is the most important case of orbit - orbit resonance in the solar system. It is well known that the orbit of Pluto has a large eccentricity of $e = 0.247$, which brings the planet at a certain moment inside the orbit of Neptune. The two planets are trapped in an orbit-orbit resonance. The period of Pluto is 3:2 times the period of Neptune.

II. Equations of Motion

We consider the planar restricted circular three-body problem in rotating synodic system (e.g .Szebehely 1967) in which the two primaries are the Sun and Neptune while the third body is Pluto. The equations of motion to be solved are

$$\dot{x} = u, \tag{1.1}$$

$$\dot{y} = v, \tag{1.2}$$

$$\dot{u} = -(1-\mu)\frac{x-\mu}{R_1^3} - \mu\frac{x+1-\mu}{R_2^3} + x + 2v, \tag{1.3}$$

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$$\dot{v} = -(1-\mu)\frac{y}{R_1^3} - \mu\frac{y}{R_2^3} + y + 2u, \tag{1.4}$$

with

$$R_1 = \sqrt{(x-\mu)^2 + y^2} \tag{2.1}$$

and

$$R_2 = \sqrt{(x+1-\mu)^2 + y^2} \quad , \tag{2.2}$$

where dot denotes differentiation with respect to the time t, (x, y) are the coordinates of the third body , μ denotes the mass of the smaller primary when the total mass of the primaries has been normalized to unity.

In these equations, the unit of length is the distance between the primaries, the unit of mass is the sum of the masses of the primaries. The unit of time is $1/n$. (n is the mean motion).Normally, n is expressed in a number of radians per second, hence in $1/\text{sec}$. Its inverse $1/n$ is therefore expressed in seconds and may be interpreted as a unit of time.

III. Orbit Determination of Pluto and Neptune

All the numerical values of the following are taken from the reference (Hellings,1994)

a) *Orbital elements*

Neptune:

$$e = 0$$
$$P = 165.62$$
$$a = 30.1584$$

Pluto:

$$e = 0.247$$
$$P = 248.43$$
$$a = 39.5187$$

and $\mu : 0.0000525$

Amplitude of the libration is known to be 38^0.

b) *Initial conditions*

The Initial conditions are

$x_0 = -0.6073955952$; $y_0 = -0.7774968265$; $u_0 = 0.1083342234$; $v_0 = -0.0863997159$.

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c) The results

It should be noted that ,all the computations are performed using *Mathematica* **7**. For clear illustrations of our analysis, the results are displayed graphically in the following manner

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Fig.1 : Basic part of the orbit of Pluto relative to the Sun and Neptune

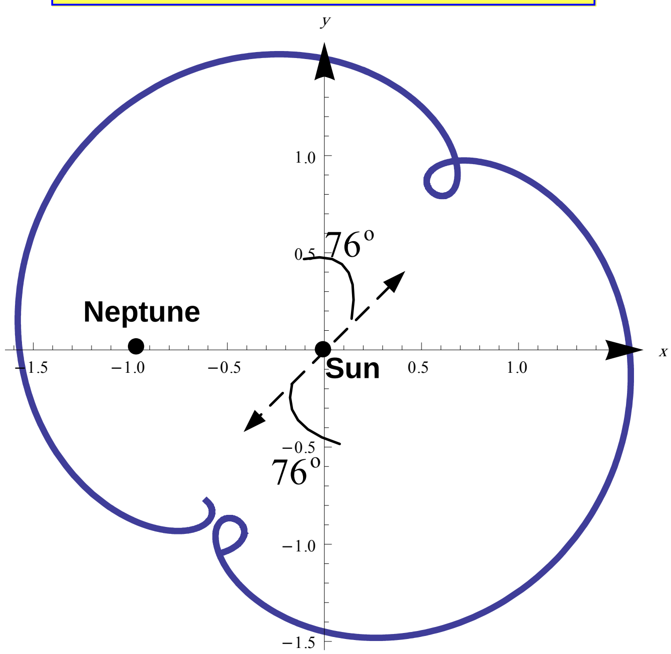

Fig.2 : Plot between u and v for the basic part of the orbit of Pluto relative to the Sun and Neptune

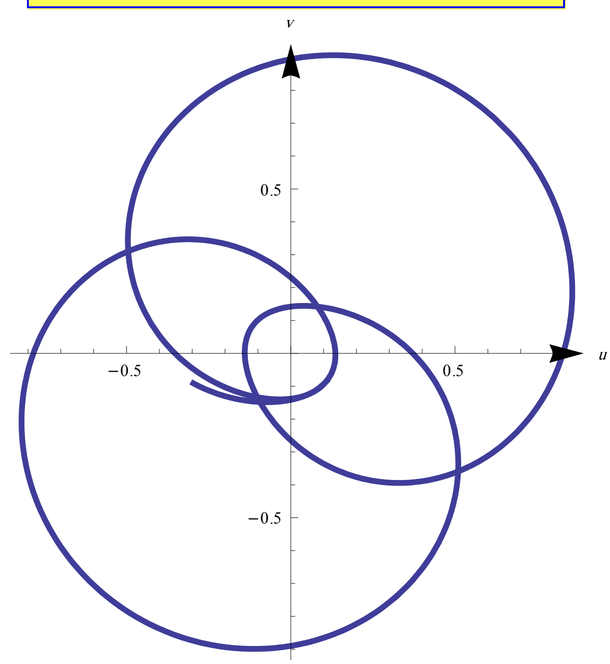

Fig.3 : Plot between x and u
for the basic part of the orbit of Pluto relative
to the Sun and Neptune

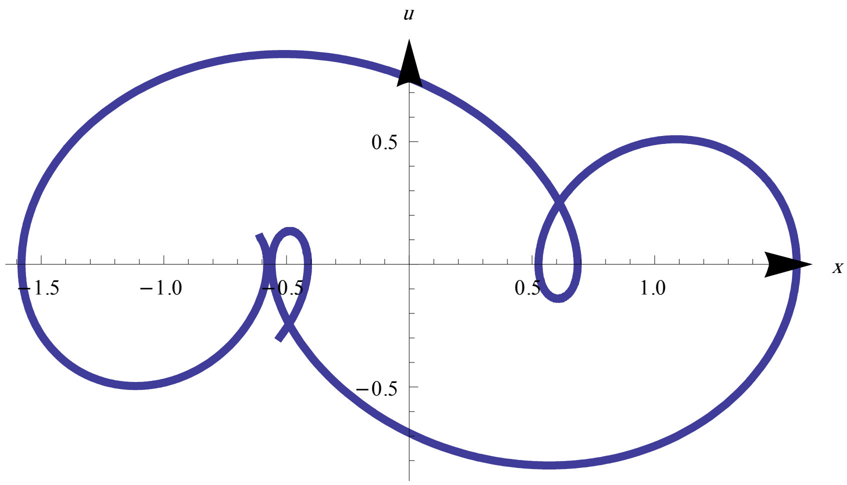

Fig.4 : Plot between y and v
for the basic part of the orbit of Pluto relative
to the Sun and Neptune

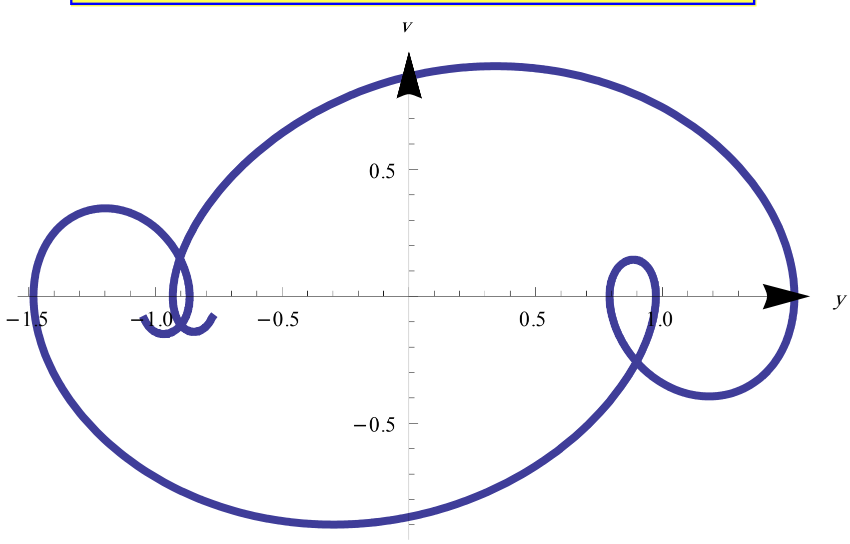

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Fig.5 : The basic orbit of Fig.1 oscillates with an amplitude of 38^o

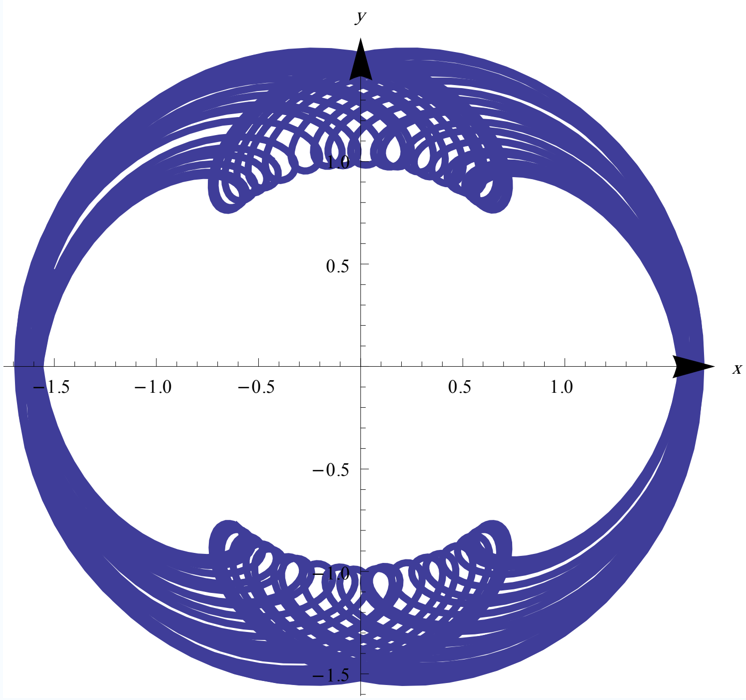


Fig.6 : The basic orbit of Fig.2 oscillates with an amplitude of 38^o

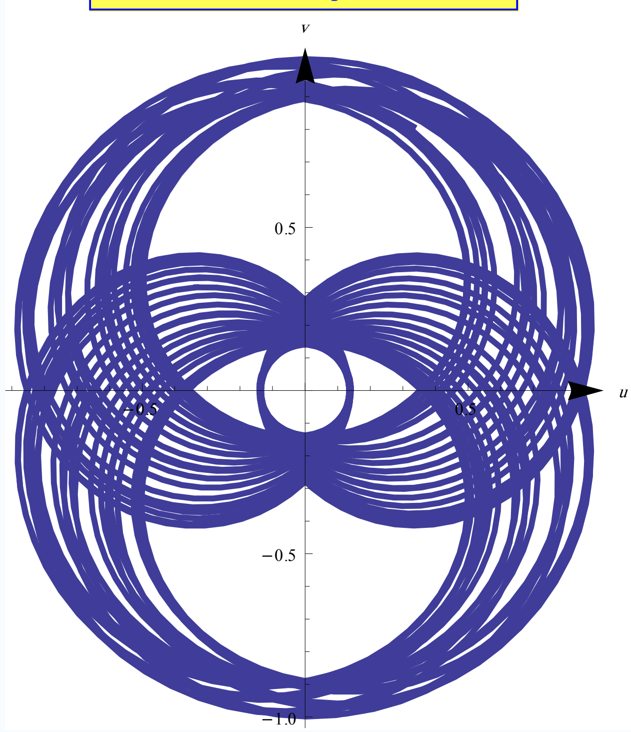


Fig.7 : The basic orbit of Fig.3 oscillates with an amplitude of 38^o

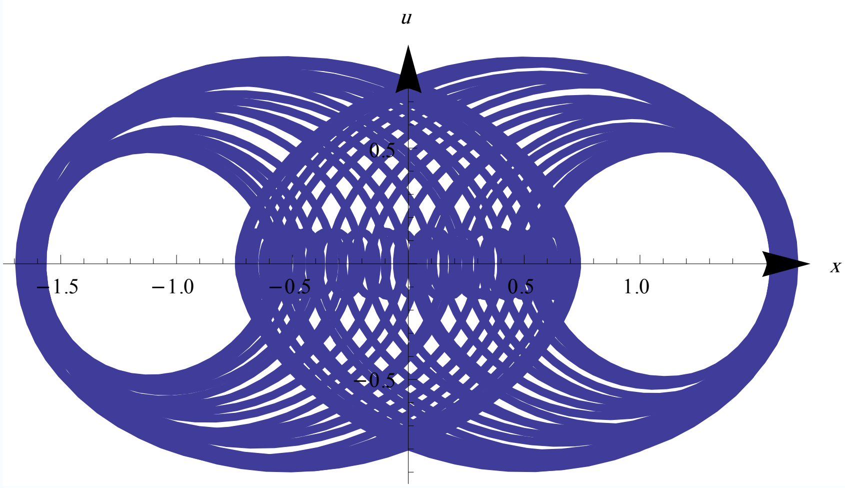
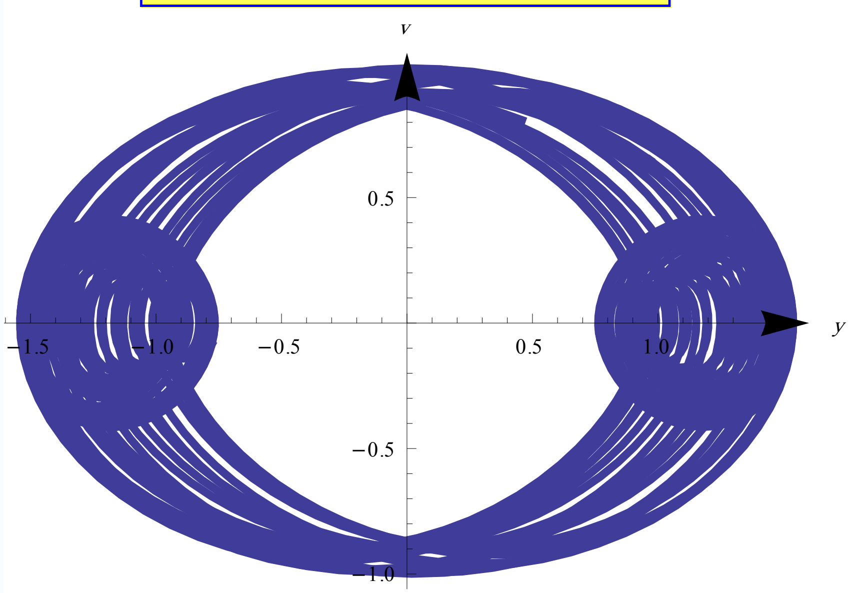


Fig.8 : The basic orbit of Fig.4 oscillates with an amplitude of 38^o



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IV. Conclusion

In this paper, general computational algorithm for the planar restricted circular three-body problem in rotating synodic system is developed in Section 2. This algorithm is applied to determinate orbit-orbit resonance of Pluto and Neptune. Finally the results are illustrate graphically in Section 3 which could be summarized as :

1-Figure 1 shows the basic part of the orbit of Pluto relative to the Sun and Neptune. This part represents two revolutions of Pluto around the Sun.

2- Pluto reaches two times a distance closer to the sun than Neptune.

3-The next two revolutions have the same shape, but the figure is rotated a little bit counter clockwise as the two perihelia approach the y- axis. This phenomenon increases until the whole figure is rotated over 76^0 .

4- A total libration, shown in Fig. 5, illustrates that Pluto will never collide with Neptune since its distance to Neptune is always larger than about 17 A.U.

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