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New Generating Fun ctions Pertaining To Generalized Mellin-Barnes Type of Contour Integrals

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NEW GENERATING FUN CTIONS PERTAINING TO GENERALIZED MELLIN-BARNES TYPE OF CONTOUR INTEGRALS

Strictly as per the compliance and regulations of :

Ref.

1. A.A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals: I. Transformation and reeducation formulae, J. Phys. A: Math. Gen.20 (1987), 4109-4117.

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Keywords : $\bar{H}$*-function, generating function, F-equations.*

I. Introduction

In 1987, Inayat-Hussain [1, 2] introduced generalization form of Fox's H-function, which is popularly known as $\bar{H}$-function. Now $\bar{H}$-function stands on fairly firm footing through the research contributions of various authors [1-3, 6, 7, 9-12]. $\bar{H}$-function is defined and represented in the following manner [7]:

$$\bar{H}_{p,q}^{m,n}[z]=\bar{H}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L z^{\xi}\bar{\phi}(\xi)d\xi\ (z\neq 0) \tag{1.1}$$

where

$$\bar{\phi}(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

It may be noted that the $\bar{\phi}(\xi)$ contains fractional powers of some of the gamma function and m,n,p,q are integers such that $1\leq m\leq q, 1\leq n\leq p$, $(\alpha_j)_{1,p},(\beta_j)_{1,q}$ are positive real numbers and $(A_j)_{1,n},(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(a_j)_{1,p}$ and $(b_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\bar{H}$-function can be seen in the papers [6, 7].

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The behavior of the $\bar{H}$-function for small values of $|z|$ follows easily from a result given by Rathie [3]:

$$\bar{H}^{m,n}_{p,q}[z] = o(|z|^{\alpha});$$

Where

$$\alpha = \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{\alpha_j}\right), |z| \to 0 \tag{1.3}$$

$$\Omega = \sum_{j=1}^{m}|\beta_j| - \sum_{j=m+1}^{q}|\beta_j B_j| + \sum_{j=1}^{n}|\alpha_j A_j| - \sum_{j=n+1}^{p}|\alpha_j| > 0\,,\, 0<|z|<\infty \tag{1.4}$$

The following function which follows as special cases of the $\bar{H}$-function will be required in the sequel [7]:

$$ {}_p\bar{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};z\right] = \bar{H}^{1,p}_{p,q+1}\left[-z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right] \tag{1.5}$$

Truesdell's F-equations are defined and represented as:

a) The function $F(z,s)$ is said to satisfy the ascending F-equations if $D_z^r F(z,s) = F(z,s+r)$ where $D_z^r = \left(\frac{d}{dz}\right)^r$.

b) The function $Y(z,s)$ is said to satisfy the descending F-equations if $D_z^r Y(z,s) = Y(z,s-r)$, where r is a positive integer.

For $F(z,s)$ satisfying ascending F-equation, Truesdell [13] and Agarwal and Saxena [4] obtained the following generating functions using Tayler's series:

$$F(z+y,s) = \sum_{n=0}^{\infty} y^n \frac{F(z,s+n)}{n!} \tag{1.6}$$

$$Y(z+y,s) = \sum_{n=0}^{\infty} y^n \frac{Y(z,s-n)}{n!} \tag{1.7}$$

In order to obtain main results of this section, we will make use of the following well known results on multiplication formulae for the Gamma functions.

$$\prod_{k=0}^{m-1}\Gamma\left(\frac{s+r+k}{m}\right) = m^{-r}(s)_r \prod_{k=0}^{m-1}\Gamma\left(\frac{s+k}{m}\right) \tag{1.8}$$

$$\prod_{k=0}^{m-1}\Gamma\left(\frac{s-r-k}{m}\right) = \frac{(-m)^{-r}}{(m-s)_r} \prod_{k=0}^{m-1}\Gamma\left(\frac{s-k}{m}\right) \tag{1.9}$$

$$\prod_{k=0}^{m-1}\Gamma\left(\frac{s-r+k}{m}\right) = \frac{(-m)^{-r}}{(m-s)_r} \prod_{k=0}^{m-1}\Gamma\left(\frac{s+k}{m}\right) \tag{1.10}$$

Ref.

3. A.K. Rathie, A new generalization of generalized hypergeometric functions, Le Mathematic he Fasc. II 52 (1997), 297-310.

$$\prod_{k=0}^{m-1}\Gamma\left(\frac{s+r-k}{m}\right)=m^{-r}(s-m+1)_r\prod_{k=0}^{m-1}\Gamma\left(\frac{s-k}{m}\right) \tag{1.11}$$

$$\{\Delta(a,s),h\}=\left(\frac{s}{a},h\right),\left(\frac{s+1}{a},h\right),\left(\frac{s+2}{a},h\right),\ldots,\left(\frac{s+a-1}{a},h\right) \tag{1.12}$$

Notes

II. Different Forms of $\bar{H}$-Function

In this section, we have different forms of $\bar{H}$-Function which satisfy Truesdell's ascending and descending F-equation.

The following forms of $\bar{H}$-Function satisfy Truesdell's ascending F-equation:

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.1}$$

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s+1/2),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(2a,2s),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.2}$$

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s+2/3),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s+1/3),h\}\\ \{\Delta(3a,3s),h\},(b_j,\beta_j)_{3a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.3}$$

$$\left(\frac{z}{a}\right)^{-(s+1)}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(2a,2s+1/2),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p-2a},\{\Delta(2a,2s+1),h\}\\ \{\Delta(4a,4s+1),h\},(b_j,\beta_j)_{4a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1),h\}\end{matrix}\right.\right] \tag{2.4}$$

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s),h\}\end{matrix}\right.\right] \tag{2.5}$$

$$\left(\frac{z}{a}\right)^{-s}2^{s}e^{i\pi s/2}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{2ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s/2),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,(s+1)/2),h\}\end{matrix}\right.\right] \tag{2.6}$$

$$\left(\frac{z}{a}\right)^{-s}e^{i\pi s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(\rho,s),h\}\ (a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s),h\}(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.7}$$

$$\left(\frac{z}{a}\right)^{-s}e^{i\pi s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.8}$$

Making use of the equation (2.1), we obtain:

$$D_z^r[A(z,s)] = D_z^r\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \quad (2.9)$$

Replacing the $\bar{H}$-Function by its definition (1.1) and then interchanging the order of integration and differentiation and (2.9) transforms to

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=0}^{a-1}\Gamma\left(\frac{s+k}{a}-h\xi\right)\prod_{j=a+1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}\left(\frac{t^{h\xi}}{a^{ha\xi-s}}\right)D_z^r(z)^{ha\xi-s}}{\prod_{j=m+1}^{q-\rho}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{k=0}^{\rho-1}\Gamma\left(1-\frac{s+k}{\rho}-h\xi\right)\prod_{j=n+1}^{p-\rho}\Gamma(a_j-\alpha_j\xi)\prod_{k=0}^{\rho-1}\Gamma\left(\frac{s+k}{\rho}-h\xi\right)}d\xi \quad (2.10)$$

Now using (1.8) and (1.9) lead to two identities:

$$\prod_{k=0}^{a-1}\Gamma\left(\frac{s+k}{a}-h\xi\right)=\frac{a^r}{(s-ha\xi)_r}\prod_{k=0}^{a-1}\Gamma\left(\frac{s+r+k}{a}-h\xi\right) \quad (2.11)$$

$$\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+k}{a}+h\xi\right)=\frac{(s-ha\xi)_r}{(-1)^r a^r}\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+r+k}{a}+h\xi\right) \quad (2.12)$$

Using these results, equation (2.10) takes the following form:

$$D_z^r[A(z,s)] = A[z,s+r]$$

Similarly, forms of $\bar{H}$-Function (2.2), (2.3), (2.4), (2.5), (2.6), (2.7) and (2.8) satisfy the Truesdell's ascending F-equation.

Also the following forms of $\bar{H}$-Function satisfy Truesdell's descending F-equation:

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \quad (2.13)$$

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(2a,2s),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+1/2),h\}\,(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (2.14)$$

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(4a,4s+1),h\},(a_j,\alpha_j;A_j)_{4a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q-2a},\{\Delta(2a,2s),h\}\end{matrix}\right.\right] \quad (2.15)$$

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(3a,3s),h\},(a_j,\alpha_j;A_j)_{3a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1/3),h\}\end{matrix}\right.\right] \quad (2.16)$$

Notes

$$\left(\frac{z}{a}\right)^{s-1} e^{i\pi s}\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.17}$$

$$\left(\frac{z}{a}\right)^{s-1} 2^{-s}e^{i\pi s/2}\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{2ha}\left|\begin{matrix}\{\Delta(a,s/2),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,(s+1)/2),h\}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.18}$$

$$\left(\frac{z}{a}\right)^{s-1}\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.19}$$

$$\left(\frac{z}{a}\right)^{s-1} e^{i\pi s}\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(\rho,s),h\},(a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\\{\Delta(\rho,s),h\}\ (b_j,\beta_j)_{\rho+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.20}$$

Making use of the equation (2.13), we obtain:

$$D_z^r[B(z,s)] = D_z^r\left(\frac{z}{a}\right)^{s-1}\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.21}$$

Replacing the $\overline{H}$-Function by its definition (1.1) and then interchanging the order of integration and differentiation and (2.21) transforms to

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+k}{a}+h\xi\right)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}\left(\frac{t^{ha\xi}}{a^{ha\xi+s-1}}\right)D_z^r(z)^{s-ha\xi-1}}{\prod_{j=m+1}^{q-\rho}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{k=0}^{\rho-1}\Gamma\left(1-\frac{s+k}{\rho}+h\xi\right)\prod_{j=n+1}^{p-\rho}\Gamma(a_j-\alpha_j\xi)\prod_{k=0}^{\rho-1}\Gamma\left(\frac{s+k}{\rho}-h\xi\right)}d\xi \tag{2.22}$$

Now using (1.10) and (1.11) lead to two identities:

$$\prod_{k=0}^{a-1}\Gamma\left(\frac{s+k}{a}-h\xi\right)=(-\lambda)^{-r}(ha\xi-s+1)_r\prod_{k=0}^{a-1}\Gamma\left(\frac{s-r+k}{a}-h\xi\right) \tag{2.23}$$

$$\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+k}{a}+h\xi\right)=\frac{a^r}{(ah\xi-s+1)_r}\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s-r+k}{a}+h\xi\right) \tag{2.24}$$

Using these results, (2.22) takes the following form:

$$D_z^r[B(z,s)] = B[z,s-r]$$

Similarly, forms of $\overline{H}$-function (2.14), (2.15), (2.16), (2.17), (2.18), (2.19) and (2.20) satisfy the Truesdell's descending F-equation.

Notes

III. Generating Functions

If $A=\left(1+\frac{h}{a}\right)$, then the generating functions obtained by employing forms (2.1) to (2.8):

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s+r),h\}\\ \{\Delta(a,s+r),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s+r),h\}\end{matrix}\right.\right] \quad (3.1)$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}\{\Delta(a,s+1/2),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(2a,2s),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}\{\Delta(a,s+r+1/2),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(2a,2s+2r),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.2)$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}\{\Delta(a,s+2/3),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s+1/3),h\}\\ \{\Delta(3a,3s),h\},(b_j,\beta_j)_{3a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}\{\Delta(a,s+r+2/3),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s+r+1/3),h\}\\ \{\Delta(3a,3s+3r),h\},(b_j,\beta_j)_{3a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.3)$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}\{\Delta(2a,2s+1/2),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p-2a},\{\Delta(2a,2s+1),h\}\\ \{\Delta(4a,4s+1),h\},(b_j,\beta_j)_{4a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}\{\Delta(2a,2s+2r+1/2),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p-2a},\{\Delta(2a,2s+2r+1),h\}\\ \{\Delta(4a,4s+4r+1),h\},(b_j,\beta_j)_{4a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+r+1),h\}\end{matrix}\right.\right] \quad (3.4)$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+r),h\}\end{matrix}\right.\right] \quad (3.5)$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{2ha}x^2\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s/2),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,(s+1)/2),h\}\end{matrix}\right.\right]$$

Notes

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$$= \sum_{r=0}^{\infty} \frac{(2h)^r (-1)^{r/2}}{r!} \bar{H}^{m,n}_{p,q} \left[x^2 \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ \{\Delta(a,(s+r)/2),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,(s+r+1)/2),h\} \end{matrix} \right. \right] \quad (3.6)$$

Notes

$$A^{-s}\bar{H}^{m,n}_{p,q} \left[A^{ha}x \left| \begin{matrix} \{\Delta(\rho,s),h\}\ (a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p} \\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} \bar{H}^{m,n}_{p,q} \left[x \left| \begin{matrix} \{\Delta(\rho,s+r),h\}\ (a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p} \\ \{\Delta(a,s+r),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s+r),h\} \end{matrix} \right. \right] \quad (3.7)$$

$$A^{-s}\bar{H}^{m,n}_{p,q} \left[A^{ha}x \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ \{\Delta(a,s),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} \bar{H}^{m,n}_{p,q} \left[x \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ \{\Delta(a,s+r),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q} \end{matrix} \right. \right] \quad (3.8)$$

To prove the above generating function, we employ the forms (2.1) to (2.8) in equation (1.6) and then on replacing z by $\frac{ya}{h}$ and $\left(\frac{yt}{h}\right)^{ha}$ by x, we get the above result from (3.1) to (3.8).

Generating functions obtained by employing forms (2.13) to (2.20):

$$A^{s-1}\bar{H}^{m,n}_{p,q} \left[A^{-ha}x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\} \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} \bar{H}^{m,n}_{p,q} \left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s-r),h\} \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s-r),h\} \end{matrix} \right. \right] \quad (3.9)$$

$$A^{s-1}\bar{H}^{m,n}_{p,q} \left[A^{-ha}x \left| \begin{matrix} \{\Delta(2a,2s),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p} \\ \{\Delta(a,s+1/2),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} \bar{H}^{m,n}_{p,q} \left[x \left| \begin{matrix} \{\Delta(2a,2s-2r),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p} \\ \{\Delta(a,s-r+1/2),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q} \end{matrix} \right. \right] \quad (3.10)$$

$$A^{s-1}\bar{H}^{m,n}_{p,q} \left[A^{-ha}x \left| \begin{matrix} \{\Delta(4a,4s+1),h\},(a_j,\alpha_j;A_j)_{4a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q-2a},\{\Delta(2a,2s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} \bar{H}^{m,n}_{p,q} \left[x \left| \begin{matrix} \{\Delta(4a,4s-4r+1),h\},(a_j,\alpha_j;A_j)_{4a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q-2a},\{\Delta(2a,2s-2r),h\} \end{matrix} \right. \right] \quad (3.11)$$

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{matrix}\{\Delta(3a,3s),h\},(a_j,\alpha_j;A_j)_{3a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1/3),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}\{\Delta(3a,3s-3r),h\},(a_j,\alpha_j;A_j)_{3a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s-r+2/3),h\}\,(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s-r+1/3),h\}\end{matrix}\right.\right] \quad (3.12)$$

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{matrix}\{\Delta(a,s),h\}\,(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}\{\Delta(a,s-r),h\}\,(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.13)$$

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-2ha}x^2\left|\begin{matrix}\{\Delta(a,s/2),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,(s+1)/2),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{(2h)^r(-1)^{r/2}}{r!}\bar{H}_{p,q}^{m,n}\left[x^2\left|\begin{matrix}\{\Delta(a,(s-r)/2),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,(s-r+1)/2),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.14)$$

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s-r),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.15)$$

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{matrix}\{\Delta(\rho,s),h\},(a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(\rho,s),h\}\,(b_j,\beta_j)_{\rho+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}\{\Delta(\rho,s-r),h\},(a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s-r),h\}\\ \{\Delta(\rho,s-r),h\}\,(b_j,\beta_j)_{\rho+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.16)$$

To prove the above generating function, we employ the forms (2.13) to (2.20) in equation (1.7) and then on replacing z by $\frac{ya}{h}$ and $\left(\frac{yt}{h}\right)^{ha}$ by x, we get the above result from (3.9) to (3.16).

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IV. Special Cases

(4.1) When $A_j = B_j = 1$, the $\bar{H}$-Function reduces to the Fox's H-function [5, p. 10, Eqn. (2.1.1)], the above results (3.1) to (3.16) reduces to the following form:

$$A^{-s} H_{p,q}^{m,n}\left[A^{ha} x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p-\rho}, \{\Delta(\rho,s),h\} \\ \{\Delta(a,s),h\}, (b_j,\beta_j)_{1,q-\rho}, \{\Delta(\rho,s),h\} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p-\rho}, \{\Delta(\rho,s+r),h\} \\ \{\Delta(a,s+r),h\}, (b_j,\beta_j)_{a+1,q-\rho}, \{\Delta(\rho,s+r),h\} \end{matrix}\right] \quad (4.1.1)$$

$$A^{-s} H_{p,q}^{m,n}\left[A^{ha} x \middle| \begin{matrix} \{\Delta(a,s+1/2),h\}, (a_j,\alpha_j)_{a+1,p} \\ \{\Delta(2a,2s),h\}, (b_j,\beta_j)_{2a+1,q} \end{matrix}\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \middle| \begin{matrix} \{\Delta(a,s+r+1/2),h\}, (a_j,\alpha_j)_{a+1,p} \\ \{\Delta(2a,2s+2r),h\}, (b_j,\beta_j)_{2a+1,q} \end{matrix}\right] \quad (4.1.2)$$

$$A^{-s} H_{p,q}^{m,n}\left[A^{ha} x \middle| \begin{matrix} \{\Delta(a,s+2/3),h\}, (a_j,\alpha_j)_{a+1,p-a}, \{\Delta(a,s+1/3),h\} \\ \{\Delta(3a,3s),h\}, (b_j,\beta_j)_{3a+1,q} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \middle| \begin{matrix} \{\Delta(a,s+r+2/3),h\}, (a_j,\alpha_j)_{a+1,p-a}, \{\Delta(a,s+r+1/3),h\} \\ \{\Delta(3a,3s+3r),h\}, (b_j,\beta_j)_{3a+1,q} \end{matrix}\right] \quad (4.1.3)$$

$$A^{-s} H_{p,q}^{m,n}\left[A^{ha} x \middle| \begin{matrix} \{\Delta(2a,2s+1/2),h\}, (a_j,\alpha_j)_{2a+1,p-2a}, \{\Delta(2a,2s+1),h\} \\ \{\Delta(4a,4s+1),h\}, (b_j,\beta_j)_{4a+1,q-a}, \{\Delta(a,s+1),h\} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \middle| \begin{matrix} \{\Delta(2a,2s+2r+1/2),h\}, (a_j,\alpha_j)_{2a+1,p-2a}, \{\Delta(2a,2s+2r+1),h\} \\ \{\Delta(4a,4s+4r+1),h\}, (b_j,\beta_j)_{4a+1,q-a}, \{\Delta(a,s+r+1),h\} \end{matrix}\right] \quad (4.1.4)$$

$$A^{-s} H_{p,q}^{m,n}\left[A^{ha} x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q-a}, \{\Delta(a,s),h\} \end{matrix}\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q-a}, \{\Delta(a,s+r),h\} \end{matrix}\right] \quad (4.1.5)$$

$$A^{-s} H_{p,q}^{m,n}\left[A^{2ha} x^2 \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ \{\Delta(a,s/2),h\}, (b_j,\beta_j)_{a+1,q-a}, \{\Delta(a,(s+1)/2),h\} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r (-1)^{r/2}}{r!} H_{p,q}^{m,n}\left[x^2 \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ \{\Delta(a,(s+r)/2),h\}, (b_j,\beta_j)_{a+1,q-a}, \{\Delta(a,(s+r+1)/2),h\} \end{matrix}\right] \quad (4.1.6)$$

$$A^{-s} H_{p,q}^{m,n}\left[A^{ha} x \middle| \begin{matrix} \{\Delta(\rho,s),h\}, (a_j,\alpha_j)_{\rho+1,p} \\ \{\Delta(a,s),h\}, (b_j,\beta_j)_{a+1,q-\rho}, \{\Delta(\rho,s),h\} \end{matrix}\right]$$

Ref.

5. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publishers, New Dehli, Madras (1982).

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(\rho,s+r),h\},(a_j,\alpha_j)_{\rho+1,p} \\ \{\Delta(a,s+r),h\},(b_j,\beta_j)_{a+1,q-\rho},\{\Delta(\rho,s+r),h\} \end{matrix} \right. \right] \quad (4.1.7)$$

$$A^{-s} H_{p,q}^{m,n}\left[A^{ha}x \left| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} (a_j,\alpha_j)_{1,q} \\ \{\Delta(a,s+r),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right. \right] \quad (4.1.8)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,\alpha_j)_{a+1,p-\rho},\{\Delta(\rho,s),h\} \\ (b_j,\beta_j)_{1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,\alpha_j)_{a+1,p-\rho},\{\Delta(\rho,s-r),h\} \\ (b_j,\beta_j)_{1,q-\rho},\{\Delta(\rho,s-r),h\} \end{matrix} \right. \right] \quad (4.1.9)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(2a,2s),h\},(a_j,\alpha_j)_{2a+1,p} \\ \{\Delta(a,s+1/2),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(2a,2s-2r),h\},(a_j,\alpha_j)_{2a+1,p} \\ \{\Delta(a,s-r+1/2),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right. \right] \quad (4.1.10)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(4a,4s+1),h\},(a_j,\alpha_j)_{4a+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j)_{2a+1,q-2a},\{\Delta(2a,2s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(4a,4s-4r+1),h\},(a_j,\alpha_j)_{4a+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,\beta_j)_{2a+1,q-2a},\{\Delta(2a,2s-2r),h\} \end{matrix} \right. \right] \quad (4.1.11)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(3a,3s),h\},(a_j,\alpha_j)_{3a+1,p} \\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j)_{a+1,q-a},\{\Delta(a,s+1/3),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(3a,3s-3r),h\},(a_j,\alpha_j)_{3a+1,p} \\ \{\Delta(a,s-r+2/3),h\},(b_j,\beta_j)_{a+1,q-a},\{\Delta(a,s-r+1/3),h\} \end{matrix} \right. \right] \quad (4.1.12)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,\alpha_j)_{a+1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,\alpha_j)_{a+1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] \quad (4.1.13)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-2ha}x^2 \left| \begin{matrix} \{\Delta(a,s/2),h\},(a_j,\alpha_j)_{a+1,p-a},\{\Delta(a,(s+1)/2),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right]$$

Notes

$$= \sum_{r=0}^{\infty} \frac{(2h)^r (-1)^{r/2}}{r!} H_{p,q}^{m,n} \left[x^2 \left| \begin{matrix} \{\Delta(a,(s-r)/2),h\},(a_j,\alpha_j)_{a+1,p-a},\{\Delta(a,(s-r+1)/2),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] \quad (4.1.14)$$

$$A^{s-1} H_{p,q}^{m,n} \left[A^{-ha} x \left| \begin{matrix} (a_j,\alpha_j)_{1,p-a},\{\Delta(a,s),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n} \left[x \left| \begin{matrix} (a_j,\alpha_j)_{1,p-a},\{\Delta(a,s-r),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] \quad (4.1.15)$$

$$A^{s-1} H_{p,q}^{m,n} \left[A^{-ha} x \left| \begin{matrix} \{\Delta(\rho,s),h\},(a_j,\alpha_j)_{\rho+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(\rho,s),h\},(b_j,\beta_j)_{\rho+1,q} \end{matrix} \right. \right]$$
$$= \sum_{r=0}^{\infty} \frac{h^r (-1)^r}{r!} H_{p,q}^{m,n} \left[x \left| \begin{matrix} \{\Delta(\rho,s-r),h\},(a_j,\alpha_j)_{\rho+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(\rho,s-r),h\},(b_j,\beta_j)_{\rho+1,q} \end{matrix} \right. \right] \quad (4.1.16)$$

(4.2) If we put $A_j = B_j = 1; \alpha_j = \beta_j = 1$, then the $\bar{H}$-function reduces to general type of G-function [8] i.e. $\bar{H}_{p,q}^{m,n} \left[z \left| \begin{matrix} (a_j,1,1)_{1,n},(a_j,1)_{n+1,p} \\ (b_j,1,1)_{1,m},(b_j,1)_{m+1,q} \end{matrix} \right. \right] = G \left[z \left| \begin{matrix} (a_j,1)_{1,p} \\ (b_j,1)_{1,q} \end{matrix} \right. \right]$, the above results (3.1) to (3.16) reduces to the following form:

$$A^{-s} G_{p,q}^{m,n} \left[A^{ha} x \left| \begin{matrix} (a_j,1)_{1,p-\rho},\{\Delta(\rho,s),h\} \\ \{\Delta(a,s),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$
$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n} \left[x \left| \begin{matrix} (a_j,1)_{1,p-\rho},\{\Delta(\rho,s+r),h\} \\ \{\Delta(a,s+r),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s+r),h\} \end{matrix} \right. \right] \quad (4.2.1)$$

$$A^{-s} G_{p,q}^{m,n} \left[A^{ha} x \left| \begin{matrix} \{\Delta(a,s+1/2),h\},(a_j,1)_{a+1,n+1,p} \\ \{\Delta(2a,2s),h\},(b_j,1)_{2a+1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n} \left[x \left| \begin{matrix} \{\Delta(a,s+r+1/2),h\},(a_j,1)_{a+1,p} \\ \{\Delta(2a,2s+2r),h\},(b_j,1)_{2a+1,q} \end{matrix} \right. \right] \quad (4.2.2)$$

$$A^{-s} G_{p,q}^{m,n} \left[A^{ha} x \left| \begin{matrix} \{\Delta(a,s+2/3),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,s+1/3),h\} \\ \{\Delta(3a,3s),h\},(b_j,1)_{3a+1,q} \end{matrix} \right. \right]$$
$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n} \left[x \left| \begin{matrix} \{\Delta(a,s+r+2/3),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,s+r+1/3),h\} \\ \{\Delta(3a,3s+3r),h\},(b_j,1)_{3a+1,q} \end{matrix} \right. \right] \quad (4.2.3)$$

$$A^{-s} G_{p,q}^{m,n} \left[A^{ha} x \left| \begin{matrix} \{\Delta(2a,2s+1/2),h\},(a_j,1)_{2a+1,p-2a},\{\Delta(2a,2s+1),h\} \\ \{\Delta(4a,4s+1),h\},(b_j,1)_{4a+1,q-a},\{\Delta(a,s+1),h\} \end{matrix} \right. \right] \quad (4.2.4)$$

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$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(2a,2s+2r+1/2),h\},(a_j,1)_{2a+1,p-2a},\{\Delta(2a,2s+2r+1),h\} \\ \{\Delta(4a,4s+4r+1),h\},(b_j,1)_{4a+1,q-a},\{\Delta(a,s+r+1),h\} \end{matrix} \right. \right]$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{ha}x \left| \begin{matrix} (a_j,1)_{1,p} \\ (b_j,1)_{1,q-a},\{\Delta(a,s),h\} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} (a_j,1)_{1,p} \\ (b_j,1)_{1,q-a},\{\Delta(a,s+r),h\} \end{matrix} \right. \right] \tag{4.2.5}$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{2ha}x^2 \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,s/2),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,(s+1)/2),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r(-1)^{r/2}}{r!} G^{m,n}_{p,q}\left[x^2 \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,(s+r)/2),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,(s+r+1)/2),h\} \end{matrix} \right. \right] \tag{4.2.6}$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{ha}x \left| \begin{matrix} \{\Delta(\rho,s),h\},(a_j,1)_{\rho+1,p} \\ \{\Delta(a,s),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(\rho,s+r),h\},(a_j,1)_{\rho+1,p} \\ \{\Delta(a,s+r),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s+r),h\} \end{matrix} \right. \right] \tag{4.2.7}$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{ha}x \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,s),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,s+r),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] \tag{4.2.8}$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,1)_{a+1,p-\rho},\{\Delta(\rho,s),h\} \\ (b_j,1)_{1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,1)_{a+1,p-\rho},\{\Delta(\rho,s-r),h\} \\ (b_j,1)_{1,q-\rho},\{\Delta(\rho,s-r),h\} \end{matrix} \right. \right] \tag{4.2.9}$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(2a,2s),h\},(a_j,1)_{2a+1,p} \\ \{\Delta(a,s+1/2),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(2a,2s-2r),h\},(a_j,1)_{2a+1,p} \\ \{\Delta(a,s-r+1/2),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] \tag{4.2.10}$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(4a,4s+1),h\},(a_j,1)_{4a+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(2a,2s+1/2),h\},(b_j,1)_{2a+1,q-2a},\{\Delta(2a,2s),h\} \end{matrix} \right. \right]$$

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$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(4a,4s-4r+1),h\},(a_j,1)_{4a+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,1)_{2a+1,q-2a},\{\Delta(2a,2s-2r),h\} \end{matrix} \right. \right] \quad (4.2.11)$$

$$A^{s-1} G_{p,q}^{m,n}\left[A^{-ha} x \left| \begin{matrix} \{\Delta(3a,3s),h\},(a_j,1)_{3a+1,p} \\ \{\Delta(a,s+2/3),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,s+1/3),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(3a,3s-3r),h\},(a_j,1)_{3a+1,p} \\ \{\Delta(a,s-r+2/3),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,s-r+1/3),h\} \end{matrix} \right. \right] \quad (4.2.12)$$

$$A^{s-1} G_{p,q}^{m,n}\left[A^{-ha} x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,1)_{a+1,p} \\ (b_j,1)_{1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,1)_{a+1,p} \\ (b_j,1)_{1,q} \end{matrix} \right. \right] \quad (4.2.13)$$

$$A^{s-1} G_{p,q}^{m,n}\left[A^{-2ha} x^2 \left| \begin{matrix} \{\Delta(a,s/2),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,(s+1)/2),h\} \\ (b_j,1)_{1,q} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r(-1)^{r/2}}{r!} G_{p,q}^{m,n}\left[x^2 \left| \begin{matrix} \{\Delta(a,(s-r)/2),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,(s-r+1)/2),h\} \\ (b_j,1)_{1,q} \end{matrix} \right. \right] \quad (4.2.14)$$

$$A^{s-1} G_{p,q}^{m,n}\left[A^{-ha} x \left| \begin{matrix} (a_j,1)_{1,p-a},\{\Delta(a,s),h\} \\ (b_j,1)_{1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} (a_j,1)_{1,p-a},\{\Delta(a,s-r),h\} \\ (b_j,1)_{1,q} \end{matrix} \right. \right] \quad (4.2.15)$$

$$A^{s-1} G_{p,q}^{m,n}\left[A^{-ha} x \left| \begin{matrix} \{\Delta(\rho,s),h\},(a_j,1)_{\rho+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(\rho,s),h\},(b_j,1)_{\rho+1,q} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(\rho,s-r),h\},(a_j,1)_{\rho+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(\rho,s-r),h\},(b_j,1)_{\rho+1,q} \end{matrix} \right. \right] \quad (4.2.16)$$

(4.3) If we put $n=p, m=1, q=q+1, b_1=0, \beta_1=1, a_j=1-a_j, b_j=1-b_j$, then the $\overline{H}$-function reduces to generalized wright hypergeometric function [12] i.e. $\overline{H}_{p,q+1}^{1,p}\left[z \left| \begin{matrix} (1-a_j,\alpha_j;A_j)_{1,p} \\ (0,1),(1-b_j,\beta_j;B_j)_{1,q} \end{matrix} \right. \right] = {}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ;-z \right]$, the above results (3.1) to (3.16) reduces to the following form:

$$A^{-s}\,{}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p-\rho}\,\{\Delta(\rho,s),h\} \\ \{\Delta(a,s),h\},(b_j,\beta_j;B_j)_{a+1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} ;-A^{ha}x \right]$$

Ref.

12. R.G. Buschman and H.M. Srivastava, The H-function associated with a certain class of Feynman integrals, J.Phys.A:Math.Gen. 23(1990), 4707-4710.

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p-\rho}, \{\Delta(\rho,s+r),h\} \\ \{\Delta(a,s+r),h\}, (b_j,\beta_j;B_j)_{a+1,q-\rho}, \{\Delta(\rho,s+r),h\} \end{matrix} ;-x \right] \quad (4.3.1)$$

$$A^{-s} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+1/2),h\}, (a_j,\alpha_j;A_j)_{a+1,p} \\ \{\Delta(2a,2s),h\}, (b_j,\beta_j;B_j)_{2a+1,q} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+r+1/2),h\}, (a_j,\alpha_j;A_j)_{a+1,p} \\ \{\Delta(2a,2s+2r),h\}, (b_j,\beta_j;B_j)_{2a+1,q} \end{matrix} ;-x \right] \quad (4.3.2)$$

$$A^{-s} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+2/3),h\}, (a_j,\alpha_j;A_j)_{a+1,p-a}, \{\Delta(a,s+1/3),h\} \\ \{\Delta(3a,3s),h\}, (b_j,\beta_j;B_j)_{3a+1,q} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+r+2/3),h\}, (a_j,\alpha_j;A_j)_{a+1,p-a}, \{\Delta(a,s+r+1/3),h\} \\ \{\Delta(3a,3s+3r),h\}, (b_j,\beta_j;B_j)_{3a+1,q} \end{matrix} ;-x \right] \quad (4.3.3)$$

$$A^{-s} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(2a,2s+1/2),h\}, (a_j,\alpha_j;A_j)_{2a+1,p-2a}, \{\Delta(2a,2s+1),h\} \\ \{\Delta(4a,4s+1),h\}, (b_j,\beta_j;B_j)_{4a+1,q-a}, \{\Delta(a,s+1),h\} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(2a,2s+2r+1/2),h\}, (a_j,\alpha_j;A_j)_{2a+1,p-2a}, \{\Delta(2a,2s+2r+1),h\} \\ \{\Delta(4a,4s+4r+1),h\}, (b_j,\beta_j;B_j)_{4a+1,q-a}, \{\Delta(a,s+r+1),h\} \end{matrix} ;-x \right] \quad (4.3.4)$$

$$A^{-s} {}_p\overline{\Psi}_q \left[\left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q-a}, \{\Delta(a,s),h\} \end{matrix} ;-A^{ha}x \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q-a}, \{\Delta(a,s+r),h\} \end{matrix} ;-x \right] \quad (4.3.5)$$

$$A^{-s} {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ \{\Delta(a,s/2),h\}, (b_j,\beta_j;B_j)_{a+1,q-a}, \{\Delta(a,(s+1)/2),h\} \end{matrix} ;-A^{2ha}x^2 \right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r(-1)^{r/2}}{r!} {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ \{\Delta(a,(s+r)/2),h\}, (b_j,\beta_j;B_j)_{a+1,q-a}, \{\Delta(a,(s+r+1)/2),h\} \end{matrix} ;-x^2 \right] \quad (4.3.6)$$

$$A^{-s} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(\rho,s),h\}, (a_j,\alpha_j;A_j)_{\rho+1,p} \\ \{\Delta(a,s),h\}, (b_j,\beta_j;B_j)_{a+1,q-\rho}, \{\Delta(\rho,s),h\} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(\rho,s+r),h\}, (a_j,\alpha_j;A_j)_{\rho+1,p} \\ \{\Delta(a,s+r),h\}, (b_j,\beta_j;B_j)_{a+1,q-\rho}, \{\Delta(\rho,s+r),h\} \end{matrix} ;-x \right] \quad (4.3.7)$$

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$$A^{-s}\,{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\ \{\Delta(a,s),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-A^{ha}x\right]=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,q}\\ \{\Delta(a,s+r),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-x\right] \tag{4.3.8}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,p-\rho},\{\Delta(\rho,s),h\}\\ (b_j,\beta_j;B_j)_{1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s-r),h\},(a_j,\alpha_j;A_j)_{a+1,p-\rho},\{\Delta(\rho,s-r),h\}\\ (b_j,\beta_j;B_j)_{1,q-\rho},\{\Delta(\rho,s-r),h\}\end{matrix};-x\right] \tag{4.3.9}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(2a,2s),h\},(a_j,\alpha_j;A_j)_{2a+1,p}\\ \{\Delta(a,s+1/2),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(2a,2s-2r),h\},(a_j,\alpha_j;A_j)_{2a+1,p}\\ \{\Delta(a,s-r+1/2),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-x\right] \tag{4.3.10}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(4a,4s+1),h\},(a_j,\alpha_j;A_j)_{4a+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j;B_j)_{2a+1,q-2a},\{\Delta(2a,2s),h\}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(4a,4s-4r+1),h\},(a_j,\alpha_j;A_j)_{4a+1,p-a},\{\Delta(a,s-r),h\}\\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,\beta_j;B_j)_{2a+1,q-2a},\{\Delta(2a,2s-2r),h\}\end{matrix};-x\right] \tag{4.3.11}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(3a,3s),h\},(a_j,\alpha_j;A_j)_{3a+1,p}\\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j;B_j)_{a+1,q-a},\{\Delta(a,s+1/3),h\}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(3a,3s-3r),h\},(a_j,\alpha_j;A_j)_{3a+1,p}\\ \{\Delta(a,s-r+2/3),h\},(b_j,\beta_j;B_j)_{a+1,q-a},\{\Delta(a,s-r+1/3),h\}\end{matrix};-x\right] \tag{4.3.12}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,p}\\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s-r),h\},(a_j,\alpha_j;A_j)_{a+1,p}\\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};-x\right] \tag{4.3.13}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s/2),h\},(a_j,\alpha_j;A_j)_{a+1,p-a},\{\Delta(a,(s+1)/2),h\}\\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};-A^{-2ha}x^2\right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r (-1)^{r/2}}{r!} \, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,(s-r)/2),h\}, (a_j,\alpha_j;A_j)_{a+1,p-a}, \{\Delta(a,(s-r+1)/2),h\} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -x^2 \right] \tag{4.3.14}$$

$$A^{s-1} \, {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p-a}, \{\Delta(a,s),h\} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -A^{-ha}x \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} \, {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p-a}, \{\Delta(a,s-r),h\} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -x \right] \tag{4.3.15}$$

$$A^{s-1} \, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(\rho,s),h\}, (a_j,\alpha_j;A_j)_{\rho+1,p-a}, \{\Delta(a,s),h\} \\ \{\Delta(\rho,s),h\}, (b_j,\beta_j;B_j)_{\rho+1,q} \end{matrix} ; -A^{-ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r (-1)^r}{r!} \, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(\rho,s-r),h\}, (a_j,\alpha_j;A_j)_{\rho+1,p-a}, \{\Delta(a,s-r),h\} \\ \{\Delta(\rho,s-r),h\}, (b_j,\beta_j;B_j)_{\rho+1,q} \end{matrix} ; -x \right] \tag{4.3.16}$$

In all the above results, it is assumed that all the parameters satisfy the conditions necessary for the existence of the $\overline{H}$-function involved.

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