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New Representations in Terms of q-product Identities for Ramanujan's Results IV

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GJSFR-F Classification : *AMS Subject Classifications: Primary 05A17, 05A15; Secondary 11P83*

NEW REPRESENTATIONS IN TERMS OF Q-PRODUCT IDENTITIES FOR RAMANUJANS RESULTS IV

Strictly as per the compliance and regulations of :

Ref.

9. B.C. Berndt; *Ramanujan's Notebooks, Part III*, Springer-Verlag, New York, 1991.

New Representations in Terms of q-product Identities for Ramanujan's Results IV

M.P. Chaudhary [α], Upendra Kumar Pandit [σ] & Ashish Arora [ρ]

Abstract - In this paper author has established seven q-product identities, which are presumably new, and not available in the literature.

Keywords : Theta functions, functions, triple product identities.

I. Introduction

For $|q| < 1$,

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1 - aq^n) \tag{1.1}$$

$$(a;q)_\infty = \prod_{n=1}^{\infty}(1 - aq^{(n-1)}) \tag{1.2}$$

$$(a_1, a_2, a_3, ..., a_k; q)_\infty = (a_1;q)_\infty(a_2;q)_\infty(a_3;q)_\infty...(a_k;q)_\infty \tag{1.3}$$

Ramanujan has defined general theta function, as

$$f(a,b) = \sum_{-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} \quad ; \quad |ab| < 1, \tag{1.4}$$

Jacobi's triple product identity [9,p.35] is given, as

$$f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty \tag{1.5}$$

Special cases of Jacobi's triple products identity are given, as

$$\Phi(q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)_\infty^2(q^2;q^2)_\infty \tag{1.6}$$

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$$\Psi(q) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.7}$$

$$f(-q) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.8}$$

Equation (1.8) is known as Euler's pentagonal number theorem. Euler's another well known identity is as

$$(q;q^2)_\infty^{-1} = (-q;q)_\infty \tag{1.9}$$

Roger-Ramanujan identities [6, p.578] are given as

$$G(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n} = \frac{1}{(q;q^5)_\infty (q^4;q^5)_\infty} = \frac{(q^2;q^5)_\infty (q^3;q^5)_\infty (q^5;q^5)_\infty}{(q;q)_\infty} \tag{1.10}$$

$$H(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q;q)_n} = \frac{1}{(q^2;q^5)_\infty (q^3;q^5)_\infty} = \frac{(q;q^5)_\infty (q^4;q^5)_\infty (q^5;q^5)_\infty}{(q;q)_\infty} \tag{1.11}$$

Roger-Ramanujan function is given by

$$R(q) = q^{\frac{1}{5}} \frac{H(q)}{G(q)} = q^{\frac{1}{5}} \frac{(q;q^5)_\infty (q^4;q^5)_\infty}{(q^2;q^5)_\infty (q^3;q^5)_\infty} \tag{1.12}$$

Throughout this paper we use the following representations

$$(q^a;q^n)_\infty (q^b;q^n)_\infty (q^c;q^n)_\infty \cdots (q^t;q^n)_\infty = (q^a,q^b,q^c \cdots q^t;q^n)_\infty \tag{1.13}$$

$$(q^a;q^n)_\infty (q^a;q^n)_\infty (q^c;q^n)_\infty \cdots (q^t;q^n)_\infty = (q^a,q^a,q^c \cdots q^t;q^n)_\infty \tag{1.14}$$

Now we can have following q-products identities, as

$$(q^2;q^2)_\infty = \prod_{n=0}^{\infty} (1-q^{2n+2})$$

$$= \prod_{n=0}^{\infty} (1-q^{2(4n)+2}) \times \prod_{n=0}^{\infty} (1-q^{2(4n+1)+2}) \times \prod_{n=0}^{\infty} (1-q^{2(4n+2)+2}) \times \prod_{n=0}^{\infty} (1-q^{2(4n+3)+2})$$

$$= \prod_{n=0}^{\infty} (1-q^{8n+2}) \times \prod_{n=0}^{\infty} (1-q^{8n+4}) \times \prod_{n=0}^{\infty} (1-q^{8n+6}) \times \prod_{n=0}^{\infty} (1-q^{8n+8})$$

$$= (q^2;q^8)_\infty (q^4;q^8)_\infty (q^6;q^8)_\infty (q^8;q^8)_\infty = (q^2,q^4,q^6,q^8;q^8)_\infty \tag{1.15}$$

Ref.

6. G.E. Andrews, R. Askey and R. Roy; *Special Functions*, Cambridge University Press, Cambridge, 1999.

Notes

$$(q^4;q^4)_\infty = \prod_{n=0}^{\infty}(1-q^{4n+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{4(3n)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+1)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+2)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{12n+4}) \times \prod_{n=0}^{\infty}(1-q^{12n+8}) \times \prod_{n=0}^{\infty}(1-q^{12n+12})$$

$$= (q^4;q^{12})_\infty(q^8;q^{12})_\infty(q^{12};q^{12})_\infty = (q^4,q^8,q^{12};q^{12})_\infty \tag{1.16}$$

$$(q^4;q^{12})_\infty = \prod_{n=0}^{\infty}(1-q^{12n+4}) = \prod_{n=0}^{\infty}(1-q^{12(5n)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+1)+4}) \times$$

$$\times \prod_{n=0}^{\infty}(1-q^{12(5n+2)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+3)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+4)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{60n+4}) \times \prod_{n=0}^{\infty}(1-q^{60n+16}) \times \prod_{n=0}^{\infty}(1-q^{60n+28}) \times \prod_{n=0}^{\infty}(1-q^{60n+40}) \times \prod_{n=0}^{\infty}(1-q^{60n+52})$$

$$= (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty = (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \tag{1.17}$$

Similarly we can compute following as

$$(q^4;q^{12})_\infty = (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$

$$= (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \tag{1.18}$$

$$(q^6;q^6)_\infty = (q^6;q^{24})_\infty(q^{12};q^{24})_\infty(q^{18};q^{24})_\infty(q^{24};q^{24})_\infty = (q^6,q^{12},q^{18},q^{24};q^{24})_\infty \tag{1.19}$$

$$(q^6;q^{12})_\infty = (q^6;q^{60})_\infty(q^{18};q^{60})_\infty(q^{30};q^{60})_\infty(q^{42};q^{60})_\infty(q^{54};q^{60})_\infty$$

$$= (q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty \tag{1.20}$$

$$(q^8;q^8)_\infty = (q^8;q^{48})_\infty(q^{16};q^{48})_\infty(q^{24};q^{48})_\infty(q^{32};q^{48})_\infty(q^{40};q^{48})_\infty(q^{48};q^{48})_\infty$$

$$= (q^8,q^{16},q^{24},q^{32},q^{40},q^{48};q^{48})_\infty \tag{1.21}$$

$$(q^8;q^{12})_\infty = (q^8;q^{60})_\infty(q^{20};q^{60})_\infty(q^{32};q^{60})_\infty(q^{44};q^{60})_\infty(q^{56};q^{60})_\infty$$

$$= (q^8,q^{20},q^{32},q^{44},q^{56};q^{60})_\infty \tag{1.22}$$

$$(q^8;q^{16})_\infty = (q^8;q^{48})_\infty(q^{24};q^{48})_\infty(q^{40};q^{48})_\infty = (q^8,q^{24},q^{40};q^{48})_\infty \tag{1.23}$$

$$(q^{10};q^{20})_\infty = (q^{10};q^{60})_\infty(q^{30};q^{60})_\infty(q^{50};q^{60})_\infty = (q^{10},q^{30},q^{50};q^{60})_\infty \tag{1.24}$$

$$(q^{12};q^{12})_\infty = (q^{12};q^{60})_\infty(q^{24};q^{60})_\infty(q^{36};q^{60})_\infty(q^{48};q^{60})_\infty(q^{60};q^{60})_\infty$$

$$= (q^{12},q^{24},q^{36},q^{48},q^{60};q^{60})_\infty \tag{1.25}$$

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$$(q^{16};q^{16})_\infty = (q^{16};q^{48})_\infty(q^{32};q^{48})_\infty(q^{48};q^{48})_\infty = (q^{16},q^{32},q^{48};q^{48})_\infty \tag{1.26}$$

$$(q^{20};q^{20})_\infty = (q^{20};q^{60})_\infty(q^{40};q^{60})_\infty(q^{60};q^{60})_\infty = (q^{20},q^{40},q^{60};q^{60})_\infty \tag{1.27}$$

The outline of this paper is as follows. In sections 2, we have recorded some well known results, those are useful to the rest of the paper. In section 3, we state and prove seven new q-product identities, which are not available in the literature of special functions.

II. Preliminaries

Let us recall the definition of cubic theta functions $A(q)$,$B(q)$ and $C(q)$ due to Borwein et al.[4], as

$$A(q) = \sum_{m,n=-\infty}^{\infty} q^{m^2+mn+n^2} \tag{2.1}$$

$$B(q) = \sum_{m,n=-\infty}^{\infty} \omega^{m-n} q^{m^2+mn+n^2}; \quad \omega = exp\left(\frac{2\pi\imath}{3}\right) \tag{2.2}$$

$$C(q) = \sum_{m,n=-\infty}^{\infty} q^{m^2+mn+n^2+m+n} \tag{2.3}$$

Borwein et al.[4] established the following relations

$$A(q) = A(q^3) + 2qC(q^3) \tag{2.4}$$

$$B(q) = A(q^3) - qC(q^3) \tag{2.5}$$

$$C(q) = \frac{3(q^3;q^3)^3_\infty}{(q;q)_\infty} \tag{2.6}$$

$$A(q)A(q^2) = B(q)B(q^2) + qC(q)C(q^2) \tag{2.7}$$

Entry-2, in Ramanujan's first note book [8, p.230], [10, p.356] is stated as

$$\Psi(q)\Psi(q^3) - \Psi(-q)\Psi(-q^3) = 2q\Phi(q^2)\Psi(q^{12}) \tag{2.8}$$

Entry-4(iv), in the chapter 20 of Ramanujan's second note book [8], [9, p.359] is stated as

$$\Phi(q)\Phi(q^{27}) - \Phi(-q)\Phi(-q^{27}) = 4qf(-q^6)f(-q^{18}) + 4q^7\Psi(q^2)\Psi(q^{54}) \tag{2.9}$$

Entry-9(i), in the chapter 20 of Ramanujan's second note book [8], [9, p.277] is stated as

$$\Psi(q^3)\Psi(q^5) - \Psi(-q^3)\Psi(-q^5) = 2q^3\Psi(q^2)\Psi(q^{30}) \tag{2.10}$$

Ref.

4. J.M. Borwein, P.B. Borwein and F.G.Garvan;*Some cubic modular identities of Ramanujan*, trans.Amer.Math.Soc.,343(1994),35-47.

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Entry-9(iii), in the chapter 20 of Ramanujan's second note book [8], [9, p.377] is stated as

$$\Phi(q^3)\Phi(q^5) = \Phi(-q^2)\Phi(-Q^2) + 2q^2\Psi(q)\Psi(Q); \quad where\ Q = q^{15} \tag{2.11}$$

Entry-9(iv), in the chapter 20 of Ramanujan's second note book [8], [9, p.377] is stated as

$$\Psi(q)\Psi(q^{15}) + \Psi(-q)\Psi(-q^{15}) = 2\Psi(q^6)\Psi(q^{10}) \tag{2.12}$$

Entry-25, in Ramanujan's note book [9, p.39] is stated as

$$\Phi(q) + \Phi(-q) = 2\Phi(q^4) \tag{2.13}$$

$$\Phi(q) - \Phi(-q) = 4q\Psi(q^8) \tag{2.14}$$

$$\Phi(q)\Phi(-q) = \Phi(-q^2) \tag{2.15}$$

III. Main Results

We have establish following

$$(q^2, q^4, q^6; q^8)_\infty[(-q; q^2)^2_\infty + (q; q^2)^2_\infty] = 2(-q^4; q^8)^2_\infty \tag{3.1}$$

$$(q^2, q^4, q^6, q^8; q^8)_\infty[(-q; q^2)^2_\infty - (q; q^2)^2_\infty] = 4q\frac{(q^{16}, q^{32}, q^{48}; q^{48})_\infty}{(q^8, q^{24}, q^{40}; q^{48})_\infty} \tag{3.2}$$

$$\frac{(-q; q^2)^2_\infty + (q; q^2)^2_\infty}{(-q; q^2)^2_\infty - (q; q^2)^2_\infty} = \frac{(-q^4; q^8)^2_\infty(q^8, q^8, q^{24}, q^{24}, q^{40}, q^{40}; q^{48})_\infty}{2q} \tag{3.3}$$

$$(-q; q^2)^2_\infty(q; q^2)^2_\infty(q^2; q^2)^2_\infty = (q^2, q^2, q^4; q^4)_\infty \tag{3.4}$$

$$\frac{(-q; q^2)_\infty(-q^3; q^6)_\infty - (q; q^2)_\infty(q^3; q^6)_\infty}{(-q; q^2)_\infty \times (-q^3; q^6)_\infty \times (q; q^2)_\infty \times (q^3; q^6)_\infty} = \frac{2q(-q^2; q^4)^2_\infty(q^4, q^8, q^{16}, q^{20}, q^{24}; q^{24})_\infty}{(q^2, q^4, q^6, q^8; q^8)_\infty(q^6, q^{12}, q^{18}; q^{24})_\infty} \tag{3.5}$$

$$\frac{(-q^3; q^6)_\infty(-q^5; q^{10})_\infty - (q^3; q^6)_\infty(q^5; q^{10})_\infty}{(-q^3; q^6)_\infty \times (-q^5; q^{10})_\infty \times (q^3; q^6)_\infty \times (q^5; q^{10})_\infty} = \frac{(q^4, q^8, q^{12}; q^{12})_\infty}{(q^6, q^{12}, q^{18}, q^{24}; q^{24})_\infty} \times$$

$$\times \frac{2q^3}{(q^2, q^6, q^{10}; q^{12})_\infty(q^{10}, q^{20}, q^{30}, q^{30}, q^{40}, q^{50}; q^{60})_\infty} \tag{3.6}$$

$$\frac{[(q; q^2)_\infty(q^{15}; q^{30})_\infty] + [(-q; q^2)_\infty(-q^{15}; q^{30})_\infty]}{[(q; q^2)_\infty(q^{15}; q^{30})_\infty][(-q; q^2)_\infty(-q^{15}; q^{30})_\infty]} = \frac{(q^{12}, q^{20}, q^{24}, q^{36}, q^{40}, q^{48}, q^{60}, q^{60}; q^{60})_\infty}{(q^{10}, q^{30}, q^{30}, q^{50}, q^{60}; q^{60})_\infty} \times$$

$$\times \frac{2}{(q^2, q^4, q^6, q^8, q^8; q^8)_\infty(q^6, q^{18}, q^{30}, q^{42}, q^{54}; q^{60})_\infty} \tag{3.7}$$

Proof of (3.1): Employing equation (1.6) in equation (2.13), we have

$$(-q; q^2)^2_\infty(q^2; q^2)_\infty + (q; q^2)^2_\infty(q^2; q^2)_\infty = 2(-q^4; q^8)^2_\infty(q^8; q^8)_\infty$$

$$(q^2;q^2)_\infty[(-q;q^2)_\infty^2+(q;q^2)_\infty^2]=2(-q^4;q^8)_\infty^2(q^8;q^8)_\infty$$
$$(q^2;q^8)_\infty(q^4;q^8)_\infty(q^6;q^8)_\infty(q^8;q^8)_\infty[(-q;q^2)_\infty^2+(q;q^2)_\infty^2]=2(-q^4;q^8)_\infty^2(q^8;q^8)_\infty$$
$$(q^2;q^8)_\infty(q^4;q^8)_\infty(q^6;q^8)_\infty[(-q;q^2)_\infty^2+(q;q^2)_\infty^2]=2(-q^4;q^8)_\infty^2$$
$$(q^2,q^4,q^6;q^8)_\infty[(-q;q^2)_\infty^2+(q;q^2)_\infty^2]=2(-q^4;q^8)_\infty^2$$

which establish the result (3.1).
Proof of (3.2): Employing equations (1.6) and (1.7) in equation (2.14), we have

Notes

$$(-q;q^2)_\infty^2(q^2;q^2)_\infty-(q;q^2)_\infty^2(q^2;q^2)_\infty=\frac{4q(q^{16};q^{16})_\infty}{(q^8;q^{16})_\infty}$$
$$(q^2;q^2)_\infty[(-q;q^2)_\infty^2-(q;q^2)_\infty^2]=4q\frac{(q^{16},q^{32},q^{48};q^{48})_\infty}{(q^8,q^{24},q^{40};q^{48})_\infty}$$
$$(q^2,q^4,q^6,q^8;q^8)_\infty[(-q;q^2)_\infty^2-(q;q^2)_\infty^2]=4q\frac{(q^{16},q^{32},q^{48};q^{48})_\infty}{(q^8,q^{24},q^{40};q^{48})_\infty}$$

which establish the result (3.2).
Proof of (3.3): Dividing equation (3.1) by (3.2), we get equation (3.3).
Proof of (3.4): Employing equation (1.6) in equation (2.15), we have

$$(-q;q^2)_\infty^2(q^2;q^2)_\infty(q;q^2)_\infty^2(q^2;q^2)_\infty=(q^2;q^4)_\infty^2(q^4;q^4)_\infty$$
$$(-q;q^2)_\infty^2(q;q^2)_\infty^2(q^2;q^2)_\infty^2=(q^2;q^4)_\infty^2(q^4;q^4)_\infty$$
$$(-q;q^2)_\infty^2(q;q^2)_\infty^2(q^2;q^2)_\infty^2=(q^2;q^4)_\infty(q^2;q^4)_\infty(q^4;q^4)_\infty$$
$$(-q;q^2)_\infty^2(q;q^2)_\infty^2(q^2;q^2)_\infty^2=(q^2,q^2,q^4;q^4)_\infty$$

which establish the result (3.4).
Proof of (3.5): Employing equations (1.6) and (1.7) in equation (2.8), we get.

$$\frac{(q^2;q^2)_\infty(q^6;q^6)_\infty}{(q;q^2)_\infty(q^3;q^6)_\infty}-\frac{(q^2;q^2)_\infty(q^6;q^6)_\infty}{(-q;q^2)_\infty(-q^3;q^6)_\infty}=\frac{2q(-q^2;q^4)_\infty^2(q^4;q^4)_\infty(q^{24};q^{24})_\infty}{(q^{12};q^{24})_\infty}$$
$$\frac{(-q;q^2)_\infty(-q^3;q^6)_\infty-(q;q^2)_\infty(q^3;q^6)_\infty}{(-q;q^2)_\infty(-q^3;q^6)_\infty(q;q^2)_\infty(q^3;q^6)_\infty}=\frac{2q(-q^2;q^4)_\infty^2(q^4;q^4)_\infty(q^{24};q^{24})_\infty}{(q^{12};q^{24})_\infty(q^2;q^2)_\infty(q^6;q^6)_\infty}$$
$$\frac{(-q;q^2)_\infty(-q^3;q^6)_\infty-(q;q^2)_\infty(q^3;q^6)_\infty}{(-q;q^2)_\infty(-q^3;q^6)_\infty(q;q^2)_\infty(q^3;q^6)_\infty}=\frac{2q(-q^2;q^4)_\infty^2(q^4,q^8,q^{16},q^{20},q^{24};q^{24})_\infty}{(q^2;q^2)_\infty(q^6,q^{12},q^{18};q^{24})_\infty}$$
$$\frac{(-q;q^2)_\infty(-q^3;q^6)_\infty-(q;q^2)_\infty(q^3;q^6)_\infty}{(-q;q^2)_\infty(-q^3;q^6)_\infty(q;q^2)_\infty(q^3;q^6)_\infty}=\frac{2q(-q^2;q^4)_\infty^2(q^4,q^8,q^{16},q^{20},q^{24};q^{24})_\infty}{(q^2,q^4,q^6,q^8;q^8)_\infty(q^6,q^{12},q^{18};q^{24})_\infty}$$

which establish the result (3.5).
Proof of (3.6): Employing equation (1.7) in equation (2.10), we get.

$$\frac{(q^6;q^6)_\infty(q^{10};q^{10})_\infty}{(q^3;q^6)_\infty(q^5;q^{10})_\infty}-\frac{(q^6;q^6)_\infty(q^{10};q^{10})_\infty}{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty}=\frac{2q^3(q^4;q^4)_\infty(q^{60};q^{60})_\infty}{(q^2;q^4)_\infty(q^{30};q^{60})_\infty}$$

$$\frac{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty-(q^3;q^6)_\infty(q^5;q^{10})_\infty}{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty(q^3;q^6)_\infty(q^5;q^{10})_\infty}=\frac{2q^3(q^4;q^4)_\infty(q^{60};q^{60})_\infty}{(q^2;q^4)_\infty(q^6;q^6)_\infty(q^{10};q^{10})_\infty(q^{30};q^{60})_\infty}$$

$$\frac{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty-(q^3;q^6)_\infty(q^5;q^{10})_\infty}{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty(q^3;q^6)_\infty(q^5;q^{10})_\infty}=\frac{(q^4,q^8,q^{12};q^{12})_\infty}{(q^6,q^{12},q^{18},q^{24};q^{24})_\infty}\times$$

$$\times\frac{2q^3}{(q^2,q^6,q^{10};q^{12})_\infty(q^{10},q^{20},q^{30},q^{30},q^{40},q^{50};q^{60})_\infty}$$

Notes

which establish the result (3.6).

Proof of (3.7): Employing equation (1.7) in equation (2.12), we get.

$$\frac{(q^2;q^2)_\infty(q^{30};q^{30})_\infty}{(q;q^2)_\infty(q^{15};q^{30})_\infty}+\frac{(q^2;q^2)_\infty(q^{30};q^{30})_\infty}{(-q;q^2)_\infty(-q^{15};q^{30})_\infty}=\frac{2(q^{12};q^{12})_\infty(q^{20};q^{20})_\infty}{(q^6;q^{12})_\infty(q^{10};q^{20})_\infty}$$

$$\frac{(-q;q^2)_\infty(-q^{15};q^{30})_\infty+(q;q^2)_\infty(q^{15};q^{30})_\infty}{(-q;q^2)_\infty(-q^{15};q^{30})_\infty(q;q^2)_\infty(q^{15};q^{30})_\infty}=\frac{2(q^{12};q^{12})_\infty(q^{20};q^{20})_\infty}{(q^2;q^2)_\infty(q^6;q^{12})_\infty(q^{10};q^{20})_\infty(q^{30};q^{30})_\infty}$$

$$\frac{(-q;q^2)_\infty(-q^{15};q^{30})_\infty+(q;q^2)_\infty(q^{15};q^{30})_\infty}{(-q;q^2)_\infty(-q^{15};q^{30})_\infty(q;q^2)_\infty(q^{15};q^{30})_\infty}=\frac{2(q^{12},q^{20},q^{24},q^{36},q^{40},q^{48},q^{60},q^{60};q^{60})_\infty}{(q^2;q^2)_\infty(q^6;q^{12})_\infty(q^{10};q^{20})_\infty(q^{30};q^{30})_\infty}$$

$$\frac{(-q;q^2)_\infty(-q^{15};q^{30})_\infty+(q;q^2)_\infty(q^{15};q^{30})_\infty}{(-q;q^2)_\infty(-q^{15};q^{30})_\infty(q;q^2)_\infty(q^{15};q^{30})_\infty}=\frac{(q^{12},q^{20},q^{24},q^{36},q^{40},q^{48},q^{60},q^{60};q^{60})_\infty}{(q^{10},q^{30},q^{30},q^{50},q^{60};q^{60})_\infty}\times$$

$$\times\frac{2}{(q^2,q^4,q^6,q^8,q^8;q^8)_\infty(q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty}$$

which establish the result (3.7).

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