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Numerical Simulation of the Effect of Distant Tsunami along the Coast of Peninsular Malaysia and Southern Thailand through an Open Boundary Condition in a Linear Model

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Abstract - The 26 December 2004 Indonesian tsunami was the third known global tsunami and reached every distant corner of the globe. An effort has been made here to evaluate the effect of this distant tsunami in a limited area model domain. The effect of distant tsunami has been simulated through an open boundary condition in a Cartesian coordinate shallow water linear model. The open boundary condition is applied to simulate the tsunami propagation when it is assumed that the tsunami is generated far away from the region of interest. The computational domain covers the region so that the 26 December 2004 Indonesian tsunami source is well within the model domain. First, the initial disturbance of the tsunami source are examined along the western open boundary of the model domain and then the boundary condition is formulated and adjusted in such a manner that the effects of tsunami due to the source along the coasts are same as the effects due to the formulated boundary condition. The response of the open boundary condition is investigated along the coastal region of Peninsular Malaysia and southern Thailand. The results are compared with the data available in the website and a very reasonable agreement is observed.

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NUMERICAL SIMULATION OF THE EFFECT OF DISTANT TSUNAMI ALONG THE COAST OF PENINSULAR MALAYSIA AND SOUTHERN THAILAND THROUGH AN OPEN BOUNDARY CONDITION IN A LINEAR MODEL

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I. Introduction

A tsunami is a natural coastal hazard generated in the deep ocean by vertical displacement of ocean water column and propagates across the ocean from the point of generation to the coast. It is usually a shallow water wave. A wave is characterized as a shallow water wave when the ratio between the water depth and its wavelength gets very small ($h \,/\, L < 0.05$). Tsunami has a very large wavelength and the speed is directly proportional to the depth of water. So it propagates in deep ocean at a very high speed with a limited loss of energy since the rate at which a tsunami wave loses its energy is inversely proportional to its wavelength. Thus the effect of a tsunami source along a particular region far away from the source position may be significant if the waves move through deep ocean.

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The response of the 2004 Indonesian tsunami wave reached every distant corner of the globe (Kowalik et al., 2005). The first known global tsunami that associated with the Krakatau Volcano explosion of 27 August 1883 (Murty, 1977) was generated in the same region where the 2004 earthquake occurred. The second known global tsunami was the Chilean tsunami of May 22, 1960 and tsunami waves observed at many far-field sites were very strong (Berkman and Symons, 1964). The 2004 Indonesian tsunami, resulting from a strong under sea earthquake that occurred off the coast of Sumatra of Indonesia, was the third known global tsunami and it was clearly recorded by a large number of tide gauges throughout the world ocean, including tide gauges located in the North Pacific and North Atlantic (Rabinovich et al., 2006). The distant effect of this tsunami was noticed as far as Struisbaai in South Africa, some 8,500 km away from the source zone, where a 1.5 m high surge was recorded about 16 hours after the earthquake. The tsunami also reached Antarctica where oscillations of up to 1 m were recorded with disturbances lasting a couple of days (Indian Ocean Tsunami at Syowa Station, Antarctica, 2007). Some of the tsunami's energy escaped into the Pacific Ocean, where it produced small but measurable tsunamis along the western coasts of North and South America, typically around 20 to 40 cm (NOAA, 2005). Mid-ocean ridges played a major role as wave guides that transferred the tsunami energy to distant regions outside the source area in the Indian Ocean (Kowalik et al., 2005; Titov et al., 2005).

The effect of a tsunami source along a particular region far away from the source position may be significant if the waves move through deep ocean. Since the response of the 2004 Indonesian tsunami reached every distant corner of the globe, it is necessary to estimate the response along a particular region of interest due to a source located far away from that region. This may be done through a global model that contains both the source and the region of interest. However a global model is very expensive in terms of both computer storage and CPU time and is not suitable for real time simulation. In hydrodynamical computations problem arises when the theoretical model applies to an infinite or semi-infinite region. In this case, where the original domain of the problem under investigation is infinite or very large, open boundaries may be used. An open boundary is an artificial boundary of a computational domain through which propagation of waves or flow should pass in order to leave the computational domain without giving rise to spurious reflection (Joolen et. al. 2003). The main purpose of using the open boundaries is to allow waves and disturbances originating within the model domain to leave the domain without affecting the interior solution.

Imamura et al. (1988) developed a shallow water model to simulate far field or distant tsunami where they used the finite difference method with the leap-frog scheme. Cho and Yoon (1998) improved the model of Imamura et al. (1988). The limitation of the models developed by Imamura et al. (1988) and Cho and Yoon (1998) is that the models should be used to the case of constant water depth with a uniform finite difference grid. Yoon (2002) again improved the model of Imamura et al. (1988) over a slowly varying topography. But as the model of Yoon (2002) has a large number of hidden grids the model is not suitable to calculate the tsunami propagation in deep sea (Cho et al., 2007).

Tidal oscillation in a limited area model domain may be generated through an open boundary (Johns et al. 1985, Roy 1995). A sinusoidal wave is allowed to propagate towards the model domain through an open boundary by using appropriate amplitude, phase and time period of the wave. The response of this type of boundary condition at every grid point is also sinusoidal. By adjusting the amplitude and phase it is possible to generate a representative tidal oscillation of specific time period in the model domain. Similarly, in computing the effect of distant tsunami source in a limited area model

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surrounding the region of interest, a boundary condition may be incorporated along the open boundary, which is facing the tsunami source. From our literature survey it is found that not many works have been carried out on formulating appropriate open boundary conditions that may be imposed to compute tsunami response due to distant source in a limited area model domain.

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Roy et al. (2006) developed a Cartesian co-ordinate non-linear shallow water model to formulate an open boundary condition to investigate the effect of distant tsunami along the coastal belt of Peninsular Malaysia and southern Thailand associated with Indonesian tsunami of 2004. The convective terms in the shallow water equations are insignificant and their effect is negligible in tsunami propagation in deep sea. However, the convective terms are weakly significant for the wave height near the coast. Thus a linear model can be applied for tsunami propagation in the deep sea, on the other hand, non-linear model should be applied to compute run-up or water level near the coast. The advantage of a linear model is that it needs less computer memory and computation time since the convective terms are excluded in the model; see for instance Zahibo et al. (2006). From the above discussion it should be conclude that it is better to use linear model instead of non-linear model for any hydrodynamical computations in deep sea. Since the west open boundary of our model domain is in the deep sea we are more interested to develop a linear model.

In this study, we describe the formulation of an open boundary condition for computing the effect of a distant tsunami in a Cartesian co-ordinate shallow water linear model. A linear Cartesian coordinate shallow water model has been developed to compute tsunami along the west coast of Peninsular Malaysia and Thailand associated with Indonesian tsunami of 2004. The analysis area of this model is a rectangular region approximately between 2° N, 14° N and 101.5° E, 91° E. First, the response of the tsunami source associated with the 2004 Indonesian tsunami is investigated along the west open boundary of the model domain. The 2004 tsunami source is incorporated as an initial condition during the computation. The linear shallow water equation with boundary conditions is applied to compute the maximum amplitudes and time series of water levels along the western open boundary. On the basis of the time series and amplitude, the open boundary condition is formulated for the western open boundary by a proper choice of the values for the amplitude, phase, period and the scale factor and at the same time the tsunami source near Sumatra is removed. The formulated boundary condition is imposed as an effect of tsunami source to compute the distant tsunami in absence of any tsunami source in the model domain.

II. Governing Equations and Boundary Conditions

The shallow water equations, which describe the inviscid flow of a thin layer of fluid in two dimensions, are a commonly accepted governing approximation for tsunami propagation in the deep ocean as well as in near-shore regions including inundation (Aizinger and Dawson, 2002). The depth averaged shallow water equations in Cartesian co-ordinates and the boundary conditions of this model are as follows:

A system of rectangular Cartesian coordinates is used in which the origin, O, is in the undisturbed sea surface (MSL), x-axis directed towards the west and y-axis directed towards the north and z-axis is directed vertically upwards. We consider the displaced position of the free surface as $z = \zeta(x, y, t)$ and the sea floor as $z = -h(x, y)$ so that the total depth of the fluid layer is $\zeta + h$. Taking into account that the characteristic wavelength exceeds the water depth, neglecting the convective terms and using the

Global Journal of Science Frontier Research (F) Volume XII Issue IX Version I Year 2012

parameterization of the bottom stress via the depth averaged velocity components, due to Karim (2006), the linear shallow water equations are:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left[(\zeta + h)u\right] + \frac{\partial}{\partial y}\left[(\zeta + h)v\right] = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} - f\,v = -g\frac{\partial \zeta}{\partial x} - \frac{C_f u(u^2+v^2)^{1/2}}{\zeta + h} \tag{2}$$

$$\frac{\partial v}{\partial t} + f\,u = -g\frac{\partial \zeta}{\partial y} - \frac{C_f v(u^2+v^2)^{1/2}}{\zeta + h} \tag{3}$$

For numerical treatment it is convenient to express the Eqs. (2) & (3) in the flux form by using the Eq. (1).

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \tag{4}$$

$$\frac{\partial \tilde{u}}{\partial t} - f\,\tilde{v} = -g(\zeta + h)\frac{\partial \zeta}{\partial x} - \frac{C_f\,\tilde{u}\left(u^2+v^2\right)^{1/2}}{\zeta + h} \tag{5}$$

$$\frac{\partial \tilde{v}}{\partial t} + f\,\tilde{u} = -g(\zeta + h)\frac{\partial \zeta}{\partial y} - \frac{C_f\,\tilde{v}\left(u^2+v^2\right)^{1/2}}{\zeta + h} \tag{6}$$

where, $(\tilde{u}, \tilde{v}) = (\zeta + h)(u, v)$

In the bottom stress terms of (5) & (6), u and v have been replaced by $\tilde{u}$ and $\tilde{v}$ in order to solve the equations in a semi-implicit manner.

In addition to the fulfillment of the surface and bottom conditions, appropriate conditions have to be satisfied along the boundaries of the model area for all time. Theoretically the only boundary condition needed in the vertically integrated system is that the normal component of the vertically integrated velocity vanishes at the coast and this may be expressed as $u\cos\alpha + v\sin\alpha = 0$, for all $t \geq 0$, where α denotes the inclination of the outward directed normal to x-axis. It follows that $u = 0$ along y-directed boundaries and $v = 0$ along the x-directed boundaries.

At the open-sea boundaries the waves and disturbance are allowed, generated within the model domain, to leave the domain without affecting the interior solution. Thus the normal component of velocity cannot vanish and so a radiation type of boundary is generally used. Following Heaps (1973), the following radiation type of condition may be used in our model:

$u\cos\alpha + v\sin\alpha = -\left(g/h\right)^{1/2}\zeta$, for all $t \geq 0$. Note that the velocity structure in a shallow water wave is described by $w = \dfrac{g\zeta}{\sqrt{gh}}$, where w is the horizontal particle velocity.

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Other than the coastal boundary of the domain the boundaries are considered as straight lines along the open sea. Thus there lie three open sea boundaries (Fig. 1). Commonly used radiation type of boundary conditions for the open sea boundaries, due to Johns et al. (1981), are:

$$u-(g/h)^{1/2}\zeta=0 \quad \text{at the west open boundary} \tag{7}$$

$$v+(g/h)^{1/2}\zeta=0 \quad \text{at the south open boundary} \tag{8}$$

$$v-(g/h)^{1/2}\zeta=0 \quad \text{at the north open boundary} \tag{9}$$

The coastal belts of the main land and islands are the closed boundaries where the normal components of the current are taken as zero.

III. Numerical Discretisation

The governing shallow water equations and the boundary conditions are discretised by finite difference (forward in time and central in space) and are solved by a conditionally stable semi-implicit method using a staggered grid system which is similar to Arakawa C system (Arakawa and Lamb, 1977). Let there are M gridlines parallel to y-axis and N gridlines parallel to the x-axis so that the total number of grid points are $M\times N$. We define the grid points (x_i, y_j) in the domain by

$$x_i = (i-1)\Delta x, \qquad i=1,2,3,\ldots,M \tag{10}$$

$$y_j = (j-1)\Delta y, \qquad j=1,2,3,\ldots,N \tag{11}$$

The sequence of discrete time instants is given by

$$t_k = k\,\Delta t, \qquad k=1,2,3,\ldots \tag{12}$$

Discretisation in time can be performed with either explicit or implicit schemes. An explicit scheme uses numerical values from current time steps only in the advanced time step computations. In an implicit scheme values from current time steps and from the advanced time step are used in advanced time step computations. Here we use the semi-implicit scheme to descretise the equation. The indexing of the horizontal coordinates is (i, j) and the time steps are indexed by the superscript k. For the purpose of discretisation the following notations are used.

For any dependent variable $\chi(x,y,t)$, let us consider:

$$\chi(x_i, y_j, t_k) = \chi_{i\,j}^k$$

$$\frac{1}{2}(\chi_{i+1\,j}^k + \chi_{i-1\,j}^k) = \overline{\chi_{i\,j}^k}^{\,x}$$

$$\frac{1}{2}(\chi_{i\,j+1}^k + \chi_{i\,j-1}^k) = \overline{\chi_{i\,j}^k}^{\,y}$$

$$\frac{1}{4}(\chi_{i+1\,j}^k + \chi_{i-1\,j}^k + \chi_{i\,j+1}^k + \chi_{i\,j-1}^k) = \overline{\chi_{i\,j}^k}^{\,xy}$$

The discretised form of continuity equation (4) is

$$\frac{\zeta_{ij}^{k+1}-\zeta_{ij}^{k}}{\Delta t}+\frac{\left(\zeta_{i+1j}^{k}+h_{i+1j}\right)u_{i+1j}^{k}-\left(\zeta_{i-1j}^{k}+h_{i-1j}\right)u_{i-1j}^{k}}{2\Delta x}+\frac{\left(\zeta_{ij+1}^{k}+h_{ij+1}\right)v_{ij+1}^{k}-\left(\zeta_{ij-1}^{k}+h_{ij-1}\right)v_{ij-1}^{k}}{2\Delta y}=0 \tag{13}$$

from which we compute ζ_{ij}^{k+1} for $i=2, 4, 6, \ldots, M-2$ and $j=3, 5, 7, \ldots, N-2$

The boundary condition (7) is discretised as

$$u_{M-1j}^{k}-(g/h_{M-1j})^{1/2}\frac{1}{2}\left(\zeta_{M-2j}^{k+1}+\zeta_{Mj}^{k+1}\right)=0 \tag{14}$$

from which we compute ζ_{Mj}^{k+1} for $j=1, 3, 5, 7, \ldots, N$

The boundary condition (8) is discretised as

$$v_{i2}^{k}+(g/h_{i2})^{1/2}\frac{1}{2}\left(\zeta_{i1}^{k+1}+\zeta_{i3}^{k+1}\right)=0 \tag{15}$$

from which we compute ζ_{i1}^{k+1} for $i=2, 4, 6, 8, \ldots, M-2$

The boundary condition (9) is discretised as

$$v_{iN-1}^{k}-(g/h_{iN-1})^{1/2}\frac{1}{2}\left(\zeta_{iN-2}^{k+1}+\zeta_{iN}^{k+1}\right)=0 \tag{16}$$

from which we compute ζ_{iN}^{k+1} for $i=2, 4, 6, 8, \ldots, M-2$

The discretised form of linear *x*-momentum equation (5) is

$$\frac{u_{ij}^{k+1}-u_{ij}^{k}}{\Delta t}-f\,\overline{v_{ij}}^{xy}=-g\frac{\zeta_{i+1j}^{k+1}-\zeta_{i-1j}^{k+1}}{2\Delta x}-\frac{C_f\,u_{ij}^{k+1}\left(\left(u_{ij}^{k}\right)^2+\left(\overline{v_{ij}^{k}}^{xy}\right)^2\right)^{1/2}}{\overline{\zeta_{ij}^{k+1}}^{x}+h_{ij}} \tag{17}$$

from which we compute u_{ij}^{k+1} for $i=3, 5, 7, \ldots, M-1$ and $j=3, 5, 7, \ldots, N-2$. Note that in the last term u_{ij}^{k+1} is in advanced time level and this ensures a semi-implicit nature of the numerical method.

Similarly, the discretised form of linear *y*-momentum equation (6) is

$$\frac{v_{ij}^{k+1}-v_{ij}^{k}}{\Delta t}+f\,\overline{u_{ij}}^{xy}=-g\frac{\zeta_{ij+1}^{k+1}-\zeta_{ij-1}^{k+1}}{2\Delta y}-\frac{C_f\,v_{ij}^{k+1}\left(\left(\overline{u_{ij}^{k}}^{xy}\right)^2+\left(v_{ij}^{k}\right)^2\right)^{1/2}}{\overline{\zeta_{ij}^{k+1}}^{y}+h_{ij}} \tag{18}$$

from which we compute v_{ij}^{k+1} for $i=2, 4, 6, \ldots, M-2$ and $j=2, 4, 6, \ldots, N-1$. As before, in the last term v_{ij}^{k+1} is in advanced time level and this ensures a semi-implicit nature of the numerical method. Here $\zeta_{ij}^{k+1}, u_{ij}^{k+1}, v_{ij}^{k+1}$ are the water elevations, velocity components in the *x* and *y* directions respectively at the advanced time level.

The model domain is a rectangular region approximately between 2^0 N, 14^O N and

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91^0 E, 101.5^0 E, which includes the tsunami source region associated with 2004 Indonesian tsunami (Fig.1). The origin of the Cartesian coordinate system is at O (3.125° N, 101.5° E), the x-axis is directed towards west at an angle 15° (anticlockwise) with the latitude line through O and the y-axis is directed towards north inclined at an angle 15° (anticlockwise) with the longitude line through O. The grid size of the rectangular mesh is given by $\Delta x = \Delta y = 4$ km and number of grids in x-direction and y-direction are respectively $M = 230$ and $N = 319$ so that there are 73370 grid points in the computational domain. The time step Δt is taken as 10 seconds and this satisfies the CFL criterion and thus ensures the stability of the numerical scheme. Following Kowalik et al. (2005), the value of the friction coefficient C_f is taken as 0.0033 through out the model area. The bathymetries for the model area are collected from the Admiralty bathymetric charts.

IV. Tsunami Source Generation and Initial Condition

Accurate initial conditions are required to obtain reasonable results from numerical simulation of tsunami. The generation of an earthquake tsunami source depends essentially on the pattern and dynamics of motions in the earthquake source zone and on the initial seafloor movements. The magnitude of the earthquake gives a relationship among the three parameters – length, width and dislocation. The generation mechanism of the 2004 Indonesian tsunami was mainly a static sea floor uplift caused by an abrupt slip at the India/Burma plate interface. A detailed description of the estimation of the extent of earthquake rupture along with the maximum uplift and subsidence of the seabed has been reported in Kowalik et al. (2005) and this estimation was based on Okada (1985). From the deformation contour, it is seen that the estimated uplift and subsidence zone is between 92° E to 97°E and 2°N to 10°N with a maximum uplift of 507 cm at the west and maximum subsidence of 474 cm at the east. Following Kowalik et al. (2005) the disturbance in the form of rise and fall of sea surface is assigned as the initial condition in the model with a maximum rise of 5 m to maximum fall of 4.75 m to generate the response along the western open boundary. The initial sea surface elevations are taken as zero everywhere except in the part of the source zone which is activated at the initial time. Also the initial x and y components of velocity are taken as zero throughout the model area.

V. Open Boundary Condition

The wave propagation from the tsunami source has been investigated along the western open boundary of the model domain. The amplitudes of tsunami wave along the western open boundary have been computed to estimate the amplitude of the boundary condition to be formulated. For generating tidal oscillation in a limited area model through a boundary, the radiation type of boundary condition along with a sinusoidal term, containing amplitude, period and phase is needed (Johns et al. 1985, Roy 1995). The amplitude of this sinusoidal term remains constant with respect to time. But in case of tsunami propagation through a particular point in the sea, the time series is also oscillatory but with damping amplitude. On the basis of time series data and amplitude of the tsunami wave due to the source, the formulated open boundary condition (due to Roy et al., 2006) that represents the effect of distant tsunami is given by

$$u-(g/h)^{1/2}\zeta = -2(g/h)^{1/2}e^{(-st)}a\sin(2\pi t/T+\varphi) \quad \text{at the west open boundary} \qquad (19)$$

where a is the amplitude, T is the period, φ is the phase of the wave and s is the scale factor used for damping the amplitude of the wave with respect to time. In Eq. (19), the following conditions are imposed:

$s = 0$ for $t \leq T$

and $s > 0$ for $t > T$.

Through this condition we are allowing one wave, with full amplitude, to enter into the domain through the open boundary before damping of the amplitude begins.

Based on the amplitudes obtained through the source of Indonesian tsunami 2004, the assigned amplitudes (a) in (19) are adjusted so that the response in model domain is similar to that associated with the source of Indonesian tsunami 2004. By trial and error method, the values of phase (φ), period (T) and the scale factor in Eq. (19) have also been adjusted and these are $\varphi = 0$, $T = 0.5$ hr and $s = 0.01$. The formulation of the open boundary condition is such that in absence of the source its response in the domain is similar to that of the original source of the Indonesian tsunami of 2004.

Figure 2 shows the time series of sea surface fluctuation at a particular grid point at the western open boundary of the model domain, where solid line indicates the computed amplitudes due to the tsunami source and dotted line indicates the amplitudes that are imposed as the boundary condition. Both the time series are found to be almost identical, which means that the boundary condition (19) is capable of generating time series which is similar to that generated by the source.

VI. Distant Tsunami Computation Through the Open Boundary Condition

We first simulate the 2004 Indonesian tsunami propagation along the west open boundary of the model domain. The simulated data are then applied to formulate the boundary condition. The effects of the formulated open boundary condition are then investigated along the west coast of Thailand and Peninsular Malaysia in absence of the tsunami source. Wave propagation from the boundary is computed and the water levels along the coastal belts of the west coast of Thailand and Penang Island are estimated.

The propagation of tsunami wave due to the imposed boundary condition from the open boundary and the arrival time at the coast have been studied. Tsunami travel time is an important parameter in the tsunami prediction and warning. We consider the 0.1 m sea level rise as the arrival of tsunami. Figure 3 shows the contour plot of time, in minutes, for attaining +0.1 m sea level rise at each grid point in the model domain. It is seen that after imposition of the boundary condition at the west open boundary of the model domain, the disturbance propagates gradually towards the coast (Fig. 3). The arrival time of the wave at Phuket is approximately 110 min and the same at Penang is approximately 240 min. If we use the tsunami source, within the model domain, in our linear model the arrival time of tsunami from the source at Phuket and Penang islands are 95 min and 230 min respectively (Fig. 4). In the present study we compute the response of the open boundary condition imposed at the west boundary, which is away from the source zone of the Indonesian tsunami 2004. This is why the computed arrival time due to the boundary condition is delayed by up to 10 to 15 min. This time difference can be estimated by measuring the total distance of the open boundary from the coast and its travel time. Thus the corrected time related to the tsunami source at Sumatra should be 10 to 15 min earlier than the present computed time.

Figure 5 depicts the maximum water level contours, along the coast from Penang Island to Phuket. The surge amplitude is increasing from south to north; the maximum

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Global Journal of Science Frontier Research (F) Volume XII Issue IX Version I Year 2012

Notes

water level at Penang Island is from 1.5 m to 3.5 m, whereas at Phuket region it is 3.5 m to 7.5 m. The surge amplitude is increasing very fast near the shoreline everywhere. The computed water levels indicate that the north coast of Penang Island is vulnerable for stronger surges. Similarly the north-west part of Phuket is at risk of highest surge due to the source at Sumatra. The times of attaining maximum elevations along these regions due to the formulated boundary condition are also computed and it is found that this time at each location is approximately 10 to 15 min later than the time of attaining +0.1 m.

The computed time histories of water surface fluctuations at different locations of the coastal belt of Phuket and Penang Island are stored at an interval of 30 seconds and are shown in Figures 6 and 7 respectively. Figure 6 depicts the time series of water levels for the Phuket region in south west Thailand. At the east coast of Phuket, the maximum water level is 4.4 m and the minimum is - 4.0 m and the water level continues to oscillate for a long time (Fig. 6a). It is important to note that at approximately 1.6 hrs after imposition of boundary condition, instead of increasing, the water level starts decreasing as the response to the boundary condition and reaches a minimum level of – 4.0 m. Then the water level increases continuously to reach a level of 4.4 m (1^{st} crest) at 2.2 hrs before going down again. At the south-west coast of Phuket the time series begins with a depression of – 6.0 m and the maximum water level reaches up to 7.4 m and the oscillation continues with low amplitudes (Fig. 6b).

Figure 7 shows the time series of water levels at two locations at the north and south coasts of Penang Island in Malaysia. At north coast the maximum elevation is approximately 4.5 m (Fig. 7a). At approximately 3.75 hrs after imposition of boundary condition, the water level starts decreasing as the response to the boundary condition and reaches a minimum level of –1.6 m. Then the water level increases continuously to reach a level of 3.6 m (1^{st} crest) at 4 hrs 30 min before going down again. The water level oscillates and this oscillation continues for several hours. At the south coast the maximum elevation is approximately 2 m and the minimum is – 2.5 m (Fig. 7b).

We have seen in previous section that the time series of sea surface fluctuation at each coastal location begins with the sea level fall (withdrawal of water from the coast) instead of rise of water as the response of imposed boundary condition (Figs. 6, 7). The sinusoidal boundary condition is imposed at the boundary so that the phase is from trough to crest. Thus from the boundary the first wave propagates so that the trough is in front of the crest. To investigate this initial withdrawal we change the phase of the imposed boundary condition. Relative to the west coast of Malaysia and Thailand, the phase of the imposed boundary condition on open boundary is in the form of trough to crest. To identify whether this phase of the boundary condition is responsible for initial withdrawal of water from the coastal belt, an investigation is undertaken by a boundary condition of same intensity, but with the reverse phase, that is, from crest to trough. Figure 8 depicts the time series of water levels at the same locations as in Fig. 6b of Phuket and Fig. 7b of Penang Island associated with the reversed boundary condition. This figure shows that, at each location, the tsunami surge is not preceded by withdrawal of water from the coast. Similar results are found along the other coastal belts of Phuket and Penang (not shown). Thus, the initial withdrawal of water from the coasts depends upon the nature of the phase of the imposed boundary condition.

The computed arrival time of wave generated by imposed boundary condition is compared with the data available in USGS website (Tab. 1). According to USGS report the tsunami waves reached at Phuket within two hours time after the earthquake and the arrival time of tsunami at Penang is between 3 hr 30 min and 4 hours. It is mentioned that the formulated boundary condition is imposed on the open boundary of the model which is far from the tsunami source. Thus the corrected arrival time related to the tsunami source at Sumatra is obtained by shifting 10 to 15 min earlier than the computed arrival times of wave of imposed boundary condition. Therefore the computed time of attaining maximum surge due to the original tsunami source is 100 min and 230 min for Phuket and Penang respectively. Thus the computed time is almost identical with the website data.

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On the other hand maximum water level surrounding Penang is 1.5 - 3.5 m and the same for Phuket is 3.5 – 7.0 m computed by the formulated boundary condition. In USGS website it is reported that wave height reached 7 to 11 m surrounding Phuket and due to Roy et al. (2006) the wave height reached 2.0 – 3.5 m surrounding Penang. From table 1 and the above discussion it is clear that computed tsunami arrival time and maximum water levels along the island boundaries of Penang and Phuket agree well with the observed data or data available in the USGS website.

VII. Conclusion

A two-dimensional linear model has been developed to formulate an appropriate boundary condition. The response of the 26 December 2004 Indonesian tsunami is computed at the west open boundary of the model domain. Then by the amplitudes of tsunami wave with the adjusted values of phase, period and scale factor of the boundary condition the appropriate boundary condition is formulated to compute the distant tsunami along the coastal boundary of west coast Peninsular Malaysia and southern Thailand. It is observed that the response of the formulated boundary condition in absence of the source is similar to that of the original source of the Indonesian tsunami of 2004. The computed water levels along the coastal belts of Penang and Phuket are found to be quite reasonable and consistent with the water level data. The arrival time of tsunami due to the boundary condition is little bit later than the arrival time of tsunami due to the source because the boundary condition is away from the source zone of the Indonesian tsunami 2004. It is found that the initial withdrawal of water from a coastal belt depends upon the phase of the boundary condition. Thus the computed results obtained by the formulated boundary condition are in good agreement with the observed data. Since this model is computationally efficient in comparison with a nonlinear model, authors believe that in real time operational distant tsunami forecast programs in a particular coastal region this open boundary technique is practically applicable.

Table 1 : Computed and observed / USGS tsunami propagation time and water levels for Penang and Phuket

	Location	Computed water level (m)	Observed/ USGS
Propagation time (min)	Penang	230	< 240
	Phuket	100	< 120
Max. water level (m)	surrounding Penang	1.5 - 3.5	2.0 – 3.5
	surrounding Phuket	3.5 – 7.0	7 - 11

Notes

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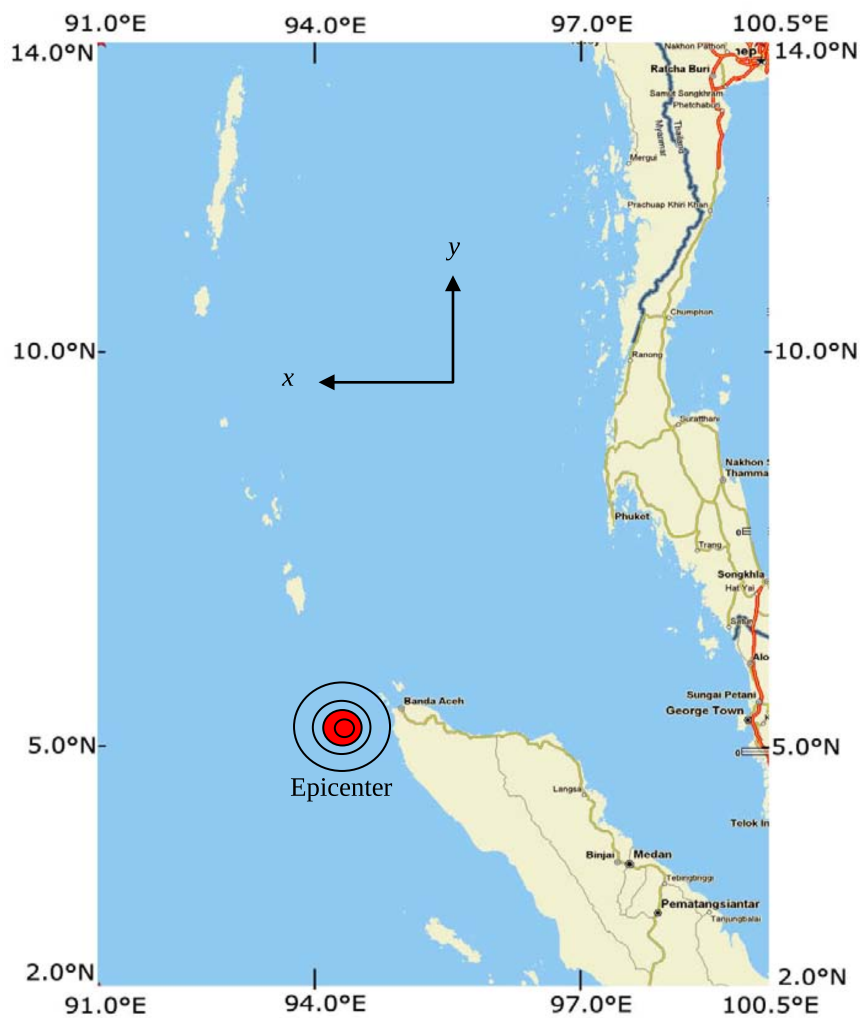

Figure 1 : Model Domain including the coastal geometry and the epicenter of the 2004 earthquake (Courtesy: Roy et al., 2006).

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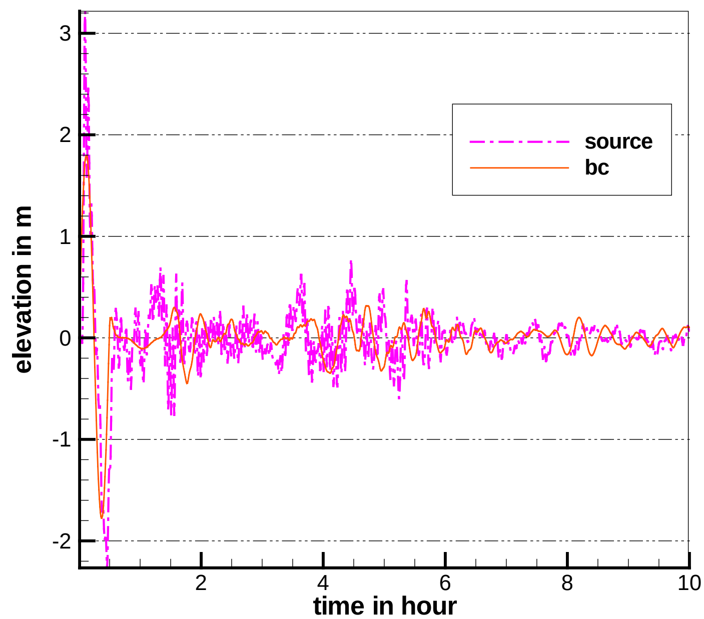

Figure 2 : Time series of sea surface fluctuation at the western open boundary

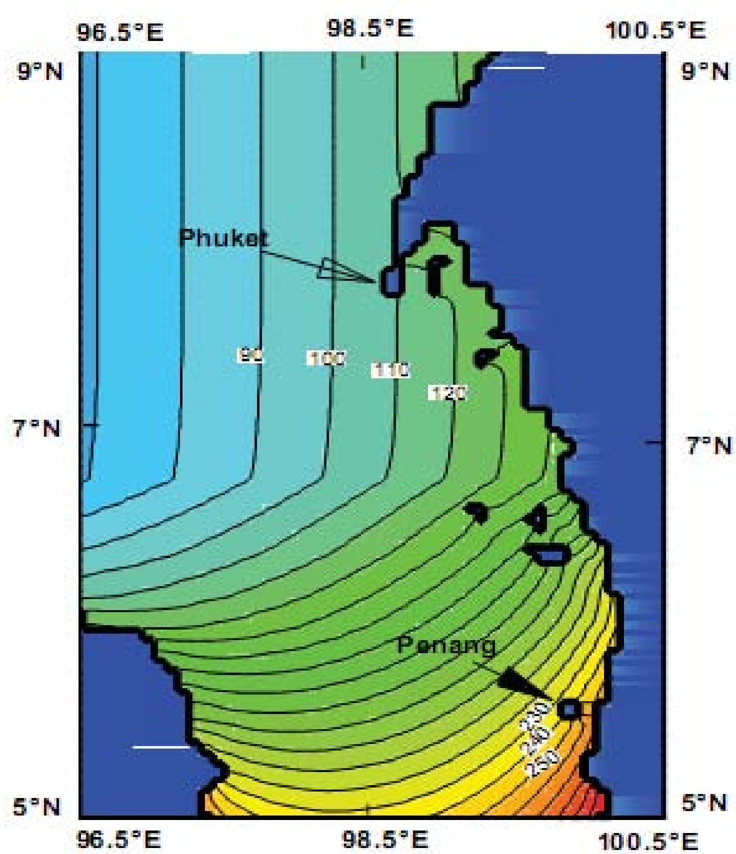

Figure 3 : Tsunami propagation time in minutes towards Phuket and Penang due to the formulated open boundary condition

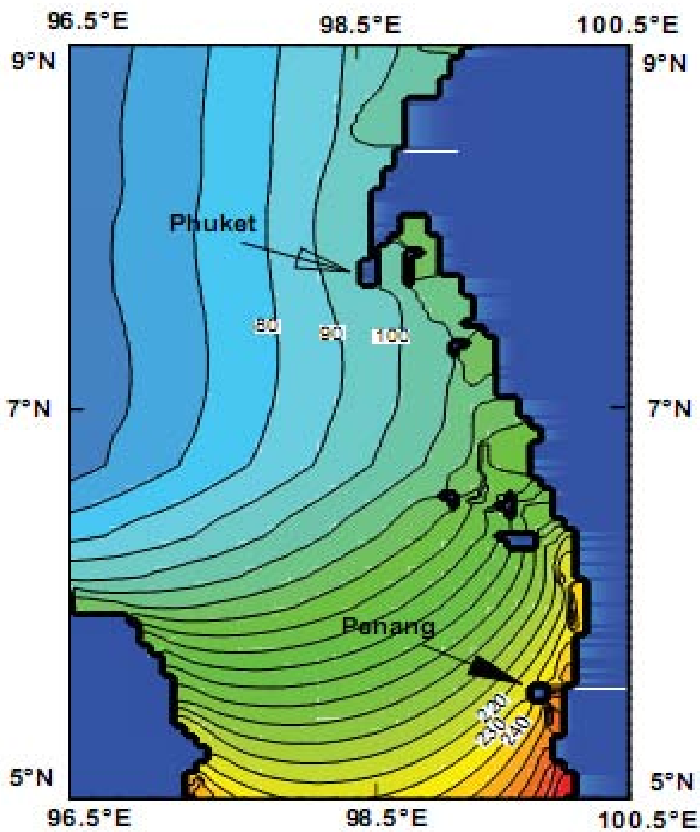

Figure 4 : Tsunami propagation time in minutes towards Phuket and Penang due to the 2004 Indonesian tsunami source

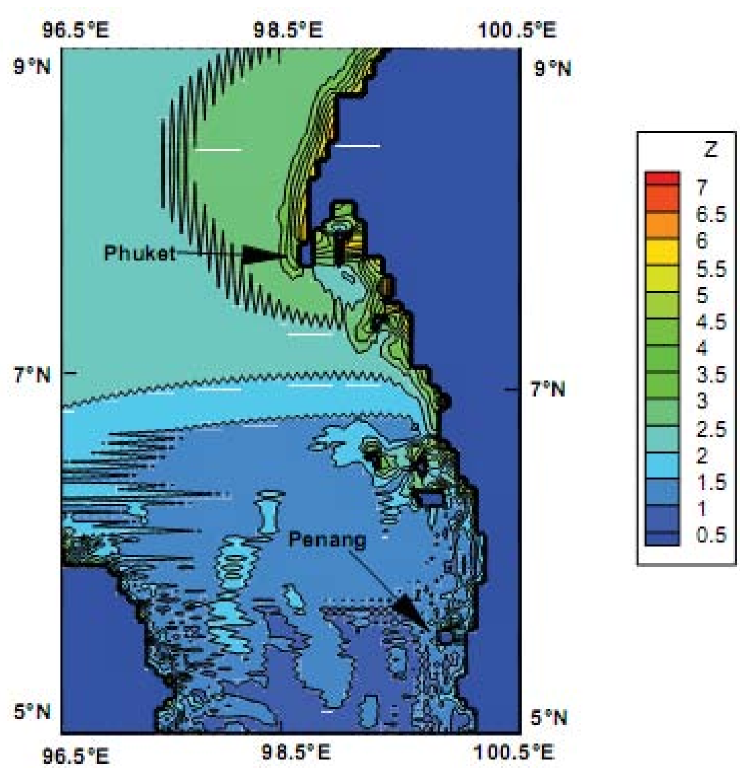

Figure 5 : Contour of maximum elevation associated with the formulated boundary condition around the west coast of Thailand and Malaysia

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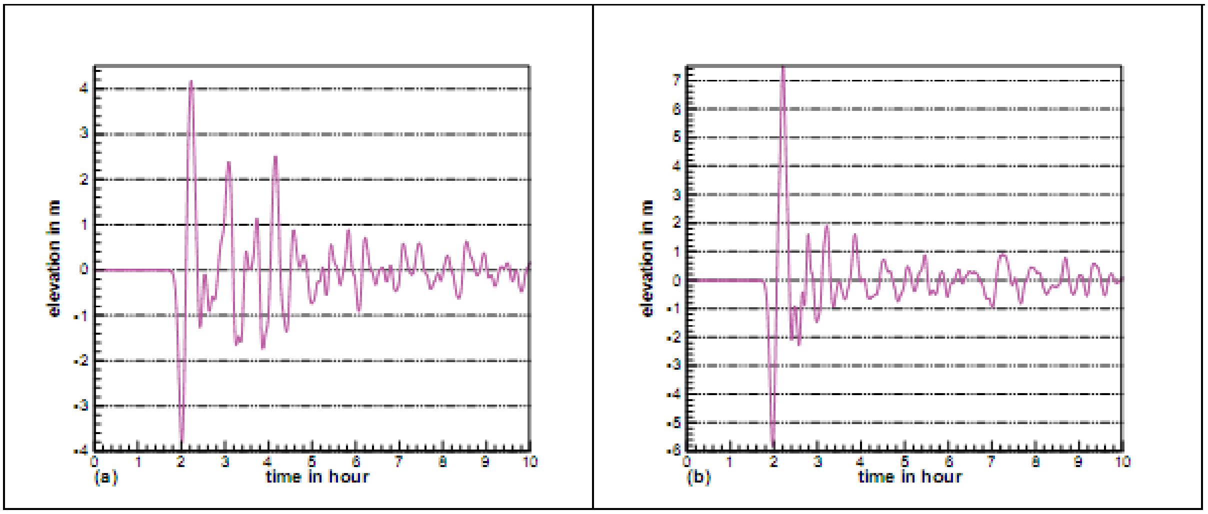

Figure 6 : Time series of computed elevation at two coastal locations of Phuket Island associated with the formulated boundary condition: (a) East Phuket, (b) South-west Phuket

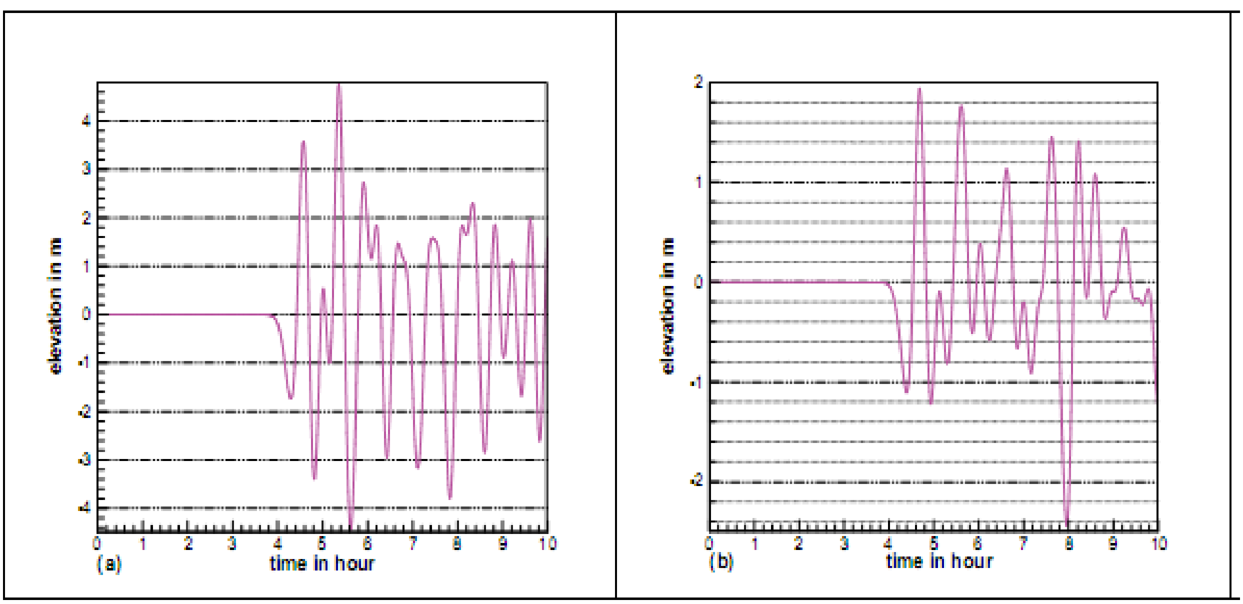

Figure 7 : Time series of computed elevation at two coastal locations of Penang Island associated with the formulated boundary condition: (a) Batu Ferringi, (b) South coast

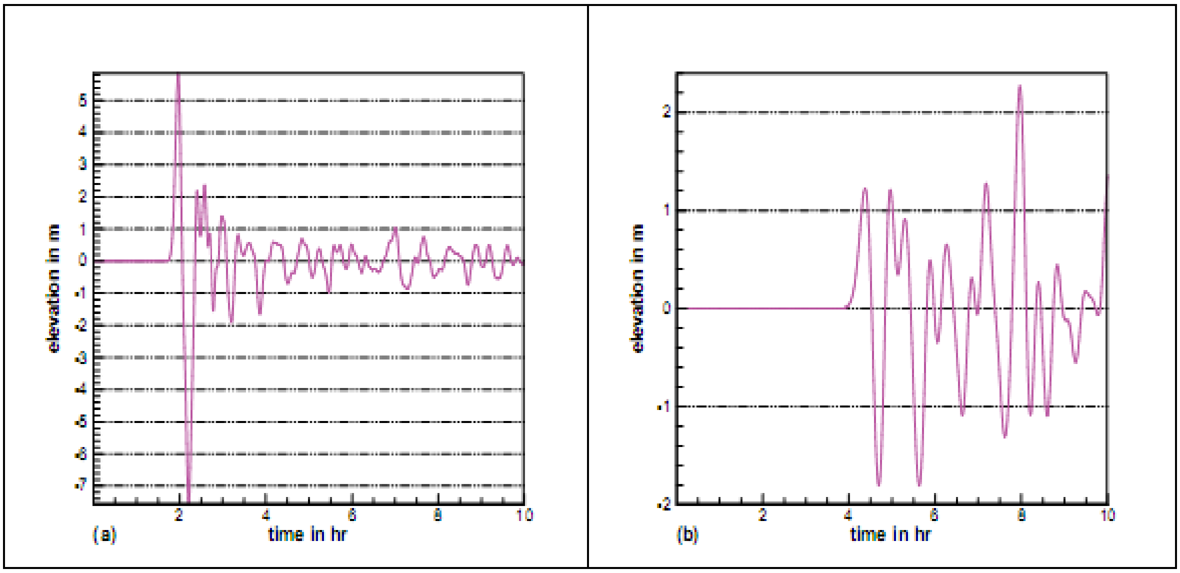

Figure 8 : Same as Figure 6b and 7b, except that the phase of the boundary condition has been reversed.

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