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On Certain Class of Difference Sequence Spaces

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Abstract - In this article we define the sequence spaces $c_0(u, \triangle_v^m, F, p), c(u, \triangle_v^m, F, p)$ and $l_\infty(u, \triangle_v^m, F, p)$ for $F = (f_k)$ a sequence of moduli, $p = (pk)$ sequence of positive reals, $v = (vk)$ is any fixed sequence of zero complex numbers, $m \in \mathbb{N}$ is a fixed number, and $u \in U$t the set of all sequences and establish some inclusion relations.

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On Certain Class of Difference Sequence Spaces

Khalid Ebadullah

Abstract - In this article we define the sequence spaces $c_0(u,\triangle_v^m,F,p), c(u,\triangle_v^m,F,p)$ and $l_\infty(u,\triangle_v^m,F,p)$ for $F=(f_k)$ a sequence of moduli, $p=(p_k)$ sequence of positive reals, $v=(v_k)$ is any fixed sequence of non zero complex numbers, $m\in\mathbb{N}$ is a fixed number, and $u\in U$ the set of all sequences and establish some inclusion relations.

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I. Introduction

Let $\mathbb{N}$,$\mathbb{R}$ and $\mathbb{C}$ be the sets of all natural,real and complex numbers respectively.
We write

$$\omega=\{x=(x_k): x_k\in\mathbb{R}\ or\ \mathbb{C}\},$$

the space of all real or complex sequences. Let l_∞,c and c_0 denote the Banach spaces of bounded,convergent and null sequences respectively.
The following subspaces of ω were first introduced and discussed by Maddox [13-15].

$$l(p):=\{x\in\omega:\sum_k|x_k|^{p_k}<\infty\},$$
$$l_\infty(p):=\{x\in\omega:\sup_k|x_k|^{p_k}<\infty\},$$
$$c(p):=\{x\in\omega:\lim_k|x_k-l|^{p_k}=0,\quad\text{for some } l\in\mathbb{C}\},$$
$$c_0(p):=\{x\in\omega:\lim_k|x_k|^{p_k}=0\},$$

where $p=(p_k)$ is a sequence of striclty positive real numbers.
The idea of Difference sequence sets

$$X_\triangle=\{x=(x_k)\in\omega:\triangle x=(x_k-x_{k+1})\in X\},$$

where $X=l_\infty,\ c$ or c_0 was introduced by Kizmaz [9].
In 1981 Kizmaz [9] defined the following sequence spaces,

$$l_\infty(\triangle)=\{x=(x_k)\in\omega:(\triangle x_k)\in l_\infty\},$$

Ref.

[13] Maddox,I.J.*Elements of Functional Analysis*,Cambridge University Press.(1970)

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$$c(\triangle) = \{x = (x_k) \in \omega : (\triangle x_k) \in c\},$$

$$c_0(\triangle) = \{x = (x_k) \in \omega : (\triangle x_k) \in c_0\},$$

where $\triangle x = (x_k - x_{k+1})$. These are Banach spaces with the norm

$$||x||_{\triangle} = |x_1| + ||\triangle x||_{\infty}.$$

After then Et[3] defined the sequence spaces

$$l_\infty(\triangle^2) = \{x = (x_k) \in \omega : (\triangle^2 x_k) \in l_\infty\}$$

$$c(\triangle^2) = \{x = (x_k) \in \omega : (\triangle^2 x_k) \in c\}$$

$$c_0(\triangle^2) = \{x = (x_k) \in \omega : (\triangle^2 x_k) \in c_0\}$$

Where $(\triangle^2 x) = (\triangle^2 x_k) = (\triangle x_k - \triangle x_{k+1})$.
The sequence spaces $l_\infty(\triangle^2), c(\triangle^2)$ and $c_0(\triangle^2)$ are Banach spaces with the norm

$$||x||_{\triangle} = |x_1| + |x_2| + ||\triangle^2 x||_{\infty}.$$

After then R. Colak and M. Et [4] defined the sequence spaces

$$l_\infty(\triangle^m) = \{x = (x_k) \in \omega : (\triangle^m x_k) \in l_\infty\},$$

$$c(\triangle^m) = \{x = (x_k) \in \omega : (\triangle^m x_k) \in c\},$$

$$c_0(\triangle^m) = \{x = (x_k) \in \omega : (\triangle^m x_k) \in c_0\},$$

where $m \in \mathbb{N}$,

$$\triangle^0 x = (x_k),$$

$$\triangle x = (x_k - x_{k+1}),$$

$$\triangle^m x = (\triangle^{m-1} x_k - \triangle^{m-1} x_{k+1}),$$

and so that

$$\triangle^m x_k = \sum_{i=0}^{m} (-1)^i \begin{bmatrix} m \\ i \end{bmatrix} \quad x_{k+i}.$$

and showed that these are Banach spaces with the norm

$$||x||_{\triangle} = \sum_{i=1}^{m} |x_i| + ||\triangle^m x||_{\infty}.$$

Let U be the set of all sequences $u = (u_k)$ such that $u_k \neq 0 (k = 1, 2, 3....)$.
Malkowsky[16] defined the following sequence spaces

Ref.

[3] Et,M..On some difference sequence spaces,$Dogra-Tr.J.Math.$,17,(1993)18-24.

$$l_\infty(u,\triangle)=\{x=(x_k)\in\omega:(u_k\triangle x_k)\in l_\infty\},$$

$$c(u,\triangle)=\{x=(x_k)\in\omega:(u_k\triangle x_k)\in c\},$$

$$c_0(u,\triangle)=\{x=(x_k)\in\omega:(u_k\triangle x_k)\in c_0\},$$

where $u\in U$.

The concept of paranorm(See[15]) is closely related to linear metric spaces.It is a generalization of that of absolute value.

Let X be a linear space. A function $g:X\longrightarrow R$ is called paranorm, if for all $x,y\in X$,

(PI) $g(x)=0\ if\ x=\theta$,

(P2) $g(-x)=g(x)$,

(P3) $g(x+y)\le g(x)+g(y)$,

(P4) If (λ_n) is a sequence of scalars with $\lambda_n\to\lambda\ (n\to\infty)$ and $x_n,a\in X$ with $x_n\to a\ \ (n\to\infty)$, in the sense that $g(x_n-a)\to 0\ \ (n\to\infty)$, in the sense that $g(\lambda_n x_n-\lambda a)\to 0\ \ (n\to\infty)$.

A paranorm g for which $g(x)=0$ implies $x=\theta$ is called a total paranorm on X, and the pair (X,g) is called a totally paranormed space.

The idea of modulus was structured in 1953 by Nakano.(See[17]).

A function $f:[0,\infty)\longrightarrow[0,\infty)$ is called a modulus if

(P1)$f(t)=0$ if and only if $t=0$,

(P2) $f(t+u)\le f(t)+f(u)$ for all $t,u\ge 0$,

(P3) f is increasing, and

(P4) f is continuous from the right at zero.

Ruckle [18-20] used the idea of a modulus function f to construct the sequence space

$$X(f)=\{x=(x_k):\sum_{k=1}^{\infty}f(|x_k|)<\infty\}$$

This space is an FK space ,and Ruckle[18-20] proved that the intersection of all such $X(f)$ spaces is ϕ, the space of all finite sequences.

The space $X(f)$ is closely related to the space l_1 which is an $X(f)$ space with $f(x)=x$ for all real $x\ge 0$.Thus Ruckle[18-20] proved that, for any modulus f.

$$X(f)\subset l_1 \text{ and } X(f)^\alpha=l_\infty$$

The space $X(f)$ is a Banach space with respect to the norm

$$||x||=\sum_{k=1}^{\infty}f(|x_k|)<\infty.\text{(See[18-20])}.$$

Ref.

[15] Maddox,I.J.Some properties of paranormed sequence spaces.,*J.London. Math.Soc.*1(1969),316-322.

Spaces of the type $X(f)$ are a special case of the spaces structured by B.Gramsch in[8].From the point of view of local convexity, spaces of the type $X(f)$ are quite pathological. Symmetric sequence spaces, which are locally convex have been frequently studied by D.J.H Garling[6-7],G.Köthe[12]and W.H.Ruckle[18-20].
After then E.Kolk [10-11] gave an extension of $X(f)$ by considering a sequence of moduli $F = (f_k)$ and defined the sequence space

$$X(F) = \{x = (x_k) : (f_k(|x_k|)) \in X\}.\text{(See[10-11])}.$$

After then Vakeel.A.Khan and Lohani[21] defined the following sequence spaces

$$l_\infty(u, \triangle, F) = \{x = (x_k) \in \omega : \sup_{k\geq 0} f_k(|u_k \triangle x_k|) < \infty\},$$

$$c(u, \triangle, F) = \{x = (x_k) \in \omega : \lim_{k\to\infty} f_k(|u_k \triangle x_k - l|) = 0, l \in \mathbb{C}\},$$

$$c_0(u, \triangle, F) = \{x = (x_k) \in \omega : \lim_{k\to\infty} f_k(|u_k \triangle x_k|) = 0\},$$

where $u \in U$.
If we take x_k instead of $\triangle x$,then we have the following sequence spaces

$$l_\infty(u, F) = \{x = (x_k) \in \omega : \sup_{k\geq 0} f_k(|u_k x_k|) < \infty\},$$

$$c(u, F) = \{x = (x_k) \in \omega : \lim_{k\to\infty} f_k(|u_k x_k - l|) = 0, l \in \mathbb{C}\},$$

$$c_0(u, F) = \{x = (x_k) \in \omega : \lim_{k\to\infty} f_k(|u_k x_k|) = 0\},$$

where $u \in U$.
After then C.Asma and R.Colak[1] defined the following sequence spaces

$$l_\infty(u, \triangle, p) = \{x = (x_k) \in \omega : (|u_k \triangle x_k|) \in l_\infty(p)\},$$

$$c(u, \triangle, p) = \{x = (x_k) \in \omega : (|u_k \triangle x_k|) \in c(p)\},$$

$$c_0(u, \triangle, p) = \{x = (x_k) \in \omega : (|u_k \triangle x_k|) \in c_0(p)\},$$

where $u \in U$,$p = (p_k)$ be any sequence of positive reals.
After then again Vakeel.A.Khan and Lohani[21] defined the following sequence spaces

$$l_\infty(u, \triangle, F, p) = \{x = (x_k) \in \omega : \sup_{k\geq 0} (f_k(|u_k \triangle x_k|))^{p_k} < \infty\},$$

$$c(u, \triangle, F, p) = \{x = (x_k) \in \omega : \lim_{k\to\infty} (f_k(|u_k \triangle x_k - l|))^{p_k} = 0, l \in \mathbb{C}\},$$

$$c_0(u, \triangle, F, p) = \{x = (x_k) \in \omega : \lim_{k\to\infty} (f_k(|u_k \triangle x_k|))^{p_k} = 0\},$$

Ref.

[6] Garling,D.J.H. *On Symmetric Sequence Spaces*,**Proc.London. Math.Soc.16(1966), 85-106.**

which are paranormed spaces paranormed with

$$Q(x) = \sup_{k\geq 0}(f_k(|u_k \triangle x_k|))^{p_k})^{\frac{1}{H}} \leq a$$

where $H = max(1, \sup_{k\geq 0} p_k)$ and $a = f_k(l), l = \sup_{k\geq 0}(|u_k \triangle x_k|)$.

Esi and Isik[2] defined the sequence spaces

$$l_\infty(\triangle_v^m, s, p) = \{x = (x_k) \in \omega : \sup \lim_k k^{-s}|\triangle_v^m x_k|^{p_k} < \infty, s \geq 0\},$$

$$c(\triangle_v^m, s, p) = \{x = (x_k) \in \omega : k^{-s}|\triangle_v^m x_k - L|^{p_k} \to 0(k \to \infty), s \geq 0, \text{forsome L}\},$$

$$c_0(\triangle_v^m, s, p) = \{x = (x_k) \in \omega : k^{-s}|\triangle_v^m x_k|^{p_k} \to 0(k \to \infty), s \geq 0\},$$

where $v = (v_k)$ is any fixed sequence of non zero complex numbers, $m \in \mathbb{N}$ is a fixed number,

$$\triangle_v^0 x_k = (v_k x_k), \quad \triangle_v x_k = (v_k x_k - v_{k+1} x_{k+1})$$

and

$$\triangle_v^m x_k = (\triangle_v^{m-1} x_k - \triangle_v^{m-1} x_{k+1})$$

and so that

$$\triangle_v^m x_k = \sum_{i=0}^{m} (-1)^i \begin{bmatrix} m \\ i \end{bmatrix} \quad v_{k+i} x_{k+i}.$$

When s=0, m=1, v=(1,1,1,.......) and $p_k = 1$ for all $k \in \mathbb{N}$, they are just $l_\infty(\triangle), c(\triangle)$ and $c_0(\triangle)$ defined by Kizmaz[9].
When s=0 and $p_k = 1$ for all $k \in \mathbb{N}$, they are the following sequence spaces defined by Et and Esi[5]

$$l_\infty(\triangle_v^m) = \{x = (x_k) \in \omega : (\triangle_v^m x_k) \in l_\infty\},$$

$$c(\triangle_v^m) = \{x = (x_k) \in \omega : (\triangle_v^m x_k) \in c\},$$

$$c_0(\triangle_v^m) = \{x = (x_k) \in \omega : (\triangle_v^m x_k) \in c_0\}.$$

II. Main Results

In this article we introduce the following classes of sequence spaces.

$$l_\infty(u, \triangle_v^m, F, p) = \{x = (x_k) \in \omega : \sup_{k\geq 0}(f_k(|u_k \triangle_v^m x_k|))^{p_k} < \infty\},$$

$$c(u, \triangle_v^m, F, p) = \{x = (x_k) \in \omega : \lim_{k\to\infty}(f_k(|u_k \triangle_v^m x_k - l|))^{p_k} = 0, l \in \mathbb{C}\},$$

$$c_0(u, \triangle_v^m, F, p) = \{x = (x_k) \in \omega : \lim_{k\to\infty}(f_k(|u_k \triangle_v^m x_k|))^{p_k} = 0\},$$

Ref.

[2] Esi.A.,Isik,M. Some generalized difference sequence spaces,$Thai.J.Math.$,3(2),(2005)241-247.

Theorem 2.1. $l_\infty(u, \triangle_v^m, F)$ is a Banach space with norm

$$||x||_{(\triangle_v^m)_u} = \sup_{k\geq 0}(f_k(|u_k\triangle_v^m x_k|)) \leq \alpha,$$

where $\alpha = f_k(l)$ and $l = \sup_{k\geq 0}(|u_k\triangle_v^m x_k|)$.

Proof. Let (x^i) be a cauchy sequence in $l_\infty(u, \triangle_v^m, F)$ for each $i \in \mathbb{N}$.
Let r, x_0 be fixed. Then for each $\frac{\epsilon}{rx_0} > 0$ there exists a positive integer $\mathbb{N}$ such that

$$||x^i - x^j||_{(\triangle_v^m)_u} < \frac{\epsilon}{rx_0} \quad \text{for all i,j} \geq \mathbb{N}$$

Using the definition of norm, we get

$$\sup_{k\geq 0} f_k\left(\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{||x^i - x^j||_{(\triangle_v^m)_u}}\right) \leq \alpha, \quad \text{for all i,j} \geq \mathbb{N}$$

ie,

$$f_k\left(\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{||x^i - x^j||_{(\triangle_v^m)_u}}\right) \leq \alpha, \quad \text{for all i,j} \geq \mathbb{N}$$

Hence we can find $r > 0$ with $f_k(\frac{rx_0}{2}) \geq \alpha$ such that

$$f_k\left(\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{||x^i - x^j||_{(\triangle_v^m)_u}}\right) \leq f_k\left(\frac{rx_0}{2}\right)$$

$$\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{||x^i - x^j||_{(\triangle_v^m)_u}} \leq \frac{rx_0}{2}$$

This implies that

$$|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)| \leq \frac{rx_0}{2}\frac{\epsilon}{rx_0} = \frac{\epsilon}{2}$$

Since $u_k \neq 0$ for all k, we have

$$|\triangle_v^m x_k^i - \triangle_v^m x_k^j| \leq \frac{\epsilon}{2} \quad \text{for all i,j} \geq \mathbb{N}$$

Hence $(\triangle_v^m x_k^i)$ is a cauchy sequence in $\mathbb{C}$
For each $\epsilon > 0$ there exists a positive integer $\mathbb{N}$ such that $|\triangle_v^m x_k^i - \triangle_v^m x_k| < \epsilon$ for all $i > \mathbb{N}$.
Using the continuity of $F = (f_k)$ we can show that

$$\sup_{k\geq \mathbb{N}} f_k(|u_k(\triangle_v^m x_k^i - \lim_{j\to\infty} \triangle_v^m x_k^j)|) \leq \alpha,$$

Thus

$$\sup_{k\geq \mathbb{N}} f_k(|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k)|) \leq \alpha,$$

Notes

Notes

since $(x^i) \in l_\infty(u, \triangle_v^m, F)$ and $F = (f_k)$ is continuous it follows that $x \in l_\infty(u, \triangle_v^m, F)$
Thus $l_\infty(u, \triangle_v^m, F)$ is complete.

Theorem 2.2. $l_\infty(u, \triangle_v^m, F, p)$ is a complete paranormed space with

$$Q_u(x) = \sup_{k\geq 0}(f_k(|u_k \triangle_v^m x_k|)^{p_k})^{\frac{1}{H}} \leq \alpha$$

where $H = max(1, \sup_{k\geq 0} p_k)$ and $\alpha = f_k(l)$, $l = \sup_{k\geq 0}(|u_k \triangle_v^m x_k|)$.

Proof. Let (x^i) be a cauchy sequence in $l_\infty(u, \triangle_v^m, F, p)$ for each $i \in \mathbb{N}$.
Let $r > 0, x_0$ be fixed.Then for each $\frac{\epsilon}{rx_0} > 0$ there exists a positive integer $\mathbb{N}$ such that

$$Q_u(x^i - x^j)_{(\triangle_v^m)_u} < \frac{\epsilon}{rx_0} \quad \text{for all i,j} \geq \mathbb{N}$$

Using the definition of paranorm, we get

$$\sup_{k\geq 0} f_k(\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{Q_u(x^i - x^j)_{(\triangle_v^m)_u}})^{\frac{p_k}{H}} \leq \alpha, \quad \text{for all i,j} \geq \mathbb{N}$$

ie,

$$f_k(\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{Q_u(x^i - x^j)_{(\triangle_v^m)_u}})^{p_k} \leq \alpha, \quad \text{for all i,j} \geq \mathbb{N}$$

Hence we can find $r > 0$ with $f_k(\frac{rx_0}{2}) \geq \alpha$ such that

$$f_k(\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{Q_u(x^i - x^j)_{(\triangle_v^m)_u}}) \leq f_k(\frac{rx_0}{2})$$

$$\frac{|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)|}{Q_u(x^i - x^j)_{(\triangle_v^m)_u}} \leq \frac{rx_0}{2}$$

This implies that

$$|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k^j)| \leq \frac{rx_0}{2}\frac{\epsilon}{rx_0} = \frac{\epsilon}{2}$$

Since $u_k \neq 0$ for all k, we have

$$|\triangle_v^m x_k^i - \triangle_v^m x_k^j| \leq \frac{\epsilon}{2} \quad \text{for all i,j} \geq \mathbb{N}$$

Hence $(\triangle_v^m x_k^i)$ is a cauchy sequence in $\mathbb{C}$
For each $\epsilon > 0$ there exists a positive integer $\mathbb{N}$ such that $|\triangle_v^m x_k^i - \triangle_v^m x_k| < \epsilon$ for all $i > \mathbb{N}$.
Using the continuity of $F = (f_k)$ we can show that

$$\sup_{k\geq \mathbb{N}} f_k(|u_k(\triangle_v^m x_k^i - \lim_{j\to\infty} \triangle_v^m x_k^j)|)^{\frac{p_k}{H}} \leq \alpha,$$

Thus

$$\sup_{k\geq N} f_k(|u_k(\triangle_v^m x_k^i - \triangle_v^m x_k)|)^{\frac{p_k}{H}} \leq \alpha,$$

since $(x^i) \in l_\infty(u, \triangle_v^m, F, p)$ and $F = (f_k)$ is continuous it follows that $x \in l_\infty(u, \triangle_v^m, F, p)$
Thus $l_\infty(u, \triangle_v^m, F, p)$ is complete.

Theorem 2.3. Let $0 < p_k \leq q_k < \infty$ for each k. Then we have

$$c_0(u, \triangle_v^m, F, p) \subseteq c_0(u, \triangle_v^m, F, q)$$

Proof. Let $x \in c_0(u, \triangle_v^m, F, p)$ that is

$$\lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|))^{p_k} = 0$$

This implies that

$$f_k(|u_k(\triangle_v^m x_k)|) \leq 1$$

for sufficiently large k, since modulus function is non decreasing.
Hence we get

$$\lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|))^{q_k} \leq \lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|))^{p_k} = 0$$

Therefore $x \in c_0(u, \triangle_v^m, F, q)$.

Theorem 2.4.(a) Let $0 < \inf p_k \leq p_k \leq 1$. Then we have

$$c_0(u, \triangle_v^m, F, p) \subseteq c_0(u, \triangle_v^m, F).$$

(b) Let $1 \leq p_k \leq \sup_k p_k < \infty$. Then we have

$$c_0(u, \triangle_v^m, F) \subseteq c_0(u, \triangle_v^m, F, p).$$

Proof.(a) Let $x \in c_0(u, \triangle_v^m, F, p)$, that is

$$\lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|))^{p_k} = 0$$

Since $0 < \inf p_k \leq p_k \leq 1$,

$$\lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|)) \leq \lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|))^{p_k} = 0$$

Hence $x \in c_0(u, \triangle_v^m, F)$.
(b)Let $p_k \geq 1$ for each k and $\sup_k p_k < \infty$.
Suppose that $x \in c_0(u, \triangle_v^m, F)$.
Then for each $\epsilon > 0$ there exists a positive integer N such that

$$f_k(|u_k(\triangle_v^m x_k)|) \leq \epsilon \quad \text{for all } k \geq N$$

Notes

Since $1 \leq p_k \leq \sup_k p_k < \infty$, we have

$$\lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|))^{p_k} \leq \lim_{k\to\infty} (f_k(|u_k(\triangle_v^m x_k)|)) \leq \epsilon < 1$$

Therefore $x \in c_0(u, \triangle_v^m, F, p)$.

Notes

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Notes