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Resolution Limits of Continuous Phase Filters Beyond the Diffraction Limit

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Resolution Limits of Continuous Phase Filters Beyond the Diffraction Limit

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I. INTRODUCTION

A detailed knowledge of the intensity profile at and near the geometrical focus of an optical system is highly desirable for the imaging and focusing optics. Focusing of light into small spots leads to high resolution and concentration of light flux. This can be achieved by introducing suitable apodizers in the optical systems as super resolving filters. Super resolving filters have found many practical applications in diverse potential fields such as image processing, confocal microscopy, spectroscopy, microscopy, laser printing, optical data storage, lithography, medical imaging and microelectronics.

Many methods [1-12] have been proposed for the design of apodizer structure to achieve superresolution based on variable transmittance, phase-only profile and both amplitude-phase profiles. In recent years, considerable attention has been drawn by phase only filters since they offer more advantages than transmittance filters such as best performance, more simplicity and easy to manufacture. Keeping in view all these things and with the aim of achieving superresolution we have designed and introduced continuous phase-only filters at the exit pupil to modify the optical field distribution that comes to focus in the geometrical focal region. Our designing procedure is based on the figures of merit that govern the superresolving performance of the final PSF of the optical system [3].

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The figures of merit that we have selected are the transverse gain, G_T , axial gain, G_A , and Strehl ratio, S . G_T and G_A gives a measure of the superresolution performance in the transverse and axial directions, respectively. These factors are unity for unmodified pupil. The filter is said to be superresolver if they are more than unity and apodizer if they are less than unity. Finally, Strehl ratio is an important image quality assessment parameter and is defined as the ratio of the central core irradiance of the superresolving pattern to that an unobstructed pupil. It is a well-known fact that the image of a point object is not a point, but the diffraction pattern of non-zero dimensions. The central part of the diffraction pattern is usually considered as an image of the object and it determines one of the principal performances of the optical imaging system. If two such image diffraction patterns overlap, it will be very difficult to detect the presence of two objects. Hence it is desirable to have criteria of resolution, which serves the purpose of image quality assessment and comparing the efficiency of various optical imaging systems.

Here we will explore the two-point resolution of optical systems in terms of intensity distribution of resultant image and the Sparrow limits. The most widely used are the Rayleigh and Sparrow resolution criteria. As the Sparrow [13] criterion is independent of the degree of coherence of illumination of the two points, it has been used very extensively. This criterion originally has been proposed for two points of equal intensity. Asakura [14] has modified the Sparrow criterion to be applicable for the more practical case of unequally bright object points. In the present study this modified Sparrow criteria has been applied to the rotationally symmetric optical systems in which superresolving filters are introduced. The two-point Sparrow limits of resolution have been obtained for designed phase filter in various situations. Our studies are very important in all imaging situations where the resolution of two-point objects has to be considered. It has practical significance also in Astronomy in the domain of resolution of visual binary stars.

II. THEORY

By adopting the line of approach of Hopkins and Barham [15], the expression for the composite image intensity distribution in the image plane of an

apodized optical system, as a function of the reduced co-ordinate Z , is given by,

$$I(Z) = |G(Z+B)|^2 + \alpha |G(Z-B)|^2 + 2\sqrt{\alpha} \gamma(Z_0) |G(Z+B)| |G(Z-B)| \quad (1)$$

Where $2B=Z_0$ is the separation between the two object points, α is the ratio of their intensities, $\gamma(Z_0)$ is the degree of spatial coherence of the illumination. Z is the dimensionless diffraction variable. $\alpha = 1$ gives the case of equal intensities while $\alpha \neq 1$ corresponds to unequally bright object points. The coherent and the incoherent extremes of illuminations are given by $\gamma = 1$ and $\gamma = 0$, respectively. The $0 < \gamma < 1$ is for the partially coherent illumination. $I(Z)$ is an image intensity distribution as a function of Z , which is measured from the axis of the optical system. Z is the dimensionless diffraction variable. $G(Z+B)$ and $G(Z-B)$ are the normalized amplitude impulse response functions of the optical imaging system corresponding to the object points, each of which is situated at equal distance $B = Z_0/2$, on either side of the optical axis and are given by,

$$G(Z \pm B) = 2 \int_0^1 J_0[(Z \pm B)r] r dr \quad (2)$$

Where J_0 is the Bessel function of first kind of order zero, r is the normalized distance of a general point on the circular exit pupil varying from 0 to 1. It should be noted that the above equations (2) give the amplitude impulse response functions at the Gaussian focal plane. However, at the defocused plane the expressions for the amplitude impulse response functions with a generalized pupil function can be expressed as

$$G(Z \pm B, u) = 2 \int_0^1 f(r) J_0[(Z \pm B)r] \exp(-iur^2/2) r dr \quad (3)$$

Where u is the defocusing parameter whose value specifies the out-of-focus plane and $f(r)$ is the generalized pupil function and it can be expressed as

$$f(r) = P(r) \exp[i\phi(r)] \quad (4)$$

Where $P(r)$ is the amplitude transmittance function and $f(r)$ is the phase function. Following Sheppard and Hegedus [12] and Juana, Oti, Canales and Cagigal [3], we have designed

The simple phase-only filters based on the definitions of a series of figures of merit. The expressions for these figures of merit are analogous to the expressions developed by them and they can be expressed in terms of moments of the pupil function, I_n as follows.

$$G_T = 2 \frac{\text{Re}(I_0 I_1^*) - u_F (I_m I_2^*)}{S} \quad (5)$$

$$G_A = 12 \frac{\text{Re}(I_2 I_0^*) - |I_1|^2}{S} \quad (6)$$

Where u_F is the displacement of focus in the axial direction from the geometrical focus, and S is the Strehl ratio. They can be expressed as

$$u_F = -2 \frac{I_m(I_0 I_1^*)}{\text{Re}(I_2 I_0^*) - |I_1|^2} \quad (7)$$

$$S = |I_0|^2 - u_F I_m(I_0 I_1^*) \quad (8)$$

The n th moment of the pupil function is defined as

$$I_n = \int_0^1 f(t) t^n dt \quad (9)$$

To reduce the spot size and subsequently to increase the resolving power of the optical system, we have introduced continuous phase-only filters as superresolvers in the optical system, and we have chosen a phase function as,

$$\phi(r) = a \sin(2\pi br) - r/2 \quad (10)$$

Where a and b are the coefficients to be fitted and controlling the degree of non-uniformity of the phase transformation over the exit pupil. These parameters will be fitted from the following system of equations:

$$|u_F(a, b)| \leq u_F^0 \quad (11)$$

$$G_T(a, b) - G_T^0 = 0 \quad (12)$$

$$S(a, b) - S^0 = 0 \quad (13)$$

To fit the filter coefficients for the desired values of the superresolution parameters G_T^0 and S^0 we used MATHEMATICA to solve this system of equations for the maximum amount of defocus u_F^0 is allowed.

Figure 1 shows the phase variation of the designed filters from the center to edge of the pupil. The figure reveals that the phase increases from center towards edge as the pupil coordinate increases from 0 to 0.8 and then decreases there onwards. This variation is very small for F1 and F2 whereas it is high in the case of F3 and F4.

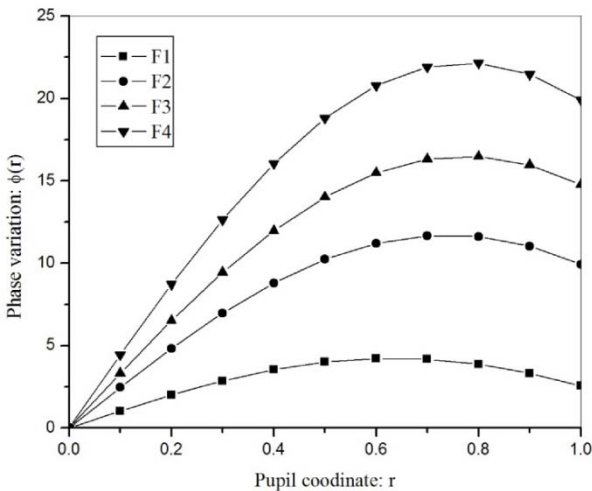


Figure 1. Phase variation as a function of pupil coordinate for different phase filters (F1 - F4).

$$I(Z) = |G(Z+B, u)|^2 + \alpha |G(Z-B, u)|^2 + 2\sqrt{\alpha} \gamma(Z_0) |G(Z+B, u)| |G(Z-B, u)| \quad (14)$$

According to the Asakura's modified Sparrow criterion [5] both the second and the first derivatives of the resultant image intensity $I(Z)$, vanish for a particular separation of objects which now gives the Sparrow limit Z_1^0 . Mathematically, the modified Sparrow criterion may be expressed as follows,

$$\left| \frac{\partial^2 I(Z)}{\partial Z^2} \right|_{Z=Z_1^0} = 0 \quad (15)$$

$$\left| \frac{\partial I(Z)}{\partial Z} \right|_{Z=Z_1^0} = 0 \quad (16)$$

III. RESULTS AND DISCUSSIONS

a) Design of phase filters

With the goal to reduce the spot size we designed four continuous phase-only filters by adopting the procedure as mentioned in previous section. In order to examine the performance of the designed filters, the intensity PSF is generated for a single point from the equation (3) as a function of diffraction parameter Z . The computational method based on the twelve point Gauss quadrature.

In addition to the consider superresolution parameters we have also calculated other parameters that measure the degree of super resolution performance of the designed filters. They are the spot size, G , the side lobe intensity, I_s , side lobe to main lobe intensity ratio, M , and the full width at half maximum, FWHM. These parameters are very much related to the superresolution performance of the filter and also image quality assessment.

They may be defined as follows:

G : is defined as the ratio of the first minimum of the superresolved pattern to that of the Airy pattern.

Expressions for the point spread functions, represented by the equation (3), equation (7) and (10) have been used in the evaluation of the intensity distribution $I(Z)$ in the composite image of a two-point object given by the equation

M : It is defined as the ratio of the first side lobe intensity to that of the corresponding main lobe.

$FWHM$: It is defined as twice the value of Z for which the intensity PSF is 0.5 times of the peak intensity. It becomes a very important image assessment parameter whenever the PSF value does not attain a zero minimum.

Table 1 shows the parameters for four filters. For each filter, the pair of coefficients (a , b) that govern the shape of the filter are obtained for the desired values of superresolution parameters G_T^0 and S^0 for the maximum amount of defocus u_F^0 is allowed. The FWHM is minimum for the filter F4 and it is 1.34 whereas in the case of Airy pupil it is 1.616. This indicates improvement in resolution.

Table 1 : Design and Performance of Phase Filters

Parameter	Filters			
	F1	F2	F3	F4
a	4.557	12.088	16.891	22.560
b	0.383	0.334	0.320	22.560
U_f	0.91	0.82	0.72	0.72
G_T	1.18	1.38	1.36	1.40
G	0.89	0.79	0.77	0.75
S	0.65	0.38	0.32	0.26
I_s	0.053	0.050	0.043	0.038
M	0.080	0.132	0.131	0.148
FWHM	1.523	1.394	1.361	1.340

Figure 2 shows the superresolving PSFs of these four filters in addition to Airy PSF. It can be observed that both the spot size and the Strehl ratio are

decreasing together with the slight increase in the side lobe intensity. It means an increase in resolution is accompanied by a decrease of the Strehl ratio and an increase of the side lobe intensity. One more thing we can observe that the full width at half maxima decreased in the intensity PSF for all the filters. For filter F5 this is 1.34

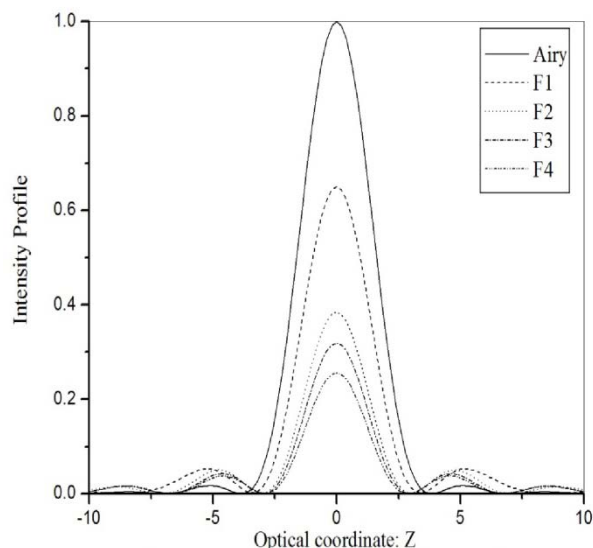


Figure 2. Superresolution PSFs of circular pupil obtained for designed phase filters (F1 - F4) in addition to Airy PSF.

b) Two-point resolution

We have examined the performance of designed filters in terms of two-point resolution. By means of Eq. (14) the intensity profiles in the composite image of two-point objects formed by the optical imaging system have been obtained as a function of diffraction parameter Z by employing a twelve-point Gauss quadrature numerical method of integration. An iterative method has been developed and applied to find the Sparrow limit $\delta_s = Z_0^{-1}$. Table 2 lists the critical Sparrow limits for different amount of the intensity ratio α and the degree of coherence γ for each filter. For widely varying intensities of the two points with $\alpha=0.2$, the limit increases from 3.822 to 5.127 for the unobstructed (Airy) case as the degree of coherence goes up from 0 to 1. These values are 2.501 and 4.3, respectively, for a filter F4. The limit increases with the degree of coherence. However, the percentage of increase of limit decreases with the phase filter. It means for any amount of degree of coherence, the limit decreases by the presence of phase filter. It should be noted that as the limit of resolution, $\delta_s = Z_0^{-1}$, decreases, the overall optical resolution increases. Similar trend is noticed for all other filters. For equally bright points ($\alpha = 1$), the Sparrow limit is lesser, hence, resolution is higher in the case of first two filters F1 and F2. For incoherent illumination the sparrows limits are 2.858 and 2.657 whereas in the case of coherent light they are 4.585 and 4.429, respectively. Filter F3 and F4 showing opposite trend by slightly increasing in their resolution limit. For these filters

incoherent limits are 2.60 and 2.566 and coherent limits are 4.351 and 4.319, respectively.

Table 2 : Sparrow critical limit for various combinations of intensity ratio (α) and degree of coherence (γ) for phase filters as well as for Airy pupil (A)

α	γ	A	Filter			
			F1	F2	F3	F4
0.2	0.000	3.822	3.266	2.701	2.581	2.501
	0.200	4.202	3.671	2.995	2.901	2.764
	0.400	4.551	4.049	3.442	3.323	3.148
	0.600	4.871	4.412	3.872	3.757	3.615
	0.800	5.111	4.750	4.329	4.224	4.086
	1.000	5.127	4.846	4.590	4.442	4.300
0.4	0.000	3.691	3.392	3.007	2.924	2.856
	0.200	4.041	3.746	3.359	3.270	3.201
	0.400	4.369	4.088	3.705	3.612	3.541
	0.600	4.678	4.426	4.063	3.966	3.893
	0.800	4.949	4.753	4.449	4.351	4.279
	1.000	5.120	4.987	4.741	4.608	4.521
0.6	0.000	3.511	3.298	2.995	2.920	2.868
	0.200	3.850	3.638	3.325	3.245	3.191
	0.400	4.172	3.971	3.655	3.570	3.513
	0.600	4.482	4.302	3.997	3.907	3.849
	0.800	4.773	4.634	4.365	4.272	4.215
	1.000	5.014	4.930	4.722	4.617	4.557
0.8	0.000	3.306	3.143	2.890	2.823	2.780
	0.200	3.639	3.476	3.211	3.138	3.092
	0.400	3.959	3.803	3.532	3.454	3.406
	0.600	4.272	4.133	3.866	3.783	3.734
	0.800	4.579	4.473	4.230	4.144	4.095
	1.000	4.868	4.815	4.632	4.541	4.495
1.0	0.000	2.974	2.858	2.657	2.600	2.566
	0.200	3.303	3.185	2.971	2.907	2.870
	0.400	3.622	3.507	3.285	3.217	3.177
	0.600	3.939	3.837	3.614	3.541	3.500
	0.800	4.265	4.187	3.978	3.900	3.860
	1.000	4.609	4.585	4.429	4.351	4.319

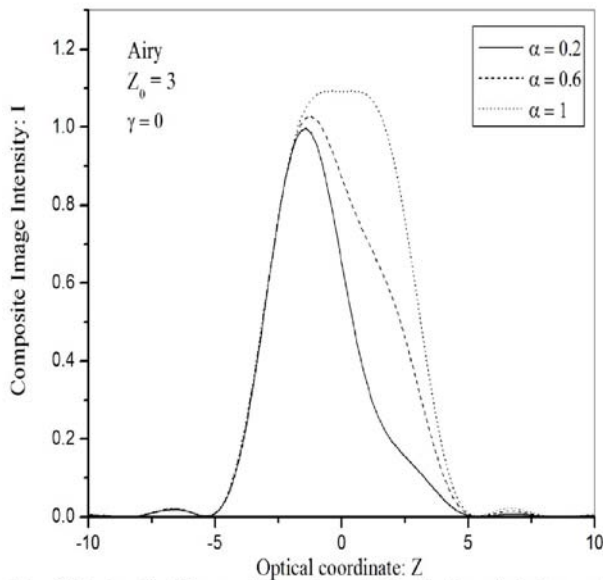


Figure 3. Intensity profile of the composite image of two point objects for Airy pupil when they are separated by the distance $Z_0 = 3$ with different intensities and illuminated with incoherent light.

Figure 3 shows the intensity profile of the incoherent composite image of the two-point objects separated by a distance $Z_0 = 3$ with different intensity ratio α . These are formed at geometrical focus by the unobstructed optical system. As figure shows the only equally bright point objects, $\alpha = 1$, are just resolved in the Sparrow sense. The dip just vanished by merging two peaks at the center. For the object separation now, the first and the second derivative of the intensity vanish.

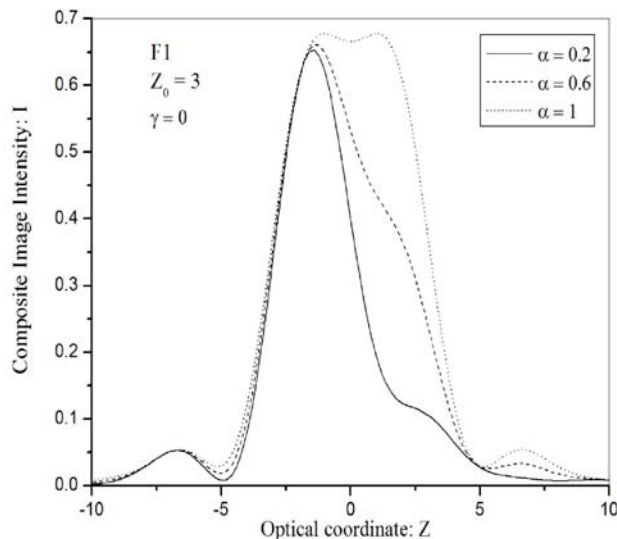


Figure 4. Intensity profile in the composite image of two point objects as a function of intensity ratio of two point objects for filter F1 when incoherent illumination is used.

A flat top to the intensity distribution curve can be considered as an absolute Sparrow limit.

The effect of the phase filters F1 and F4 on the intensity profile of the incoherent composite image of the same two-point objects, under the same conditions as in previous case of Figure 3 but the focal regions are

$U_F = 0.91$ and 0.72 , respectively, has been depicted in figure 4 and 5. In figure 4 the clear dip shows that the equally bright points, $\alpha = 1$, are well resolved whereas that of the object points with widely varying intensities, $\alpha = 0.2$, are in the neighborhood of being resolved while there is no dip or flat top and hence no resolution of the object points with intensity ratio $\alpha = 0.6$. An interesting observation may be made in this figure that the resolution limit first increasing and then decreasing with the intensity ratio α . Hence, this filter F1 is comfortable to resolve the equally bright or widely varying intensity objects under these conditions. The dip in each curve of the intensity profiles in figure 5 indicating the degree of superresolution nature of the filter F5. The object points are well resolved whatever the intensity ratio may be.

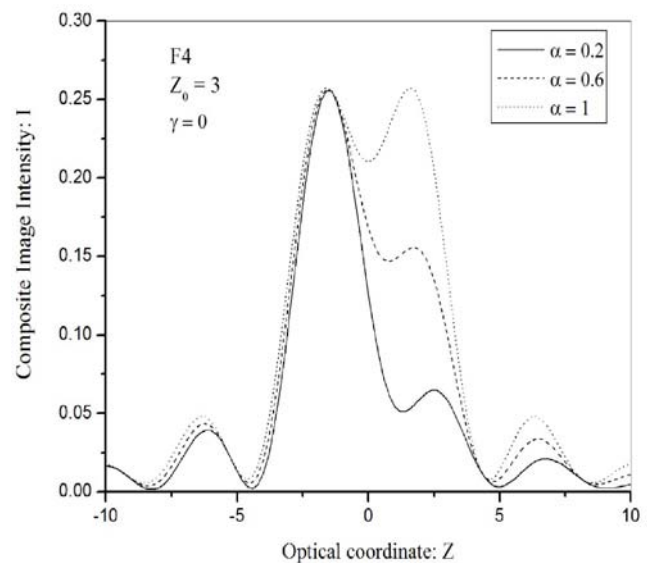


Figure 5. Intensity profile in the composite image of two point objects obtained for filter F5.

For all designed filters, the variation of Sparrow limit $\delta_s = Z_0^{-1}$ with the degree of coherence γ for unequal and the equally bright object points has been shown in figs. 6(a) and 6(b). Almost a linear relation is seen as γ is varying from 0 to 0.8 which can be used to find the degree of coherence. The resolution is degraded with coherence. For $\alpha = 0.2$, the variation of limit is very small as γ is increasing from 0.8 to 1. For any amount of degree of coherence, filter F4 has lesser limit and hence a higher resolution as compared to Airy pupil as well as other filters. This can be seen in more detail from figure 6(a). Figure 6(b) reveals that filters F3 and F4 have almost same limits for equally bright object points. Unlike unequal bright points, in this case there is a large variation in the resolution limit for small change in γ as it increasing from 0.8 to 1. However, the resolution is high for all these filters than Airy pupil even in this region of coherence.

Figure 7(a) and (b) show the variation of Sparrow limit $\delta_s = Z_0^{-1}$ with the intensity ratio α for fully

incoherent and coherent illumination, respectively. Both the figures reveal the anomalous behavior of Sparrow limit with the intensity ratio α . In both the cases, It is noticed that the filters F3 and F4 have lesser limit for unequal bright object points, $\alpha = 0.2$, than that of the equally bright object points $\alpha = 1$.

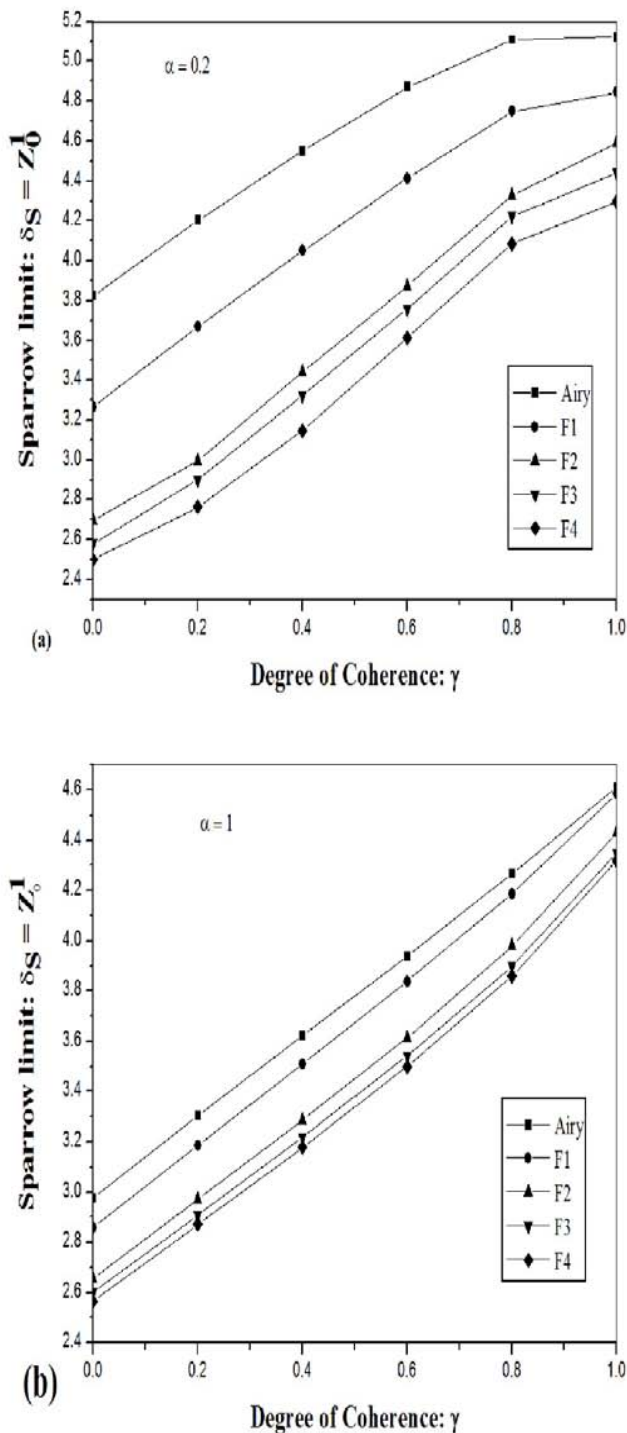


Figure 6. Variation of Sparrow limit for filters F1 - F4. (a) as a function of degree of coherence γ when the

point objects are unequal bright, $\alpha = 0.2$. (b) when the point objects are equally bright, $\alpha = 1$.

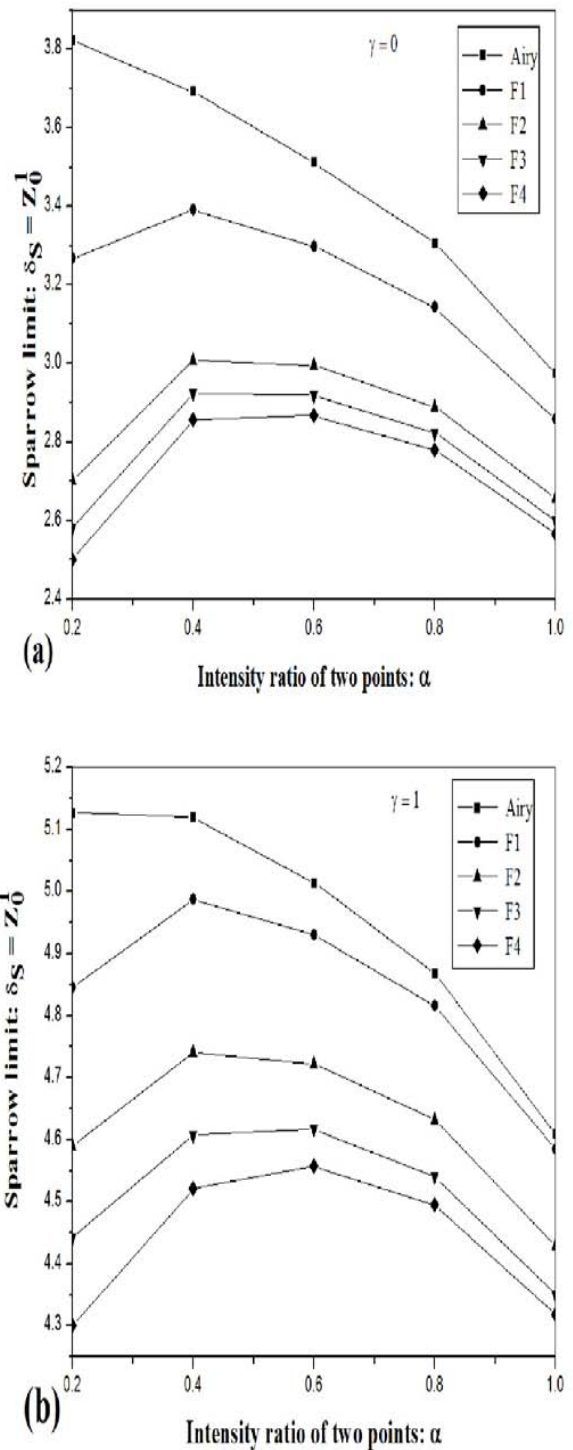


Figure 7. Variation of Sparrow limit for filters F1 - F4. (a) as a function of intensity ratio α for when the point objects are illuminated with incoherent light, $\gamma = 0$. (b) when the point objects are illuminated with coherent light, $\gamma = 1$.

IV. CONCLUSION

In conclusion we may emphasize that the continuous phase filters are efficient in reducing spot size thereby enhancing the resolution of an optical system for desired values of superresolution parameters. In addition to that they are simple and high tolerance to errors at low cost and also suitable for mass production. The critical Sparrow limit is found to let down by the presence of continuous phase filters compared to Airy pupil whatever the degree of coherence. This means that the presence of phase pupil functions leads to an improvement in the resolution. For two extreme cases of equally, $\alpha = 1$, and widely varying, $\alpha = 0.2$, bright object points the resolution is high. Increase in the degree of coherence lower the resolution with and without modified pupil. When single peak is obtained in the composite image, the peak is not at $Z = 0$. It is asymmetrically situated. Finally, we notice that the highest resolution is possible for filter F5 when widely varying intensity object points are illuminated by the incoherent light. The same is true for equally bright object points also but with slight degradation in the resolution.

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