Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 13 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

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GJSFR-F Classification : *1991 MSC: Primary 54C40, 14E20; Secondary 46E25, 20C20*

TRIGONOMETRICANDWAVELETTRANSFORMSFORCERTAINCLASSOFGENERALIZEDFUNCTIONS

Trigonometric and Wavelet Transforms for Certain Class of Generalized Functions

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Abstract - In this article, we discuss certain class of generalized functions for sine and cosine transforms. we, also obtain a new relationship between Fourier sine transform (Fourier cosine transform) and wavelet transform. Other related theorems are also established in concern.

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I. Introduction

A function $g(t)$ is said to be a mother wavelet (or simply, a wavelet) if it satisfies the following conditions:

$$(i)\int_{\mathcal{R}} g(t)\,dt=0;\,(ii)\int_{\mathcal{R}} |g(t)|^2\,dt=0;\,(iii)\int_{\mathcal{R}} \frac{|g(t)|^2}{t}dt<\infty.$$

The continuous wavelet transform of $f(t)$ with respect to a mother wavelet $g(t)$ is defined by $[2,8]$

$$\mathcal{W}_g f(b,a)=\int_{\mathcal{R}} f(t)\frac{1}{\sqrt{|a|}}\overline{g}\left(\frac{t-b}{a}\right)dt.$$

Let $\mathcal{S}(\mathcal{R})$ denote the space of rapid descent and $\mathcal{S}'(\mathcal{R})$ its strong dual of tempered distributions over $\mathcal{R}$, the set of real numbers $[5]$. Due to $[6]$, $\mathcal{S}_+(\mathcal{R})$ is the set of all those functions in $\mathcal{S}(\mathcal{R})$ whose Fourier transform is supported by the positive frequencies and $\mathcal{S}_0(\mathcal{R})$ is the space of all those functions from $\mathcal{S}(\mathcal{R})$ for which all moments vanish. That is, $g\in\mathcal{S}_0(\mathcal{R})$ if $g\in\mathcal{S}(\mathcal{R})$ and $\left|x^\alpha g^{(\beta)}(x)\right|<\infty$ and $\int_{\mathcal{R}} x^n g(x)\,dx=0$. Further, if g is a wavelet in $\mathcal{S}_0(\mathcal{R})$ and $\Delta=\{(b,a):b\in\mathcal{R},a>0\}$, then the wavelet transform for $f\in\mathcal{S}(\mathcal{R})$ is defined by $[6]$

$$\mathcal{W}_g f(b,a)=\left\langle f(t),\overline{g_{b,a}(t)}\right\rangle,(b,a)\in\Delta,t\in\mathcal{R}.$$

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where,

$$g_{b,a}(t) = \mathcal{T}_b \mathcal{D}_a g(t) = \frac{1}{a} g\left(\frac{t-b}{a}\right), t, b \in \mathcal{R}, a > 0,$$

$\mathcal{T}_b$ and $\mathcal{D}_a$ stands for the translation and dilation of the wavelet g .

The Fourier cosine transform of a complex-valued absolutely integrable function $f(t)$ over $(-\infty, \infty)$ is the function of the variable ξ given by [9, p.42]

$$\mathcal{F}_{cn}(f(t))(\xi) = \int_{\mathcal{R}} f(t) \cos t\xi dt.$$

Fourier sine transform $\mathcal{F}_{sn}$ has similar representation.

Fourier sine and cosine transforms being the imaginary and real parts of the complex Fourier transform, their properties for sine and cosine transform can be obtained from the known properties of the Fourier transform. Furthermore, the product of different combinations of even and odd functions is an odd function and the product of similar combinations of even and odd functions is an even function, establishes that the Fourier transform of an even function is, indeed, a Fourier cosine transform and similarly, the Fourier transform of an odd function in is a Fourier sine transform.

Our approach in this work is infact a motivation of [8] which describe a relationship between the cited trigonometric transforms, Fourier sine and Fourier cosine transforms, and the wavelet transform in the sense of generalized functions. However, the approach in this paper seems different and interesting.

II. Wavelet Transform of Ultradifferentiable Functions

By an ultradifferentiable function we mean a C^{∞} function whose derivatives satisfy certain growth conditions as the order of the derivatives increase [1, 2, 3, 4].

Denote by $\mathcal{U}_o(n_i, n, \mathcal{R})$ and $\mathcal{U}_e(n_i, n, \mathcal{R})$ the spaces of all odd (resp. even) C^{∞} functions $\phi(x)$ on $\mathcal{R}$ such that $|x^i \mathcal{D}_x^j \phi(x)| < N n^i n_j$, $j = 1, 2, 3, ...$ for some positive constant N.

Functions in $\mathcal{U}_o(n_i, n, \mathcal{R})$ and $\mathcal{U}_e(n_i, n, \mathcal{R})$ are , indeed, ultradifferentiable functions and decrease to zero faster than every power of $1/x$. Denoting by $\acute{\mathcal{U}_o}(n_i, n, \mathcal{R})$ and $\acute{\mathcal{U}_e}(n_i, n, \mathcal{R})$ the duals of the corresponding spaces. The resulting space is called the odd (resp. even) tempered ultradistribution spaces which contain the space $\mathcal{S}'(\mathcal{R})$ properly [5].

In view of definitions, $\mathcal{U}_o(n_i, n, \mathcal{R}) \subset \mathcal{S}(\mathcal{R})$ and $\mathcal{U}_e(n_i, n, \mathcal{R}) \subset \mathcal{S}(\mathcal{R})$, we write $\mathcal{S}'(\mathcal{R}) \subset \acute{\mathcal{U}_o}(n_i, n, \mathcal{R})$ and $\mathcal{S}'(\mathcal{R}) \subset \acute{\mathcal{U}_e}(n_i, n, \mathcal{R})$, see [9, Theorem 2.1] .

Denote by $\mathcal{U}_o(n_i, n, \mathcal{R}, +)$ and $\mathcal{U}_e(n_i, n, \mathcal{R}, +)$ the subsets of $\mathcal{U}_o(n_i, n, \mathcal{R})$ and $\mathcal{U}_e(n_i, n, \mathcal{R})$ whose Fourier transforms are supported by the positive frequencies. Whereas, $\acute{\mathcal{U}_o}(n_i, n, \mathcal{R}, +)$ and $\acute{\mathcal{U}_e}(n_i, n, \mathcal{R}, +)$ are the respective duals of $\mathcal{U}_o(n_i, n, \mathcal{R}, +)$ and $\mathcal{U}_e(n_i, n, \mathcal{R}, +)$. In notations

$$\mathcal{U}_o(n_i, n, \mathcal{R}, +) = \left\{\phi : \phi \in \mathcal{U}_o(n_i, n, \mathcal{R}),\ \operatorname{supp} \widehat{\phi} \subset \mathcal{R}_+\right\}$$

Ref.

[9] Sneddon, I.N. *The Use of Integral Transforms*, Tata McGraw-Hill Publications Company Ltd, New Delhi, 1974.

Notes

and

$$\mathcal{U}_e(n_i,n,\mathcal{R},+)=\left\{\phi:\phi\in\mathcal{U}_e(n_i,n,\mathcal{R}),\ \operatorname{supp}\widehat{\phi}\subset\mathcal{R}_+\right\}.$$

Denoting by $\mathcal{U}_o(n_i,n,\mathcal{R},*)$ $(\mathcal{U}_e(n_i,n,\mathcal{R},*))$ the set of all $g\in\mathcal{U}_o(n_i,n,\mathcal{R})$ $(\mathcal{U}_e(n_i,n,\mathcal{R}))$ where,

$$\left|x^\alpha g^{(\beta)}(n)\right|<\infty \text{ and } \int_{\mathcal{R}} x^n g(x)\,dx=0,$$

the following propostion holds true:

Proposition 1: *Let $f\in\mathcal{U}_o(n_i,n,\mathcal{R})$ $(\mathcal{U}_e(n_i,n,\mathcal{R}))$ and $g\in\mathcal{U}_o(n_i,n,\mathcal{R},*)$ $(\mathcal{U}_e(n_i,n,\mathcal{R},*))$, then the wavelet transform of f is given by*

(1.1) $$\mathcal{W}_g f(b,a)=\left\langle f(t),\overline{g}_{b,a}(t)\right\rangle,(b,a)\in\Delta,t\in\mathcal{R},$$

where Δ has the usual meaning.

Proof : We have $\mathcal{U}_o(n_i,n,\mathcal{R})\subset\mathcal{S}(\mathcal{R})$ and $\mathcal{U}_e(n_i,n,\mathcal{R})\subset\mathcal{S}(\mathcal{R})$. Also, by the assumption that $\left|x^\alpha g^\beta(x)\right|<\infty$ and $\int\limits_{\mathcal{R}} x^n g(x)\,dx=0$ we deduce that the wavelet $g\in\mathcal{U}_o(n_i,n,\mathcal{R},*)$ and $g\in\mathcal{U}_e(n_i,n,\mathcal{R},*)$ which justifies the proposition.

Theorem 1: *Let $(b,a)\in\Delta$ and $g\in\mathcal{U}_o(n_i,n,\mathcal{R},*)$ then*

$$\mathcal{F}_{sn}(g_{b,a}(x))(\xi)=0,\ \textit{for}\ \xi\notin\mathcal{R}_+.$$

Proof : Let $g\in\mathcal{U}_o(n_i,n,\mathcal{R},*)$ then

$$\begin{aligned}\mathcal{F}_{sn}(g_{b,a}(x))(\xi)&=\int\limits_{\mathcal{R}} g_{b,a}(x)\sin\xi x\,dx\\&=\int\limits_{\mathcal{R}}\frac{1}{a}g\left(\frac{x-b}{a}\right)\sin\xi x\,dx\\&=\int\limits_{\mathcal{R}} g(y)\sin\xi(ay+b)\,dy,\ \frac{x-b}{a}=y\\&=\int\limits_{\mathcal{R}} g(y)(\sin\xi ay\cos b\xi+\sin b\xi\cos\xi ay)\,dy.\end{aligned}$$

In view of the fact that multiplication of different combinations of even and odd functions yield odd functions, we have

$$\mathcal{F}_{sn}(g_{b,a}(x))(\xi)=\cos b\xi\int\limits_{\mathcal{R}} g(y)\sin\xi ay\,dy.$$

Hence, $\mathcal{F}_{sn}(g_{b,a}(x))(\xi)=0$, for $\xi\notin\mathcal{R}_+$.

Theorem 2 : *Let $(b,a)\in\Delta$* and *$g\in\mathcal{U}_e(n_i,n,\mathcal{R},*)$. Then*

$$\mathcal{F}_{cn}(g_{b,a}(x))(\xi)=0,\ \textit{for}\ \xi\notin\mathcal{R}_+.$$

Proof is similar to the proof employed above and, thus, avoided.

Theorem 3: *Let $(b,a)\in\Delta$ and $g\in\mathcal{U}_o(n_i,n,\mathcal{R},*)$. Then*

$$\mathcal{F}_{sn}(\mathcal{W}_g\phi)=\mathcal{W}_{\breve{g}_{sn}}\phi,$$

for all $\phi \in \mathcal{U}_o(n_i, n, \mathcal{R})$, *where* $\breve{g}_{sn} = \mathcal{F}_{sn}^{-1}(g_{s,t}(x))$

Proof : We have

$$\begin{aligned}
\mathcal{F}_{sn}(\mathcal{W}_g\phi)(s,t) &= \int_{\mathcal{R}} \left(\int_{\Delta} \phi(x)\, \overline{g_{b,a}}(x)\, dx \right) \sin((b,a).(s,t))\, d(b\times a) \\
&= \int_{\Delta} \phi(x) \left(\int_{\mathcal{R}} \overline{g_{b,a}(x)} \sin((b,a).(s,t))\, d(b\times a) \right) dx \\
&= \int_{\Delta} \phi(x)\, \overline{\breve{g}_{sn}(x)} dx \\
&= \mathcal{W}_{\breve{g}_{sn}}\phi.
\end{aligned}$$

This proves the theorem.

Theorem 4: *Let* $(b,a) \in \Delta$ *and* $g \in \mathcal{U}_e(n_i, n, \mathcal{R}, *)$ *. Then*

$$\mathcal{F}_{cn}(\mathcal{W}_g\phi) = \mathcal{W}_{\breve{g}_{cn}}\phi,$$

for all $\phi \in \mathcal{U}_e(n_i, n, \mathcal{R})$ *,where* $\breve{g}_{cn} = \mathcal{F}_{cn}^{-1}(g_{s,t}(x))$.

Proof being similar to that of Theorem 2 we avoid details.

Theorem 5: *Let* $(b,a) \in \Delta, g \in \mathcal{U}_o(n_i, n, \mathcal{R}, *)$ *and* $\phi, \boldsymbol{\psi} \in \mathcal{U}_o(n_i, n, \mathcal{R})$ *then*

$$\mathcal{W}_g\left(\mathcal{F}_{sn}^{-1}\phi\mathcal{F}_{sn}^{-1}\boldsymbol{\psi}\right) = \mathcal{W}_{\breve{g}_{sn}}(\phi * \boldsymbol{\psi}),$$

where $\breve{g}_{sn} = \mathcal{F}_{sn}^{-1}\left(\overline{g_{b,a}(y)}\right)(x) = \int \overline{g_{b,a}(y)} \sin yx\, dy$, and $*$ *is the convolution product.*

Proof : Let $\phi, \boldsymbol{\psi} \in \mathcal{U}_o(n_i, n, \mathcal{R})$. Then, we have

$$\begin{aligned}
\mathcal{W}_{\breve{g}_{sn}}(\phi * \boldsymbol{\psi})(y) &= \int_{\Delta} (\phi * \boldsymbol{\psi})(y)\, \overline{\breve{g}_{b,a}(y)} dy \\
&= \int_{\Delta} \left(\int_{\mathcal{R}} \phi(y)\, \boldsymbol{\psi}(y-\eta)\, d\eta \right) \overline{\breve{g}_{b,a}(y)} dy
\end{aligned}$$

Let $y - \eta = \xi$ then $dy = d\xi$ .Then ,

$$\begin{aligned}
\mathcal{W}_{\breve{g}_{sn}}(\phi * \boldsymbol{\psi})(y) &= \int_{\Delta} \left(\int_{\mathcal{R}} \phi(y)\, \boldsymbol{\psi}(\xi) \right) \overline{\breve{g}_{b,a}(\xi+\eta)} d\eta d\xi \\
&= \int_{\Delta} \left(\int_{\mathcal{R}} \phi(y)\, \boldsymbol{\psi}(\xi) \right) \left(\int_{\mathcal{R}} \overline{g_{b,a}(x)} \sin(\xi+\eta)\, x dx \right) d\eta d\xi.
\end{aligned}$$

Now, using

$$\sin(\xi+\eta)\, x = \sin\xi x \cos\eta x + \sin\eta x \cos\xi x$$

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and rules of integration for even and odd functions, we have

$$\begin{aligned}\mathcal{W}_{\breve{g}_{sn}}(\phi*\boldsymbol{\psi})(y) &= \int\limits_{\Delta}\overline{g_{b,a}(x)}\left(\int\limits_{\mathcal{R}}\boldsymbol{\psi}(\xi)\cos\xi x\int\limits_{\mathcal{R}}\phi(\eta)\sin\eta x d\eta d\xi\right)dx\\ &= \int_{\Delta}\overline{g_{b,a}(x)}\left(\int_{\mathcal{R}}\boldsymbol{\psi}(\xi)\cos\xi x d\xi\int_{\mathcal{R}}\phi(\eta)\sin\eta x d\eta\right)dx\\ &= \int\limits_{\Delta}\overline{g_{b,a}(x)}\hat{\boldsymbol{\psi}}_c(\xi)\hat{\phi}_{sn}(\eta)dx\\ &= \mathcal{W}_g\left(\hat{\boldsymbol{\psi}}_{cn}\hat{\phi}_{sn}\right),\end{aligned}$$

where $\hat{\boldsymbol{\psi}}_{cn}=\mathcal{F}_{cn}\boldsymbol{\psi}$ and $\hat{\phi}_{sn}=\mathcal{F}_{sn}\boldsymbol{\psi}$. This completes the proof of the theorem.

Theorem 6: *Let* $(b,a)\in\Delta$ *and* $g\in\mathcal{U}_e(n_i,n,\mathcal{R},*)$. *Let* ϕ, $\boldsymbol{\psi}\in\mathcal{U}_e(n_i,n,\mathcal{R})$ *then*

$$\mathcal{W}_g\left(\hat{\phi}_{cn}\hat{\boldsymbol{\psi}}_{cn}\right)=\mathcal{W}_{\breve{g}_{cn}}(\phi*\boldsymbol{\psi}),$$

where $*$ *is the convolution product and* $\breve{g}_{cn}$ is the inverse Fourier cosine function of g .

Analysis of the proof of above theorem being similar to that employed for Theorem 5, details are avoided.

Theorem 7: *Given* $\mathcal{T}_1,\mathcal{T}\in\mathcal{U}'_o(n_i,n,\mathcal{R})$ *,we have*

$$\mathcal{F}_{sn}((\mathcal{W}_g\mathcal{T})(\mathcal{W}_g\mathcal{T}_1))=(\mathcal{W}_{\breve{g}_{sn}}\mathcal{T})*(\mathcal{W}_{\breve{g}_{sn}}\mathcal{T}_1).$$

Proof : Let $\phi\in\mathcal{U}_o(n_i,n,\mathcal{R})$. Then in view of Theorem 3, we have

$$\begin{aligned}\langle(\mathcal{W}_g\mathcal{T})(\mathcal{W}_g\mathcal{T}_1),\phi\rangle &= \mathcal{F}_{sn}^{-1}(\mathcal{F}_{sn}(\mathcal{W}_g\mathcal{T}_1)\mathcal{F}_{sn}(\mathcal{W}_g\mathcal{T}_1)),\phi\rangle\\ &= \mathcal{F}_{sn}^{-1}((\mathcal{W}_{\breve{g}_{sn}}\mathcal{T})*(\mathcal{W}_{\breve{g}_{sn}}\mathcal{T}_1)),\phi\rangle.\end{aligned}$$

Hence,

$$(\mathcal{W}_g\mathcal{T})(\mathcal{W}_g\mathcal{T}_1)=\mathcal{F}_{sn}^{-1}((\mathcal{W}_{\breve{g}}\mathcal{T})*(\mathcal{W}_{\breve{g}}\mathcal{T}_1)).$$

Equivalently,

$$\mathcal{F}_{sn}(\mathcal{W}_g\mathcal{T})(\mathcal{W}_g\mathcal{T}_1)=(\mathcal{W}_{\breve{g}}\mathcal{T})*(\mathcal{W}_{\breve{g}}\mathcal{T}_1).$$

The proof of the theorem ,thus, completes. Following theorems are natural consequence of the above theorem and proofs being almost similar, we omit details.

Theorem 8: *Given* $\mathcal{T}_1,\mathcal{T}\in\mathcal{U}'_e(n_i,n,\mathcal{R})$, *we have*

$$\mathcal{F}_{cn}((\mathcal{W}_g\mathcal{T})(\mathcal{W}_g\mathcal{T}_1))=(\mathcal{W}_{\breve{g}cn}\mathcal{T})*(\mathcal{W}_{\breve{g}_{cn}}\mathcal{T}_1).$$

Theorem 9: *Let* $\mathcal{T}_1,\mathcal{T}\in\mathcal{U}'_o(n_i,n,\mathcal{R})$ *.Then*

$$\mathcal{F}_{sn}(\mathcal{W}_g\mathcal{T})*(\mathcal{W}_g\mathcal{T}_1)=(\mathcal{W}_{\breve{g}_{sn}}\mathcal{T})(\mathcal{W}_{\breve{g}_{sn}}\mathcal{T}_1).$$

Theorem 10: *Let* $\mathcal{T}_1,\mathcal{T}\in\mathcal{U}'_e(n_i,n,\mathcal{R})$. *Then*

$$\mathcal{F}_{cn}(\mathcal{W}_g\mathcal{T})*(\mathcal{W}_g\mathcal{T}_1)=(\mathcal{W}_{\breve{g}_{cn}}\mathcal{T})(\mathcal{W}_{\breve{g}_{cn}}\mathcal{T}_1).$$

III. Asymptote Properties of the Wavelet Transform

Let $(c_m)_{m=1}$ be a sequence of continuous positive functions defined on $(0, a_m)$, $a_m > 0$ and $\lim_{t\to 0^+} \frac{c_{m+1}(t)}{c_m(t)} = 0$. Let $(u_m)_{m=1}^{\infty}$be a sequence in $\acute{\mathcal{U}_o}(n_i, n, \mathcal{R}, +)$ and $\acute{\mathcal{U}_e}(n_i n, \mathcal{R}, +)$, respectively, such that $u_m \neq 0, m = 1, 2, ..., p < \infty$ and $u_m = 0, m > p$.The set of pairs $(c_m(t), u_m)$ is denoted by Ω.

A function $f \in \acute{\mathcal{U}_o}(n_i, n, \mathcal{R}, +)$ and $\acute{\mathcal{U}_e}(n_i, n, \mathcal{R}, +)$, respectively, is said to have a quasiasymptotic behaviour($f \sim h$) at 0^+ related to $c(t)$ if there is a non-zero $h \in \acute{\mathcal{U}_o}(n_i, n, \mathcal{R}, +)$ and $\acute{\mathcal{U}_e}(n_i, n, \mathcal{R}, +)$ such that

$$\lim_{t\to 0^+} \left\langle \frac{f(tx)}{c(t)}, \phi(x) \right\rangle = \langle h(x), \phi(x) \rangle, \ \phi \in \mathcal{U}_o(n_i, n, \mathcal{R}), \mathcal{U}_e(n_i, n, \mathcal{R}).$$

A positive function $\mathcal{L}(x)$, which is continuous on $(0, \infty)$, is said to be slowly varying at 0^+ if

$$\lim_{t\to 0^+} \frac{\mathcal{L}(tx)}{\mathcal{L}(t)} = 1, x \in (0, \infty).$$

By Ω_1 denote the subset of Ω of all $(c_m(t), u_m) \in \Omega$ such that $u_m \sim v_m$ at 0^+related to $c_m(t)$ and $v_m \neq 0$ when $u_m \neq 0, m = 1, 2, ..., \infty$ and $v_m \in \acute{\mathcal{U}_o}(n_i, n, \mathcal{R}, +)$ and $\acute{\mathcal{U}_e}(n_i, n, \mathcal{R}, +)$. Let $(c_m(t), u_m) \in \Omega_1$.

An ultradistribution $f \in \acute{\mathcal{U}_o}(n_i, n, \mathcal{R}, +)$ (resp. $\acute{\mathcal{U}_e}(n_i, n, \mathcal{R}, +)$) satisfying

$$\lim_{t\to 0^+} \left\langle \frac{\left(f(.) - \sum_{i=1}^{m} u_i(.)\right)(tx)}{c_m(t)}, \phi(x) \right\rangle = 0$$

for all $\phi \in \mathcal{U}_o(n_i, n, \mathcal{R})$ resp. $\mathcal{U}_e(n_i, n, \mathcal{R})$, is said to have a quasi-asymptotic expansion at 0^+ related to $(c_m(t), u_m)$. In notations, we write

$$f \sim \sum u_i / c_m(t), \text{ at } 0^+.$$

Next, we state without proof the following theorems, namely, theorem 11,12 and 13. Detailed proof is analoguous to that of Theorem 3.1 and Theorem 3.2 of [8] and hence avoided.

Theorem 11: *Let $a_m, m = 1, 2, ...,$ be a sequence of strictly increasing real numbers and $\mathcal{L}_m, m = 1, 2, ...,$ be a sequence of slowly varying functions at 0^+. Let $f \in \acute{\mathcal{U}_o}(n_i, n, \mathcal{R}, +)$ has a quasiasymptotic expansion at 0^+ with respect to $(t^{a_m}\mathcal{L}_m(t), u_m), t > 0$. Then*

$$Lim_{t\to 0^+} \frac{\mathcal{W}_g f(tb, ta) - \sum_{i=1}^{m} \mathcal{W}_g u_i(tb, ta)}{t^{a_m}\mathcal{L}_m(t)} = 0, m = 1, 2, ...$$

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Theorem 12: *Let $a_m, m = 1, 2, ...,$ be a sequence of strictly increasing real numbers and $\mathcal{L}_m, m = 1, 2, ...$be a sequence of slowly varying functions at 0^+. Let $f \in \acute{\mathcal{U}}_e (n_i, n, \mathcal{R}, +)$ has a quasiasymptotic expansion at 0^+ with respect to $(t^{a_m}\mathcal{L}_m(t), u_m), t > 0$. Then*

$$Lim_{t\to 0^+} \frac{\mathcal{W}_g f(tb, ta) - \sum_{i=1}^{m} \mathcal{W}_g u_i (tb, ta)}{t^{a_m}\mathcal{L}_m(t)} = 0, m = 1, 2, ...$$

Theorem 13: *Let $a_m, m = 1, 2...., \infty$ be a sequence of strictly real numbers and $(\mathcal{L}_m)_{m=1}^{\infty}$ be a sequence of slowly varying functions at 0^+. Let $f \in \acute{\mathcal{U}}_{b_0^+} (n_i, n, \mathcal{R})$ has a quasiasymptotic expansion at b_0^+ with respect to $(t^{a_m}\mathcal{L}_m(t), u_m)$. Then*

$$Lim_{t\to 0^+} \frac{\mathcal{W}_g f(b_0, ta) - \sum_{i=1}^{m} \mathcal{W}_g u_i (b_0, ta)}{t^{a_m}\mathcal{L}_m(t)} = 0, m = 1, 2, ...$$

IV. Acknowledgement

Author is thankful to Prof. P. K. Banerji, Emeritus Fellow, UGC (India), and Dr. Deshna Loonker, DST (SERC), India, Young Scientist, Department of Mathematics, JNV University, Jodhpur, India, for revising and formatting the paper.

References Références Referencias

[1] Al-Omari, S.K.Q. , Loonker D. , Banerji P. K. and Kalla, S. L. . *Fourier Sine(Cosine) Transform for Ultradistributions and their Extensions to Tempered and UltraBoehmian Spaces*, Integral Transforms Spec. Funct. 19(6) (2008), 453 – 462.

[2] Al-Omari, S.K.Q. *Certain Class of Kernels for Roumieu-Type Convolution Transform of Ultradistribution of Compact Growth*, J. Concr. Appl. Math.7(4) (2009), 310-316.

[3] Al-Omari, S.K.Q. *Cauchy and Poison Integrals of Tempered Ultradistributions of Roumieu and Beurling Types*, J. Concr. Appl. Math. 7(1) (2009), 36-47.

[4] Al-Omari, S.K.Q. and Al-Omari, J. M. *Operations on Multipliers and Certain Spaces of Tempered Ultradistributions of Roumieu and Beurling types for the Hankel-type Transformations*, J. Appl. Funct. Anal. (2010) (to Appear) .

[5] Banerji, P.K., Al-Omari,S.K.Q. and Debnath,L. *Tempered Distributional Fourier Sine(Cosine)Transform*, Integral Transforms Spec. Funct. 17(11) (2006), 759-768.

[6] Holschneider, M. *Wavelets,an Analysis Tool*, Oxford,Clarendon Press, 1995.

[7] Raghuveer, M. and Ajit, S.,*Wavelet Transforms,Introduction to Theory and Application*, Addison-Wesley, 1998.

[8] S.K.Q. Al-Omari, *On the Continuous-time Wavelet Transform of Generalized Functions*, Int. Journal of Math. Analysis, 4(21), 2010, 1027-1034.

[9] Sneddon, I.N. *The Use of Integral Transforms*, Tata McGraw-Hill Publications Company Ltd, New Delhi, 1974.