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## A Summation Formula Tangled with Hypergeometric Function and Recurrence Relation

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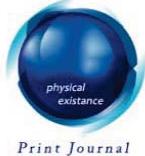
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A SUMMATION FORMULA TANGLED WITH HYPERGEOMETRIC FUNCTION AND RECURRENCE RELATION

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# A Summation Formula Tangled with Hypergeometric Function and Recurrence Relation

Salahuddin <sup>a</sup>, M. P. Chaudhary <sup>o</sup> & Rahul Singh <sup>p</sup>

**Abstract** - The main object of the present paper is to establish a summation formula tangled with Hypergeometric function and recurrence relation.

**Keywords and Phrases** : Contiguous relation, Gauss second summation theorem, Recurrence relation .

## I. INTRODUCTION

### a) Generalized Hypergeometric Functions

A generalized hypergeometric function  ${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$  is a function which can be defined in the form of a hypergeometric series, i.e., a series for which the ratio of successive terms can be written

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)} = \frac{(k+a_1)(k+a_2)\dots(k+a_p)}{(k+b_1)(K+b_2)\dots(k+b_q)(k+1)} z. \quad (1)$$

Where  $k + 1$  in the denominator is present for historical reasons of notation, and the resulting generalized hypergeometric function is written

$${}_pF_q \left[ \begin{array}{c} a_1, a_2, \dots, a_p \\ b_1, b_2, \dots, b_q \end{array}; z \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \dots (a_p)_k z^k}{(b_1)_k (b_2)_k \dots (b_q)_k k!} \quad (2)$$

or

$${}_pF_q \left[ \begin{array}{c} (a_p) \\ (b_q) \end{array}; z \right] \equiv {}_pF_q \left[ \begin{array}{c} (a_j)_{j=1}^p \\ (b_j)_{j=1}^q \end{array}; z \right] = \sum_{k=0}^{\infty} \frac{((a_p))_k z^k}{((b_q))_k k!} \quad (3)$$

where the parameters  $b_1, b_2, \dots, b_q$  are neither zero nor negative integers and  $p, q$  are non-negative integers.

The  ${}_pF_q$  series converges for all finite  $z$  if  $p \leq q$ , converges for  $|z| < 1$  if  $p \neq q+1$ , diverges for all  $z, z \neq 0$  if  $p > q+1$ .

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The  ${}_pF_q$  series absolutely converges for  $|z| = 1$  if  $R(\zeta) < 0$ , conditionally converges for  $|z| = 1, z \neq 0$  if  $0 \leq R(\zeta) < 1$ , diverges for  $|z| = 1$ , if  $1 \leq R(\zeta)$ ,  $\zeta = \sum_{i=1}^p a_i - \sum_{i=0}^q b_i$ .

The function  ${}_2F_1(a, b; c; z)$  corresponding to  $p = 2, q = 1$ , is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function (Gauss 1812, Barnes 1908). To confuse matters even more, the term "hypergeometric function" is less commonly used to mean closed form, and "hypergeometric series" is sometimes used to mean hypergeometric function.

The hypergeometric functions are solutions of Gaussian hypergeometric linear differential equation of second order

$$z(1-z)y'' + [c - (a+b+1)z]y' - aby = 0 \quad (4)$$

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The solution of this equation is

$$y = A_0 \left[ 1 + \frac{ab}{1! c} z + \frac{a(a+1)b(b+1)}{2! c(c+1)} z^2 + \dots \right] \quad (5)$$

This is the so-called regular solution, denoted

$${}_2F_1(a, b; c; z) = \left[ 1 + \frac{ab}{1! c} z + \frac{a(a+1)b(b+1)}{2! c(c+1)} z^2 + \dots \right] = \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!} \quad (6)$$

which converges if  $c$  is not a negative integer for all of  $|z| < 1$  and on the unit circle  $|z| = 1$  if  $R(c-a-b) > 0$ .

It is known as Gauss hypergeometric function in terms of Pochhammer symbol  $(a)_k$  or generalized factorial function.

Many of the common mathematical functions can be expressed in terms of the hypergeometric function, or as limiting cases of it. Some typical examples are

$$(1-z)^{-a} = z {}_2F_1(1, 1; 2; -z) \quad (7)$$

$$\sin^{-1} z = z {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z^2\right) \quad (8)$$

The special case of (1.3.4) when  $a = c$  and  $b = 1$ , or  $a = 1$  and  $b = c$ , yields the elementary geometric series

$$\sum_{n=0}^{\infty} z^n = 1 + z + z^2 + z^3 + \dots + z^n + \dots \quad (9)$$

Hence the term "Hypergeometric" is given. The term hypergeometric was first used by Wallis in his work "Arithmetica Infinitorum". Hypergeometric series or more precisely Gauss series is due to Carl Friedrich Gauss(1777-1855) who in year 1812 introduced and studied this series in his thesis presented at Gottingen and gave the  $F$ -notation for it.

Here  $z$  is a real or complex variable. If  $c$  is zero or negative integer, the series (6) does not exist and hence the function  ${}_2F_1(a, b; c; z)$  is not defined unless one of the parameters

$a$  or  $b$  is also a negative integer such that  $-c < -a$ . If either of the parameters  $a$  or  $b$  is a negative integer, say  $-m$  then in this case (6) reduce to the hypergeometric polynomial defined as

$${}_2F_1(-m, b; c; z) = \sum_{n=0}^m \frac{(-m)_n (b)_n z^n}{(c)_n n!} \quad (10)$$

## Notes

### b) Generalized Ordinary Hypergeometric Function of One Variable

The generalized Gaussian hypergeometric function of one variable is defined as follows

$${}_A F_B \left[ \begin{array}{c} a_1, a_2, a_3, \dots, a_A \\ b_1, b_2, b_3, \dots, b_B \end{array} ; z \right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n \cdots (a_A)_n z^n}{(b_1)_n (b_2)_n (b_3)_n \cdots (b_B)_n n!} \quad (11)$$

$$\text{or, } {}_A F_B \left[ \begin{array}{c} (a_A) \\ (b_B) \end{array} ; z \right] = \sum_{n=0}^{\infty} \frac{[(a_A)]_n z^n}{[(b_B)]_n n!} \quad (12)$$

where for the sake of convenience (in the contracted notation),  $(a_A)$  denotes the array of “ $A$ ” number of parameters given by  $a_1, a_2, a_3, \dots, a_A$ . The denominator parameters are neither zero nor negative integers. The numerator parameters may be zero and negative integers.  $A$  and  $B$  are positive integers or zero. Empty sum is to be interpreted as zero and empty product as unity.

$$\sum_{n=a}^b \text{ and } \prod_{n=a}^b \text{ are empty if } b < a.$$

$$[(a_A)]_{-n} = \frac{(-1)^{nA}}{[1 - (a_A)]_n} \quad (13)$$

$$[(a_A)]_n = (a_1)_n (a_2)_n (a_3)_n \cdots (a_A)_n = \prod_{m=1}^A (a_m)_n = \prod_{m=1}^A \frac{\Gamma(a_m + n)}{\Gamma(a_m)} \quad (14)$$

where  $a_1, a_2, a_3, \dots, a_A; b_1, b_2, b_3, \dots, b_B$  and  $z$  may be real and complex numbers.

$$\begin{aligned} {}_3 F_2 \left[ \begin{array}{c} a, b, 1 \\ c, 2 \end{array} ; z \right] &= \frac{(c-1)}{(a-1)(b-1)z} \times \\ &\times \left\{ {}_2 F_1 \left[ \begin{array}{c} a-1, b-1 \\ c-1 \end{array} ; z \right] - 1 \right\} \end{aligned} \quad (15)$$

The convergence conditions of  ${}_A F_B$  are given below

Suppose that numerator parameters are neither zero nor negative integers (otherwise question of convergence will not arise).



- (i) If  $A \leq B$ , then series  ${}_A F_B$  is always convergent for all finite values of  $z$  (real or complex) i.e.,  $|z| < \infty$ .
- (ii) If  $A = B + 1$  and  $|z| < 1$ , then series  ${}_A F_B$  is convergent.
- (iii) If  $A = B + 1$  and  $|z| > 1$ , then series  ${}_A F_B$  is divergent.
- (iv) If  $A = B + 1$  and  $|z| = 1$ , then series  ${}_A F_B$  is absolutely convergent, when

## Notes

$$\operatorname{Re} \left\{ \sum_{m=1}^B b_m - \sum_{n=1}^A a_n \right\} > 0$$

- (v) If  $A = B + 1$  and  $z = 1$ , then series  ${}_A F_B$  is convergent, when

$$\operatorname{Re} \left\{ \sum_{m=1}^B b_m - \sum_{n=1}^A a_n \right\} > 0$$

- (vi) If  $A = B + 1$  and  $z = 1$ , then series  ${}_A F_B$  is divergent, when

$$\operatorname{Re} \left\{ \sum_{m=1}^B b_m - \sum_{n=1}^A a_n \right\} \leq 0$$

- (vii) If  $A = B + 1$  and  $z = -1$ , then series  ${}_A F_B$  is convergent, when

$$\operatorname{Re} \left\{ \sum_{m=1}^B b_m - \sum_{n=1}^A a_n \right\} > -1$$

- (viii) If  $A = B + 1$  and  $|z| = 1$ , but  $z \neq 1$ , then series  ${}_A F_B$  is conditionally convergent, when

$$-1 < \operatorname{Re} \left\{ \sum_{m=1}^B b_m - \sum_{n=1}^A a_n \right\} \leq 0$$

- (ix) If  $A > B + 1$ , then series  ${}_A F_B$  is convergent, when  $z = 0$ .

- (x) If  $A = B + 1$  and  $|z| \geq 1$ , then it is defined as an analytic continuation of this series.

- (xi) If  $A = B + 1$  and  $|z| = 1$ , then series  ${}_A F_B$  is divergent, when

$$\operatorname{Re} \left\{ \sum_{m=1}^B b_m - \sum_{n=1}^A a_n \right\} \leq -1$$

(xii) If  $A > B + 1$ , then a meaningful independent attempts were made to define MacRobert's  $E$ -function, Meijer's  $G$ -function, Fox's  $H$ -function and its related functions.

(xiii) If one or more of the numerator parameters are zero or negative integers, then series  ${}_A F_B$  terminates for all finite values of  $z$  i.e.,  ${}_A F_B$  will be a hypergeometric polynomial and the question of convergence does not enter the discussion.

**Contiguous Relation is defined by**

[ E. D. p.51(10)]

$$(a - b) {}_2 F_1 \left[ \begin{matrix} a, b \\ c \end{matrix} ; z \right] = a {}_2 F_1 \left[ \begin{matrix} a+1, b \\ c \end{matrix} ; z \right] - b {}_2 F_1 \left[ \begin{matrix} a, b+1 \\ c \end{matrix} ; z \right] \quad (16)$$

**Gauss second summation theorem is defined by** [Prud., 491(7.3.7.8)]

$${}_2 F_1 \left[ \begin{matrix} a, b \\ \frac{a+b+1}{2} \end{matrix} ; \frac{1}{2} \right] = \frac{\Gamma(\frac{a+b+1}{2}) \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2}) \Gamma(\frac{b+1}{2})} \quad (17)$$

$$= \frac{2^{(b-1)} \Gamma(\frac{b}{2}) \Gamma(\frac{a+b+1}{2})}{\Gamma(b) \Gamma(\frac{a+1}{2})} \quad (18)$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud., p.491(7.3.7.8)]

$${}_2 F_1 \left[ \begin{matrix} a, b \\ \frac{a+b-1}{2} \end{matrix} ; \frac{1}{2} \right] = \sqrt{\pi} \left[ \frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2}) \Gamma(\frac{b+1}{2})} + \frac{2 \Gamma(\frac{a+b-1}{2})}{\Gamma(a) \Gamma(b)} \right] \quad (19)$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$${}_2 F_1 \left[ \begin{matrix} a, b \\ \frac{a+b-1}{2} \end{matrix} ; \frac{1}{2} \right] = \frac{2^{(b-1)} \Gamma(\frac{a+b-1}{2})}{\Gamma(b)} \left[ \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)} \Gamma(\frac{a}{2}) \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})} \right] \quad (20)$$

**Recurrence relation is defined by**

$$\Gamma(z+1) = z \Gamma(z) \quad (21)$$

## II. MAIN SUMMATION FORMULA

$$\begin{aligned}
 {}_2F_1 \left[ \begin{matrix} a, & b \\ \frac{a+b+41}{2} & \end{matrix} ; \quad \frac{1}{2} \right] = & \frac{2^b \Gamma(\frac{a+b+41}{2})}{(a-b) \Gamma(b)} \times \\
 & \times \left[ \frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})} \left\{ \frac{524288a(-8200794532637891559375 + 20125013723397976152375a)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \right. \right. \\
 & + \frac{524288a(-19688993487602867898225a^2 + 10792700030471840300745a^3)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(-3824294822931302783964a^4 + 946995223404049011324a^5)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(-171930790626988570804a^6 + 23615262213846406804a^7)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(-2505874787291646498a^8 + 208251057899323218a^9 - 13663776163658478a^{10})}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(710084079834558a^{11} - 29186718196012a^{12} + 942715036492a^{13})}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(-23625216132a^{14} + 449681892a^{15} - 6278151a^{16} + 60591a^{17} - 361a^{18} + a^{19})}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(33222453094521656744625b - 26784014367886904649150ab)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(66569416113060226275165a^2b - 18197261858418397376400a^3b)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(11907649593190511368500a^4b - 1732720204487419142472a^5b)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(507724860074808912468a^6b - 44575549851700633584a^7b)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]} + \\
 & + \frac{524288a(7096841648258109774a^8b - 394251249479137908a^9b + 37227237877945830a^{10}b)}{\left[ \prod_{\Theta=1}^{19} \{a-b-(2\Theta-1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a-b+(2\Upsilon-1)\} \right]}
 \end{aligned}$$

Notes

## Notes

$$\begin{aligned}
& + \frac{524288a(-1316694562355952a^{11}b + 76320137288772a^{12}b - 1663027017288a^{13}b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(58788536196a^{14}b - 716920464a^{15}b + 14574729a^{16}b - 75582a^{17}b + 741a^{18}b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(2464947339460964078175b^2 + 89709154927079146338555ab^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-16481181023683218686376a^2b^2 + 40357651170352314922968a^3b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-4673222221480836168652a^4b^2 + 3176905101637503805348a^5b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-237022087220428430648a^6b^2 + 72616236512301230216a^7b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-3554713715058825462a^8b^2 + 588835800871070610a^9b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-18785390190548696a^{10}b^2 + 1828505864702504a^{11}b^2 - 36535526629420a^{12}b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(2155023393796a^{13}b^2 - 24404420040a^{14}b^2 + 861332472a^{15}b^2 - 4194801a^{16}b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(82251a^{17}b^2 + 28718225937835914827295b^3 + 3318894504681786671472ab^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(50291362269874511578728a^2b^3 - 3438152189587572233712a^3b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(8208505768506397623204a^4b^3 - 479471198586317093520a^5b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(322177752843393342168a^6b^3 - 13517910048426401904a^7b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

$$\begin{aligned}
& + \frac{524288a(4118567530121081466a^8b^3 - 117765498111209520a^9b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(19370822163507672a^{10}b^3 - 356401367234640a^{11}b^3 + 34220151840420a^{12}b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-363586707120a^{13}b^3 + 20899430760a^{14}b^3 - 96946512a^{15}b^3 + 3262623a^{16}b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(3743527666786355832228b^4 + 23735039336168466505836ab^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(1065598075457801482740a^2b^4 + 9808909042980361520700a^3b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-307540391879642734540a^4b^4 + 711859291630188684892a^5b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-22863326090812082876a^6b^4 + 14834228962017812204a^7b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-362192635506367380a^8b^4 + 106961337355063620a^9b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-1770421086614180a^{10}b^4 + 281900758731956a^{11}b^4 - 2778732507460a^{12}b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(256594423540a^{13}b^4 - 1127935380a^{14}b^4 + 61523748a^{15}b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(3405266444028472415652b^5 + 1592826112836059973560ab^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(5499330172367303710204a^2b^5 + 127747922024587372144a^3b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(828745355724256566596a^4b^5 - 13244326294690683192a^5b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

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$$\begin{aligned}
& + \frac{524288a(29612710418417746620a^6b^5 - 541711449908579808a^7b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(335036903394444108a^8b^5 - 4701727850267448a^9b^5 + 1325553122001108a^{10}b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-11743720135056a^{11}b^5 + 1781300804556a^{12}b^5 - 7282174536a^{13}b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(635745396a^{14}b^5 + 307340423319633457676b^6 + 1340384188957112471692ab^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(211246684907825219016a^2b^6 + 512593323491544520680a^3b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(6855294547498745348a^4b^6 + 33871331518519795300a^5b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-289382296218006992a^6b^6 + 623272853264512880a^7b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-6430101458336556a^8b^6 + 3760722206829588a^9b^6 - 28033056115064a^{10}b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(7469623102760a^{11}b^6 - 27375582052a^{12}b^6 + 3910797436a^{13}b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(108269327415353435916b^7 + 70179445128686011664ab^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(162371581902467831608a^2b^7 + 11873184948454395280a^3b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(22318167470495565812a^4b^7 + 178509989944434720a^5b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(703969753138114320a^6b^7 - 3211397396599392a^7b^7 + 6647435147415348a^8b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

$$\begin{aligned}
& + \frac{524288a(-35833247976240a^9b^7 + 19709827528248a^{10}b^7 - 60338017584a^{11}b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(15084504396a^1b^7 + 6586460453221363806b^8 + 23694863813913400290ab^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(5175623316897888426a^2b^8 + 8294628633401516406a^3b^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(316411422061594860a^4b^8 + 484586470941488916a^5b^8 + 2291343736653972a^6b^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(7438485518649900a^7b^8 - 16894761676650a^8b^8 + 33539087889450a^9b^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(-73204212510a^{10}b^8 + 37711260990a^{11}b^8 + 1190397299268527454b^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(882649351319057036ab^9 + 1644205273478553214a^2b^9 + 162921387014111440a^3b^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(200537843674548380a^4b^9 + 4108083073246152a^5b^9 + 5284711076616972a^6b^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(13647662542800a^7b^9 + 37267793684550a^8b^9 - 32820602100a^9b^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(62359143990a^{10}b^9 + 48653715410164722b^{10} + 154157906385590250ab^{10})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(37967523155613480a^2b^{10} + 48186270011142120a^3b^{10} + 2366386284722460a^4b^{10})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(2360734480596492a^5b^{10} + 24584628748680a^6b^{10} + 27105250989960a^7b^{10})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(29538541890a^8b^{10} + 68923264410a^9b^{10} + 5046299073566322b^{11})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

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$$\begin{aligned}
& + \frac{524288a(3908213830318096ab^{11} + 6287173301234072a^2b^{11} + 671998070250416a^3b^{11})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(647217596189164a^4b^{11} + 15334926887280a^5b^{11} + 12807631555992a^6b^{11})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(53239427280a^7b^{11} + 51021117810a^8b^{11} + 136044645566804b^{12})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(390969013904092ab^{12} + 96486711472788a^2b^{12} + 104212054616124a^3b^{12})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(4990312383420a^4b^{12} + 3859957069332a^5b^{12} + 35197176924a^6b^{12})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(25140840660a^7b^{12} + 8455024465236b^{13} + 6364613182648ab^{13})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(9131066181020a^2b^{13} + 886583500880a^3b^{13} + 718310791660a^4b^{13})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(12634884024a^5b^{13} + 8122425444a^6b^{13} + 143249607228b^{14} + 377940383964ab^{14})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(82385891640a^2b^{14} + 78284308440a^3b^{14} + 2600776620a^4b^{14} + 1676056044a^5b^{14})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(5307418428b^{15} + 3532333168ab^{15} + 4577615432a^2b^{15} + 300782768a^3b^{15})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(211915132a^4b^{15} + 50652537b^{16} + 122581407ab^{16} + 18177471a^2b^{16})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(15380937a^3b^{16} + 1047033b^{17} + 493506ab^{17} + 575757a^2b^{17})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{524288a(4199b^{18} + 9139ab^{18} + 39b^{19})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$



$$\begin{aligned}
& + \frac{524288b(-8200794532637891559375 + 33222453094521656744625a)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(2464947339460964078175a^2 + 28718225937835914827295a^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(3743527666786355832228a^4 + 3405266444028472415652a^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(307340423319633457676a^6 + 108269327415353435916a^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(6586460453221363806a^8 + 1190397299268527454a^9 + 48653715410164722a^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(5046299073566322a^{11} + 136044645566804a^{12} + 8455024465236a^{13})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(143249607228a^{14} + 5307418428a^{15} + 50652537a^{16} + 1047033a^{17} + 4199a^{18})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(39a^9 + 20125013723397976152375b - 26784014367886904649150ab)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(89709154927079146338555a^2b + 3318894504681786671472a^3b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(23735039336168466505836a^4b + 1592826112836059973560a^5b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(1340384188957112471692a^6b + 70179445128686011664a^7b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(23694863813913400290a^8b + 882649351319057036a^9b + 154157906385590250a^{10}b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(3908213830318096a^{11}b + 390969013904092a^{12}b + 6364613182648a^{13}b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes

$$\begin{aligned}
& + \frac{524288b(377940383964a^{14}b + 3532333168a^{15}b + 122581407a^{16}b + 493506a^{17}b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(9139a^18b - 19688993487602867898225b^2 + 66569416113060226275165ab^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-16481181023683218686376a^2b^2 + 50291362269874511578728a^3b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(1065598075457801482740a^4b^2 + 5499330172367303710204a^5b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(211246684907825219016a^6b^2 + 162371581902467831608a^7b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(5175623316897888426a^8b^2 + 1644205273478553214a^9b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(37967523155613480a^{10}b^2 + 6287173301234072a^{11}b^2 + 96486711472788a^{12}b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(9131066181020a^{13}b^2 + 82385891640a^{14}b^2 + 4577615432a^{15}b^2 + 18177471a^{16}b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(575757a^{17}b^2 + 10792700030471840300745b^3 - 18197261858418397376400ab^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(40357651170352314922968a^2b^3 - 3438152189587572233712a^3b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(9808909042980361520700a^4b^3 + 127747922024587372144a^5b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(512593323491544520680a^6b^3 + 11873184948454395280a^7b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(8294628633401516406a^8b^3 + 162921387014111440a^9b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes

$$\begin{aligned}
& + \frac{524288b(48186270011142120a^{10}b^3 + 671998070250416a^{11}b^3 + 104212054616124a^{12}b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(886583500880a^{13}b^3 + 78284308440a^{14}b^3 + 300782768a^{15}b^3 + 15380937a^{16}b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-3824294822931302783964b^4 + 11907649593190511368500ab^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-4673222221480836168652a^2b^4 + 8208505768506397623204a^3b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-307540391879642734540a^4b^4 + 828745355724256566596a^5b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(6855294547498745348a^6b^4 + 22318167470495565812a^7b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(316411422061594860a^8b^4 + 200537843674548380a^9b^4 + 2366386284722460a^{10}b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(647217596189164a^{11}b^4 + 4990312383420a^{12}b^4 + 718310791660a^{13}b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(2600776620a^{14}b^4 + 211915132a^{15}b^4 + 946995223404049011324b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-1732720204487419142472ab^5 + 3176905101637503805348a^2b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-479471198586317093520a^3b^5 + 711859291630188684892a^4b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-13244326294690683192a^5b^5 + 33871331518519795300a^6b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(178509989944434720a^7b^5 + 484586470941488916a^8b^5 + 4108083073246152a^9b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes

## Notes

$$\begin{aligned}
& + \frac{524288b(2360734480596492a^{10}b^5 + 15334926887280a^{11}b^5 + 3859957069332a^{12}b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(12634884024a^{13}b^5 + 1676056044a^{14}b^5 - 171930790626988570804b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(507724860074808912468ab^6 - 237022087220428430648a^2b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(322177752843393342168a^3b^6 - 22863326090812082876a^4b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(29612710418417746620a^5b^6 - 289382296218006992a^6b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(703969753138114320a^7b^6 + 2291343736653972a^8b^6 + 5284711076616972a^9b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(24584628748680a^{10}b^6 + 12807631555992a^{11}b^6 + 35197176924a^{12}b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(8122425444a^{13}b^6 + 23615262213846406804b^7 - 44575549851700633584ab^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(72616236512301230216a^2b^7 - 13517910048426401904a^3b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(14834228962017812204a^4b^7 - 541711449908579808a^5b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(623272853264512880a^6b^7 - 3211397396599392a^7b^7 + 7438485518649900a^8b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(13647662542800a^9b^7 + 27105250989960a^{10}b^7 + 53239427280a^{11}b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(25140840660a^{12}b^7 - 2505874787291646498b^8 + 7096841648258109774ab^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

$$\begin{aligned}
& + \frac{524288b(-3554713715058825462a^2b^8 + 4118567530121081466a^3b^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-362192635506367380a^4b^8 + 335036903394444108a^5b^8 - 6430101458336556a^6b^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(6647435147415348a^7b^8 - 16894761676650a^8b^8 + 37267793684550a^9b^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(29538541890a^{10}b^8 + 51021117810a^{11}b^8 + 208251057899323218b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-394251249479137908ab^9 + 588835800871070610a^2b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-117765498111209520a^3b^9 + 106961337355063620a^4b^9 - 4701727850267448a^5b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(3760722206829588a^6b^9 - 35833247976240a^7b^9 + 33539087889450a^8b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-32820602100a^9b^9 + 68923264410a^{10}b^9 - 13663776163658478b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(37227237877945830ab^{10} - 18785390190548696a^2b^{10} + 19370822163507672a^3b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-1770421086614180a^4b^{10} + 1325553122001108a^5b^{10} - 28033056115064a^6b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(19709827528248a^7b^{10} - 73204212510a^8b^{10} + 62359143990a^9b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(710084079834558b^{11} - 1316694562355952ab^{11} + 1828505864702504a^2b^{11})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-356401367234640a^3b^{11} + 281900758731956a^4b^{11} - 11743720135056a^5b^{11})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes

$$\begin{aligned}
& + \frac{524288b(7469623102760a^6b^{11} - 60338017584a^7b^{11} + 37711260990a^8b^{11})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-29186718196012b^{12} + 76320137288772ab^{12} - 36535526629420a^2b^{12})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(34220151840420a^3b^{12} - 2778732507460a^4b^{12} + 1781300804556a^5b^{12})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-27375582052a^6b^{12} + 15084504396a^7b^{12} + 942715036492b^{13})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-1663027017288ab^{13} + 2155023393796a^2b^{13} - 363586707120a^3b^{13})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(256594423540a^4b^{13} - 7282174536a^5b^{13} + 3910797436a^6b^{13} - 23625216132b^{14})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(58788536196ab^{14} - 24404420040a^2b^{14} + 20899430760a^3b^{14} - 1127935380a^4b^{14})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(635745396a^5b^{14} + 449681892b^{15} - 716920464ab^{15} + 861332472a^2b^{15})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-96946512a^3b^{15} + 61523748a^4b^{15} - 6278151b^{16} + 14574729ab^{16})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-4194801a^2b^{16} + 3262623a^3b^{16} + 60591b^{17} - 75582ab^{17} + 82251a^2b^{17})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{524288b(-361b^{18} + 741ab^{18} + b^{19})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} \Big\} - \\
& - \frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})} \left\{ \frac{1048576(8200794532637891559375 + 33222453094521656744625a)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \right. \\
& + \frac{1048576(-2464947339460964078175a^2 + 28718225937835914827295a^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} +
\end{aligned}$$

## Notes

$$\begin{aligned}
& + \frac{1048576(-3743527666786355832228a^4 + 3405266444028472415652a^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-307340423319633457676a^6 + 108269327415353435916a^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-6586460453221363806a^8 + 1190397299268527454a^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-48653715410164722a^{10} + 5046299073566322a^{11} - 136044645566804a^{12})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(8455024465236a^{13} - 143249607228a^{14} + 5307418428a^{15} - 50652537a^{16})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(1047033a^{17} - 4199a^{18} + 39a^{19} + 20125013723397976152375b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(26784014367886904649150ab + 89709154927079146338555a^2b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-3318894504681786671472a^3b + 23735039336168466505836a^4b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-1592826112836059973560a^5b + 1340384188957112471692a^6b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-70179445128686011664a^7b + 23694863813913400290a^8b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-882649351319057036a^9b + 154157906385590250a^{10}b - 3908213830318096a^{11}b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(390969013904092a^{12}b - 6364613182648a^{13}b + 377940383964a^{14}b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-3532333168a^{15}b + 122581407a^{16}b - 493506a^{17}b + 9139a^{18}b)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

## Notes

## Notes

$$\begin{aligned}
& + \frac{1048576(19688993487602867898225b^2 + 66569416113060226275165ab^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(16481181023683218686376a^2b^2 + 50291362269874511578728a^3b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-1065598075457801482740a^4b^2 + 5499330172367303710204a^5b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-211246684907825219016a^6b^2 + 162371581902467831608a^7b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-5175623316897888426a^8b^2 + 1644205273478553214a^9b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-37967523155613480a^{10}b^2 + 6287173301234072a^{11}b^2 - 96486711472788a^{12}b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(9131066181020a^{13}b^2 - 82385891640a^{14}b^2 + 4577615432a^{15}b^2 - 18177471a^{16}b^2)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(575757a^{17}b^2 + 10792700030471840300745b^3 + 18197261858418397376400ab^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(40357651170352314922968a^2b^3 + 3438152189587572233712a^3b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(9808909042980361520700a^4b^3 - 127747922024587372144a^5b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(512593323491544520680a^6b^3 - 11873184948454395280a^7b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(8294628633401516406a^8b^3 - 162921387014111440a^9b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(48186270011142120a^{10}b^3 - 671998070250416a^{11}b^3 + 104212054616124a^{12}b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$



$$\begin{aligned}
& + \frac{1048576(-886583500880a^{13}b^3 + 78284308440a^{14}b^3 - 300782768a^{15}b^3 + 15380937a^{16}b^3)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(3824294822931302783964b^4 + 11907649593190511368500ab^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(4673222221480836168652a^2b^4 + 8208505768506397623204a^3b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(307540391879642734540a^4b^4 + 828745355724256566596a^5b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-6855294547498745348a^6b^4 + 22318167470495565812a^7b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-316411422061594860a^8b^4 + 200537843674548380a^9b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-2366386284722460a^{10}b^4 + 647217596189164a^{11}b^4 - 4990312383420a^{12}b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(718310791660a^{13}b^4 - 2600776620a^{14}b^4 + 211915132a^{15}b^4)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(946995223404049011324b^5 + 1732720204487419142472ab^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(3176905101637503805348a^2b^5 + 479471198586317093520a^3b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(711859291630188684892a^4b^5 + 13244326294690683192a^5b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(33871331518519795300a^6b^5 - 178509989944434720a^7b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(484586470941488916a^8b^5 - 4108083073246152a^9b^5 + 2360734480596492a^{10}b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

## Notes

$$\begin{aligned}
& + \frac{1048576(-15334926887280a^{11}b^5 + 3859957069332a^{12}b^5 - 12634884024a^{13}b^5)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(1676056044a^{14}b^5 + 171930790626988570804b^6 + 507724860074808912468ab^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(237022087220428430648a^2b^6 + 322177752843393342168a^3b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(22863326090812082876a^4b^6 + 29612710418417746620a^5b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(289382296218006992a^6b^6 + 703969753138114320a^7b^6 - 2291343736653972a^8b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(5284711076616972a^9b^6 - 24584628748680a^{10}b^6 + 12807631555992a^{11}b^6)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-35197176924a^{12}b^6 + 8122425444a^{13}b^6 + 23615262213846406804b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(44575549851700633584ab^7 + 72616236512301230216a^2b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(13517910048426401904a^3b^7 + 14834228962017812204a^4b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(541711449908579808a^5b^7 + 623272853264512880a^6b^7 + 3211397396599392a^7b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(7438485518649900a^8b^7 - 13647662542800a^9b^7 + 27105250989960a^{10}b^7)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(-53239427280a^{11}b^7 + 25140840660a^{12}b^7 + 2505874787291646498b^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(7096841648258109774ab^8 + 3554713715058825462a^2b^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

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$$\begin{aligned}
& + \frac{1048576(335036903394444108a^5b^8 + 6430101458336556a^6b^8 + 6647435147415348a^7b^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(16894761676650a^8b^8 + 37267793684550a^9b^8 - 29538541890a^{10}b^8)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(51021117810a^{11}b^8 + 208251057899323218b^9 + 394251249479137908ab^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(588835800871070610a^2b^9 + 117765498111209520a^3b^9 + 106961337355063620a^4b^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(4701727850267448a^5b^9 + 3760722206829588a^6b^9 + 35833247976240a^7b^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(33539087889450a^8b^9 + 32820602100a^9b^9 + 68923264410a^{10}b^9)}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(13663776163658478b^{10} + 37227237877945830ab^{10} + 18785390190548696a^2b^{10})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(19370822163507672a^3b^{10} + 1770421086614180a^4b^{10} + 1325553122001108a^5b^{10})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(28033056115064a^6b^{10} + 19709827528248a^7b^{10} + 73204212510a^8b^{10})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(62359143990a^9b^{10} + 710084079834558b^{11} + 1316694562355952ab^{11})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(1828505864702504a^2b^{11} + 356401367234640a^3b^{11} + 281900758731956a^4b^{11})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(11743720135056a^5b^{11} + 7469623102760a^6b^{11} + 60338017584a^7b^{11})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(37711260990a^8b^{11} + 29186718196012b^{12} + 76320137288772ab^{12})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]}
\end{aligned}$$

Notes

## Notes

$$\begin{aligned}
& + \frac{1048576(36535526629420a^2b^{12} + 34220151840420a^3b^{12} + 2778732507460a^4b^{12})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(1781300804556a^5b^{12} + 27375582052a^6b^{12} + 15084504396a^7b^{12})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(942715036492b^{13} + 1663027017288ab^{13} + 2155023393796a^2b^{13})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(363586707120a^3b^{13} + 256594423540a^4b^{13} + 7282174536a^5b^{13} + 3910797436a^6b^{13})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(23625216132b^{14} + 58788536196ab^{14} + 24404420040a^2b^{14} + 20899430760a^3b^{14})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(1127935380a^4b^{14} + 635745396a^5b^{14} + 449681892b^{15} + 716920464ab^{15})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(861332472a^2b^{15} + 96946512a^3b^{15} + 61523748a^4b^{15} + 6278151b^{16})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(14574729ab^{16} + 4194801a^2b^{16} + 3262623a^3b^{16} + 60591b^{17} + 75582ab^{17})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(82251a^2b^{17} + 361b^{18} + 741ab^{18} + b^{19})}{\left[ \prod_{\Theta=1}^{19} \{a - b - (2\Theta - 1)\} \right] \left[ \prod_{\Upsilon=1}^{20} \{a - b + (2\Upsilon - 1)\} \right]} + \\
& + \frac{1048576(8200794532637891559375 + 20125013723397976152375a)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(19688993487602867898225a^2 + 10792700030471840300745a^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(3824294822931302783964a^4 + 946995223404049011324a^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(171930790626988570804a^6 + 23615262213846406804a^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

$$\begin{aligned}
& + \frac{1048576(2505874787291646498a^8 + 208251057899323218a^9 + 13663776163658478a^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(710084079834558a^{11} + 29186718196012a^{12} + 942715036492a^{13})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(23625216132a^{14} + 449681892a^{15} + 6278151a^{16} + 60591a^{17} + 361a^{18} + a^{19})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(33222453094521656744625b + 26784014367886904649150ab)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(66569416113060226275165a^2b + 18197261858418397376400a^3b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(11907649593190511368500a^4b + 1732720204487419142472a^5b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(507724860074808912468a^6b + 44575549851700633584a^7b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(7096841648258109774a^8b + 394251249479137908a^9b + 37227237877945830a^{10}b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(1316694562355952a^{11}b + 76320137288772a^{12}b + 1663027017288a^{13}b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(58788536196a^{14}b + 716920464a^{15}b + 14574729a^{16}b + 75582a^{17}b + 741a^{18}b)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-2464947339460964078175b^2 + 89709154927079146338555ab^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(16481181023683218686376a^2b^2 + 40357651170352314922968a^3b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(4673222221480836168652a^4b^2 + 3176905101637503805348a^5b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes



$$\begin{aligned}
& + \frac{1048576(237022087220428430648a^6b^2 + 72616236512301230216a^7b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(3554713715058825462a^8b^2 + 588835800871070610a^9b^2 + 18785390190548696a^{10}b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(1828505864702504a^{11}b^2 + 36535526629420a^{12}b^2 + 2155023393796a^{13}b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(24404420040a^{14}b^2 + 861332472a^{15}b^2 + 4194801a^{16}b^2 + 82251a^{17}b^2)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(28718225937835914827295b^3 - 3318894504681786671472ab^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(50291362269874511578728a^2b^3 + 3438152189587572233712a^3b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(8208505768506397623204a^4b^3 + 479471198586317093520a^5b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(322177752843393342168a^6b^3 + 13517910048426401904a^7b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(4118567530121081466a^8b^3 + 117765498111209520a^9b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(19370822163507672a^{10}b^3 + 356401367234640a^{11}b^3 + 34220151840420a^{12}b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(363586707120a^{13}b^3 + 20899430760a^{14}b^3 + 96946512a^{15}b^3 + 3262623a^{16}b^3)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-3743527666786355832228b^4 + 23735039336168466505836ab^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-1065598075457801482740a^2b^4 + 9808909042980361520700a^3b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes

$$\begin{aligned}
& + \frac{1048576(307540391879642734540a^4b^4 + 711859291630188684892a^5b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(22863326090812082876a^6b^4 + 14834228962017812204a^7b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(362192635506367380a^8b^4 + 106961337355063620a^9b^4 + 1770421086614180a^{10}b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(281900758731956a^{11}b^4 + 2778732507460a^{12}b^4 + 256594423540a^{13}b^4)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(1127935380a^{14}b^4 + 61523748a^{15}b^4 + 3405266444028472415652b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-1592826112836059973560ab^5 + 5499330172367303710204a^2b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-127747922024587372144a^3b^5 + 828745355724256566596a^4b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(13244326294690683192a^5b^5 + 29612710418417746620a^6b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(541711449908579808a^7b^5 + 335036903394444108a^8b^5 + 4701727850267448a^9b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(1325553122001108a^{10}b^5 + 11743720135056a^{11}b^5 + 1781300804556a^{12}b^5)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(7282174536a^{13}b^5 + 635745396a^{14}b^5 - 307340423319633457676b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(1340384188957112471692ab^6 - 211246684907825219016a^2b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(512593323491544520680a^3b^6 - 6855294547498745348a^4b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes

$$\begin{aligned}
& + \frac{1048576(33871331518519795300a^5b^6 + 289382296218006992a^6b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(623272853264512880a^7b^6 + 6430101458336556a^8b^6 + 3760722206829588a^9b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(28033056115064a^{10}b^6 + 7469623102760a^{11}b^6 + 27375582052a^{12}b^6)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(3910797436a^{13}b^6 + 108269327415353435916b^7 - 70179445128686011664ab^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(162371581902467831608a^2b^7 - 11873184948454395280a^3b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(22318167470495565812a^4b^7 - 178509989944434720a^5b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(703969753138114320a^6b^7 + 3211397396599392a^7b^7 + 6647435147415348a^8b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(35833247976240a^9b^7 + 19709827528248a^{10}b^7 + 60338017584a^{11}b^7)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(15084504396a^1b^7 - 6586460453221363806b^8 + 23694863813913400290ab^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-5175623316897888426a^2b^8 + 8294628633401516406a^3b^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-316411422061594860a^4b^8 + 484586470941488916a^5b^8 - 2291343736653972a^6b^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(7438485518649900a^7b^8 + 16894761676650a^8b^8 + 33539087889450a^9b^8)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(73204212510a^{10}b^8 + 37711260990a^{11}b^8 + 1190397299268527454b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
\end{aligned}$$

## Notes



$$\begin{aligned}
 & + \frac{1048576(-882649351319057036ab^9 + 1644205273478553214a^2b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(-162921387014111440a^3b^9 + 200537843674548380a^4b^9 - 4108083073246152a^5b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(5284711076616972a^6b^9 - 13647662542800a^7b^9 + 37267793684550a^8b^9)}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(32820602100a^9b^9 + 62359143990a^{10}b^9 - 48653715410164722b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(154157906385590250ab^{10} - 37967523155613480a^2b^{10} + 48186270011142120a^3b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(-2366386284722460a^4b^{10} + 2360734480596492a^5b^{10} - 24584628748680a^6b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(27105250989960a^7b^{10} - 29538541890a^8b^{10} + 68923264410a^9b^{10})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(5046299073566322b^{11} - 3908213830318096ab^{11} + 6287173301234072a^2b^{11})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(-671998070250416a^3b^{11} + 647217596189164a^4b^{11} - 15334926887280a^5b^{11})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(12807631555992a^6b^{11} - 53239427280a^7b^{11} + 51021117810a^8b^{11})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(-136044645566804b^{12} + 390969013904092ab^{12} - 96486711472788a^2b^{12})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(104212054616124a^3b^{12} - 4990312383420a^4b^{12} + 3859957069332a^5b^{12})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
 & + \frac{1048576(-35197176924a^6b^{12} + 25140840660a^7b^{12} + 8455024465236b^{13})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]}
 \end{aligned}$$

Notes

## Notes

$$\begin{aligned}
& + \frac{1048576(-6364613182648ab^{13} + 9131066181020a^2b^{13} - 886583500880a^3b^{13})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(718310791660a^4b^{13} - 12634884024a^5b^{13} + 8122425444a^6b^{13} - 143249607228b^{14})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(377940383964ab^{14} - 82385891640a^2b^{14} + 78284308440a^3b^{14} - 2600776620a^4b^{14})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(1676056044a^5b^{14} + 5307418428b^{15} - 3532333168ab^{15} + 4577615432a^2b^{15})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-300782768a^3b^{15} + 211915132a^4b^{15} - 50652537b^{16} + 122581407ab^{16})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-18177471a^2b^{16} + 15380937a^3b^{16} + 1047033b^{17} - 493506ab^{17} + 575757a^2b^{17})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} + \\
& + \frac{1048576(-4199b^{18} + 9139ab^{18} + 39b^{19})}{\left[ \prod_{\Xi=1}^{20} \{a - b - 2\Xi\} \right] \left[ \prod_{\Psi=1}^{19} \{a - b + 2\Psi\} \right]} \Bigg\} \quad (22)
\end{aligned}$$

## III. DERIVATION OF THE MAIN SUMMATION FORMULA

Proceeding on the same way of Ref[8], we get the main result.

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