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New Finite Integrals of Generalized Meliin-Barnes Type of Contour Integrals

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Abstract - In the present paper, we obtain three new finite integral formulas. These formulas involve the product of a general class of polynomials and the generalized Meliin-Barnes type of contour integrals. Mainly we are using series representation of the $\overline{H}$-function given by Agarwal [14], Agarwal and Jain [13]. These integral formulas are unified in nature and act as the key formulas from which we can obtain as their special cases. By giving suitable values to the parameters, our main integral formulas are reduces to the Fox H-function, the G-function and generalized wright hypergeometric function.

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NEW FINITE INTEGRALS OF GENERALIZED MELIIN-BARNES TYPE OF CONTOUR INTEGRALS

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1. A.A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals: I. Transformation and reeducation formulae, J.Phys.A:Math.Gen.20 (1987), 4109-4117.
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New Finite Integrals of Generalized Meliin-Barnes Type of Contour Integrals

Praveen Agarwal [α] & Mehar Chand [σ]

Abstract - In the present paper, we obtain three new finite integral formulas. These formulas involve the product of a general class of polynomials and the generalized Meliin- Barnes type of contour integrals. Mainly we are using series representation of the $\bar{H}$-function given by Agarwal [14], Agarwal and Jain [13]. These integral formulas are unified in nature and act as the key formulas from which we can obtain as their special cases. By giving suitable values to the parameters, our main integral formulas are reduces to the Fox H-function, the G-function and generalized wright hypergeometric function.

Keywords : $\bar{H}$ -function, general class of polynomial, generalized wright hypergeometric function.

I. Introduction

In 1987, Inayat-Hussain [1,2] was introduced generalization form of Fox's H-function, which is popularly known as $\bar{H}$-function. Now $\bar{H}$-function stands on fairly firm footing through the research contributions of various authors [1-3, 9- 10, 13-15]. $\bar{H}$-function is defined and represented in the following manner [10].

$$\bar{H}^{m,n}_{p,q}[z] = \bar{H}^{m,n}_{p,q}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L z^{\xi}\bar{\phi}(\xi)\,d\xi \qquad (z\neq 0) \tag{1.1}$$

where

$$\bar{\phi}(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

It may be noted that the $\bar{\phi}(\xi)$ contains fractional powers of some of the gamma function and m,n,p,q are integers such that $1\le m\le q, 1\le n\le p$ $(\alpha_j)_{1,p},(\beta_j)_{1,q}$ are positive real numbers and $(A_j)_{1,n},(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(\alpha_j)_{1,p}$ and $(\beta_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\bar{H}$-function can be seen in the papers [9, 10]. The behavior of the $\bar{H}$-function for small values of $|z|$ follows easily from a result given by Rathie [3]:

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$\overline{H}_{p,q}^{m,n}[z]=o(|z|^{\alpha})$; Where

$$\alpha=\min_{1\le j\le m}\operatorname{Re}\left(\frac{b_j}{\alpha_j}\right),|z|\to 0 \tag{1.3}$$

$$\mu_1=\sum_{j=1}^{m}|B_j|+\sum_{j=m+1}^{q}|b_jB_j|-\sum_{j=1}^{n}|a_jA_j|-\sum_{j=n+1}^{q}|A_j|>0\,,\,0<|z|<\infty \tag{1.4}$$

The following function which follows as special cases of the $\overline{H}$-function will be required in the sequel [10]

$${}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};z\right]=\overline{H}_{p,q+1}^{1,p}\left[-z\left|\begin{matrix}(1-a_j,\alpha_j,A_j)_{1,p}\\(0,1),(1-b_j,\beta_j,B_j)_{1,q}\end{matrix}\right.\right] \tag{1.5}$$

The general class of polynomials $S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]$ will be defined and represented as follows [6, p.185, eqn. (7)]:

$$S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]=\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}x^{l_i} \tag{1.6}$$

where $n_1,\dots,n_r=0,1,2,\dots;m_1,\dots m_r$ are arbitrary positive integers, the coefficients $A_{n_i,l_i}(n_i,l_i\ge 0)$ are arbitrary constants, real or complex. $S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]$ yields a number of known polynomials as its special cases. These includes, among other, the Jacobi polynomials, the Bessel Polynomials, the Lagurre Polynomials, the Brafman Polynomials and several others [8, p. 158-161].

The following formulas [11, p.77, Eqs. (3.1), (3.2) & (3.3)] will be required in our investigation.

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)}\,,\quad(a>0;b\ge 0;c+4ab>0;\operatorname{Re}(p)+1/2>0) \tag{1.7}$$

$$\int_0^\infty\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)}\,,\quad(a\ge 0;b>0;c+4ab>0;\operatorname{Re}(p)+1/2>0) \tag{1.8}$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)}\,,\quad(a>0;b>0;c+4ab>0;\operatorname{Re}(p)+1/2>0) \tag{1.9}$$

II. Main Integrals

First Integral

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}$$

Ref.

6. H.M.Srivastava, A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials, Paci c J.Math.117, (1985), 183-191.

10. K.C. Gupta, R. Jain and R. Agarwal, On existence conditions for a generalized Mellin-Barnes type integral Natl Acad Sci Lett. 30(5-6) (2007), 169-172.

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$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{w}}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),\left(a_j,\alpha_j,A_j\right)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \quad (2.1)$$

The above result will be converge under the following conditions

i. $a>0; b\geq 0; c+4ab>0$ and v_i, w are positive integers.

ii. $\mathrm{Re}\left[u+w\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z|<\frac{1}{2}\mu_1\pi$, where μ_1 is given by equation (1.4)

Second Integral

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{w}}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),\left(a_j,\alpha_j,A_j\right)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \quad (2.2)$$

The above result will be converge under the following conditions

i. $a\geq 0; b>0; c+4ab>0$ and v_i, w are positive integers.

ii. $\mathrm{Re}\left[u+w\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z|<\frac{1}{2}\mu_1\pi$, where μ_1 is given by equation (1.4)

Third Integral

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{w}}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),\left(a_j,\alpha_j,A_j\right)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \quad (2.3)$$

The above result will be converge under the following conditions

i. $a>0; b>0; c+4ab>0$ and v_i, w are positive integers.

ii. $\mathrm{Re}\left[u+w\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

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iii. $|\arg z| < \frac{1}{2}\mu_1\pi$, where μ_1 is given by equation (1.4)

Proof : To prove the first integral, we express $\bar{H}$-function occurring on the L.H.S. of equation (2.1) in terms of Mellin-Barnes type of contour integral given by equation (1.1) and the general class of polynomials $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ in series form with the help of equation (1.6) and then interchanging the order of integration and summation, we get:

$$\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{2\pi i}\int_L\bar{\phi}(\xi)\left\{\int_0^\infty\left[\left(ax+\frac{b}{x^2}\right)^2+c\right]^{-u-\sum_{i=1}^{r}v_il_i-w\xi-1}dx\right\}z^{\xi}d\xi \qquad (2.4)$$

Further using the result (1.7) the above integral becomes

$$\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}\frac{1}{2\pi i}\int_L\bar{\phi}(\xi)\frac{\left\{\Gamma\left(1/2+u+\sum_{i=1}^{r}v_il_i+w\xi\right)\right\}^1}{\left\{\Gamma\left(1+u+\sum_{i=1}^{r}v_il_i+w\xi\right)\right\}^1}\left[\frac{z}{(4ab+c)^w}\right]^{\xi}d\xi \qquad (2.5)$$

Then interpreting with the help of (1.1) and (2.5) provides first integral.

The proof of second and third integral can be developing on the lines similar to those given with first integral with the help of the result (1.8) and (1.9) respectively.

(3.1) If we put $A_j = B_j = 1$, $\bar{H}$ - function reduces to Fox's H-function [7, p. 10, Eqn. (2.1.1)], then the equation (2.1), (2.2) and (2.3) takes the following form.

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]H_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \qquad (3.1.1)$$

$$\int_0^\infty\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]H_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \qquad (3.1.2)$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]H_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \qquad (3.1.3)$$

Ref.

7. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publishers, New Dehli, Madras (1982).

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The Conditions of validity of (3.1.1), (3.1.2) and (3.1.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.2) By applying the results given in (2.1), (2.2) and (2.3) to the case of Hermite polynomials [4, 5] by setting $S_n^2(x)\to x^{n/2}H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $m_1,...,m_r=2;n_1,...,n_r=n;r=1;v_i=v,y_i=y,A_{n_i,l_i}=(-1)^l$, we have the following interesting results.

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{1}{y}\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{v}}\right]\bar{H}^{m,n}_{p,q}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{vl}}\bar{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^w}\middle|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right] \tag{3.2.1}$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{1}{y}\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{v}}\right]\bar{H}^{m,n}_{p,q}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{vl}}\bar{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^w}\middle|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right] \tag{3.2.2}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{1}{y}\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{v}}\right]\bar{H}^{m,n}_{p,q}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{vl}}\bar{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^w}\middle|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right] \tag{3.2.3}$$

The Conditions of validity of (3.2.1), (3.2.2) and (3.2.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.3) By applying the our results given in (2.1), (2.2) and (2.3) to the case of Lagurre polynomials [4, 5] by setting $S_n^2(x)\to L_n^{(\alpha)}[x]$ in which $m_1,...,m_r=1;n_1,...,n_r=n;r=1;v_i=v,y_i=y,A_{n_i,l_i}=\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}$, we have the following interesting results.

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} L_n^{(\alpha)}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]\bar{H}^{m,n}_{p,q}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}\frac{(y)^l}{(4ab+c)^{vl}}$$

$$\bar{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^w}\middle|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right] \tag{3.3.1}$$

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$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} L_n^{(\alpha)}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}\frac{(y)^l}{(4ab+c)^{vl}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \quad (3.3.2)$$

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$$\int_0^\infty\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} L_n^{(\alpha)}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}\frac{(y)^l}{(4ab+c)^{vl}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \quad (3.3.3)$$

The Conditions of validity of (3.3.1), (3.3.2) and (3.3.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.4) If we put $A_j=B_j=1;\alpha_j=\beta_j=1$, in (1.1) then the $\overline{H}$-function reduces to the general type of G-function [12] i.e. $\overline{H}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,1,1)_{1,n},(a_j,1)_{n+1,p}\\(b_j,1,1)_{1,m},(b_j,1)_{m+1,q}\end{matrix}\right.\right]=G\left[z\left|\begin{matrix}(a_j,1)_{1,p}\\(b_j,1)_{1,q}\end{matrix}\right.\right]$, So using same assumptions in the equations (2.1), (2.2) and (2.3) then they takes the following form.

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^r y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]G_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^r\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^r v_il_i,w;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-u-\sum_{i=1}^r v_il_i,w;1\right)\end{matrix}\right.\right]$$

(3.4.1)

$$\int_0^\infty\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^r y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]G_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^r\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^r v_il_i,w;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-u-\sum_{i=1}^r v_il_i,w;1\right)\end{matrix}\right.\right]$$

(3.4.2)

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^r y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]G_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

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$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}G^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \quad (3.4.3)$$

The Conditions of validity of (3.4.1), (3.4.2) and (3.4.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.5) If we put $n=p, m=1, q=q+1, b_1=0, \beta_1=1, a_j=1-a_j, b_j=1-b_j$, in (1.1) then the $\overline{\mathrm{H}}$-function reduces to generalized wright hypergeometric function [16] i.e. $\overline{H}^{1,p}_{p,q+1}\left[z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right]={}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\right]$, using same assumptions in the equations (2.1), (2.2) and (2.3) then they takes the following form.

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}\,{}_{p+1}\overline{\Psi}_{q+1}\left[\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix};-\frac{z}{(4ab+c)^w}\right] \quad (3.5.1)$$

$$\int_0^\infty\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}\,{}_{p+1}\overline{\Psi}_{q+1}\left[\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix};-\frac{z}{(4ab+c)^w}\right] \quad (3.5.2)$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}\,{}_{p+1}\overline{\Psi}_{q+1}\left[\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix};-\frac{z}{(4ab+c)^w}\right] \quad (3.5.3)$$

The Conditions of validity of (3.5.1), (3.5.2) and (3.5.3) easily follow from those given in (2.1), (2.2) and (2.3).

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Notes

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