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An Integral Transformation Involving a Certain Product of Special Functions

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I. Introduction

Integrals with Fox's H-function, M-series and multi variable H-function were studied by many authors. We have the following series representation of the H-function by Skibińki [4]

$$H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p,e_p)\\(b_q,f_q)\end{matrix}\right.\right]=\sum_{N=1}^{m}\sum_{s=0}^{\infty}\frac{(-1)^s z^{\eta_s}}{f_N\, N!}\phi(\eta_s), \qquad (1)$$

where

$$\phi(\eta_s)=\prod_{i=1}^{m}\Gamma(b_i-f_i\eta_s)\prod_{i=1}^{n}\Gamma(1-a_i+e_i\eta_s)$$

$$\left\{\prod_{i=m+1}^{q}\Gamma(1-b_i+f_i\eta_s)\prod_{i=n+1}^{p}\Gamma(a_i-e_i\eta_s)\right\}^{-1}$$

and

$$\eta_s=\frac{b_N+s}{f_N}.$$

The following results of Srivastava and Daoust [1, eq.(1.2), p.15], Slater [2, p.79, eq. (2.5.27) and Chaurasia [3, p.194, eq. (2.3)] respectively also required in our investigations:

(a)

$$F_{\sigma\,:N';\ldots;N^{(s)};1,1}^{v:M';\ldots;M^{(s)};0,0}\left(\begin{matrix}[(\alpha_v):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:(m'):\rho];\ldots;\\ [(\beta_\sigma):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

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Ref.

1. **Srivastava, H.M. and Daoust, Martha, C.** Certain generalized Neuman expansion associated with the Kampé de Fériet function, Ned. Akad. Wet. Proc. Ser. A – 72 = Indag. Math. 31 (1969), 449-457.
2. **Slater, L.J.** Generalized hypergeometric functions, Cambridge University Press (1966).

$$\begin{matrix}[(m^{(s)}:\rho^{(s)}];\ ---\ ;----\ ; \\ [(\ell^{(s)}:\tau^{(s)}];[\alpha+1,1];[\beta+1,1)];\end{matrix}\ z_1',\ldots,z_s',-xt,(1-x)t\Bigg)$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}}P_n^{(\alpha,\beta)}(1-2x)$$

$$F^{\nu:M';\ldots;M^{(s)}}_{\sigma:N',\ldots,N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_u):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots; & [(m^{(s)}):\rho^{(s)}]; \\ [(\beta_\sigma+n\mu_\sigma):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots; & [(\ell^{(s)}):\tau^{(s)}];\end{matrix}\ z_1',\ldots,z_s'\right)t^n \qquad (2)$$

(b)

$${}_4F_3\left[\begin{matrix}a,b,\frac{m+d}{2},\frac{m+b-1}{2} \\ a+b,m,d\end{matrix};4y(1-y)\right]$$

$$=\sum_{k=0}^{\infty}\frac{(m+d-1)_k}{(a+b)_k}m_k\,y^k, \qquad (3)$$

where m_k is given by

$${}_2F_1(a,b;m;y)\ {}_2F_1(a,b;d;y)=\sum_{k=0}^{\infty}m_k\,y^k, \qquad (4)$$

(c)

$$\int_0^1 y^k H\left(y^{h_1}z_1,\ldots,y^{h_r}z_r\right)H^{M_1,N_1}_{P_1,Q_1}\left(xy^{L_1}\middle|\begin{matrix}(a_p,e_p)\\(b_q,f_q)\end{matrix}\right){}_{P_2}M^{\alpha'}_{Q_2}(\tau_1y^{L_2})\ S_V^U[\tau_2y^{L_3}]\,dy$$

$$=\sum_{\sigma=0}^{M_1}\sum_{k_1,k_2=0}^{\infty}\sum_{k_3=0}^{[V/U]}\frac{(-1)^{k_1}x^{\eta k_1}}{f_\sigma'}\phi(\eta k_1)\frac{(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_1)^{k_2}(-V)_{Uk_3}A_{V,k_3}}{(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\Gamma(\alpha'k_2+1)\ k_2!\,k_3!}\tau_2^{k_3}$$

$$.\ H^{0,\lambda+1\ :(u',v')\ ;\ldots;(u^{(r)},v^{(r)})}_{A+1,C+1:(B',D');\ldots;(B^{(r)},D^{(r)})}\left[\begin{matrix}[-k-L_1\eta_{k_1}-L_2k_2-k_3L_3:h_1,\ldots,h_r], \\ [-k_1-1-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}] \\ [(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]\end{matrix},z_1,\ldots,z_r\right], \qquad (5)$$

where $h_i>0,\mathrm{Re}\left(1+L_1\frac{b_{j'}}{f_{j'}}+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)}\right)>0,|\arg(z_i)|<\frac{T_i\pi}{2},T_i>0,\ i=1,\ldots,r;$

$j=1,\ldots,u^{(i)},j'=1,\ldots,P_1,P_2\le Q_2,|\tau_2|<1,$ U is an arbitrary positive integer, the

Notes

coefficients A_{V,k_3} (V, k_3 ¿ 0) are arbitrary constants, real or complex. L_1, L_2, L_3 ¿ 0 ,

$$|\arg \tau_1| < \frac{1}{2}\pi T', T' = \sum_{i=1}^{N_1} e_i - \sum_{i=N_1+1}^{P_1} e_i + \sum_{i=1}^{M_1} f_i - \sum_{i=M_1+1}^{Q_1} f_i \tag{6}$$

The result (5) is a generalization of a result of Chaurasia [2, p.194, eq. (2.3)]. Proof process used is the same.

Ref.

2. **Slater, L.J.** Generalized hypergeometric functions, Cambridge University Press (1966).

II. Main Results

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta}\, F^{v:M';\ldots;M^{(s)};0,0}_{\in:N';\ldots;N^{(s)};1,1}\left(\begin{matrix}[(\alpha_v):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:(m'):\rho];\ldots;\\ [(\beta_{\in}):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}:\rho^{(s)}];\ ---\ ;----\ ;\\ [(\ell^{(s)}:\tau^{(s)}];[\alpha+1,1];[\beta+1,1)];\end{matrix}\ z_1',\ldots,z_s',-xt,(1-x)t\right) H(z_1 x^{h_1},\ldots,z_r x^{h_r})$$

$$\cdot H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2 x^{L_2}]S^U_V[\tau_3 x^{L_3}]dx$$

$$= \sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{k_1,k_2,n=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})}{\tau_4 f_{\tau_4} k_1!k_3!k_2!}$$

$$\cdot\frac{(a_1)_{k_2}\cdots(a_{P_2})_{k_2}(\tau_2)^{k_2}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}}{(b_1)_{k_2}\cdots(b_{Q_2})_{k_2}\Gamma(\alpha' k_2+1)\,k_3!}$$

$$\cdot F^{\nu:M';\ldots;M^{(s)}}_{\sigma:N',\ldots,N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_\sigma+n\mu_\sigma):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau']\ ;\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\ z_1',\ldots,z_s'\right)$$

$$\frac{(-t)^{\eta}\Gamma(\beta+n+1)}{n!}H^{0,\lambda+2\ :(u',v')\ ;\ldots;(u^{(r)},v^{(r)})}_{A+2,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}\left(\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-k_3L_3:h_1,\ldots,h_r],\\ [-\sigma-1-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]\\ [(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]\end{matrix},z_1,\ldots,z_r\right) \tag{7}$$

where $\mathrm{Re}(\beta)>-1, \mathrm{Re}\left(\sigma+L_1\frac{b_{j'}}{f_{j'}}+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)}\right)>0, |\arg(z_i)|<\frac{T_i\pi}{2}, T_i>0,$ $i=1,\ldots,r;$ $j=1,\ldots,u^{(i)}, j'=1,\ldots,P_1, P_2\le Q_2, |\tau_2|<1,$ U is an arbitrary positive integer, the coefficients A_{V,k_3} (V, k_3 ¿ 0) are arbitrary constants, real or complex. L_1, L_2, L_3 ¿ 0 , $|\arg\tau_1|<\frac{1}{2}\pi T'$

$$\left[T'=\sum_{i=1}^{N_1}e_i-\sum_{i=N_1+1}^{P_1}e_i+\sum_{i=1}^{M_1}f_i-\sum_{i=M_1+1}^{Q_1}f_i\right],$$ and the series on the right is convergent.

Proof of main result

Multiplying both sides of (2) by $x^{\sigma-1}(1-x)^{\beta}\,H(z_1x^{h_1},\dots,z_rx^{h_r})$. $H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]\,{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2x^{L_2}]S^U_V[\tau_3x^{L_3}]$ and integrating it with respect to x from 0 to 1. Evaluating the right hand side thus obtained by interchanging the order of integration and summations (which is justified due to the absolute convergence of the integral involved in the process) and then integrating the inner integral with the help of the following result [5, eq. 2.2, p.131]

$$\int_0^1 x^{\epsilon}(1-x)^{\beta}P_n^{(\alpha,\beta)}(1-2x)\,H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2x^{L_2}]S^U_V[\tau_3x^{L_3}]$$

$$\cdot\, H(z_1x^{\sigma_1},\dots,z_rx^{\sigma_r})\,dx$$

$$=\sum_{\tau_4}^{M_1}\sum_{k_1,k_2=0}^{\infty}\sum_{k_3=0}^{[V/U]}\frac{(-1)^n\,\Gamma(\beta+n+1)(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\dots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{n!\,\tau_4!\,f_{\tau_4}\,k_1!k_2!k_3!(b_1)_{k_2}\dots(b_{Q_2})_{k_2}\,\Gamma(\alpha'k_2+1)}$$

$$\cdot(-V)_{Uk_3}\,A_{V,k_3}(\tau_3)^{k_3}\,H^{0,\lambda:(u',v');\dots;(u^{(r)},v^{(r)})}_{A,C:(B',D');\dots;(B^{(r)},D^{(r)})}\left(\begin{matrix}[-\epsilon-L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\dots,\sigma_r],\\ [-\beta-\epsilon-n-1-L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\dots,\sigma_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[\alpha-\epsilon-L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\dots,\sigma_r],[(a):\theta',\dots,\theta^{(r)}]:[(b'):\phi']\;;\dots;[(b^{(r)}):\phi^{(r)}]:\\ [\alpha+n-\epsilon L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\dots,\sigma_r],[(c):\psi',\dots,\psi^{(r)}]:[(d'):\delta'];\dots;[(d^{(r)}):\delta^{(r)}]:\end{matrix}\;z_1,\dots,z_r\right), \tag{8}$$

$$\text{where } Re(\beta)>-1, Re\left(\epsilon+L_1\frac{b'_j}{f'_j}+\sum_{i=1}^{r}\sigma_i\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>-1, \sigma_i>0, L_1,L_2,L_3>0, |\arg(z_i)|$$

$<\frac{1}{2}T_i\pi, \tau_1,\tau_2>0, |\arg\tau_1|<\frac{1}{2}T'\pi, i=1,\dots,r;\ j=1,\dots,u^{(i)},\ j'=1,\dots,Q_2,$ we arrive the required result (7).

1.8 SPECIAL CASES OF (7)

(i) Letting $\lambda=A, u^{(i)}=1, V^{(i)}=B^{(i)}, D^{(i)}=D^{(i)}+1\,\forall\, i=1,\dots,r$ in (7), we find

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta}F^{\nu:M',\dots,M^{(s)};0;0}_{\epsilon:N',\dots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\dots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\dots;\\ [(\beta_\epsilon):\zeta',\dots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\dots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]:\;-\;;-\;;\\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix}\;z'_1,\dots,z'_s,-xt,(1-x)t\right)$$

$$.F^{A:B',\dots,B^{(r)}}_{C:D',\dots,D^{(r)}}\left(\begin{matrix}[1-(a):\theta',\dots,\theta^{(r)}]\;:[1-(b'):\phi'];\dots;[1\text{-}(b^{(r)}):\phi^{(r)}]\\ [1-(c):\psi',\dots,\psi^{(r)}]:[1-(d'):=\delta'];\dots;[1\text{-}(d^{(r)}):\delta^{(r)}]\end{matrix}\;-z_1X^{h_1},\dots,z_rX^{h_r}\right)$$

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5. **Srivastava, H.M. and Panda, R.** Expansion theorem for the H-function of several complex variables, J. Reine Angew. Math. **288**, (1976), 129-145.

$$\cdot H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right] {}_{P_2}\overset{\alpha'}{M}_{Q_2}[\tau_2 x^{L_2}] S^U_V[\tau_3 x^{L_3}]\, dx$$

$$= \sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty} \frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}$$

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4! f_{\tau_4} k_1! k_2! k_3!(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\Gamma(\alpha' k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F^{\nu:M';\ldots;M^{(s)}}_{\in:N';\ldots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_t+\eta\mu_t):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right)$$

$$\frac{(-t)^n\Gamma(\beta+n+1)}{n!} F^{A+2:B';\ldots;B^{(r)}}_{C+2:D';\ldots;D^{(r)}}\left(\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],\\ [1-(c):\psi',\ldots,\psi^{(r)}],\end{matrix}\right.$$

$$[1+\alpha+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,\ldots,h_r],[1-(a):\theta',\ldots,\theta^{(r)}];$$

$$[1-\sigma\text{-}L\,\eta_{k_1}-L_2k_2-L_3k_3+\alpha+n:h_1,\ldots,h_r],[-\beta-h-\sigma-L\,\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],$$

$$\left.\begin{matrix}[1-(b'):\phi'];\ldots;[1-(b^{(r)}):\phi^{(r)}];\\ [(1-(d'):\delta'];\ldots;[1\text{-}d^{(r)}):\delta^{(r)}];\end{matrix}\; -z_1,\ldots,-z_r\right) \tag{9}$$

provided that $\mathrm{Re}(\sigma)>0, \mathrm{Re}(\beta)<-1, h_i>0, L_1, L_2, L_3, \tau_2, \tau_3>0, i=1,\ldots,r, |t|<1$ and the series on the right is convergent.

(ii) Taking r = 2, the result in (9) reduces to the following integral

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta} F^{\nu:M',\ldots,M^{(s)};0;0}_{\in:N',\ldots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\ldots;\\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]:[\;-\;;\;-\;];\\ [(\ell^{(s)}):\tau^{(s)}];[\alpha+1,1];[\beta+1,1];\end{matrix}\; z_1',\ldots,z_s',-xt,(1-x)t\right)$$

$$.F^{A:B',B''}_{C:D',D''}\left(\begin{matrix}[1-(a):\theta',\theta'']\;:[1-(b'):\phi'];[1\text{-}(b''):\phi'']\\ [1-(c):\psi',\psi'']:[1-(d'):=d'];[1\text{-}(d''):\delta'']\end{matrix}\; -z_1x^{h_1},-z_2x^{h_2}\right)$$

$$\cdot\, H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right] {}_{P_2}\overset{\alpha'}{M}_{Q_2}[\tau_2 x^{L_2}] S^U_V[\tau_3 x^{L_3}]\, dx$$

$$= \sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty} \frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}$$

Notes

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\dots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}\,k_1!\,k_2!\,k_3!(b_1)_{k_2}\dots(b_{Q_2})_{k_2}\,\Gamma(\alpha' k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$.F^{\nu:M';\dots;M^{(s)}}_{\in:N';\dots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\dots,\eta^{(s)}]:[(m'):\rho'];\dots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_\in+\eta\mu_\in):\zeta',\dots,\zeta^{(s)}]:[(\ell'):\tau'];\dots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\dots,z_s'\right).\frac{(-t)^n\,\Gamma(\beta+n+1)}{n!}$$

$$.F^{A+2:B',B''}_{C+2:D',D''}\left[\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2],\\ [1-(c):\psi',\psi''],\end{matrix}\right.$$

$$[1+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,h_2],[1-(a):\theta',\theta''];$$

$$[1-\sigma-L\,\eta_{k_1}-L_2k_2-L_3k_3+\alpha+n:h_1,h_2],[-\beta-h-\sigma-L\,\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2],$$

$$\left.\begin{matrix}[1-(b'):\phi'];\dots;[1-(b''):\phi''];\\ [(1-(d'):\delta'];\dots;[1-d''):\delta''];\end{matrix}\;-z_1,-z_2\right), \tag{10}$$

where $\mathrm{Re}(\sigma)>0, \mathrm{Re}(\beta)<-1, h_i>0, L_1, L_2, L_3, \tau_2, \tau_3>0, i=1,\dots,r, |t|<1$ and the multiple series on the right of (10) converges absolutely.

(iii) Putting A = C = λ = 0 in (9), we have

$$\int_0^1 x^{\sigma-1}(1-x)^\beta F^{\nu:M',\dots,M^{(s)};0;0}_{\in:N',\dots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\dots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\dots;\\ [(\beta_\in):\zeta',\dots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\dots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]:\; - \;;-\;;\\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix}\; z_1',\dots,z_s',-xt,(1-x)t\right)$$

$$\cdot H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\ (b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}\overset{\alpha'}{M}{}_{Q_2}[\tau_2x^{L_2}]S^U_V[\tau_3x^{L_3}]$$

$$\cdot\prod_{i=1}^{r}\left\{H^{(u^{(i)},v^{(i)})}_{B^{(i)},D^{(i)}}\left[z_1x^{h_i}\left|\begin{matrix}[(b^{(i)}):\phi^{(i)}]\\ [(d^{(i)}):\delta^{(i)}]\end{matrix}\right.\right]\right\}dx$$

$$=\sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}\frac{(-t)^n\Gamma(\beta+n+1)}{n!}$$

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\dots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}\,k_1!\,k_2!\,k_3!(b_1)_{k_2}\dots(b_{Q_2})_{k_2}\,\Gamma(\alpha' k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F^{\nu:M';\dots;M^{(s)}}_{\in:N';\dots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\dots,\eta^{(s)}]:[(m'):\rho'];\dots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_t+\eta\mu_t):\zeta',\dots,\zeta^{(s)}]:[(\ell'):\tau'];\dots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\dots,z_s'\right)$$

$$H^{0,2:(u',v');\dots;(u^{(r)},v^{(r)})}_{2,2:(B',D');\dots;(B^{(r)},D^{(r)})}\left[\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\dots,h_r],\\ [(c):\psi',\dots,\psi^{(r)}],\end{matrix}\right.$$

Notes

$$[1+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,\ldots,h_r],[1-(a):\theta',\ldots,\theta^{(r)}];$$
$$[1+\sigma+\alpha+n:h_1,\ldots,h_r],[-\beta-n-L\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],$$
$$\left.\begin{matrix}[(b'):\phi'];\ldots;[(b^{r)}'):\phi^{(r)}'];\\ [((d'):\delta'];\ldots;[d^{(r)}):\delta^{(r)}'];\end{matrix}\; -z_1,\ldots,z_r\right], \qquad (11)$$

Notes

valid under the conditions obtainable from (9).

(iv) Taking r = 2 in (11), we have

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta}\, F^{\nu:M',\ldots,M^{(s)};0;0}_{\in:N',\ldots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\ldots;\\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]: \;\; - \;\;;- \;\;;\\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix}\; z_1',\ldots,z_s',-xt,(1-x)t\right)$$

$$\cdot H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2 x^{L_2}]S_V^U[\tau_3 x^{L_3}]$$

$$\cdot H^{0,\lambda:(u',v');(u'',v'')}_{A,C:(B',D');(B'',D'')}\left[\begin{matrix}[(a):\theta',\theta'']:[(b'):\phi'];[(b''):\phi''];\\ [(c):=\psi',\psi'']:[(d'):\delta'];[(d''):\delta''];\end{matrix}\; z_1x^{h_1},z_2x^{h_2}\right]dx$$

$$=\sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}\frac{(-t)^n\Gamma(\beta+n+1)}{n!}$$

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}\,k_1!\,k_2!\,k_3!(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\,\Gamma(\alpha'k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F^{\nu:M';\ldots;M^{(s)}}_{\in:N';\ldots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_t+n\mu_t):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right)$$

$$H^{0,\lambda+2:(u',v');(u'',v'')}_{A+2,C+2:(B',D');(B'',D'')}\left[\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2],\\ [(c):\psi',\psi''],\end{matrix}\right.$$

$$[1+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,\ldots,h_r],[1-(a):\theta',\theta''];$$

$$[1+\sigma+\alpha+n:h_1,h_2],[-\beta-n-L\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2],$$

$$\left.\begin{matrix}[(b'):\phi'];[(b''):\phi''];\\ [((d'):\delta'];[d''):\delta''];\end{matrix}\; z_1,z_2\right], \qquad (12)$$

where $\operatorname{Re}(\beta) > -1, \operatorname{Re}\left(\alpha + h_1 \frac{d'_j}{\delta'_j} + h_2 \frac{d''_j}{\delta''_j} + L \frac{b_{j'''}}{f_{j'''}}\right) > 0, j' = 1,\ldots,u^{(i)}, j'' = 1,\ldots,u'',$

$j''' = 1,\ldots,Q_2, T_1 > 0, T_2 > 0, |\arg(z_1)| < \frac{1}{2}T_1\pi, |\arg(z_2)| < \frac{1}{2}T_2\pi, |t| < 1$ and the series on the right of (12) absolutely convergent.

(v) putting r=1 in (11),we find

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta} F^{\nu:M',\ldots,M^{(s)};0;0}_{\in:N',\ldots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\ldots;\\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]: \; - \; ; - \; ;\\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix} z'_1,\ldots,z'_s, -xt, (1-x)t\right)$$

$$.H^{u',v'}_{B',D'}\left[z_1 x^{h_1}\left|\begin{matrix}(b'):\phi'\\(d'):\delta'\end{matrix}\right.\right] H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}\overset{\alpha'}{M}_{Q_2}[\tau_2 x^{L_2}]S^U_V[\tau_3 x^{L_3}]dx$$

$$= \sum_{\tau_4=1}^{M_1} \sum_{k_3=0}^{[V/U]} \sum_{n,k_1,k_2=0}^{\infty} \frac{\prod_{j=1}^{\nu}(\alpha_j)_{m\gamma_j}}{(\alpha+1)_n(\beta+1)_n \prod_{j=1}^{\in}(\beta_j)_{n\mu_j}} \frac{(-t)^n\Gamma(\beta+n+1)}{n!(\alpha+1)_n(\beta+1)_n}$$

$$\cdot \frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4! f_{\tau_4} k_1! k_2! k_3! (b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\Gamma(\alpha' k_2+1)}(-V)_{U,k_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F^{\nu:M';\ldots;M^{(s)}}_{\in:N';\ldots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_t+n\mu_t):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix} z'_1,\ldots,z'_s\right)$$

$$H^{u',v'+2}_{B'+2,D'+2}\left[z_1\left|\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1],\\ [(d'):\delta'],\end{matrix}\right.\right.$$

$$\left.\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3+\alpha:h_1],[(b'):\phi']\\ [1-\sigma+\alpha+n+L\eta_{k_1}+L_2k_2+Lk_3:h_1],[-\sigma-\beta-n-L_1\eta_{k_1}-L_2k_2-Lk_3:h_1]\end{matrix}\right] \qquad (13)$$

where $\operatorname{Re}(\beta) > -1, h_1 > 0, \tau_2, \tau_3 > 0, L_1, L_2, L_3 > 0, \operatorname{Re}\left(\sigma + h_1 d'_j/\delta'_j + L_1 \frac{b'_j}{f'_j}\right)$ ¿ 0,

$j = 1,\ldots,u', j' = 1,\ldots,Q_2, T_1 > 0, T_2 > 0, |t| < 1, |\arg(z_1)| < \frac{1}{2}T_1\pi, |\arg \tau_1| < \frac{1}{2}T_2\pi$ and the series on the right of (13) is absolutely convergent.

Notes

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References Références Referencias

Notes

1. **Srivastava, H.M. and Daoust, Martha, C.** Certain generalized Neuman expansion associated with the Kampé de Fériet function, Ned. Akad. Wet. Proc. Ser. A – 72 = Indag. Math. 31 (1969), 449-457.
2. **Slater, L.J.** Generalized hypergeometric functions, Cambridge University Press (1966).
3. **Chaurasia, V.B.L.** A theorem concerning the multivariable H-function, Bull. Inst. Math. Acad. Sinica, **13**(2) (1985), 193-194.
4. **Skinbińki, P.** Some expansion theorem for the H-function, Ann. Polon. Math., **23** (1970), 125-138.
5. **Srivastava, H.M. and Panda, R.** Expansion theorem for the H-function of several complex variables, J. Reine Angew. Math. **288**, (1976), 129-145.
6. **Srivastava, H.M.**A contour integral involving Fox's H-function, Indian J. Math., **14**, (1972), 1-6.
7. **Srivastava, H.M. and Singh, N.P.** The integration of certain products of the multivariable H-function with a general class of polynomials, Rend. Circ. Mat. Di Palermo 32(2), (1983), 157-187.
8. **Mittal, P.K. and Gupta, K.C.** An integral involving generalized function of two variables, Proc. Ind. Akad. Sci. **75A** (1972), 117-123.

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