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# Bianchi Type-$VI_0$ Dark Energy Cosmological Models in General Relativity

By Priyanka, S. Chandel, M.K. Singh & Shri Ram
*Banaras Hindu University, India*

*Abstract* - Bianchi type-$VI_0$ cosmological models of the universe filled with dark energy with constant and time-dependent equation of state parameters are investigated in general relativity. We obtain exact solutions of Einstein's field equations using the condition that the shear scalar is proportional to the expansion scalar, which represent singular and non-singular cosmological models of the universe. The physical behavior of the models are discussed. We conclude that the universe models do not approach isotropy through the evolution of the universe.

*Keywords :* *Bianchi type-VI. Dark energy. Cosmological models.*

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BIANCHI TYPE-VI0 DARK ENERGY COSMOLOGICAL MODELS IN GENERAL RELATIVITY

*Strictly as per the compliance and regulations of :*

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# Bianchi Type-$VI_0$ Dark Energy Cosmological Models in General Relativity

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## I. Introduction

Recent observations on expansion history of the universe indicate that the universe is currently experiencing a phase of accelerated expansion. This was first observed from high red shift supernova Ia (Reiss et al. [1-2], Perlmutter et al. [3], Astier et al. [4], Spergel et al.[5] etc.) and confirmed later by cross checks from the cosmic microwave background radiation (Bennett et al. [6], Abazajian et al.[7-9], Hawkins et al. [10] etc.). The current accelerating expansion of the universe attributed to the fact that our universe is dominated by an unknown dark energy DE an exotic energy with negative pressure.

The simplest dark energy candidate is the vacuum energy density which is mathematically equivalent to the cosmological constant $\Lambda$. As per Copeland et al. [11] "fine tuning "and the cosmic "coincidence "are the two well known difficulties of the cosmological constant problems. There are several alternative theories for the dynamical DE scenario which have been proposed by scientists to interpret the accelerating universe. Wang and Tegmark [12] have shown that the universe is actually undergoing an acceleration with repulsive gravity of some strange energy-form i.e. DE at work. Dark energy is a mysterious substance with negative pressure and accounts for nearly 70% of total matter-energy of universe, but has no clear explanation. Karami et al. [13] introduced a polytropic gas model of DE as an alternative model to explain the accelerated expansion of the universe. Gupta and Pradhan [14] proposed a new candidate known as cosmological nuclear-energy as a possible candidate for the dark energy.

*Authors : Department of Applied Mathematics, Indian Institute of Technology, Banaras Hindu University, Varanasi 221 005, India.*
*E-mail : srmathitbhu@rediffmail.com*

Bianchi types I-IX cosmological models are important in the sense that these are homogeneous and anisotropic, from which the process of isotropization of the universe is studied through the passage of time. Moreover, from the theoretical point of view anisotropic universe have a greater generality than FRW isotropic models. The simplicity of the field equations made Bianchi space-times useful in constructing models of spatially homogeneous and anisotropic cosmologies. Considerable works have been done in obtaining various Bianchi type cosmological models and their inhomogeneous generalization. Bianchi type-VI$_0$ space-time is of special interest in anisotropic cosmology. Barrow [15] pointed out that Bianchi type-VI$_0$ models of the universe give a better explanation of some of the cosmological problems like primordial helium abundance and they also isotropize in a special sense. Looking to the importance of Bianchi type-VI$_0$ universes, many authors [16-20] have studied it in different context. Shri Ram[21, 22] has presented Bianchi type-VI$_0$ cosmological models filled with dust and perfect fluid in modified Brans-Dicke theory respectively.

Adhav et al. [23] studied Bianchi type-VI$_0$ cosmological models with anisotropic dark energy. Abdussattar and Prajapati [24] obtained a class of bouncing non-singular FRW models by constraining the deceleration parameter (DP) in the presence of an interacting dark energy represented by a time-varying cosmological constant. They have also discussed the role of deceleration parameter and interacting dark energy in singularity avoidance. Bisabr [25] has shown that an accelerating expansion is possible in a spatially flat universe for large values of the Brans-Dicke parameter consistent with the local gravity experiments. Yadav and Saha [26] studied DE models with variable equation of state (EoS) parameter. Recently, Saha and Yadav [27] presented a general relativistic cosmological model with time-dependent DP in LRS Bianchitype-II space-time which can be described by isotopic and variable EoS parameter. In this paper, We present general relativistic cosmological models with constant and time-dependent DP in Bianchi type-VI$_0$ space-time which can be described by isotropic constant and variable EoS parameters. This paper is organized as follows: We present the metric and field equations in Sect.2. In Sect.3, we obtain the solutions of the field equations representing Bianchi type-VI$_0$ cosmological models with perfect fluid by imposing the condition that the shear scalar is proportional to expansion scalar. We also discuss the physical behaviors of the cosmological models with dark energy. Concluding remarks are given in Sect.4.

## II. The Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-VI$_0$ space-time in the form

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)e^{-2mx}dy^2 + C^2(t)e^{2mx}dz^2 \tag{1}$$

where $A$, $B$ and $C$ are functions of the cosmic time $t$ and $m$ is a constant

The Einstein's field equations, in natural limits $(8\pi G = c = 1)$are

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = -T_{\mu\nu} \tag{2}$$

Ref.

15. Barrow, J.D.: Mon. Not. Astron. Soc. **211**, 221(1984)

where $R_{\mu\nu}$ is the Ricci tensor, $R$ is the Ricci scalar curvature and $T_{\mu\nu}$ is the energy-momentum tensor of matter. For a perfect fluid distribution, the tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \tag{3}$$

where $\rho$ is the energy density of the cosmic matter $p$ is the isotropic pressure and $u^\mu$ is the four-velocity vector. In comoving coordinate system $u^\mu = (0, 0, 0, 1)$, the Einstein's field equation (2) together with (3), for the metric (1), yield

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{m^2}{A^2} = -\omega\rho, \tag{4}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -\omega\rho, \tag{5}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -\omega\rho, \tag{6}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = \rho, \tag{7}$$

$$(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}) = 0 \tag{8}$$

where $\omega$ is the EoS parameter given by

$$p = \omega\rho \tag{9}$$

and a dot denotes ordinary differentiation with respect to t.

The average scalar factor $a$ and volume scalar $V$ are given by

$$a^3 = V = ABC. \tag{10}$$

The generalized mean Hubble parameter $H$ is defined by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3) \tag{11}$$

where the directional Hubble parameters $H_1$, $H_2$ and $H_3$ are given by

$$H_1 = \frac{\dot{A}}{A}, H_2 = \frac{\dot{B}}{B}, H_3 = \frac{\dot{C}}{C}. \tag{12}$$

The expansion scalar $\theta$ and shear scalar $\sigma$ are given by

$$\theta = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{13}$$

$$\sigma^2 = \frac{1}{2}[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}] - \frac{1}{6}\theta^2. \tag{14}$$

The deceleration parameter $q$ is defined by

$$q = -1 + \frac{d}{dt}(H). \tag{15}$$

The sign of $q$ indicates whether the model inflates or not. A positive sign of $q$ corresponds to the standard decelerating model whereas the negative sign of $q$ indicates inflation. The recent observations of SN Ia (Reiss et al.[1], Perlmutter et al.[3]) reveal that the present universe is accelerating and the value of DP lies somewhere in the range $-1 < q < 0$.

## III. Solution of Field Equations

Equation (8), on integration, gives

$$B = C \tag{16}$$

where the constant of integration is absorbed in $B$ or $C$. Using (16), equations (4) – (7) reduce to

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{m^2}{A^2} = -\omega\rho, \tag{17}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -\omega\rho, \tag{18}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{m^2}{A^2} = \rho. \tag{19}$$

These are three equations connecting four unknown functions $A$, $B$, $\rho$ and $\omega$. In order to solve the above equations we use the physical condition that expansions scalar is proportional to shear scalar, which in our case leads to

$$A = B^n \tag{20}$$

where $n$ is a constant. Roy and Banerjee [28], Bali and Singh [29] have proposed this condition to find exact solutions of cosmological models.

Here we use the procedure of Saha and Yadav [27] to find exact solutions of (17) – (19) combining (10) and (20), we obtain

$$A = V^{\frac{n}{n+1}}, \qquad B = V^{\frac{1}{n+1}}. \tag{21}$$

Subtraction of (18) from (17) gives

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}\dot{B}}{AB} + \frac{2m^2}{A^2} = 0. \tag{22}$$

Substituting (21) into (22), we obtain

$$\ddot{V} = \frac{2m^2(n+2)}{n-1} V^{\frac{2-n}{n+2}}. \tag{23}$$

The first integral of (23) is

$$\int \frac{dV}{V^{\frac{4}{n+2}+C}} = \frac{m(n+2)t}{\sqrt{n-1}} \tag{24}$$

Ref.

3. Perlmutter, S., et al.: Astrophys. J. **483**, 565 (1997)

where $C$ is an arbitrary constant. Clearly (24) imposes some restriction on the choice of $n$ namely, $n > 1$. It is not possible to solve equation (24) in general. So, in order to solve the problem completely, we have to choose either $C$ or $n$ in such a way that (24) be integrable. Therefore we consider the following cases.

Notes

*Case 3.1 When C=0*

In this case the solution of (24) is

$$V = \left(\frac{mn}{\sqrt{n-1}}\right)^{\frac{n+2}{n}}(t+k_1) \tag{25}$$

where $k_1$ is an arbitrary constant. From (21) and (25) we obtain the scale factor as

$$A = \frac{mn}{\sqrt{n-1}}(t+k), \tag{26}$$

$$B = (\frac{mn}{\sqrt{n-1}})^{\frac{1}{n}}(t+k)^{\frac{1}{n}}. \tag{27}$$

With these scale factors, the metric (1) can be written in form

$$ds^2 = -dT^2 + (\frac{mn}{\sqrt{n-1}})^2 dx^2 + (\frac{mn}{\sqrt{n-1}})^{\frac{2}{n}} T^{\frac{2}{n}} (e^{-2mx}dy^2 + e^{2mx}dz^2) \tag{28}$$

where T=t+k.
The expressions for the energy density $\rho$ and the EOS $\omega$ for the model (28) are obtained as

$$\rho = \frac{1+n}{n^2T^2}, \tag{29}$$

$$\omega = \frac{n-2}{n+1}. \tag{30}$$

The other physical and kinematical parameters are given by

$$nH_1 = H_2 = H_3 = \frac{1}{T}, \tag{31}$$

$$\theta = 3H = \frac{n+2}{nT}, \tag{32}$$

$$\sigma = \frac{1}{\sqrt{3}}\frac{n-1}{nT}, \tag{33}$$

$$q = -\frac{2}{n+2}. \tag{34}$$

The deceleration parameter $q$ is always negative. The EoS parameter is positive when $n > 2$ and is negative if $1 < n < 2$. Thus, the metric (28) represents as ever power-law accelerated expansion universe filled with a perfect fluid. If $1 < n < 2$, $\omega < 0$, we obtain DE cosmological model of Bianchi type-VI$_0$.

The spatial volume $V$ is zero and all physical parameters diverge at $T = 0$. Therefore, the model has a point-type singularity at $T = 0$. For $0 < T < \infty$, the spatial volume is an increasing function of time. The physical parameters are monotonically decreasing function of time and ultimately tend to zero for large $T$. The anisotropy in the model is maintained throughout the passage of time. For the physical reality of the model we will have to choose $n$, greater than 1,in such a way that $\left|\frac{n-2}{n+2}\right| \leq 1$. It deserves mention that we are unable to find $n$ for which $\omega$=$\pm$ 1

Notes

*Case 3.2* When C$\neq$ 0

When $C \neq o$ equation (24) is not integrable for general values of $n$. However, for $n = 2$, it becomes

$$\int \frac{dV}{\sqrt{V + C}} = 4mt \tag{35}$$

which, after integration, yields

$$V = 4m^2t^2 + 2\beta t + \gamma \tag{36}$$

where $\beta$ and $\gamma$ are arbitrary constants. The constant $C$ is absorbed in $\gamma$. From (21) and (36), we obtain the scale factors as

$$A = (4m^2t^2 + 2\beta t + \gamma)^{\frac{1}{2}}, \tag{37}$$

$$B = (4m^2t^2 + 2\beta t + \gamma)^{\frac{1}{4}}. \tag{38}$$

Therefore, the metric (1) of our solutions can be written in the form

$$ds^2 = -dt^2 + (4m^2t^2 + 2\beta t + \gamma)dx^2 + (4m^2t^2 + 2\beta t + \gamma)^{\frac{1}{2}}(e^{-2mx}dy^2 + e^{2mx}dz^2) \tag{39}$$

The expressions for $(H_1, H_2, H_3)$, $H$, $\rho$, $\theta$ and $\sigma$ are obtained as

$$H_1 = \frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}, \tag{40}$$

$$H_2 = H_3 = \frac{1}{2}(\frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}), \tag{41}$$

$$H = \frac{2}{3}(\frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}), \tag{42}$$

$$\theta = 2(\frac{4m^2t + \beta}{4m^2t^2 + 2\beta t + \gamma}), \tag{43}$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{4m^2t+\beta}{4m^2t^2+2\beta t+\gamma}\right). \tag{44}$$

The energy density, DP and $\omega$ are obtained as

$$\rho = \frac{(8m^2t+2\beta)^2-(4m^2\gamma-\beta^2)}{4(4m^2t^2+2\beta t+\gamma)^2}, \tag{45}$$

Notes

$$\omega = -\frac{5(4m^2\gamma-\beta^2)}{(8m^2t+2\beta)^2-(4m^2\gamma-\beta^2)}, \tag{46}$$

$$q = -\frac{2m^2(4m^2t^2+2\beta t+\gamma)}{(4m^2t+\beta)^2}. \tag{47}$$

The value of DP is always negative since $V$ is never negative. The EoS parameter $\omega$ is negative if $\gamma > \frac{\beta^2}{4m^2}$. If this condition holds, the model (39) corresponds to a Bianchi type-VI$_0$ energy cosmological model with variable $q$ and $\omega$.

If $\gamma > \frac{\beta^2}{4m^2}$, the model (39) has no finite singularity. The physical and kinematical parameters are all decreasing function of time and ultimately tend to zero for large time. The model essentially gives an empty space-time for large time. The anisotropy in the model never dies out.

## IV. Conclusion

In this paper, we have presented exact solutions of Einstein's field equations for a Bianchi-type VI$_0$ space-time filled with perfect fluid satisfying the barotropic equation of state under the assumption that the expansion scalar is proportional to shear scalar. Under some specific choice of problem parameters, the present consideration yields singular and non-singular models of the accelerated expansion universe filled with perfect fluid and dark energy. Models with negative EoS parameter $\omega$ may be attributed to the current accelerated expansion of universe. The physical and kinematical parameters are all decreasing function of time and ultimately tend to zero for large time. The universe models do not approach to isotropy. The models presented in this paper can be potential tools to describe the present universe as well as the early universe.

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