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New Theorems Involving the I-function and General Class of Polynomials

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Abstract – In the present paper we first establish three new theorems, which involves I-function and general class of polynomials. Next, we obtain certain new integrals and expansion formulas by the application of our theorems. By giving suitable values to the parameters, main integral reduces to Fox's H-function, G-function and generalized wright hypergeometric function, etc.

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I. Introduction

The I-function, introduced in 1982, is a byproduct of V.P. Saxena's work on higher transcendental function. Now I-function stands on fairly firm footing through the research contributions of various authors [1, 10, 11, 12]:

I-function is defined and represented in the following manner [11]:

$$I^{M,N}_{P_i,Q_i;R}[z] = I^{M,N}_{P_i,Q_i;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,N},(a_{ji},\alpha_{ji})_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_{ji},\beta_{ji})_{M+1,Q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi} d\xi \tag{1.1}$$

where

$$\phi(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\Gamma(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{R}\left[\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\prod_{j=N+1}^{P_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\right]} \tag{1.2}$$

and M,N,P_i,Q_i are integers satisfying $1\le N\le P_i, 1\le M\le Q_i (i=1,\ldots,R)$ and R is finite. $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji}$ are positive integers and a_j,b_j,a_{ji},b_{ji} are complex numbers. I-function, which is a generalized form of the well known Fox's H-function [5, p.10, Eqn. (2.1.1)]. In the sequel the I-function is studied under the following conditions of existence:

(I) $\quad \Omega_i > 0, |\arg z| < \frac{\Omega_i \pi}{2}$ (1.3)

(II) $\quad \Omega_i \ge 0, |\arg z| \le \frac{\Omega_i \pi}{2}$ and $\mathrm{Re}(B+1) < 0$ (1.4)

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Where

$$\Omega_i = \sum_{j=1}^{N} \alpha_j - \sum_{j=N+1}^{P_i} \alpha_{ji} + \sum_{j=1}^{M} \beta_j - \sum_{j=M+1}^{Q_i} \beta_{ji}, \forall i = (1,\ldots,R) \tag{1.5}$$

and

$$B = \sum_{j=1}^{M} b_j + \sum_{j=M+1}^{Q_i} b_{ji} - \sum_{j=1}^{N} a_j - \sum_{j=N+1}^{P_i} a_{ji} + \frac{1}{2}(P_i - Q_i), \forall i = (1,\ldots,R) \tag{1.6}$$

Notes

The general class of polynomials $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ will be defined and represented as follow [2, p.185, eqn. (7)]:

$$S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x] = \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} x^{l_i} \tag{1.7}$$

where $n_1,\ldots,n_r = 0,1,2,\ldots;m_1,\ldots m_r$ are arbitrary positive integers, the coefficients $A_{n_i,l_i}(n_i,l_i \geq 0)$ are arbitrary constants, real or complex. $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ yield a number of known polynomials as its special cases. These include, among other, the Jacobi polynomials, the Bessel Polynomials, the Lagurre Polynomials, the Brafman Polynomials and several others [6, p. 158-161].

The following formulas [8, p.77, Ens. (3.1), (3.2) & (3.3)] will be required in our investigation:

$$\int_0^{\infty} \left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)}, \ (a>0; b\geq 0; c+4ab>0; \mathrm{Re}(p)+1/2>0) \tag{1.8}$$

$$\int_0^{\infty} \frac{1}{x^2}\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)}, \ (a\geq 0; b>0; c+4ab>0; \mathrm{Re}(p)+1/2>0) \tag{1.9}$$

$$\int_0^{\infty} \left(a + \frac{b}{x^2}\right)\left[\left(ax + \frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)}, \ (a>0; b>0; c+4ab>0; \mathrm{Re}(p)+1/2>0) \tag{1.10}$$

II. Main Theorems

Let X stands for $\left(ax + \frac{b}{x}\right)^2 + c$

First Theorem:

If $$(1-y)^{\alpha+\beta-\gamma} \, {}_2F_1(2\alpha,2\beta;2\gamma;y) = \sum_{r=0}^{\infty} a_r y^r \tag{2.1}$$

then

$$\int_0^{\infty} X^{-\lambda-1} \, {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right] dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times \tag{2.2}$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right]$$

Notes

The above result will be converge under the following conditions:

(I) $a>0; b\geq 0; c+4ab>0$ and $\mu_i>0, \delta\geq 0$.

(II) $\mathrm{Re}\left[\lambda+\delta \min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

(III) $|\arg z|<\frac{1}{2}\Omega_i\pi$, where Ω_i is given by equation (1.5)

(IV) $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Second Theorem:

If $$(1-y)^{\alpha+\beta-\gamma}\ {}_2F_1(2\alpha,2\beta;2\gamma;y)=\sum_{r=0}^{\infty}a_r y^r \tag{2.3}$$

then

$$\int_0^{\infty}\frac{1}{x^2}X^{-\lambda-1}\ {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right)\ {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\ldots,m_k}_{n_1,\ldots,n_k}\left[\prod_{i=1}^{k}y_i X^{-\mu_i}\right]I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{2.4}$$

The above result will be converge under the following conditions:

(I) $a\geq 0; b>0; c+4ab>0$ and $\mu_i>0, \delta\geq 0$

(II) $\mathrm{Re}\left[\lambda+\delta \min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

(III) $|\arg z|<\frac{1}{2}\Omega_i\pi$, where Ω_i is given by equation (1.5)

(IV) $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Third Theorem:

If $$(1-y)^{\alpha+\beta-\gamma}\ {}_2F_1(2\alpha,2\beta;2\gamma;y)=\sum_{r=0}^{\infty}a_r y^r \tag{2.5}$$

then

$$\int_0^{\infty}\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta\right);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta\right)\end{matrix}\right.\right] \tag{2.6}$$

The above result will be converge under the following conditions

(I) $a>0;b>0;c+4ab>0$ and $\mu_i>0,\delta\ge 0$

(II) $\operatorname{Re}\left[\lambda+\delta\min_{1\le j\le m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

(III) $|\arg z|<\frac{1}{2}\Omega_i\pi$, where Ω_i is given by equation (1.5)

(IV) $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Proof: In our investigation following result [7, p. 75] is also required:

$${}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)=\sum_{r=0}^{\infty}\frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r}a_rX^r \tag{2.7}$$

Where a_r is given by (2.1).

To prove the first theorem, using the result given by equation (2.7) and express I-function occurring on the L.H.S. of equation (2.2) in terms of contour integral given by equation (1.1) and the general class of polynomials $S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]$ in series form with the help of equation (1.7) and then interchanging the order of integration and summation we get:

$$\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\frac{1}{2\pi i}\int_L\phi(\xi)z^{\xi}\left[\int_0^{\infty}\left[\left(ax+\frac{b}{x}\right)+c\right]^{-\lambda+r-\sum_{i=1}^{k}\mu_il_i-\delta\xi-1}dx\right]d\xi \tag{2.8}$$

Further using the formulae (1.8) the above integral becomes

$$\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

Notes

$$\frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi} \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1/2}} \frac{\Gamma\left(\lambda-r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1/2\right)}{\Gamma\left(\lambda-r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1\right)} d\xi \tag{2.9}$$

Then interpreting with the help of (1.1) and (2.9) provides first integral.

Proceeding on the same parallel lines, theorems second and third given by equation (2.4) and (2.6) can be obtained by using the results (1.9) and (1.10) respectively.

Notes

III. Special Cases

(3.1) If we put $R=1$, I-function reduces to Fox's H-function [5, p. 10, Eqn. (2.1.1)], then the equation (2.2), (2.4) and (2.6) takes the following form:

$$\int_0^\infty X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\ (b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{3.1.1}$$

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\ (b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{3.1.2}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\ (b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{3.1.3}$$

The Conditions of validity of (3.1.1), (3.1.2) and (3.1.3) easily follow from those given in (2.2), (2.4) and (2.6).

(3.2) By applying the our results given in (2.2), (2.4) and (2.6) to the case of Hermite polynomials [2, 3] by setting $S_n^2(x) \to x^{n/2} H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $m_1,\dots,m_k = 2; n_1,\dots,n_k = n; k = 1; v_i = v, y_i = y, A_{n_i,l_i} = (-1)^l$, we have the following interesting results:

Notes

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right] I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]} \frac{(-n)_{2l}}{l!}(-1)^l \frac{(y)^l}{(4ab+c)^{-r+\mu l}} \frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \qquad (3.2.1)$$

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right] I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]} \frac{(-n)_{2l}}{l!}(-1)^l \frac{(y)^l}{(4ab+c)^{-r+\mu l}} \frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \qquad (3.2.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right] I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]} \frac{(-n)_{2l}}{l!}(-1)^l \frac{(y)^l}{(4ab+c)^{-r+\mu l}} \frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \qquad (3.2.3)$$

The Conditions of validity of (3.2.1), (3.2.2) and (3.2.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.3) By applying the our results given in (2.2), (2.4) and (2.6) to the case of Lagurre polynomials [2, 3] by setting $S_n^2(x) \to L_n^{(\alpha')}[x]$ in which $m_1,\dots,m_k = 1; n_1,\dots,n_k = n; k = 1, v_i = v, y_i = y, A_{n_i,l_i} = \binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}$, we have the following interesting results:

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \quad (3.3.1)$$

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \quad (3.3.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \quad (3.3.3)$$

The Conditions of validity of (3.3.1), (3.3.2) and (3.3.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.4) If we put $R=1;\alpha_j=\beta_j=1$, then the I-function reduces to general type of G-function [9] i.e. $I_{P_i,Q_i;1}^{M,N}\left[z\left|\begin{matrix}(a_j,1)_{1,n};(a_{j1},1)_{n+1,p_1}\\(b_j,1)_{1,m};(b_{j1},1)_{m+1,q_1}\end{matrix}\right.\right]=G\left[z\left|\begin{matrix}(a_j,1)_{1,p}\\(b_j,1)_{1,q}\end{matrix}\right.\right]$, the equation (2.2), (2.4) and (2.6) takes the following form:

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]G_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

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$$G^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,1)_{1,p}\\ (b_j,1)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.4.1}$$

$$\int_0^{\infty}\frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\dots,m_k}_{n_1,\dots,n_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]G^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$G^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,1)_{1,p}\\ (b_j,1)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.4.2}$$

$$\int_0^{\infty}\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\dots,m_k}_{n_1,\dots,n_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]G^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$G^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,1)_{1,p}\\ (b_j,1)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.4.3}$$

The Conditions of validity of (3.4.1), (3.4.2) and (3.4.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.5) If we put $R=1,M=1,N=P_i=P,Q_i=Q+1,b_1=0,\beta_1=1,a_j=1-a_j,b_{ji}=1-b_j,\beta_{ji}=\beta_j$, then the I-function reduces to generalized wright hypergeometric function [12, p.33, Eq. (2.3.8)] i.e. $I^{1,P}_{P,Q+1;1}\left[z\left|\begin{matrix}(1-a_j,\alpha_j)_{1,p}\\(0,1),(1-b_j,\beta_j)_{1,q}\end{matrix}\right.\right]={}_p\psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix};-z\right]$, the equation (2.2), (2.4) and (2.6) takes the following form:

$$\int_0^{\infty}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\dots,m_k}_{n_1,\dots,n_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\psi_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$${}_{p+1}\psi_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\ (b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \tag{3.5.1}$$

Notes

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$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] {}_p\psi_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^\infty \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i}(y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$${}_{p+1}\Psi_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right)\end{matrix};\frac{-z}{(4ab+c)^\delta}\right] \tag{3.5.2}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] {}_p\psi_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^\infty \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i}(y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$${}_{p+1}\Psi_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right)\end{matrix};\frac{-z}{(4ab+c)^\delta}\right] \tag{3.5.3}$$

The Conditions of validity of (3.5.1), (3.5.2) and (3.5.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.6) If we put $\alpha=\gamma$, in the main theorem, the value of a_r in (2.1) comes out to be equal to $\frac{\beta_r}{r!}$ and the result (2.2), (2.4) and (2.6) gives the following interesting integral:

$$\int_0^\infty X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^\infty \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i}(y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!} \times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^\delta}\middle|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right)\end{matrix}\right] \tag{3.6.1}$$

$$\int_0^\infty \frac{1}{X^2} X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^\infty \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^k \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i}(y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!} \times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^\delta}\middle|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta\right)\end{matrix}\right] \tag{3.6.2}$$

$$\int_0^\infty \left(a+\frac{b}{X^2}\right) X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.6.3)$$

The Conditions of validity of (3.6.1), (3.6.2) and (3.6.3) easily follow from those given in (2.2), (2.4) and (2.6).

(3.7) If we put $\beta=\alpha+\frac{1}{2}$ and $\alpha=-f$ (f is non-negative integer) in (3.6.1), (3.6.2) and (3.6.3), we have:

$$\int_0^\infty X^{-\lambda-1}(1-X)^f S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}\sqrt{}}\sum_{r=0}^{f}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.7.1)$$

$$\int_0^\infty \frac{1}{X^2} X^{-\lambda-1}(1-X)^f S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.7.2)$$

$$\int_0^\infty \left(a+\frac{b}{X^2}\right) X^{-\lambda-1}(1-X)^f S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.7.3)$$

Notes

Notes

The Conditions of validity of (3.7.1), (3.7.2) and (3.7.3) easily follow from those given in (2.2), (2.4) and (2.6).

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