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Some Summation Theorems Involving Bailey Theorem

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Keywords : *Bailey theorem, Gaussian hypergeometric function.*

MSC : *33C20; 33C80; 39A10*

SOME SUMMATION THEOREMS INVOLVING BAILEYTHEOREM

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Some Summation Theorems Involving Bailey Theorem

Salahuddin [α], M.P. Chaudhary [σ] & Ashish Arora [ρ]

Abstract - Authors obtain five new summations theorems involving Gamma functions, Bailey theorem and recurrence relation of Gamma functions, which are not available in the literature of special functions.

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I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{ccc} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{ccc} (a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relation is defined as follows: [E. D. p.51(10), Andrews p.363(9.16), H.T. F. I p.103(32)]

$$ (a-b)\ {}_2F_1\left[\begin{array}{ccc} a,\ b & ; & \\ c & ; & z \end{array}\right] = a\ {}_2F_1\left[\begin{array}{ccc} a+1,\ b & ; & \\ c & ; & z \end{array}\right] - b\ {}_2F_1\left[\begin{array}{ccc} a,\ b+1 & ; & \\ c & ; & z \end{array}\right] \tag{2} $$

Recurrence relation of gamma function is defined as follows:

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{3} $$

Legendre duplication formula is defined, as

$$ \sqrt{\pi}\ \Gamma(2z) = 2^{(2z-1)}\ \Gamma(z)\ \Gamma\left(z+\frac{1}{2}\right) \tag{4} $$

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$$\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}=\frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{5}$$

$$=\frac{2^{(a-1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{6}$$

Bailey summation theorem [Prud, p.491(7.3.7.8)]is as follows:

$${}_2F_1\left[\begin{array}{cc} a, 1-a & ; \frac{1}{2} \\ c & ; 2 \end{array}\right]=\frac{\Gamma(\frac{c}{2})\ \Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})}=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-1}\ \Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} \tag{7}$$

II. Main Summation Theorems

Theorem-1:

$${}_2F_1\left[\begin{array}{ccc} a & , & -a-11 \; ; \frac{1}{2} \\ & c & ; \; 2 \end{array}\right]$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}}\times\left[\frac{-122760a+35546a^2+7161a^3+23a^4-33a^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+\right.$$

$$+\frac{-a^6+122880c-175560ac+5820a^2c+3960a^3c+180a^4c+140288c^2-70356ac^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-4218a^2c^2+396a^3c^2+18a^4c^2+57600c^3-10560ac^3-960a^2c^3+10880c^4-528ac^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-48a^2c^4+960c^5+32c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{4(15120-15510a+405a^2+330a^3+15a^4+27024c-11902ac-719a^2c+66a^3c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+$$

$$\left.+\frac{4(3a^4c+15200c^2-2640ac^2-240a^2c^2+3680c^3-176ac^3-16a^2c^3+400c^4+16c^5)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}\right] \tag{8}$$

Theorem-2:

$${}_2F_1\left[\begin{array}{ccc} a & , & -a-12 \; ; \frac{1}{2} \\ & c & ; \; 2 \end{array}\right]$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+12}}\times\left[\frac{-245640a+96002a^2+8301a^3-769a^4-69a^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+\right.$$

$$+\frac{-a^6+245760c-376304ac+40100a^2c+7246a^3c+84a^4c-6a^5c+280576c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-163376ac^2-376a^2c^2+1104a^3c^2+24a^4c^2+115200c^3-27776ac^3-1184a^2c^3+32a^3c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

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$$+\frac{21760c^4-1840ac^4-80a^2c^4+1920c^5-32ac^5+64c^6}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{665280-690744a+14030a^2+19587a^3+1211a^4-3a^5-a^6+1249536c-588880ac}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{-44500a^2c+5426a^3c+444a^4c+6a^5c+776896c^2-156688ac^2-19384a^2c^2+48a^3c^2}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{24a^4c^2+222720c^3-14464ac^3-2336a^2c^3-32a^3c^3+32320c^4-80ac^4-80a^2c^4}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}+$$

$$\left.+\frac{2304c^5+32ac^5+64c^6}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}\right] \quad (9)$$

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Theorem-3:

$${}_2F_1\left[\begin{matrix} a\ , & -a-13 & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+13}}\times\left[\frac{-2948400a+1178604a^2+123942a^3-12978a^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})}+\right.$$

$$+\frac{-1638a^5-42a^6+2949120c-4789512ac+547218a^2c+125489a^3c+1869a^4c-273a^5c}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{-7a^6c+3612672c^2-2332512ac^2-9072a^2c^2+26208a^3c^2+1008a^4c^2+1662976c^3}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{-479024ac^3-27384a^2c^3+1456a^3c^3+56a^4c^3+376320c^4-43680ac^4-3360a^2c^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{44800c^5-1456ac^5-112a^2c^5+2688c^6+64c^7}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{-2(-665280+752856a-74246a^2-18135a^3-275a^4+39a^5+a^6-1249536c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{-2(691392ac+4512a^2c-7488a^3c-288a^4c-776896c^2+207376ac^2+11896a^2c^2)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{-2(-624a^3c^2-24a^4c^2-222720c^3+24960ac^3+1920a^2c^3-32320c^4+1040ac^4)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}+$$

$$\left.+\frac{-2(80a^2c^4-2304c^5-64c^6)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+13}{2})}\right] \quad (10)$$

Theorem-4:

$${}_2F_1\left[\begin{matrix} a\ , & -a-14 & ; & \frac{1}{2} \\ c & & ; & \end{matrix}\right]$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+14}} \times \left[\frac{-5897520a + 2856228a^2 + 68104a^3 - 39225a^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})} + \right.$$

$$+ \frac{-2225a^5 - 3a^6 + a^7 + 5898240c - 10079808ac + 1788240a^2c + 186120a^3c - 6280a^4c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{-648a^5c - 8a^6c + 7225344c^2 - 5180672ac^2 + 245040a^2c^2 + 53080a^3c^2 + 720a^4c^2 - 24a^5c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{3325952c^3 - 1143840ac^3 - 16400a^2c^3 + 4320a^3c^3 + 80a^4c^3 + 752640c^4 - 117600ac^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{-4560a^2c^4 + 80a^3c^4 + 89600c^5 - 5184ac^5 - 192a^2c^5 + 5376c^6 - 64ac^6 + 128c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{17297280 - 19716432a + 1898236a^2 + 587096a^3 + 11665a^4 - 2143a^5 - 101a^6 - a^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})} +$$

$$+ \frac{33818496c - 19565024ac - 242880a^2c + 293240a^3c + 15560a^4c - 24a^5c - 8a^6c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})} +$$

$$+ \frac{22698368c^2 - 6656608ac^2 - 479040a^2c^2 + 34280a^3c^2 + 2400a^4c^2 + 24a^5c^2 + 7344512c^3}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})} +$$

$$+ \frac{-977440ac^3 - 103760a^2c^3 + 160a^3c^3 + 80a^4c^3 + 1285760c^4 - 57120ac^4 - 7920a^2c^4}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})} +$$

$$\left. + \frac{-80a^3c^4 + 124544c^5 - 192ac^5 - 192a^2c^5 + 6272c^6 + 64ac^6 + 128c^7}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})} \right] \quad (11)$$

Theorem-5:

$$ {}_2F_1 \left[\begin{matrix} a \ , & -a-15 & ; & \frac{1}{2} \\ c & & ; & 2 \end{matrix} \right] $$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+15}} \times \left[\frac{-82570320a + 40531212a^2 + 1372620a^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+16}{2})} + \right.$$

$$+ \frac{-697871a^4 - 50040a^5 - 62a^6 + 60a^7 + a^8 + 82575360c - 147571200ac + 27457920a^2c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+16}{2})} +$$

$$+ \frac{3460800a^3c - 136640a^4c - 20160a^5c - 448a^6c + 107053056c^2 - 82606080ac^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+16}{2})} +$$

$$+ \frac{4212928a^2c^2 + 1188000a^3c^2 + 21600a^4c^2 - 1440a^5c^2 - 32a^6c^2 + 53788672c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+16}{2})} +$$

$$+ \frac{-20832000ac^3 - 380800a^2c^3 + 134400a^3c^3 + 4480a^4c^3 + 13862912c^4 - 2625600ac^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+16}{2})} +$$

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$$+\frac{-139040a^2c^4+4800a^3c^4+160a^4c^4+2007040c^5-161280ac^5-10752a^2c^5+164864c^6}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+16}{2})}+$$

$$+\frac{-3840ac^6-256a^2c^6+7168c^7+128c^8}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+16}{2})}+$$

$$+\frac{-16(-2162160+2633820a-422912a^2-56175a^3+2065a^4+315a^5+7a^6-4227312c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})}+$$

$$+\frac{-16(2745900ac-122940a^2c-37425a^3c-685a^4c+45a^5c+a^6c-2837296c^2)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})}+$$

$$+\frac{-16(1001700ac^2+19530a^2c^2-6300a^3c^2-210a^4c^2-918064c^3+165300ac^3)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})}+$$

$$+\frac{-16(8770a^2c^3-300a^3c^3-10a^4c^3-160720c^4+12600ac^4+840a^2c^4-15568c^5)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})}+$$

$$\left.+\frac{-16(360ac^5+24a^2c^5-784c^6-16c^7)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})}\right] \tag{12}$$

Notes

III. Derivations of The Summation Theorems(1) to (5)

Proof of theorem-1:

putting $b=-a-11, z=\frac{1}{2}$ in known result (2),we get

$$(2a+11)\ {}_2F_1\left[\begin{matrix} a\ , & -a-11 & ; \frac{1}{2} \\ c & & ; \end{matrix}\right]$$

$$=a\ {}_2F_1\left[\begin{matrix} a+1\ , & -a-11 & ; \frac{1}{2} \\ c & & ; \end{matrix}\right]+(a+6)\ {}_2F_1\left[\begin{matrix} a\ , & -a-10 & ; \frac{1}{2} \\ c & & ; \end{matrix}\right]$$

Now using Bailey theorem, we get

$$\text{L.H.S} = \text{a}\ \frac{\sqrt{\pi}\ \Gamma(\text{c})}{2^{\text{c}+10}}\times\left[\frac{-8304-3790\text{a}+5067\text{a}^2+551\text{a}^3-3\text{a}^4-\text{a}^5-3152\text{c}-14484\text{ac}}{\Gamma(\frac{\text{c+a+11}}{2})\,\Gamma(\frac{\text{c-a}}{2})}+\right.$$

$$+\frac{1202a^2c+252a^3c+6a^4c+7080c^2-5948ac^2-216a^2c^2+12a^3c^2+3840c^3-672ac^3}{\Gamma(\frac{c+a+11}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-32a^2c^2+624c^4-16ac^4+32c^5}{\Gamma(\frac{c+a+11}{2})\,\Gamma(\frac{c-a}{2})}+$$

$$+\frac{-8304a-3790a^2+5067a^3+551a^4-3a^5-a^6-3152ac-14484a^2c+1202a^3c}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{252a^4c+6a^5c+7080ac^2-5948a^2c^2-216a^3c^2+12a^4c^2+3840ac^3-672a^2c^3}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{-32a^3c^3+624ac^4-16a^2c^4+32ac^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]+$$

$$(a+11)\ \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+10}}\times\left[\frac{-1226a+3406a^2+553a^3+2a^4-a^5+12288c-16156ac+482a^2c}{\Gamma(\frac{c+a+10}{2})\ \Gamma(\frac{c-a+1}{2})}+\right.$$

$$+\frac{228a^3c+6a^4c+12800c^2-5480ac^2-252a^2c^2+12a^3c^2+4480c^3-608ac^3-32a^2c^3}{\Gamma(\frac{c+a+10}{2})\ \Gamma(\frac{c-a+1}{2})}+$$

$$+\frac{640c^4-16ac^4+32c^5}{\Gamma(\frac{c+a+10}{2})\ \Gamma(\frac{c-a+1}{2})}+$$

$$+\frac{30240-27516a-1984a^2+527a^3+a^5+54048c-18604ac-2758a^2c+12a^3c+6a^4c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+$$

$$\left.+\frac{30400c^2-612a^2c^2-12a^3c^2+7860c^3-32ac^3-32a^2c^3+800c^4+16ac^4+32c^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}}\times\left[\frac{-16608a-7580a^2+10134a^3+1102a^4-6a^5-2a^6-6304ac-28968a^2c}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+\right.$$

$$+\frac{2404a^3c+504a^4c+12a^5c+14160ac^2-11896a^2c^2-432a^3c^2+24a^4c^2+7680ac^3}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1344a^2c^3-64a^3c^3+1248ac^4-32a^2c^4+64ac^5}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1320a+28370a^2+29771a^3-664a^4-802a^5-58a^6-a^7-31384ac-38334a^2c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{26665a^3c+3395a^4c+15a^5c-5a^5c-5a^6c+6080ac^2-46256a^2c^2+2382a^3c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{696a^4c^2+180a^5c^2+19320ac^3-11620a^2c^3-520a^3c^3+20a^4c^3+6480ac^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{-928a^2c^4-48a^3c^4+784ac^5-16a^2c^5+32ac^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]+$$

$$+\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}}\times\left[\frac{-1349040a+117116a^2+120092a^3+15239a^4+485a^5-196a^6-a^7}{\Gamma(\frac{c+a+12}{2})\ \Gamma(\frac{c-a+1}{2})}+\right.$$

$$+\frac{1351680c-1654016ac-248766a^2c+28535a^3c+6505a^4c+345a^5c+5a^6c+1543168c^2}{\Gamma(\frac{c+a+12}{2})\ \Gamma(\frac{c-a+1}{2})}+$$

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$$+\frac{-499428ac^2-140854a^2c^2-6462a^3c^2+294a^4c^2+18a^5c^2+633600c^3-20280ac^3}{\Gamma(\frac{c+a+12}{2})\ \Gamma(\frac{c-a+1}{2})}+$$

$$+\frac{-20060a^2c^3-1400a^3c^3-20a^4c^3+119680c^4+9472ac^4-656a^2c^4-48a^3c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{665280-544872a-98680a^2+7626a^3+2198a^4+126a^5+2a^6+1189056c-301192ac}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{-97884a^2c-5252a^3c+156a^4c+12a^5c+668800c^2-8720ac^2-19784a^2c^2-1488a^3c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-24a^4c^2+161920c^3+14016ac^3-768a^2c^3-64a^3c^3+17600c^4+1952ac^4+32a^2c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{704c^5+64ac^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}}\times\left[\frac{665280-561480a-106260a^2+17760a^3+3300a^4+120a^5+1189056c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+\right.$$

$$+\frac{-307496ac-126852a^2c-2848a^3c+660a^4c+24a^5c-668800c^2+5440ac^2-31680a^2c^2}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{-1920a^3c^2+161920c^3+21696ac^3-2112a^2c^3-128a^3c^3+17600c^4+3200ac^4}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{+704c^5+128ac^5}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{-1350360a+145486a^2+149863a^3+14575a^4-317a^5-77a^6-2a^7+1351680c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-1685400ac-287100a^2c+55200a^3c+9900a^4c+360a^5c+1543168c^2-493340ac^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-187110a^2c^2-4080a^3c^2+990a64c^2+36a^5c^2+633600c^3-960ac^3-31680a^2c^3-1920a^3c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{+119680c^4+15952ac^4-1584a^2c64-96a^3c^4+10560c^5+1920ac^5+352c66+64ac^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]$$

On simplification , we get

$${}_2F_1\left[\begin{matrix} a\ , & -a-11 & ; & \frac{1}{2} \\ c & & ; & 2\end{matrix}\right]$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}}\times\left[\frac{-122760a+35546a^2+7161a^3+23a^4-33a^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+\right.$$

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$$+\frac{-a^6+122880c-175560ac+5820a^2c+3960a^3c+180a^4c+140288c^2-70356ac^2}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-4218a^2c^2+396a^3c^2+18a^4c^2+57600c^3-10560ac^3-960a^2c^3+10880c^4-528ac^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-48a^2c^4+960c^5+32c^6}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{4(15120-15510a+405a^2+330a^3+15a^4+27024c-11902ac-719a^2c+66a^3c+3a^4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}$$

$$\left.+\frac{4(15200c^2-2640ac^2-240a^2c^2+3680c^3-176ac^3-16a^2c^3+400c^4+16c^5)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}\right]$$

which proves the theorem -1.

Proof of theorems 2-5: On the similar lines of proof of theorem-1, we can prove other theorems 2-5.

References Références Referencias

1. Arora, Asish, Singh, Rahul , Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems *Journal of Rajasthan Academy of Physical Sciences.*, 7(2008), 335-342.
2. Choi, J., Harsh, H. and Rathie, A. K.; Some summation formulae for the Apple's function F_1, *East Asian Math. Journal*, 17(2001), 233-237.
3. Erdélyi, A., Magnus, W., Okerhettinger, F. and Tricomi, F. G.; *Higher transcendental functions* Vol.1 (Bateman Manuscript Project) McGraw-Hill book P. Inc. New York, Toronto and London, 1953.
4. Krupnikov, E. D., Kölbig, K. S.; Some special cases of the generalized hypergeometric function ${}_{q+1}F_q$, *Journal of computational and Applied Math.*, 78(1997), 79-95.
5. Lavoie, J. L.; Notes on a paper by J. B. Miller, *J. Austral. Math. Soc. Ser. B*, 29(1987), 216-220.
6. Lavoie, J. L.; Some summation formulae for the series ${}_3F_2$, *Math. Comput.*, 49(1987), 269-274.
7. Lavoie, J. L., Grondin, F. and Rathie, A.K.; Generalizations of Watson's theorem on the sum of a ${}_3F_2$, *Indian J. Math.*, 34(1992), 23-32.
8. Lavoie, J. L., Grondin, F. and Rathie, A.K.; Generalizations of Whipple's theorem on the sum of a ${}_3F_2$, *J. Comput. Appl. Math.*, 72(1996), 293-300.
9. Lavoie, J. L., Grondin, F. Rathie, A. K. and Arora, K.; Generalizations of Dixon's theorem on the sum of a ${}_3F_2$, *Math. Comput.*, 62, 267-276.
10. Mitra, C. S.; *J. Indian Math. Soc.* (N.S.), 7(1943), 102-109.
11. Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; *Integrals and Series Vol. 3: More Special Functions.* Nauka, Moscow, 1986. Translated from the Russian

Notes

Notes

by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne, 1990.

12. Rainville, E. D.; The contiguous function relations for ${}_pF_q$ with applications to Bateman's $J_n^{u,v}$ and Rice's $H_n(\zeta, p, \nu)$, *Bull. Amer. Math. Soc.*, 51(1945), 714-723.
13. Shashikant, Sharma, S. and Rathie, A. K.; Some summation formulae for the Apple's function F_1, *Proc. of the fourth Int. Conf. SSFA*, 4(2003), 81-84.