Effect of Imperfectness on Reflection and Transmission Coefficients in Swelling Porous Elastic Media at an Imperfect Boundary

By Rajneesh Kumar, Divya & Kuldeep Kumar
Kurukshetra University, Kurukshetra, Haryana, India

Abstract - The present investigation is concerned with the reflection and transmission of plane waves at an imperfect interface between two different swelling porous elastic media. The expression for various amplitude ratios due to the incidence of longitudinal wave in solid (PS), transverse wave in solid (SVS) are obtained for imperfect boundary and are deduced for normal stiffness, transversal stiffness and welded contact. The resulting amplitude ratios are computed and depicted graphically for a specific model. The present investigation has immense application in structural problems, geophysics etc.

Keywords: longitudinal waves, transversal waves, normal stiffness, transversal stiffness, welded contact.

GJSFR-F Classification: FOR Code: 019999.

Strictly as per the compliance and regulations of:

© 2012. Rajneesh Kumar, Divya & Kuldeep Kumar. This is a research/review paper, distributed under the terms of the Creative Commons Attribution-Noncommercial 3.0 Unported License (http://creativecommons.org/licenses/by-nc/3.0/), permitting all non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.
Effect of Imperfectness on Reflection and Transmission Coefficients in Swelling Porous Elastic Media at an Imperfect Boundary

Rajneesh Kumar^a, Divya^b & Kuldeep Kumar^c

Abstract - The present investigation is concerned with the reflection and transmission of plane waves at an imperfect interface between two different swelling porous elastic media. The expression for various amplitude ratios due to the incidence of longitudinal wave in solid (PS), transverse wave in solid (SVS) are obtained for imperfect boundary and are deduced for normal stiffness, transversal stiffness and welded contact. The resulting amplitude ratios are computed and depicted graphically for a specific model. The present investigation has immense application in structural problems, geophysics etc.

Keywords : longitudinal waves, transversal waves, normal stiffness, transversal stiffness, welded contact.

I. INTRODUCTION

Dynamic analysis of theories of porous media is a subject with application in various branches of geophysics, civil and mechanical engineering. Based on the work of Von Terzaghi [1,2], Biot [3] proposed a general theory of three dimensional deformations of fluid saturated porous elastic solids. Subsequently, Biot [4,5,6,7] presented the models for describing the dynamic behaviour of fluid saturated porous media. He examines both high and low frequency limits and shows the existence of two longitudinal waves and one shear wave, which are dispersive and dissipative. Biot theory was based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and the basis for subsequent analysis in acoustic, geophysics and other fields. Based on the Fillunger model [8], (which is further based on the concept of volume fractions combined with surface porosity coefficients), Bowen [9], Boer and Ehlers[10,11] and Ehlers[12] develop and use another interesting theory in which all the constituents of a porous medium are assumed as soil; solid constituents are incompressible and liquid constituents which are generally water or oils are also incompressible.


A perfectly bonded interface is a surface across which both traction and displacement are continuous. Thus when solving harmonic wave problem in the neighborhood of a perfectly bonded interface
between two different elastic media, wave solution in one medium must be matched with those in the second medium through interface condition. The generalization of the concept is that of an imperfectly bonded interface for which the displacement and temperature distribution across a surface need not be continuous. Debonding and imperfect contact however are known to exist in composites, in the domain of electrical, thermal conduction or elasticity.

Kumar and Singh[18,19] study some problems on propagation of plane waves at an imperfect surface. Kumar et al [20] studied some problems on reflection and transmission of waves at an imperfect boundary.

The exact nature beneath the earth surface is not known. For the purpose of theoretical investigation about the earth interior one has to consider various appropriate model. The problem of waves and their reflection is very useful to understand the internal structure of earth and to explore various useful material in form of rocks buried inside the earth, for example mineral and crystals etc.

The spring like model has been adopted in the present work between two swelling porous elastic half space as has been represented by the boundary conditions in the text. \( K_n, K_s, K_{nf}, K_{tf} \) used in the boundary conditions are spring constant type material parameters. \( K_n \rightarrow \infty, K_s \rightarrow \infty, K_{nf} \rightarrow \infty, K_{tf} \rightarrow \infty \) implies the continuity of displacement components in case of solid and fluid respectively and therefore the two solids are perfectly bonded together or to say that the two solids are in welded contact. Reflection and transmission of plane waves in swelling porous elastic field at the imperfect boundary surface have been studied due to incidence of longitudinal and transversal waves. The amplitude ratios of various reflected and transmitted waves are computed and shown graphically. As such a model may be found in the earth’s crust, the results of the problem can be applicable to engineering, seismology and geophysics problem.

II. BASIC EQUATIONS

Following Eringen [13], the field equations in linear theory of swelling porous elastic soils are

\[
\begin{align*}
\mu u_{s,j}^i + (\lambda + \mu) u_{s,j}^i - \sigma^s u_{j,i}^s + \xi^s (\ddot{u}_s^i - \dddot{u}_s^i) + f_s^i &= \rho_0^s \ddot{u}_s^i, \\
\mu u_{f,j}^i + (\lambda + \mu) u_{f,j}^i - \sigma^f u_{j,i}^f + \xi^f (\ddot{u}_f^i - \dddot{u}_f^i) + f_f^i &= \rho_0^f \ddot{u}_f^i, \\
t_{ij}^s &= (-\sigma^s u_{i,r}^s + \lambda u_{i,s}^s) \delta_{ij} + \mu (u_{i,j}^s + u_{j,i}^s), \\
t_{ij}^f &= (-\sigma^f u_{i,r}^f + \lambda u_{i,s}^f) \delta_{ij} + \mu (u_{i,j}^f + u_{j,i}^f),
\end{align*}
\]

where, the superscripts \( s \) and \( f \) denote respectively, the elastic solid and the fluid; \( u_s^i \) and \( u_f^i \) are the displacement components of solid and fluid respectively. The functions \( (f_s^i, f_f^i) \) are the body forces, \( \rho_0^s, \rho_0^f \) are the densities of each constituent and \( \lambda, \mu, \lambda_s, \mu_s, \sigma^s, \sigma^f, \xi^s, \xi^f \) are constitutive constants. Subscripts preceded by a comma denote partial differentiation with respect to the corresponding Cartesian coordinate, and a superposed dot denotes time differentiation, \( t_{ij}^s, t_{ij}^f \) are the partial stress tensors.

III. FORMULATION OF THE PROBLEM AND SOLUTION

We consider two homogeneous swelling porous elastic half spaces in contact with each other at a plane surface which we designate as the plane \( z = 0 \) of a rectangular Cartesian co-ordinate system \( xyz \). We consider plane waves in the \( xz \) - plane with wave front parallel to the \( yz \) - plane and all the field variables depend only on \( x, z \) and \( t \).

For two dimensional problem, we assume the displacement vector

\[
\vec{u} = (u_s^i, 0, u_f^i)
\]

We define the non - dimensional quantities as

\[
\begin{align*}
\tilde{x}' = & \frac{\omega_s^s x}{c_s^s}, \quad \tilde{z}' = \frac{\omega_s^s z}{c_s^s}, \\
\tilde{u}'_s = & \frac{\omega_s^s}{c_s^s} u_s^i, \quad \tilde{u}'_f = \frac{\omega_s^s}{c_s^s} u_f^i, \\
t_{ij}' = & \frac{t_{ij}^s}{\mu}, \quad \phi^s = \frac{\xi^s}{\rho_0^s}, \quad \tilde{c}_s^2 = \frac{\mu}{\rho_0^s},
\end{align*}
\]

Expressing the displacement components \( u_s^i, u_f^i, u_s^f, u_f^f \) by the scalar potential functions \( \phi^i (x, z, t) \) and \( \psi^i(x, z, t) \) in dimensionless form.
Effect of Imperfectness on Reflection and Transmission Coefficients in Swelling Porous Elastic Media at an Imperfect Boundary

\[ u_i' = \frac{\partial \phi_i'}{\partial x} - \frac{\partial \psi_i'}{\partial z}, \quad u_i'' = \frac{\partial \phi_i''}{\partial z} + \frac{\partial \psi_i''}{\partial x} \]  \hspace{1cm} (7)

Using equations (1)-(2), (5)-(7) we obtain two coupled system of equations in absence of body forces

\[
\begin{bmatrix}
(1 + a_1) \nabla^2 - a_3 \frac{\partial}{\partial t} - \frac{\partial^2}{\partial t^2} & -a_2 \nabla^2 + a_3 \frac{\partial}{\partial t} \\
-h_2 \nabla^2 + h_4 \frac{\partial}{\partial t} & ((1 + h_1) \frac{\partial}{\partial t} - h_3) \nabla^2 - h_4 \frac{\partial}{\partial t} - h_5 \frac{\partial^2}{\partial t^2}
\end{bmatrix}
\begin{bmatrix}
\phi' \\
\phi''
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  \hspace{1cm} (8)

\[
\begin{bmatrix}
-\nabla^2 + a_3 \frac{\partial}{\partial t} + \frac{\partial^2}{\partial t^2} & -a_3 \frac{\partial}{\partial t} \\
-h_4 \frac{\partial}{\partial t} & (-\nabla^2 + h_3) \frac{\partial}{\partial t} + h_5 \frac{\partial^2}{\partial t^2}
\end{bmatrix}
\begin{bmatrix}
\psi' \\
\psi''
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]  \hspace{1cm} (9)

where, \( \Delta^2 \) is the Laplacian operator and

\[ a_1 = \frac{\lambda + \mu}{\mu}, \quad a_2 = \frac{\sigma_f}{\mu}, \quad a_3 = \frac{\xi f c_2^2}{\omega \mu}, \quad h_1 = \frac{\lambda + \mu}{\mu}, \quad h_2 = \frac{\sigma_f}{\mu}, \quad h_3 = \frac{\sigma_f c_2^2}{\omega \mu}, \quad h_4 = \frac{\xi f c_2^2}{\omega \mu}, \quad h_5 = \frac{\rho_f c_2^2}{\mu \omega^3}, \]

IV. REFLECTION AND TRANSMISSION

We consider a longitudinal wave in solid (PS)/longitudinal wave in fluid (PF) /transverse wave in solid (SVS)/ transverse wave in fluid(SVF) propagating through the medium \( M_1 \) which is designated as the region \( z=0 \) and incident at the plane \( z=0 \), with its direction of propagating with angle \( \theta_0 \) normal to the surface. Corresponding to each incident wave, we get reflected PS, PF, SVS, SVF waves and transmitted PS, PF, SVS, SVF waves in medium \( \bar{M} \) as shown in Fig. 1.

![Fig. 1: Geometry of the problem](image-url)
We assume the solutions of the system of equations (8)-(9) in the form

$$\left[\phi, \phi', \psi, \psi'\right] = \left[\phi_i, \phi_i', \psi_i, \psi_i'\right] e^{i[k(|\sin \theta - \cos \theta| - \omega t)]}$$

(10)

where \(k\) is the wave number and \(\omega\) is the complex circular frequency.

Making use of equation (10) in (8)-(9) we obtain two quadratic equations in \(V^2\) given by

$$AV^4 + BV^2 + C = 0, \quad A_iV^4 + B_iV^2 + C_i = 0$$

(11)

where \(V = \omega / k\) is the velocity of the waves; \(V_1, V_2\) are the velocities of the reflected longitudinal PS and PF waves respectively, given by equation (11)_1, and \(V_3, V_4\) are the velocities of transverse SVS and SVF waves respectively given by equation (11)_2.

V. BOUNDARY CONDITIONS

We consider two-bonded swelling porous elastic half-spaces as shown in Fig. 1. Imperfect bonding considered here means that the traction is continuous across the interface but that the small displacement is assumed to depend linearly on the traction vector. If the size and spacing between the imperfections is much smaller than the wave-length at the interface, we can use spring boundary conditions at \(z=0\) [21] as

\[
(i) \quad T_{33}^i = K_n(u_{1i}^i - \bar{u}_{1i}^i) \quad (ii) \quad T_{33}^f = K_n(u_{3f}^f - \bar{u}_{3f}^f) \quad (iii) \quad T_{33}^i = K_f(u_{1i}^i - \bar{u}_{1i}^i) \quad (iv) \quad T_{33}^f = K_f(u_{3f}^f - \bar{u}_{3f}^f)
\]

\[
(v) \quad T_{33}^i = T_{33}^i \quad (vi) \quad T_{33}^f = T_{33}^f \quad (vii) \quad T_{33}^i = T_{33}^i \quad (viii) \quad T_{33}^f = T_{33}^f
\]

where \(K_n, K_f\) are normal stiffness in case of solid and fluid respectively and \(K_i, K_f\) are transversal stiffness in case of solid and fluid respectively.

In view of (10), we assume the values of \(\phi^i, \phi^f, \psi^i, \psi^f\) for medium \(M_1\) and \(\phi^i, \phi^f, \psi^i, \psi^f\) for medium \(M\) satisfying the boundary conditions as

$$\phi^i, \phi^f = \sum_{i=1}^{2} \{1, \eta_i\} \left[ A_i e^{i[k(|\sin \theta - \cos \theta| - \omega t)]} + P_i \right], \quad \left[ \phi^i, \phi^f \right] = \sum_{i=1}^{2} \{1, \eta_i\} \left[ A_i e^{i[k(|\sin \theta - \cos \theta| - \omega t)]} \right]$$

$$\psi^i, \psi^f = \sum_{j=3}^{4} \{1, \eta_j\} \left[ B_j e^{i[k(|\sin \theta - \cos \theta| - \omega t)]} + P_j \right], \quad \left[ \psi^i, \psi^f \right] = \sum_{j=3}^{4} \{1, \eta_j\} \left[ B_j e^{i[k(|\sin \theta - \cos \theta| - \omega t)]} \right]$$

(12)

where,

$$P_i = A_i e^{i[k(|\sin \theta + \cos \theta| - \omega t)]}, \quad P_j = B_j e^{i[k(|\sin \theta + \cos \theta| - \omega t)]}, \quad \eta_i = \frac{\omega(1 + a_i) - i\alpha_i V_i - \omega V_i^2}{\alpha_i \omega - i\alpha_i V_i^2}, \quad \eta_j = -\frac{-\omega + (i\alpha_i + \omega)V_j^2}{i\alpha_i V_j^2}, \quad \eta_{ij} = \left[ \begin{array}{c} \frac{\omega(1 + \bar{a}_i) - i\bar{\alpha}_i V_i - \omega V_i^2}{\bar{\alpha}_i \omega - i\bar{\alpha}_i V_i^2} \\
-\omega + (i\bar{\alpha}_i + \omega)V_j^2 \\
\end{array} \right], \quad (i=1,2 \quad \& \quad j=3,4)$$

\(A_i\) are the amplitudes of the incident PS wave, PF wave and \(B_{ij}\) are the amplitudes of the incident SVS wave, SVF wave respectively. \(A_i\) are the amplitudes of the reflected PS wave (PSR), PF wave (PFR) and \(B_j\) are the amplitudes of the reflected SVS wave (SVSR) and SVF wave (SVFR), \(A\) are the amplitudes of the transmitted PS wave (PST), transmitted PF wave (PFT), \(A\) are the amplitudes of transmitted SVS wave (SVST) and transmitted SVF wave (SVFT) respectively.
In order to satisfy the boundary conditions, the extension of the Snell’s law will be

\[
\frac{\sin \theta_0}{V_0} = \frac{\sin \theta_1}{V_1} = \frac{\sin \theta_2}{V_2} = \frac{\sin \theta_3}{V_3} = \frac{\sin \theta_4}{V_4} = \frac{\sin \tilde{\theta}_1}{\tilde{V}_1} = \frac{\sin \tilde{\theta}_2}{\tilde{V}_2} = \frac{\sin \tilde{\theta}_3}{\tilde{V}_3} = \frac{\sin \tilde{\theta}_4}{\tilde{V}_4}
\]  
(13)

Where, \( k_1V_1 = k_2V_2 = k_3V_3 = k_4V_4 = \tilde{k}_1\tilde{V}_1 = \tilde{k}_2\tilde{V}_2 = \tilde{k}_3\tilde{V}_3 = \tilde{k}_4\tilde{V}_4 = \omega \) at \( z=0 \)

Making use of potentials given by (12) in boundary conditions, we obtain a system of eight non-homogeneous equations which can be written as

\[
\sum_{i,j=1}^{8} a_{ij} Z_j = Y_i
\]  
(15)

where,

\[
a_{1p} = -liK_nk_p s_p, \quad a_{1e} = -liK_nk_e \sin \theta_e, \quad a_{1r} = \left(\frac{\sigma_f}{\mu} \right) \tilde{\eta}_p - \frac{\tilde{\lambda}}{\mu} - 2\tilde{\pi}^2 \tilde{k}_p^2 - K_n i_k p \tilde{\sigma}_p \frac{\mu}{\mu}, \quad a_{1d} = (2\tilde{k}_e^2 \tilde{s}_e + K_n i_k p \tilde{\sigma}_p) \sin \theta_e, \\
a_{2p} = lK_n\eta_p i_k p s_p, \quad a_{2e} = -\frac{\mu}{\mu} K_n \eta_e i_k e \sin \theta_e, \quad a_{2r} = \left(\frac{\sigma_f}{\mu} + \left(\frac{\tilde{\lambda}}{\mu} + 2\tilde{\pi}^2 \tilde{s}_e^2\right) i \tilde{\omega} \tilde{\sigma}_p \right) \tilde{\eta}_p, \\
a_{3p} = -(2\tilde{k}_p^2 \tilde{s}_p + K_n i_k p) \sin \theta_p, \quad a_{3d} = (-2\tilde{\pi} \tilde{\omega} \tilde{k}_p^2 \tilde{s}_p + K_n i_k p \tilde{\sigma}_p) \sin \theta_p, \quad a_{3s} = K_n i_k p \sin \theta_p l, \quad a_{3e} = (1) K_n i_k e s_p l, \\
a_{3d} = l i_k e \tilde{\sigma}_e \tilde{\eta}_p s_p l, \\
a_{4p} = -(\tilde{k}_e^2 (\tilde{s}_e^2 + \sin^2 \theta_e) - i k e^2 K_p l), \quad a_{4d} = (-2 \tilde{\pi} \tilde{\omega} \tilde{k}_e^2 (\tilde{s}_e^2 - \sin^2 \theta_e) - i \tilde{k}_e K_p \tilde{\sigma}_e) \tilde{\eta}_e, \\
a_{5p} = (m \eta_p n - n + 2\tilde{s}_e^2) \tilde{k}_p^2, \quad a_{5e} = -2\tilde{k}_e^2 \sin \theta_e \tilde{s}_e, \quad a_{5r} = (-m \tilde{\eta}_p n + n + 2\tilde{s}_e^2) \tilde{k}_p^2, \quad a_{5d} = -2\tilde{k}_e^2 \sin \theta_e \tilde{s}_e, \\
a_{6p} = (m + n \eta_p + \frac{i \omega \sigma_f}{\mu} \left(\lambda_e + 2\mu_s \tilde{s}_p^2\right) \tilde{k}_p^2, \quad a_{6e} = \frac{2 \mu_s \omega}{\mu} (i \tilde{k}_e^2 \sin \theta_e s_e \omega \eta_e), \\
a_{6d} = -(\tilde{m} + (n + \tilde{n} \tilde{\omega}) i \tilde{\omega} - 2\tilde{\pi} \tilde{\omega} i \tilde{\omega} \tilde{s}_e \tilde{\eta}_p) \tilde{k}_p^2, \quad a_{6d} = (\tilde{m} \tilde{\omega} i \tilde{\alpha} \tilde{\omega} \tilde{s}_e \tilde{\eta}_p) \sin \theta_e \tilde{s}_e \tilde{k}_p^2, \quad a_{6e} = -2\tilde{k}_e \sin \theta_e s_e, \\
a_{7p} = K_n i_k p \tilde{\sigma}_p, \quad a_{7r} = -2\tilde{k}_e \tilde{\sigma}_e \sin \theta_e \tilde{s}_e, \quad a_{7d} = -2\tilde{k}_e \tilde{\sigma}_e \sin \theta_e \tilde{s}_e, \\
a_{8p} = l i_k e \tilde{\omega} \eta_p \tilde{k}_e (\sin^2 \theta_e - \tilde{s}_e^2), \quad a_{8e} = -2\tilde{\omega} \tilde{k}_p \tilde{\sigma}_p \tilde{\sigma}_e \tilde{\eta}_p, \quad a_{8d} = -2\tilde{\omega} \tilde{k}_p \tilde{\sigma}_p \tilde{\sigma}_e \tilde{\eta}_p, \quad a_{8r} = -2\tilde{\omega} \tilde{k}_p \tilde{\sigma}_p \tilde{\sigma}_e \tilde{\eta}_p.
\]

Where,

\[
s_e = \left(\frac{V_1}{V_0}\right) \left[\frac{(V_0}{V_d})^2 - \sin^2 \theta_0 \right]^{1/2}, \quad s_d = \left(\frac{V_d}{V_0}\right) \left[\frac{(V_0}{V_d})^2 - \sin^2 \theta_0 \right]^{1/2} = \frac{l}{\mu}, \quad \tilde{n} = \frac{n}{\mu}, \quad \lambda = \frac{\lambda}{\mu}, \quad m = \frac{\sigma_f}{\mu}, \quad \tilde{m} = \frac{\sigma_f}{\mu}, \quad n = \frac{\lambda}{\mu}.
\]

where, \( d=p,e,r,d \); \( p=1,2; e=3,4; r=5,6; d=7,8 \)
Effect of Imperfection on Reflection and Transmission Coefficients in Swelling Porous Elastic Media at an Imperfect Boundary

and

\[ Z_1 = \frac{A_1}{A}, \quad Z_2 = \frac{A_2}{A}, \quad Z_3 = \frac{B_3}{A}, \quad Z_4 = \frac{B_4}{A}, \quad Z_5 = \frac{A_5}{A}, \quad Z_6 = \frac{A_6}{A}, \quad Z_7 = \frac{B_7}{A}, \quad Z_8 = \frac{B_8}{A}, \]

(i) For incident PS-wave:
\[ A^* = A_{01}, \quad A_{02} = B_{03} = B_{04} = 0 \]
\[ Y_1 = a_{11}, \quad Y_2 = a_{21}, \quad Y_3 = -a_{31}, \quad Y_4 = -a_{41}, \quad Y_5 = -a_{51}, \quad Y_6 = -a_{61}, \quad Y_7 = a_{71}, \quad Y_8 = a_{81} \]

(ii) For incident PF-wave:
\[ A^* = A_{02}, \quad A_{01} = B_{03} = B_{04} = 0 \]
\[ Y_1 = a_{12}, \quad Y_2 = a_{22}, \quad Y_3 = -a_{32}, \quad Y_4 = -a_{42}, \quad Y_5 = -a_{52}, \quad Y_6 = -a_{62}, \quad Y_7 = a_{72}, \quad Y_8 = a_{82} \]

(iii) For incident SVS-wave:
\[ A^* = B_{03}, \quad A_{01} = A_{02} = B_{04} = 0 \]
\[ Y_1 = -a_{13}, \quad Y_2 = -a_{23}, \quad Y_3 = -a_{33}, \quad Y_4 = a_{43}, \quad Y_5 = a_{53}, \quad Y_6 = a_{63}, \quad Y_7 = -a_{73}, \quad Y_8 = -a_{83} \]

(iv) For incident SVF-wave:
\[ A^* = B_{04}, \quad A_{01} = A_{02} = B_{03} = 0 \]
\[ Y_1 = -a_{14}, \quad Y_2 = -a_{24}, \quad Y_3 = -a_{34}, \quad Y_4 = a_{44}, \quad Y_5 = a_{54}, \quad Y_6 = a_{64}, \quad Y_7 = -a_{74}, \quad Y_8 = -a_{84} \]

where, \( Z_1, Z_2, Z_3, Z_4 \) are the amplitude ratios of reflected PS-, PF-, SVS-, SVF-waves and \( \bar{Z}_1, \bar{Z}_2, \bar{Z}_3, \bar{Z}_4 \), are the amplitude ratios of transmitted PS-, PF-, SVS-, SVF-waves.

Case-I: Normal Stiffness (NS):
\[ K_n \neq 0, \quad K_{nf} \neq 0, \quad K_i \rightarrow \infty, \quad K_{tf} \rightarrow \infty \]
Correspond to the case of normal stiffness and we obtain a system of eight non-homogeneous equations with the changed values of \( a_{ij} \) as
\[ a_{3p} = ik_p l \sin \theta_p, \quad a_{4p} = (1) \cdot ik_p l \cos \theta_p, \quad a_{5r} = -ik_r l \sin \theta_r, \quad a_{6r} = -ik_r l \cos \theta_r, \quad a_{7d} = -\eta_{ik} l \sin \theta_d, \quad a_{8d} = -\eta_{ik} l \cos \theta_d, \]
\[ a_{4r} = (-1) \cdot \eta_{ik} l \sin \theta_r, \quad a_{4d} = -\eta_{ik} l \cos \theta_r, \]

Case-II: Transverse Stiffness (TS):
\[ K_n \rightarrow \infty, \quad K_{nf} \rightarrow \infty, \quad K_i \neq 0, \quad K_{tf} \neq 0 \]
Correspond to the case of transverse stiffness. We obtain a system of eight non-homogeneous equations with the changed values of \( a_{ij} \) as
\[ a_{1p} = -\eta_{ik} l \sin \theta_p, \quad a_{2p} = -\eta_{ik} l \cos \theta_p, \quad a_{3r} = -\eta_{ik} l \sin \theta_r, \quad a_{4r} = -\eta_{ik} l \cos \theta_r, \quad a_{5d} = -\eta_{ik} l \sin \theta_d, \quad a_{6d} = -\eta_{ik} l \cos \theta_d, \]
\[ a_{5r} = -\eta_{ik} l \sin \theta_r, \quad a_{6r} = -\eta_{ik} l \cos \theta_r, \]

Case-III: Welded Contact (WC):
\[ K_n \rightarrow \infty, \quad K_{nf} \rightarrow \infty, \quad K_i \rightarrow \infty, \quad K_{tf} \rightarrow \infty \]
Correspond to the case of transverse stiffness. We obtain a system of eight non-homogeneous equations with the changed values of \( a_{ij} \) as
\[ a_{1p} = -\eta_{ik} l \sin \theta_p, \quad a_{2p} = -\eta_{ik} l \cos \theta_p, \quad a_{3r} = -\eta_{ik} l \sin \theta_r, \quad a_{4r} = -\eta_{ik} l \cos \theta_r, \quad a_{5d} = -\eta_{ik} l \sin \theta_d, \quad a_{6d} = -\eta_{ik} l \cos \theta_d, \]
\[ a_{5r} = -\eta_{ik} l \sin \theta_r, \quad a_{6r} = -\eta_{ik} l \cos \theta_r, \]

Notes

© 2012 Global Journals Inc. (US)
VI. NUMERICAL RESULTS AND DISCUSSION

In order to illustrate theoretical results obtained in the preceding sections, we now present some numerical results. For numerical computation, the physical data is given below:

\[ \lambda = 2.238 \times 10^9 \text{ N/m}^2, \mu = 2.992 \times 10^9 \text{ N/m}^2, \lambda_c = 2.05 \times 10^9 \text{ NSec/m}^2, \mu_c = 2.5 \times 10^9 \text{ NSec/m}^2, \]
\[ \sigma^f = 1.42 \times 10^9 \text{ N/m}^2, \sigma^f = 1.75 \times 10^9 \text{ N/m}^2, \rho^f_0 = 2.65 \times 10^3 \text{ NS ec/m}^4, \rho^f_0 = 1.92 \times 10^3 \text{ NS ec/m}^4, \]
\[ \xi^f = 1.745 \times 10^3 \text{ NSec/m}^4, \bar{\lambda} = 0.91 \times 10^9 \text{ N/m}^2, \bar{\mu} = 1.11 \times 10^9 \text{ N/m}^2, \bar{\lambda}_c = 1.5 \times 10^9 \text{ NSec/m}^2, \]
\[ \bar{\rho}_c = 1.29 \times 10^9 \text{ NSec/m}^2, \bar{\rho}^f_0 = 1.25 \times 10^3 \text{ NS ec/m}^4, \bar{\rho}^f_0 = 0.12 \times 10^3 \text{ NS ec/m}^4, \sigma^f = 0.7 \times 10^9 \text{ N/m}^2, \]
\[ \bar{\sigma}^f = 0.5 \times 10^9 \text{ N/m}^2, \xi^f = 0.1 \times 10^3 \text{ NSec/m}^4 \]

A computer programme has been developed and amplitude ratios of various reflected and transmitted waves have been computed. The variations of amplitude ratios for swelling porous elastic solid with stiffness (ST), normal stiffness (NS), transversal stiffness (TS), welded contact (WC) with angle of incidence \( \theta_0 \) of the incident PS wave, incident SVS wave are shown graphically in Figures 2-3.

VII. INCIDENT PS-WAVE

Fig. 2(a)-2(h) depicts the variation in values of amplitude ratios \( |Z_s|, s = 1,2,3,4,5,6,7,8 \) when PS wave is incident.
From Fig. 2(a), we notice that the values of amplitude ratios $|Z_1|$ for PSR, NS, TS, and WC are of oscillatory behavior. Amplitude ratio for TS remains greater than the values of amplitude ratio for PSR, NS, and WC in range $\theta_0 \geq 3$, whereas values of amplitude ratio for WC remains less than the values of amplitude ratio for PSR, NS, and TS in range $\theta_0 \geq 5$. The values of amplitude ratio for PSR, NS, and WC oscillate in the whole range whereas for WC it decrease in range $1 \leq \theta_0 \leq 70$ and then for $\theta_0 \geq 71$ it starts increasing.

Fig. 2(b) shows that the values of amplitude ratio $|Z_2|$ for PFR, TS, and WC decreases with increase in angle of incidence, whereas the values of amplitude ratio $|Z_2|$ for NS initially oscillates and then decrease with angle of incidence. The values of amplitude ratios for WC remains greater than the values obtained for PFR, NS, and TS in whole range. The values of amplitude ratio for NS remain less than the values of amplitude ratio for PFR, TS, and WC in whole range.

Fig. 2(c) depicts the variation in amplitude ratio $|Z_3|$ due to incidence of PS wave. From the figure, we notice that amplitude ratio for SVSR oscillates in the region $1 \leq \theta_0 \leq 50$. Then for $\theta_0 \geq 51$ it decrease. The values of amplitude ratio for TS oscillate in the region $1 \leq \theta_0 \leq 30$ then for $31 \leq \theta_0 \leq 80$ it decrease, for $\theta_0 \geq 81$ it is of oscillatory behavior. The values of amplitude ratio for NS decreases for $1 \leq \theta_0 \leq 5$, for $6 \leq \theta_0 \leq 45$ it increase and for $\theta_0 \geq 46$ it decreases with angle of incidence. The values of amplitude ratio for WC remain less than the values obtain for SVSR, NS, and TS in whole range and keeps increasing with increase in angle of incidence.

Fig. 2(d) depicts the variation in amplitude ratio $|Z_4|$ due to the incidence of PS wave. From the figure, we notice that amplitude ratio for SVFR initially oscillates, then decreases in range $\theta_0 \geq 4$. The values of amplitude ratio for NS, TS, and WC decrease in whole range. For the $\theta_0 \geq 4$ values of amplitude ratio for WC remains greater than the values obtain for SVFR, NS, and WC. The values of amplitude ratio for TS remain less than the values of amplitude ratio for SVFR, NS, and WC in whole range.

From Fig. 2(e) we notice that the values of amplitude ratio $|Z_5|$ for PST initially oscillate then decreases in range $\theta_0 \geq 4$. The values of amplitude ratio for NS oscillates in the region $1 \leq \theta_0 \leq 3$, then decreases in the range $\theta_0 \geq 4$, whereas for TS it initially oscillates, then decrease in the range $4 \leq \theta_0 \leq 80$ and remains greater than the values obtain for PST, NS, and WC in whole range. The values of amplitude ratio for WC remain less than the values of amplitude ratio for PST, NS, and WC in whole range.

From Fig. 2(f) we notice that the values of amplitude ratio $|Z_6|$ for PFT, NS, TS, and WC decreases with increase in angle of incidence. The values of amplitude ratio for WC remain greater than the values of amplitude ratio for PFT, NS, and TS. The values of amplitude ratios for NS remain less than the values of amplitude ratios for PFT, TS, and WC in whole range.

From Fig. 2(g), we notice that values of amplitude ratio $|Z_7|$ for SVST and WC decreases with increase in angle of incidence. In range $1 \leq \theta_0 \leq 35$ the values of amplitude ratio for SVST remains greater than the values of amplitude ratio for NS, TS, and WC. The values of amplitude ratio for WC remain less than the values of amplitude ratio for SVST, NS, and TS.
From Fig. 2(h), we notice that values of amplitude ratio $|Z_s|$ for SVFT, NS, TS and WC decreases with angle of incidence. The values of amplitude ratio for WC remain greater than the values of amplitude ratio for SVFT, NS and TS, whereas for SVFT remains less than the values obtain for NS, TS and WC.

VIII. INCIDENCE OF SVS-WAVE

Fig. 3(a)-3(h) depicts the variation in values of amplitude ratios $|Z_s|$, $s = 1,2,3,4,5,6,7,8$ when SVS wave is incident.
The values of amplitude ratios $|Z_1|$ for PSR and TS are of oscillatory behavior. They attain peak value at $\theta_0 = 37$ and then for $\theta_0 \geq 38$ it decrease, whereas for WC it attains peak value at $\theta_0 = 36$ and then starts decreasing. For $\theta_0 \geq 5$ the values of amplitude ratio for WC remains less than the values for PSR, NS and TS.

From Fig. 3(b) we notice that values of amplitude ratio $|Z_1|$ for PFR, NS, TS and WC decreases in whole range. The values of amplitude ratio $|Z_2|$ for TS remain greater than the values of amplitude ratio $|Z_2|$ for PFR, NS and WC.

From Fig. 3(c) we notice, that values of amplitude ratio $|Z_1|$ for SVSR, NS, TS and WC is of oscillatory behavior. For $\theta_0 \geq 36$ the values of amplitude ratio for WC remains less than the values of amplitude ratio for SVSR, NS, and TS. For $5 \leq \theta_0 \leq 50$ the values of amplitude ratio for TS remains greater than the values of amplitude ratio for SVSR, NS, and WC.

Fig. 3(d), depicts that values of amplitude ratio $|Z_1|$ for SVFR oscillates in the region $1 \leq \theta_0 \leq 5$ then decrease with angle of incidence. The values of amplitude ratio for NS and WC decrease with angle of incidence. For $\theta_0 \geq 3$ the values of amplitude ratio for NS remains less than the values obtain for SVFR, TS and WC. The values of amplitude ratio for TS initially increase then decrease with angle of incidence. For $\theta_0 \geq 5$ the values of amplitude ratio for TS remains greater than the values of amplitude ratio for SVFR, NS, and WC.

From Fig. 3(e) we notice, that values of amplitude ratios $|Z_1|$ for PST, NS, TS and WC are of oscillatory behavior. For $10 \leq \theta_0 \leq 35$ the values of amplitude ratio for NS remains greater than the values of amplitude ratio for PST, NS and WC. The values of amplitude ratio for WC remains less than the values obtain for PST, NS, and TS in whole range. The values of amplitude ratio for NS and WC attains maximum value at $\theta_0 = 35$.

From Fig. 3(f) we notice that values of amplitude ratio $|Z_1|$ for PFT initially oscillates in region $1 \leq \theta_0 \leq 4$, then starts decreasing. The values of amplitude ratio for NS decrease with angle of incidence. The values of amplitude ratio for WC and TS initially oscillates then decrease with angle of incidence. The values of amplitude ratio for TS remains greater than the values of amplitude ratio obtain for PFT, NS and WC for $\theta_0 \geq 4$.

From Fig. 3(g), we notice that values of amplitude ratio $|Z_1|$ for SVST, NS, TS and WC are of oscillatory behavior. In range $5 \leq \theta_0 \leq 30$ the values of amplitude ratio for NS remains greater than the values of amplitude ratio for SVST, TS and WC. For $31 \leq \theta_0 \leq 64$ the values of amplitude ratio for TS remain greater than the values of amplitude ratio obtain for SVST, NS and WC.

From Fig. 3(h), we notice that values of amplitude ratio $|Z_1|$ for NS and TS are of oscillatory behavior, whereas for SVFT and WC it decrease with angle of incidence. The values of amplitude ratio for SVFT remain less than the values of amplitude ratio for NS, TS and WC in whole range.

**IX. Conclusion**

When PS wave is incident the values of amplitude ratio for $|Z_1|, |Z_2|, |Z_3|, |Z_4|$ decrease with angle of incidence, whereas for $|Z_5|, |Z_6|, |Z_7|, |Z_8|$ are of oscillatory behavior. When SVS wave is incident the values of amplitude ratio for $|Z_1|, |Z_2|, |Z_3|, |Z_4|, |Z_5|, |Z_6|$ are of oscillatory behavior, whereas $|Z_7|$ decrease with angle of incidence.

**References Références Referencias**

This page is intentionally left blank