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Air Traffic Control
Sweden, Europe

HIGHLIGHTS

- Reaction and Rotation
- Intuitionistic L-Fuzzy
- Curvature Tensor
- SEIR Epidemic Model

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The Effects of Thermal Radiation, Chemical Reaction and Rotation on Unsteady MHD Viscoelastic Slip Flow

By Aarti Manglesh & M. G. Gorla
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Aarti Manglesh [α] & M. G. Gorla [σ]

Abstract - This paper investigates the unsteady flow of an electrically conducting incompressible non-Newtonian viscoelastic fluid through a porous medium filled in a vertical porous channel in the presence of transverse magnetic field. The fluid and the channel rotate as a solid body with constant angular velocity, $\mathbf{\Omega}^*$, about an axis perpendicular to the planes of the plates. The effects of thermal radiation and chemical reaction are taken into account embedded with slip boundary condition. The closed-form analytical solutions are obtained for momentum, energy and concentration equations. The influences of the various parameters entering into the problem in the velocity, temperature and concentration field are discussed with the help of graphs. Also, numerical values of physical quantities, such as skin friction coefficient, Nusselt number and Sherwood number are presented in tabular form.

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I. Introduction

Many transport processes exist in nature and industrial application in which the transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. In the last few decades several efforts have been made to solve the problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. Chemical reaction can be codified either heterogeneous or homogeneous processes. Its effect depends on the nature of the reaction whether the reaction is heterogeneous or homogeneous. A reaction is of order n, if the reaction rate is proportional to the nth power of concentration. In particular, a reaction is of first order, if the rate of reaction is directly proportional to concentration itself. In nature, the presence of pure air or water is not possible. Some foreign mass may be present naturally mixed with air or water. The presence of foreign mass in air or water causes some kind of chemical reaction. The study of such type of chemical reaction processes is useful for improving a number of chemical technologies, such as food processing, polymer production and manufacturing of ceramics or glassware. Chambre and Young [5] analyzed the effect of homogeneous first order chemical reactions in the neighborhood of a flat plate for destructive and generative reactions. Das *et al* [9] studied the effect of first order reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Anjalidevi and Kandasamy [2] investigated the effect of chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy and Ganesan [14] studied the effect of chemical

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reaction on unsteady flow past an impulsively started infinite vertical plate. Raptis and Perdikis [15] studied numerically the steady two-dimensional flow in the presence of chemical reaction over a non-linearly semi-infinite stretching sheet. Moreover chemical reaction effects on heat and mass transfer in laminar boundary layer flow have been studied by several scholars e.g. Chamkha [6], Kandasamy *et al.* [12], Afify [1], Takhar *et al.* [20] and Mansour *et al.* [13]etc.

The study of the interaction of the Coriolis force with the electromagnetic force is of great importance. In particular, rotating MHD flows in porous media with heat transfer is one of the important current topics due to its applications in thermofluid transport modeling in magnetic geosystems [3], meteorology, MHD power generators, turbo machinery, solidification process in metallurgy, and in some astrophysical problems. It is generally thought that the existence of the geomagnetic field is due to finite amplitude instability of the Earth's core. Since most cosmic bodies are rotators, the study of convective motions in a rotating electrically conducting fluid is essential in understanding better the magnetohydrodynamics of the interiors of the Earth and other planets. It has motivated a number of studies on convective motions in hydromagnetic rotating systems, which can provide explanations for the observed variations in the geomagnetic field. The rotating flow subjected to different physical effects has been studied by many authors, such as, Vidyanidhu and Nigam [21], Jana and Datta [11], Singh [16, 17, and 18] etc.

Viscoelastic fluid flow through porous media has attracted the attention of scientists and engineers because of its importance notably in the flow of oil through porous rock, the extraction of energy from geothermal regions, the filtration of solids from liquids and drug permeation through human skin. The knowledge of flow through porous media is useful in the recovery of crude oil efficiently from the pores of reservoir rocks by displacement with immiscible fluid. The flow through porous media occurs in the ground-water hydrology, irrigation, drainage problems and also in absorption and filtration processes in chemical engineering. This subject has wide spread applications to specific problems encountered in the civil engineering and agriculture engineering, and many industries. Thus the diffusion and flow of fluids through ceramic materials as bricks and porous earthenware has long been a problem of the ceramic industry. The Scientific treatment of the problem of irrigation, Soil erosion and tile drainage are present developments of porous media. In hydrology, the movement of trace pollutants in water systems can be studied with the knowledge of flow through porous media. The principles of this subject are useful in recovering the water for drinking and irrigation purposes. Thurson was the earliest to recognize the viscoelastic nature of blood and that the viscoelastic behavior is less prominent with increasing shear rate. A series of investigations have been made by different scholars viz: Choudhary and Deb [7] and Gbadeyan *et al* [10], Attia [4] etc.

The objective of above paper is to analyze radiation and chemical reaction effects on an unsteady MHD flow of a viscoelastic, incompressible, electrically conducting fluid through an infinite vertical porous channel with simultaneous injection and suction, embedded in a uniform porous medium, in the presence of transverse magnetic field. The entire system rotates about an axis perpendicular to the plane of the plates.

II. Mathematical Formulation

The geometry of the problem is shown in Fig. 1. The fluid is assumed to be incompressible, viscoelastic, electrically conducting and flows between two infinite vertical parallel non-conducting plates located at the $y = \pm\frac{d}{2}$ planes and extend from $X^* \rightarrow -\infty$ to ∞ and from $Z^* \rightarrow -\infty$ to ∞. A Cartesian co-ordinate system is introduced such that

Ref.

1. Afify A A (2004). Effects of radiation and chemical reaction on MHD free convective flow past a vertical isothermal cone. Canadian Journal of Physics 82, 447-458.

X^*-axis lies vertically upward along the centreline of the channel, in the direction of flow and Y^*-axis is perpendicular to the wall of the channel. The channel and the fluid rotate in unision with the uniform angular velocity Ω^* about Y^* axis. A constant magnetic field of strength B_0 is applied perpendicular to the axis of the channel and the effect of induced magnetic field is neglected, which is a valid assumption on laboratory scale under the assumption of small magnetic Reynolds number [19]. The flow field is exposed to the influence of constant injection and suction velocity, thermal and mass buoyancy effect, thermal radiation and chemically reactive species. The temperature and concentration at one of the wall is oscillating. Viscous and Darcy's resistance terms are taken into account with constant permeability of the medium. Further due to the infinite plane surface assumption, the flow variables are functions of y^* and t^* only. Thus the velocity of the fluid, in general, is given by

$$\vec{V}(y,t) = u(y,t)\hat{\imath} + v(y,t)\hat{\jmath} + w(y,t)\hat{k}$$

It is because of conservation of mass i.e. $\nabla.\vec{V} = 0$ and due to uniform suction the velocity component $\vec{v}(y,t)$ is assumed to have a constant value v_0.

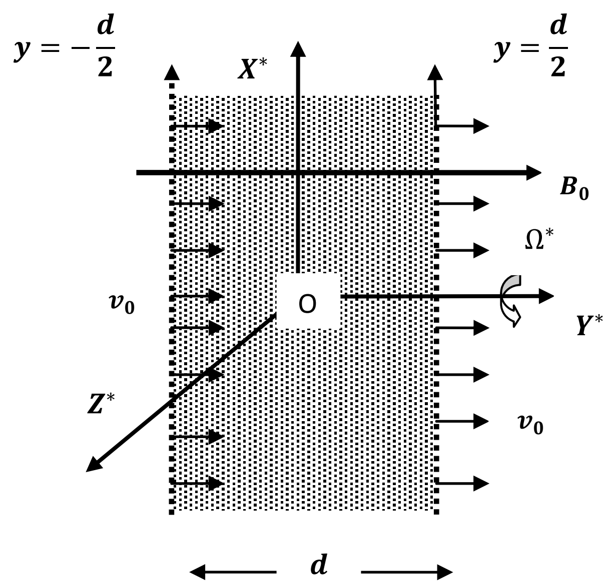

Fig.1 : Schematic presentation of the physical problem

Under the usual Boussinesq's approximation and in the absence of pressure gradient, the unsteady equations governing the MHD flow of viscoelastic fluid are:

$$\frac{\partial u^*}{\partial t^*} + v_0 \frac{\partial u^*}{\partial y^*} = \vartheta \frac{\partial^2 u^*}{\partial y^{*2}} - K_0 \frac{\partial^3 u^*}{\partial t^* \partial y^{*2}} - \frac{\sigma B_0^2 u^*}{\rho} + 2\Omega^* w^* + g_T \beta T^* + g_C \beta^* C^* - \frac{\vartheta u^*}{K_p^*} \quad (1)$$

$$\frac{\partial w^*}{\partial t^*} + v_0 \frac{\partial w^*}{\partial y^*} = \vartheta \frac{\partial^2 w^*}{\partial y^{*2}} - K_0 \frac{\partial^3 w^*}{\partial t^* \partial y^{*2}} - \frac{\sigma B_0^2 w^*}{\rho} - 2\Omega^* u^* - \frac{\vartheta w^*}{K_p^*} \quad (2)$$

$$\frac{\partial T^*}{\partial t^*} + v_0 \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho P_r} \frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho C_p} \frac{\partial q}{\partial y^*} \quad (3)$$

Ref.
19. Sutton G W, Sherman A (1965). Engineering Magnetohydrodynamics, McGraw Hill, New York.

$$\frac{\partial C^*}{\partial t^*} + v_0 \frac{\partial C^*}{\partial y^*} = D_m \frac{\partial^2 C^*}{\partial y^{*2}} - K_1 C^* \tag{4}$$

Boundary conditions of the problem are:

$$\left.\begin{aligned} u^* = L^* \frac{\partial u^*}{\partial y^*}, w^* = L^* \frac{\partial w^*}{\partial y^*}, T^* = 0, C^* = 0 \ \text{at} \ y^* = -\frac{d}{2} \\ u^* = 0, w^* = 0, T^* = \mathrm{T}_0 \cos \omega^* t^*, C^* = \mathrm{C}_0 \cos \omega^* t^* \ \text{at} \ y^* = \frac{d}{2} \end{aligned}\right\} \tag{5}$$

where $L^* = \left(\frac{2-m_1}{m_1}\right) L$, with m_1 is Maxwell's reflexion coefficient, L mean free path and is a constant for an incompressible fluid, T* is the temperature, C^* is concentration, t^* is the time, ρ is the density, ϑ is the kinematic viscosity, K_0 is the viscoelasticity, σ is the electric conductivity, Ω^* is rotation , g the acceleration due to gravity, β_T is coefficient of thermal expansion, β_C is coefficient of concentration expansion, K_p^*, is the permeability of the porous medium, κ is thermal conductivity, P_r is Prandtl number, C_p is the specific heat at constant pressure, D_m is chemical molecular diffusivity, K_1 is chemical reaction, ω^* is the frequency of oscillations. Here '*' stands for the dimensional quantities.

At this point, we limit ourselves to the condition of optically thin with relatively low-density fluid such as the one would find in the intergalactic layers where the plasma gas is assumed to be of low density. Thus, in the spirit of Cogley *et al* [8] the radiative heat flux for the present problem become

$$\frac{\partial q}{\partial y^*} = 4\alpha' T^* \tag{6}$$

Where α' is the mean radiation absorption coefficient.
Equations can be made dimensionless by introducing the following dimensionless variables:

$$u = \frac{u^*}{v_0} \quad w = \frac{w^*}{v_0} \quad x = \frac{x^*}{d} \quad y = \frac{y^*}{d} \quad \theta = \frac{T^*}{T_0} \quad C = \frac{C^*}{C_0} \quad t = \frac{t^* \vartheta}{d^2} \quad \omega = \frac{\omega^* d^2}{\vartheta}$$

We also define the following dimensionless parameters:

$\lambda = \frac{v_0 d}{\vartheta}$, the suction parameter,

$\alpha = \frac{K_0}{d^2}$, the viscoelastic parameter,

$M = B_0 d \sqrt{\frac{\sigma}{\mu}}$, the Hartmann number,

$\Omega = \frac{\Omega^*}{d^2}$, the rotation parameter,

$G_r = \frac{g\beta T_0 d^2}{v_0 \vartheta}$, the Grashoff number,

$G_m = \frac{g\beta^* C_0 d^2}{v_0 \vartheta}$, the modified Grashoff number,

Ref.

8. Cogley A C L, Vinvent W G, Giles E S (1968). Differential approximation for radiative transfer in a non-gray near equilibrium, American Institute of Aeronautics and Astronautics, 6: 551-553.

$K_p = \frac{K_p^*}{d^2}$, the permeability parameter,

$P_r = \frac{\mu c_p}{k}$, the Prandtl number,

$S_C = \frac{\vartheta}{D_m}$, the Schmidt number,

Notes

$N = \frac{2\alpha' d}{\sqrt{\kappa}}$, the radiation parameter,

$\chi = \frac{K_1 d^2}{\upsilon}$, the chemical reaction parameter,

In terms of these dimensionless quantities equations (1) to (4), written as

$$\frac{\partial u}{\partial t} + \lambda \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \alpha \frac{\partial^3 u}{\partial t \partial y^2} - M^2 u + 2\Omega w + G_r \theta + G_m C - \frac{u}{K_p} \tag{7}$$

$$\frac{\partial w}{\partial t} + \lambda \frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \alpha \frac{\partial^3 u}{\partial t \partial y^2} - M^2 w - 2\Omega u - \frac{w}{K_p} \tag{8}$$

$$\frac{\partial \theta}{\partial t} + \lambda \frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - \frac{N^2}{P_r} \theta \tag{9}$$

$$\frac{\partial C}{\partial t} + \lambda \frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} - \chi C \tag{10}$$

The relevant boundary conditions in non-dimensional form are given by:

$$\left.\begin{aligned} u = h\frac{\partial u}{\partial y}, w = h\frac{\partial w}{\partial y}, \theta = 0, C = 0 \ \text{at} \ y = -\frac{1}{2} \\ u = 0, w = 0, \theta = \cos\omega t, C = \cos\omega t \ \text{at}\ y = \frac{1}{2} \end{aligned}\right\} \tag{11}$$

Where h is velocity slip parameter.

Introducing the complex velocity $F = u + iw$, we find that equation (7) and (8) can be combined into a single equation of the form:

$$\frac{\partial F}{\partial t} + \lambda \frac{\partial F}{\partial y} = \frac{\partial^2 F}{\partial y^2} - \alpha \frac{\partial^3 F}{\partial t \partial y^2} - M^2 F - 2i\Omega F + G_r \theta + G_m C - \frac{F}{K_p} \tag{12}$$

The corresponding boundary conditions reduce to:

$$\left.\begin{aligned} F = h\frac{\partial F}{\partial y}, \theta = 0, C = 0, at\ y = -\frac{1}{2} \\ F = 0, \theta = \cos\omega t, C = \cos\omega t, \ \text{at} \ y = \frac{1}{2} \end{aligned}\right\} \tag{13}$$

In order to solve the system of equation (9), (10), (12) subject to the boundary condition (13) we assume

$$\left.\begin{aligned} F(y,t) &= F_0(y)e^{i\omega t} \\ \theta(y,t) &= \theta_0(y)e^{i\omega t} \\ C(y,t) &= C_0(y)e^{i\omega t} \end{aligned}\right\} \quad (14)$$

Notes

Substituting (14) in equations (9), (10), (12) we get,

$$(1-iA)F_0'' - \lambda F_0' - l^2 F_0 = -G_r\theta_0 - G_m C_0 = 0 \quad (15)$$

$$\theta_0'' - \lambda P_r \theta_0' - a_0\theta_0 = 0 \quad (16)$$

$$C_0'' - S_c\lambda C_0' - a_1 C_0 \quad (17)$$

Where $l^2 = M^2 + 2i\Omega + i\omega + \frac{1}{K_p}$, $A = \alpha\omega$, $a_0 = N^2 + i\omega P_r$ and $a_1 = \chi + i\omega$

Corresponding boundary condition becomes:

$$\left.\begin{aligned} F_0 = h\frac{\partial F_0}{\partial y}, \theta = 0, C = 0, at\ y = -\frac{1}{2} \\ F_0 = 0, \theta_0 = 1, C_0 = 1 \text{ at } y = \frac{1}{2} \end{aligned}\right\} \quad (18)$$

The solution of equation (15), (16) and (17) under boundary condition (18) is

$$F(y,t) = (A_7e^{r_2y} + A_8e^{s_2y} + A_5e^{r_1y} + A_6e^{s_1y} + A_3e^{ry} + A_4e^{sy})e^{i\omega t} \quad (19)$$

$$\theta(y,t) = (A_0e^{ry} + B_0e^{sy})e^{i\omega t} \quad (20)$$

$$C(y,t) = (A_1e^{r_1y} + A_2e^{s_1y})e^{i\omega t} \quad (21)$$

Where

$$r = \frac{\lambda P_r + \sqrt{\lambda^2P_r^2 + 4a_0}}{2} \qquad s = \frac{\lambda P_r - \sqrt{\lambda^2P_r^2 + 4a_0}}{2}$$

$$r_1 = \frac{S_c\lambda + \sqrt{S_c^2\lambda^2 + 4S_ca_1}}{2} \qquad s_1 = \frac{S_c\lambda - \sqrt{S_c^2\lambda^2 + 4S_ca_1}}{2}$$

$$r_2 = \frac{\lambda + \sqrt{\lambda^2 + 4l^2(1-iA)}}{2} \qquad s_2 = \frac{\lambda - \sqrt{\lambda^2 + 4l^2(1-iA)}}{2}$$

$$A_0 = -\frac{e^{\frac{-s}{2}}}{2\sin h\left(\frac{s-r}{2}\right)} \qquad B_0 = \frac{e^{\frac{-r}{2}}}{2\sin h\left(\frac{s-r}{2}\right)}$$

$$A_1 = -\frac{e^{\frac{-s1}{2}}}{2\sin h\left(\frac{s1-r1}{2}\right)} \qquad A_2 = \frac{e^{\frac{-r1}{2}}}{2\sin h\left(\frac{s1-r1}{2}\right)}$$

$$A_3 = -\frac{G_r A_0}{(1-iA)r^2 - \lambda r - l^2} \qquad A_4 = -\frac{G_r B_0}{(1-iA)s^2 - \lambda s - l^2}$$

$$A_5 = -\frac{G_m A_1}{(1-iA){r_1}^2 - \lambda r_1 - l^2} \qquad A_6 = -\frac{G_m A_2}{(1-iA)s_1 - \lambda s_1 - l^2}$$

Notes

$$A_7 = \frac{-1}{\left\{(1-hr_2)e^{\frac{(s_2-r_2)}{2}} - (1-hs_2)e^{\frac{-(s_2-r_2)}{2}}\right\}} \begin{bmatrix} A_3(1-hr)e^{\frac{(s_2-r)}{2}} - (1-hs_2)e^{\frac{-(s_2-r)}{2}} \\ A_4(1-hs)e^{\frac{(s_2-s)}{2}} - (1-hs_2)e^{\frac{-(s_2-s)}{2}} \\ A_5(1-hr_1)e^{\frac{(s_2-r_1)}{2}} - (1-hs_2)e^{\frac{-(s_2-r_1)}{2}} \\ A_6(1-hs_1)e^{\frac{(s_2-s_1)}{2}} - (1-hs_2)e^{\frac{-(s_2-s_1)}{2}} \end{bmatrix}$$

$$A_8 = \frac{1}{\left\{(1-hr_2)e^{\frac{(s_2-r_2)}{2}} - (1-hs_2)e^{\frac{-(s_2-r_2)}{2}}\right\}} \begin{bmatrix} A_3(1-hr)e^{\frac{(r_2-r)}{2}} - (1-hr_2)e^{\frac{-(r_2-r)}{2}} \\ A_4(1-hs)e^{\frac{(r_2-s)}{2}} - (1-hr_2)e^{\frac{-(r_2-s)}{2}} \\ A_5(1-hr_1)e^{\frac{(r_2-r_1)}{2}} - (1-hr_2)e^{\frac{-(r_2-r_1)}{2}} \\ A_6(1-hs_1)e^{\frac{(r_2-s_1)}{2}} - (1-hr_2)e^{\frac{-(s_2-s_1)}{2}} \end{bmatrix}$$

The shear stress, Nusselt number and Sherwood number can now be obtained easily from equations (19), (20) and (21).

Skin friction coefficient τ_L at the left plate in terms of its amplitude and phase is:

$$\tau_L = \left(\frac{\partial F}{\partial y}\right)_{y=-\frac{1}{2}} = \left(\frac{\partial F_0}{\partial y}\right)_{y=-\frac{1}{2}} e^{i\omega t} = |D|\cos(\omega t + \alpha) \tag{22}$$

With $\quad |D| = \sqrt{D_r^2 + D_i^2}$ and $\alpha = \tan^{-1}\left(\frac{D_i}{D_r}\right)$

where $\quad D_r + iD_i = r_2 A_7 e^{\frac{-r_2}{2}} + s_2 A_8 e^{\frac{-s_2}{2}} + r_1 A_5 e^{\frac{-r_1}{2}} + s_1 A_6 e^{\frac{-s_1}{2}} + rA_3 e^{\frac{-r}{2}} + sA_4 e^{\frac{-s}{2}}$

Heat transfer coefficient Nu (Nusselt number) at the left plate in terms of its amplitude and phase is:

$$Nu = \left(\frac{\partial\theta}{\partial y}\right)_{y=-\frac{1}{2}} = \left(\frac{\partial\theta_0}{\partial y}\right)_{y=-\frac{1}{2}} e^{i\omega t} = |H|\cos(\omega t + \beta) \tag{23}$$

with $|H| = \sqrt{H_r^2 + H_i^2}$ and $\beta = \tan^{-1}\left(\frac{H_i}{H_r}\right)$

where $$H_r + iH_i = rA_0 e^{\frac{-r}{2}} + sB_0 e^{\frac{-s}{2}}$$

Mass transfer coefficient Sh (Sherwood number) at the left plate in term of amplitude and phase is:

$$Sh = \left(\frac{\partial C}{\partial y}\right)_{y=-\frac{1}{2}} = \left(\frac{\partial C_0}{\partial y}\right)_{y=-\frac{1}{2}} e^{i\omega t} = |G|\cos(\omega t + \gamma) \tag{24}$$

with $$|G| = \sqrt{G_r^2 + G_i^2} \text{ and } \gamma = \tan^{-1}\left(\frac{G_i}{G_r}\right)$$

where $$G_r + iG_i = r_1 A_1 e^{\frac{-r_1}{2}} + s_1 A_2 e^{\frac{-s_1}{2}}$$

III. Result and Discussion

Numerical evaluation for the analytical solution of this problem is performed and the results are illustrated graphically in Figs. 2-16 to show the interesting features of significant parameters on velocity, temperature and concentration distribution in rotating channel. Throughout the computation we employ $t = 0,\ \lambda = 0.5,\ \omega = 5, M = 1, K_p = 0.5, N = 1,\ G_r = 2,\ G_m = 2,\ \alpha = 0.05,\ \chi = 0.2,\ P_r = 3$ and $h = 0.2$ unless otherwise stated. The effect of rotation on the velocity profile is shown in Fig.2. The rotation parameter defines the relative magnitude of the Coriolis force and the viscous force in the regime; therefore it is clear that high magnitude Coriolis forces are counter- productive for the flow. The velocity profiles initially remain parabolic with maximum at the centre of the channel for small values of rotation parameter Ω and then as rotation increases the velocity profiles flatten. For further increase in Ω (= 25) the maximum of velocity profiles no longer occurs at the centre but shift towards the right wall of the channel. It means that for large rotation there arise boundary layers on the walls of the channel.

The effect of different parameters on velocity profile for small rotation ($\Omega = 1$) and large rotation ($\Omega = 25$) are illustrated in Figs. 3-14 with the help of solid and dotted lines respectively. Figure -3 represents that the increase in slip parameter has the tendency to reduce the frictional forces which increase the fluid velocity in case of small rotation but for large rotation there is very small change in the velocity profile. Increase in thermal and solutal Grashoff numbers significantly increase the boundary layer thickness which resulted into rapid enhancement of fluid velocity for both cases, which is displayed in Figs 4 and 6. The rate of radiative heat transferred to fluid is decreased and consequently the velocity decreases as radiation parameter increases, for both cases of rotation, is represented in Fig. 5. It is obvious that the increase in the frequency of oscillation decrease the velocity for small and large rotation and that is presented in Fig. 7. Fig. 8 illustrate that the presence of transverse magnetic field produces a resistive force on the fluid flow. This force is called Lorentz force, which slows down the motion of the fluid for small as well large rotation.

Fig. 9 shows the effect of viscoelastic parameter on fluid velocity. Increasing viscoelastic parameter the hydrodynamic boundary layer adheres strongly to the surface which in term retards the flow in the left half of channel, but accelerates the flow in right half with no slip boundary condition. The pattern is same for small and large rotation. Increase in Schmidt number and chemical reaction parameter decrease the concentration. This causes the concentration buoyancy effect to decrease yielding a reduction in the fluid

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velocity, which is displayed in Figs. 10 and 11. It can be interpreted from Fig. 12 that velocity decreases with increase of suction parameter indicating the usual fact that suction stabilize the boundary layer growth. Sucking decelerated fluid particle through the porous wall reduces the growth of fluid boundary layer and hence velocity. Fig. 13 displays that the increase in the permeability coefficient of porous medium act against the porosity of the porous medium which increase the fluid velocity for small as well as large rotation. Fig. 14 represents that increase in Prandtl number is due to increase in viscosity of the fluid which makes the fluid thick and causes a decrease in velocity for small and large rotation.

a) Temperature profile

Fig. 15 illustrate that fluid temperature decreases with an increase in radiation parameter. This result qualitatively agrees with expectations, since the effect of radiation decrease the rate of energy transport to the fluid, thereby decreasing the fluid temperature. It is also clear from the figure that as Prandtl number increases, the temperature profile decreases. This is because the fluid is highly conductive for small value of Prandtl number. Physically, if Prandtl number increases, the thermal diffusivity decreases and this phenomenon lead to the deceasing manner of the energy transfer ability that reduces the thermal boundary layer.

b) Concentration Profile

Fig. 16 shows that we obtain a destructive type chemical reaction because the concentration decreases for increasing chemical reaction parameter which indicates that the diffusion rates can be tremendously changed by chemical reaction. Also with the increase in Schmidt number concentration profile also decreases.

Table-1 shows the effect of different parameters in skin friction at the left wall. From the table it is clear that skin friction(τ), decreases with an increase in $\omega, \lambda, \alpha, M, N, \chi$ and P_r and increases with an increase in G_r and G_m, for large as well as small rotation. But in case of permeability parameter and Schmidt number skin friction coefficient decreases for small rotation and increases for large rotation, while a reverse effect is found with increase of slip parameter. From Table-2 it is clear that Nusselt number increases with an increase in Prandtl number and frequency of oscillation, but decreases with radiation and suction parameter. Numerical values of Sherwood number at the left wall is given in Table-3. Table shows that Sherwood number decreases for an increase in chemical reaction parameter, Schmidt number suction parameter and frequency of oscillations.

IV. Conclusions

This paper investigates the effect of heat and mass transfer on MHD slip flow in a vertical porous channel with rotation, chemical reaction and thermal radiation under the effect of transversely applied magnetic field. The resulting partial differential equations are transformed into a set of ordinary differential equation using normalisation and solved in closed-form. Numerical evaluations of the closed- form results are performed and graphical results are obtained to illustrate the details of the flow and heat and mass transfer characteristics and their dependence on some physical parameter. It is observed that the velocity profile is increasing with increasing slip parameter, Grashof number and mass Grashof number, viscoelastic parameter and permeability of porous medium. Also, velocity reducing with increasing rotation, frequency of oscillation, radiation parameter, magnetic parameter, Schmidt number, suction parameter, chemical reaction parameter and Prandtl number. The fluid temperature is reduced by increases in the values of the Prandtl number and radiation parameters. Concentration is reducing with increase in

Schmidt number and chemical reaction parameter. In addition, it is found that skin friction coefficient decreases with frequency of oscillation, suction parameter, viscoelastic parameter, magnetic parameter, radiation parameter, chemical reaction parameter and Prandtl number but increases with thermal and mass Grashof number. However, the Nusselt number increases with an increase in Prandtl number and frequency of oscillation, but decreases with radiation and suction parameter.

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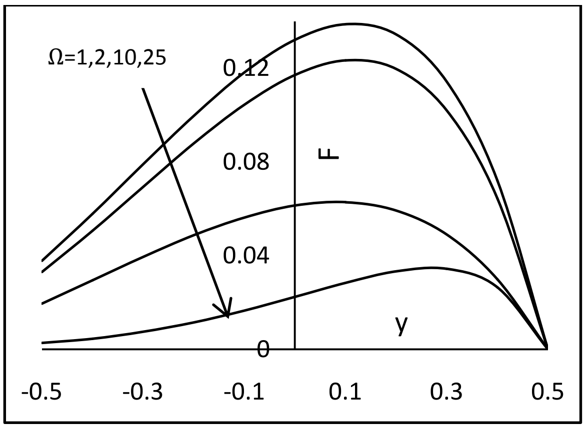

Fig. 2 : Velocity profile for different values of Ω.

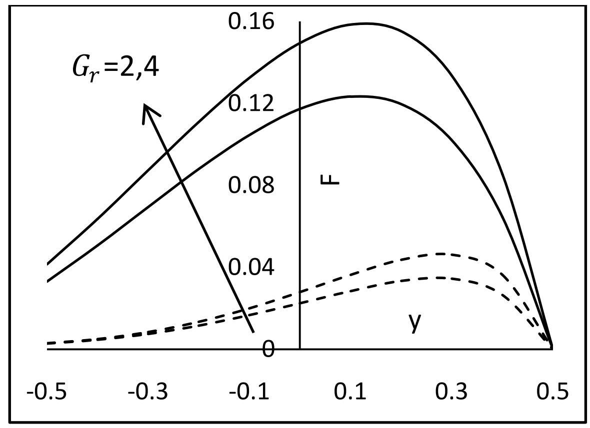

Fig. 4 : Velocity profile for different values of G_r.

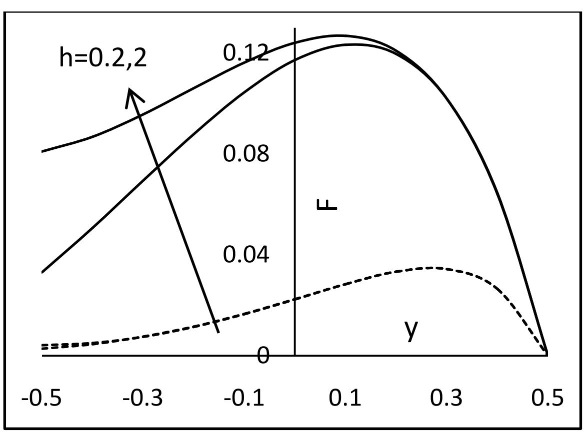

Fig. 3 : Velocity profile for different values of h.

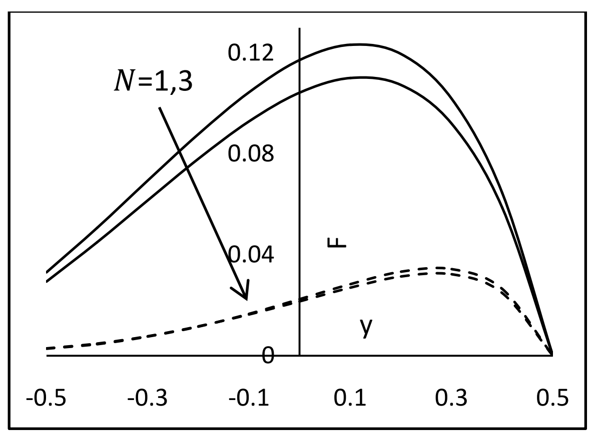

Fig. 5 : Velocity profile for different values of N.

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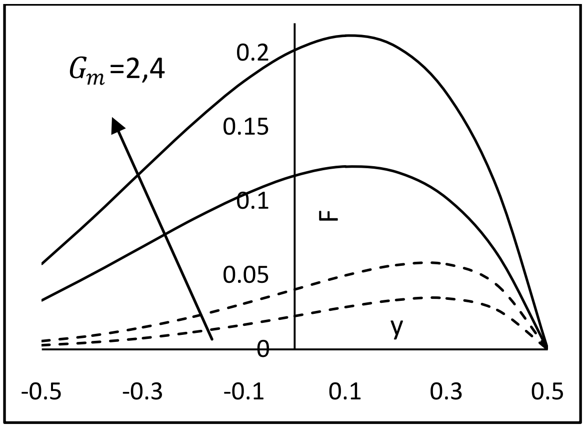

Fig. 6 : Velocity profile for different values of G_m.

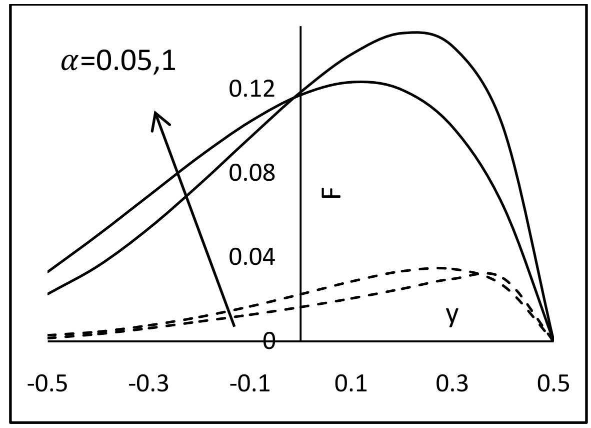

Fig. 9 : Velocity profile for different values of α.

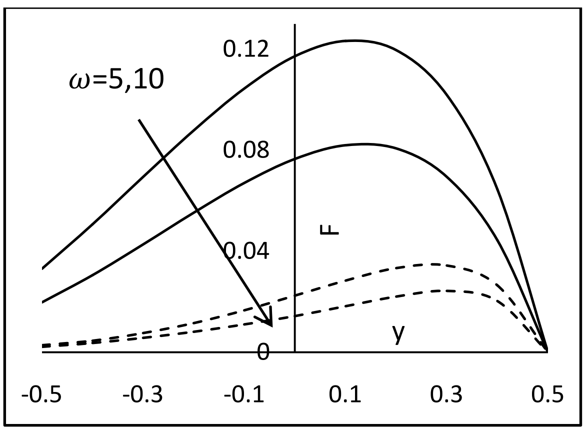

Fig. 7 : Velocity profile for different values of ω.

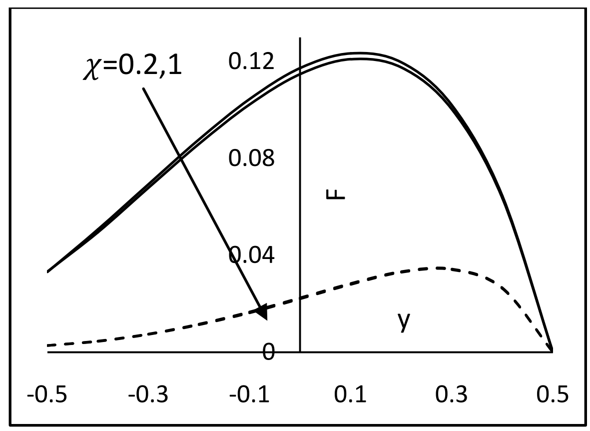

Fig. 10 : Velocity profile for different values of χ.

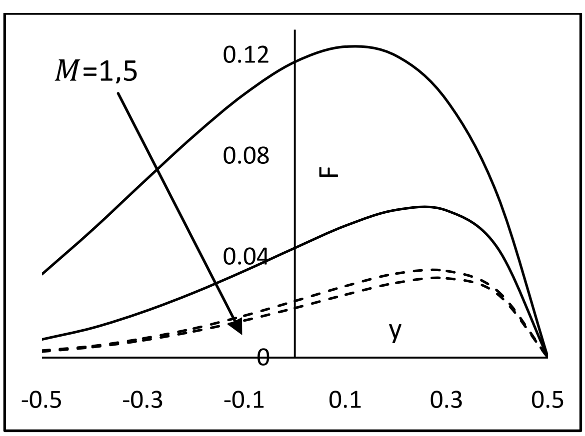

Fig. 8 : Velocity profile for different values of M.

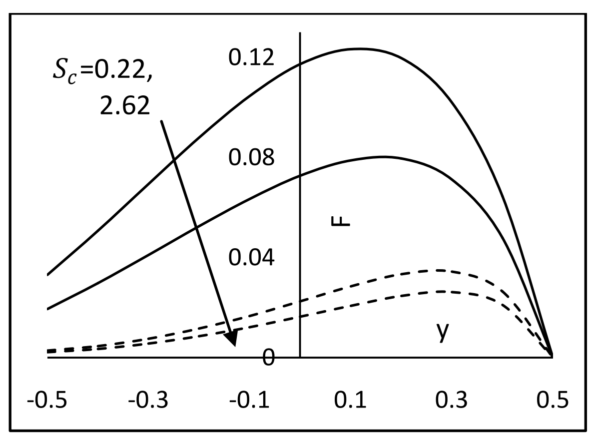

Fig. 11 : Velocity profile for different values of S_c.

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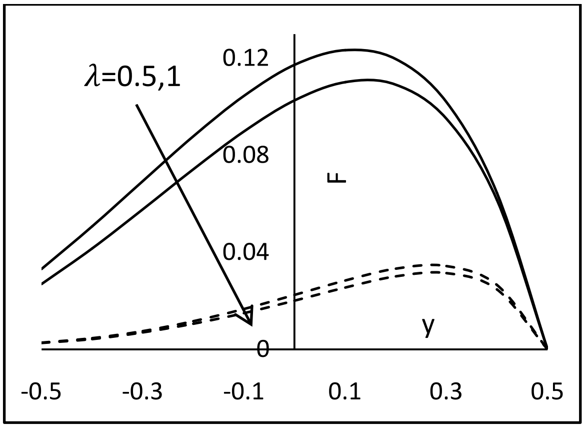

Fig. 12 : Velocity profile for different values of λ.

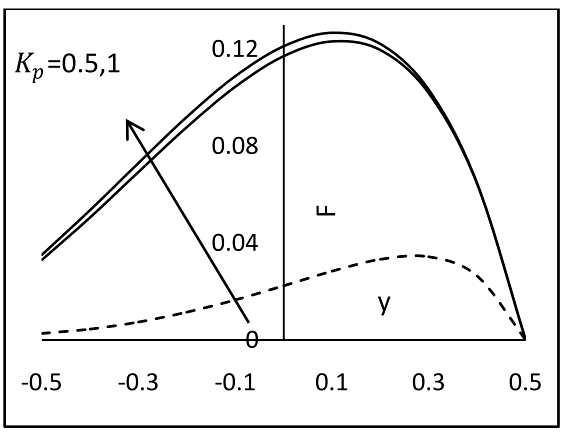

Fig. 13 : Velocity profile for different values of K_p.

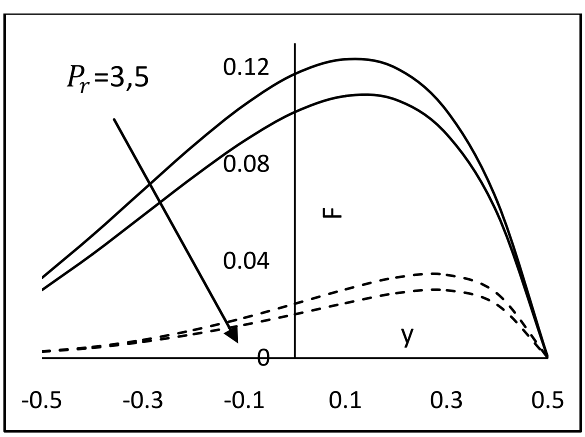

Fig. 14 : Velocity profile for different values of P_r.

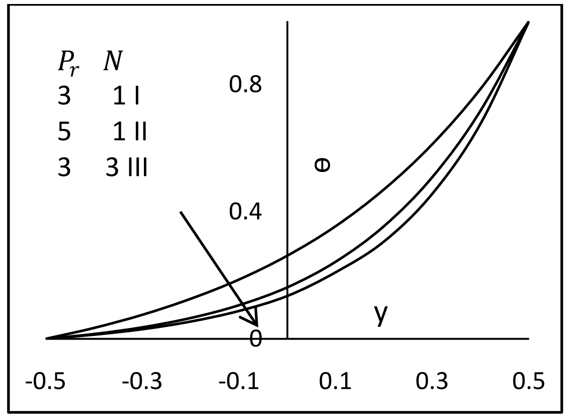

Fig. 15 : Temperature distribution for $\omega = 5, \lambda = 0.5$ and $t = 0$.

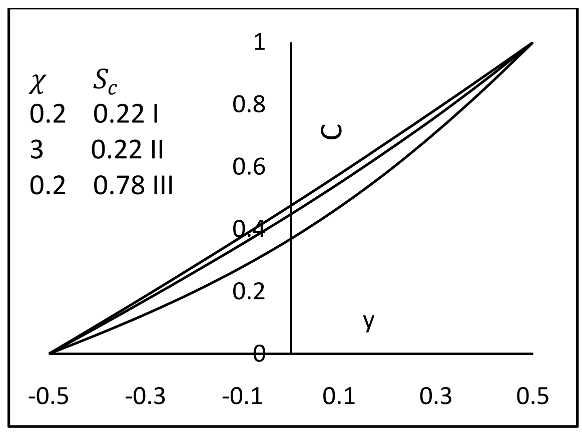

Fig. 16 : Concentration profile for $\omega = 1, \lambda = 0.5$ and $t = 0$.

Notes

G_r	G_m	ω	λ	K_p	h	α	M	N	χ	S_c	P_r	$\tau_{-\frac{1}{2}}$ Ω=1	$\tau_{-\frac{1}{2}}$ Ω=25
2	2	5	0.5	0.5	0.2	0.05	1	1	0.2	0.22	3	0.079424	0.010192
4	2	5	0.5	0.5	0.2	0.05	1	1	0.2	0.22	3	0.096165	0.013390
2	4	5	0.5	0.5	0.2	0.05	1	1	0.2	0.22	3	0.14211	0.017186
2	2	10	0.5	0.5	0.2	0.05	1	1	0.2	0.22	3	0.0054732	0.0059049
2	2	5	1	0.5	0.2	0.05	1	1	0.2	0.22	3	0.072659	0.0072924
2	2	5	0.5	1	0.2	0.05	1	1	0.2	0.22	3	0.072397	0.010455
2	2	5	0.5	0.5	2	0.05	1	1	0.2	0.22	3	0.101011	0.001816
2	2	5	0.5	0.5	0.2	1	1	1	0.2	0.22	3	0.013681	0.0021440
2	2	5	0.5	0.5	0.2	0.05	3	1	0.2	0.22	3	0.064965	0.0078971
2	2	5	0.5	0.5	0.2	0.05	1	3	0.2	0.22	3	0.074226	0.0088919
2	2	5	0.5	0.5	0.2	0.05	1	1	2	0.22	3	0.073474	0.0098028
2	2	5	0.5	0.5	0.2	0.05	1	1	0.2	0.78	3	0.004753	0.013572
2	2	5	0.5	0.5	0.2	0.05	1	1	0.2	0.22	5	0.060887	0.0065786

Table 1 : Values of skin-friction coefficient for small and large rotation.

P_r	N	λ	ω	$(N_u)_{-\frac{1}{2}}$
3	1	0.5	5	0.05925
5	1	0.5	5	0.060498
3	3	0.5	5	0.0044209
3	1	1	5	0.017097
3	1	0.5	10	0.096453

Table 2 : Values of Nusselt number.

χ	S_c	λ	ω	$(Sh)_{-\frac{1}{2}}$
0.2	0.22	0.5	5	0.91742
2	0.22	0.5	5	0.86087
0.2	0.78	0.5	5	0.59933
0.2	0.22	1	5	0.86707
0.2	0.22	0.5	10	0.85568

Table 3 : Values of Sherwood number.

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Intuitionistic L-Fuzzy Rings

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Abstract - In this paper we study some generalized properties of Intuitionistic L-fuzzy subrings. In this direction the concept of image and inverse image of an Intuitionistic L-fuzzy set under ring homomorphism are discussed. Further the concept of Intuitionistic L-fuzzy quotient subring and Intuitionistic L-fuzzy ideal of an Intuitionistic L-fuzzy subring are studied. Finally, weak homomorphism, weak isomorphism, homomorphism and isomorphism of an Intuitionistic L-fuzzy subring are introduced and some results are established in this direction.

Keywords : intuitionistic l-fuzzy quotient subring, intuitionistic l-fuzzy ideal of an intuitionistic l-fuzzy subring.

GJSFR-F Classification : MSC 2010: 16P70

INTUITIONISTIC L-FUZZY RINGS

Strictly as per the compliance and regulations of :

Intuitionistic L-Fuzzy Rings

K. Meena [α] & K. V. Thomas [σ]

Abstract - In this paper we study some generalized properties of Intuitionistic L-fuzzy subrings. In this direction the concept of image and inverse image of an Intuitionistic L-fuzzy set under ring homomorphism are discussed. Further the concept of Intuitionistic L-fuzzy quotient subring and Intuitionistic L-fuzzy ideal of an Intuitionistic L-fuzzy subring are studied. Finally, weak homomorphism, weak isomorphism, homomorphism and isomorphism of an Intuitionistic L-fuzzy subring are introduced and some results are established in this direction.

Keywords : intuitionistic l-fuzzy quotient subring, intuitionistic l-fuzzy ideal of an intuitionistic l-fuzzy subring.

I. Introduction

The theory of Intuitionistic fuzzy sets plays an important role in modern mathematics. The idea of Intuitionistic L-fuzzy set (ILFS) was introduced by Atanassov (1986) [3–5] as a generalisation of Zadeh's (1965) [15] fuzzy sets. Rosenfeld (1971) [14] has applied the concept of fuzzy sets to the theory of groups. Many researchers [1, 2, 6–10] applied the notion of Intuitionistic fuzzy concepts to set theory, relation, group theory, topological space, knowledge engineering, natural language, neural network etc. This paper is a continuation of our earlier paper [13]. Along with some basic results, we introduce and study Intuitionistic fuzzy quotient subrings and Intuitionistic fuzzy ideal of an Intuitionistic fuzzy subring of a ring. Further we define homomorphism and isomorphism of Intuitionistic fuzzy subrings of any two rings. Using this we establish the fundamental theorem of ring homomorphism and the third isomorphism theorem of rings for Intuitionistic fuzzy subrings. Infact, we emphasize the truth of the results relating to the non-membership function of an Intuitionistic fuzzy subring on a lattice $(L, \leq, \wedge, \vee)$. The proof of the results on the membership function of Intuitionistic fuzzy subrings, Intuitionistic fuzzy ideals of an Intuitionistic fuzzy subring and Intuitionistic fuzzy Quotient ring are omitted to avoid repetitions which are already done by researchers Malik D.S and Mordeson J.N. [11, 12].

II. Preliminaries

In this section we list some basic concepts and well known results of Intuitionistic L-fuzzy sets, Intuitionistic L-fuzzy subrings and Intuitionistic L-fuzzy ideals [13].

Ref.

[1] Adam Niewiadomski and Eulalia Szmidt, Handling Uncertainty in natural sentences via IFS, *Notes on Intuitionistic Fuzzy Sets*, **7** (2001), No.1, 22–29.

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Throughout this paper $(L, \leq, \wedge, \vee)$ denotes a complete distributive lattice with maximal element 1 and minimal element 0, respectively. Let R and S be commutative rings with binary operations + and $\cdot$.

Definition 2.1. *Let X be a non-empty set. A L-fuzzy set μ of X is a function $\mu : X \to L$.*

Definition 2.2. *Let $(L, \leq)$ be the lattice with an involutive order reversing operation $N : L \to L$. Let X be a non-empty set. An Intuitionistic L-fuzzy set (ILFS) A in X is defined as an object of the form*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$$

where $\mu_A : X \to L$ and $\nu_A : X \to L$ define the degree of membership and the degree of non membership for every $x \in X$ satisfying $\mu_A(x) \leq N(\nu_A(x))$.

Definition 2.3. *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in X\}$ be two Intuitionistic L-fuzzy sets of X. Then we define*

(i) $A \subseteq B$ iff for all $x \in X$, $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$
(ii) $A = B$ iff for all $x \in X$, $\mu_A(x) = \mu_B(x)$ and $\nu_A(x) = \nu_B(x)$
(iii) $A \cup B = \{\langle x, (\mu_A \cup \mu_B)(x), (\nu_A \cap \nu_B)(x)\rangle / x \in X\}$ where $\mu_A \cup \mu_B = \mu_A \vee \mu_B$, $\nu_A \cap \nu_B = \nu_A \wedge \nu_B$
(iv) $A \cap B = \{\langle x, (\mu_A \cap \mu_B)(x), (\nu_A \cup \nu_B)(x)\rangle / x \in X\}$ where $\mu_A \cap \mu_B = \mu_A \wedge \mu_B$, $\nu_A \cup \nu_B = \nu_A \vee \nu_B$.

Definition 2.4. *An Intuitionistic L-fuzzy subset $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ of R is said to be an Intuitionistic L-fuzzy subring of R (ILFSR) if for all $x, y \in R$,*
(i) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$
(ii) $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y)$
(iii) $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$
(iv) $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y)$.

Proposition 2.5. *If $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ is an ILFSR. Then*
(i) $\mu_A(0) \geq \mu_A(x)$ and $\nu_A(0) \leq \nu_A(x)$ for all $x \in R$
(ii) if R is a ring with identity 1 then $\mu_A(1) \leq \mu_A(x)$ and $\nu_A(1) \geq \nu_A(x)$, for all $x \in R$.

Theorem 2.6. *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ be two ILFSR. Then $A \cap B$ is an ILFSR of R.*

Definition 2.7. *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ be an ILFSR of R. Then A is called an Intuitionistic L-fuzzy ideal of R (ILFI) if,*

Notes

(i) $\mu_A(x-y) \geq \mu_A(x) \wedge \mu_A(y)$
(ii) $\mu_A(xy) \geq \mu_A(x)$
(iii) $\nu_A(x-y) \leq \nu_A(x) \vee \nu_A(y)$
(iv) $\nu_A(xy) \leq \nu_A(x)$*, for all* $x, y \in R$*.*

Notes

Definition 2.8. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFI of* R*. Then we define*

$$(\mu_A)_* = \{x \in R / \mu_A(x) = \mu_A(0)\}$$

$$(\nu_A)_* = \{x \in R / \nu_A(x) > \nu_A(0)\}.$$

Proposition 2.9. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFI. If* $\mu_A(x-y) = \mu_A(0)$ *then* $\mu_A(x) = \mu_A(y)$ *and if* $\nu_A(x-y) = \nu_A(0)$ *then* $\nu_A(x) = \nu_A(y)$*, for all* $x, y \in R$*.*

Proposition 2.10. *Every ILFI is an ILFSR.*

Definition 2.11. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFSR. Let* $x \in R$*, then*

$$C = \{\langle x, (\mu_A(0)_{\{x\}} + \mu_A)(x), (\nu_A(0)_{\{x\}} + \nu_A)(x)\rangle / x \in R\}$$

is called an Intuitionistic L*-fuzzy coset (ILFC) of* A *and is denoted as*

$$C = \{\langle x, (x+\mu_A)(x), (x+\nu_A)(x)\rangle / x \in R\}.$$

Definition 2.12. Let R/A={(x+μ_A),(x+ν_A)/x belongs to R} *be an ILFI.*
Let $R/A = \{(x+\mu_A, x+\nu_A)/x \in R\}$*. Define* $+$ *and* $\cdot$ *on* R/A *by*

(i) $(x+\mu_A) + (y+\mu_A) = x+y+\mu_A$*,*
(ii) $(x+\nu_A) + (y+\nu_A) = x+y+\nu_A$*, for all* $x, y \in R$*, and*
(iii) $(x+\mu_A) \cdot (y+\mu_A) = xy+\mu_A$*,*
(iv) $(x+\nu_A) \cdot (y+\nu_A) = xy+\nu_A$*, for all* $x, y \in R$*.*

Then R/A *is a ring with respect to* $+$ *and* $\cdot$ *and is called Quotient ring of* R *by* μ_A *and* ν_A*.*

III. CORRESPONDENCE THEOREM FOR INTUITIONISTIC L-FUZZY IDEAL

Here we define the image and inverse image of ILFS under ring homomorphism and study their elementary properties. Using this, Correspondence Theorem for ILFI is established. This section also provides a definition for an Intuitionistic L-fuzzy quotient subring of an ILFSR relative to an ordinary ideal of a ring.

Definition 3.1. *Let $f : R \to S$ be a ring homomorphism.*
Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in S\}$ be ILFS. Then $C = \{\langle y, f(\mu_A)(y), f(\nu_A)(y)\rangle / y \in S\}$ is called Intuitionistic Image of A, where

$$f(\mu_A)(y) = \begin{cases} \vee\{\mu_A(x)/x \in R, f(x) = y\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

$$f(\nu_A)(y) = \begin{cases} \wedge\{\nu_A(x)/x \in R, f(x) = y\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{otherwise} \end{cases}$$

for all $y \in S$;
and $D = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle / x \in R\}$ is called Intuitionistic Inverse Image of B, where

$$f^{-1}(\mu_B)(x) = \mu_B(f(x))$$

$$f^{-1}(\nu_B)(x) = \nu_B(f(x))$$

for all $x \in R$.
Here $f(\mu_A)$ and $f(\nu_A)$ are called the image of μ_A and ν_A under f. Also $f^{-1}(\mu_B)$ and $f^{-1}(\nu_B)$ are called the inverse image of μ_B and ν_B under f.

The proof of the following result is direct.

Lemma 3.2. *Let $f : R \to S$ be a ring homomorphism.*
Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in S\}$ be ILFI of R and S respectively. Then

(i) $f(\nu_A)(0') = \nu_A(0)$ *where $0'$ is the zero element of S and 0 is the zero element of R.*
(ii) $f(\nu_A)_* \subseteq (f(\nu_A))_*$;
(iii) If ν_A has the infimum property, then $f(\nu_A)_* = (f(\nu_A))_*$;
(iv) If ν_A is constant on Ker f, then $(f(\nu_A))(f(x)) = \nu_A(x)$ for all $x \in R$.

Theorem 3.3. *Let $f : R \to S$ be a ring homomorphism.*
Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in S\}$ be ILFI of R and S. Then

(i) $D = \{\langle x, f^{-1}(\mu_B)(x), f^{-1}(\nu_B)(x)\rangle / x \in R\}$ is an ILFI of R which is a constant on Ker f;
(ii) $f^{-1}(\nu_B)_* = (f^{-1}(\nu_B))_*$;
(iii) If f is onto then $(f \circ f^{-1})(\nu_B) = \nu_B$;
(iv) If ν_A is constant on Ker f, then $(f^{-1} \circ f)(\nu_A) = \nu_A$.

Notes

Notes

Proof.

(i) Let $x, y \in R$. Then

$$\begin{aligned} f^{-1}(\nu_B)(x-y) &= \nu_B(f(x-y)) \\ &= \nu_B(f(x)-f(y)) \\ &\leq \nu_B(f(x)) \vee \nu_B(f(y)) \\ &= f^{-1}(\nu_B)(x) \vee f^{-1}(\nu_B)(y) \end{aligned}$$

and
$$\begin{aligned} f^{-1}(\nu_B)(xy) &= \nu_B(f(xy)) \\ &= \nu_B(f(x)f(y)) \\ &\leq \nu_B(f(x)) \wedge \nu_B(f(y)) \\ &= f^{-1}(\nu_B)(x) \wedge f^{-1}(\nu_B)(y). \end{aligned}$$

Hence D is an ILFI of R.

Let $x \in \text{Ker } f$. Then

$$\begin{aligned} f^{-1}(\nu_B)(x) &= \nu_B(f(x)) \\ &= \nu_B(f(0)) \\ &= \nu_B(0'). \end{aligned}$$

Hence $f^{-1}(\nu_B)$ is constant on Ker f.

(ii) Let $x \in R$. Then

$$\begin{aligned} x \in f^{-1}(\nu_B)_* &\Leftrightarrow \nu_B(f(x)) > \nu_B(0') = \nu_B(f(0)) \\ &\Leftrightarrow f^{-1}(\nu_B)(x) > f^{-1}(\nu_B)(0) \\ &\Leftrightarrow x \in (f^{-1}(\nu_B))_*. \end{aligned}$$

Hence $f^{-1}(\nu_B)_* = (f^{-1}(\nu_B))_*$.

(iii) Let $y \in S$. Then $y = f(x)$ for some $x \in R$, so that

$$\begin{aligned} (f \circ f^{-1})(\nu_B)(y) &= f(f^{-1}(\nu_B))(y) \\ &= f(f^{-1}(\nu_B))(f(x)) \\ &= f^{-1}(\nu_B)(x) \\ &= \nu_B(f(x)) \\ &= \nu_B(y). \end{aligned}$$

Hence $(f \circ f^{-1})(\nu_B) = \nu_B$.

(iv) Let $x \in R$. Then

$$\begin{aligned}(f^{-1} \circ f)(\nu_A)(x) &= f^{-1}(f(\nu_A))(x)\\ &= f(\nu_A)(f(x))\\ &= \nu_A(x).\end{aligned}$$

Hence $(f^{-1} \circ f)(\nu_A) = \nu_A$.

Theorem 3.4. *Let $f : R \to S$ be an onto ring homomorphism. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ be an ILFI of R. Then $C = \{\langle y, f(\mu_A)(y), f(\nu_A)(y)\rangle / y \in S\}$ is an ILFI of S. If ν_A is a constant on Ker f, then $f(\nu_A)_* = (f(\nu_A))_*$.*

Proof. Let $s_1, s_2 \in S$. Then $s_1 = f(r_1)$, $s_2 = f(r_2)$ for some $r_1, r_2 \in R$. Now

$$\begin{aligned}f(\nu_A)(s_1 - s_2) &= \wedge\{\nu_A(x)/x \in R, f(x) = s_1 - s_2\}\\ &\leq \wedge\{\nu_A(r_1 - r_2)/r_1, r_2 \in R, f(r_1) = s_1, f(r_2) = s_2\}\\ &\leq \wedge\{\nu_A(r_1) \vee \nu_A(r_2)/r_1, r_2 \in R, f(r_1) = s_1, f(r_2) = s_2\}\\ &= (\wedge\{\nu_A(r_1)/r_1 \in R, f(r_1) = s_1\}) \vee (\wedge\{\nu_A(r_2)/r_2 \in R, f(r_2) = s_2\})\\ &= f(\nu_A)(s_1) \vee f(\nu_A)(s_2).\end{aligned}$$

Also

$$\begin{aligned}f(\nu_A)(s_1 s_2) &= \wedge\{\nu_A(x)/x \in R, f(x) = s_1 s_2\}\\ &\leq \wedge\{\nu_A(r_1 r_2)/r_1, r_2 \in R, f(r_1) = s_1, f(r_2) = s_2\}\\ &\leq \wedge\{\nu_A(r_1) \wedge \nu_A(r_2)/r_1, r_2 \in R, f(r_1) = s_1, f(r_2) = s_2\}\\ &= (\wedge\{\nu_A(r_1)/r_1 \in R, f(r_1) = s_1\}) \wedge (\wedge\{\nu_A(r_2)/r_2 \in R, f(r_2) = s_2\})\\ &= f(\nu_A)(s_1) \wedge f(\nu_A)(s_2).\end{aligned}$$

Hence $C = \{\langle y, f(\mu_A)(y), f(\nu_A)(y)\rangle / y \in S\}$ is an ILFI(S).

Next, if ν_A is a constant on Ker f, then for $y \in (f(\nu_A))_*$, we have

$$f(\nu_A)(y) > f(\nu_A)(0') = \nu_A(0).$$

Since f is onto, $y = f(x)$ for some $x \in R$. Hence $f(\nu_A)(f(x)) = \nu_A(x) > \nu_A(0)$. Thus $\nu_A(x) > \nu_A(0)$ or $x \in (\nu_A)_*$. Hence $y = f(x) \in f(\nu_A)_*$. The remaining part of the proof follows from Lemma 3.2 (ii).

Notes

Let R be a ring. Let $R/A = \{(x + \mu_A), (x + \nu_A)/x \in R\}$ be a quotient ring by μ_A and ν_A where $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle/x \in R\}$ is an ILFI.

Define $A^{(*)} = \{\langle x, \mu_A^{(*)}(x), \nu_A^{(*)}(x)\rangle/x \in R/A\}$ as follows:

$$\mu_A^{(*)}(x + \mu_A) = \mu_A(x)$$

Notes

and

$$\nu_A^{(*)}(x + \nu_A) = \nu_A(x), \text{ for all } x \in R.$$

Obviously, $\nu_A^{(*)}$ and $\mu_A^{(*)}$ are well-defined. Also $A^{(*)}$ is an ILFS(R/A).

Theorem 3.5. *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle/x \in R\}$ be an ILFI. Then $A^{(*)}$ is an ILFI of R/A, where $A^{(*)} = \{\langle x, \mu_A^{(*)}(x), \nu_A^{(*)}(x)\rangle/x \in R/A\}$ is defined by*

$$\mu_A^{(*)}(x + \mu_A) = \mu_A(x) \text{ and}$$

$$\nu_A^{(*)}(x + \nu_A) = \nu_A(x), \text{ for all } x \in R.$$

Proof. Let $x, y \in R$. Then

$$\begin{aligned}\nu_A^{(*)}((x + \nu_A) + (y + \nu_A)) &= \nu_A^{(*)}(x + y + \nu_A)\\ &= \nu_A(x + y)\\ &\leq \nu_A(x) \vee \nu_A(y)\\ &= \nu_A^{(*)}(x + \nu_A) \vee \nu_A^{(*)}(y + \nu_A)\end{aligned}$$

and

$$\begin{aligned}\nu_A^{(*)}((x + \nu_A)(y + \nu_A)) &= \nu_A^{(*)}(xy + \nu_A)\\ &= \nu_A(xy)\\ &\leq \nu_A(x) \wedge \nu_A(y)\\ &= \nu_A^{(*)}(x + \nu_A) \wedge \nu_A^{(*)}(y + \nu_A).\end{aligned}$$

Hence $A^{(*)}$ is an ILFI(R/A).

Theorem 3.6. (Correspondence Theorem for ILFI) *Let $f : R \to S$ be a ring epimorphism. Then there is a one-to-one order preserving correspondence between ILFI of S and the ILFI of R, which are constant on Ker f.*

Proof. Let $F(R)$ denote the set of ILFI of R which are constant on Ker f and $F(S)$ denote the set of ILFI of S. Define $\Phi : F(R) \to F(S)$ by $\Phi(A) = f(A)$, for all $A \in F(R)$ and $\Psi : F(S) \to F(R)$ by $\Psi(B) = f^{-1}(B)$ for all $B \in F(S)$.

Then Φ and Ψ are well-defined and are inverses of each other, thus giving the one to one correspondence. It can easily be verified that this correspondence preserves the order too.

Theorem 3.7. *Let* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ *be an ILFSR and* C *be any ideal of* R*. Let*

$$D = \{\langle [x], \mu_D[x], \nu_D[x]\rangle / [x] \in R/C\}$$

be an ILFS of R/C*, where*

$$\mu_D[x] = \vee\{\mu_B(z)/z \in [x]\}, \quad \nu_D[x] = \wedge\{\nu_B(z)/z \in [x]\}, \textit{ for all } x \in R,$$

and $[x] = x + C$*. Then* D *is an ILFSR of* R/C*.*

Proof. Let $x, y \in R$. Then

$$\begin{aligned}
\nu_D([x]-[y]) &= \nu_D([x-y]) \\
&= \wedge\{\nu_B(x-y+z)/z \in C\} \\
&\leq \wedge\{\nu_B(x-y+a-b)/a, b \in C\} \\
&= \wedge\{\nu_B((x+a)-(y+b))/a, b \in C\} \\
&\leq (\wedge\{\nu_B(x+a)/a \in C\}) \vee (\wedge\{\nu_B(y+b)/b \in C\}) \\
&= \nu_D[x] \vee \nu_D[y].
\end{aligned}$$

Also

$$\begin{aligned}
\nu_D([x][y]) &= \nu_D([xy]) \\
&= \wedge\{\nu_B(xy+z)/z \in C\} \\
&\leq \wedge\{\nu_B(xy+(xv+uy+uv))/u, v \in C\} \\
&= \wedge\{\nu_B(x(y+v)+u(y+v))/u, v \in C\} \\
&= \wedge\{\nu_B((y+v)(x+u))/u, v \in C\} \\
&\leq \wedge\{(\nu_B(y+v)) \vee (\nu_B(x+u))/u, v \in C\} \\
&= (\wedge\{\nu_B(x+u)/u \in C\}) \vee (\wedge\{\nu_B(y+v)/v \in C\}) \\
&= \nu_D[x] \vee \nu_D[y].
\end{aligned}$$

Hence D is an ILFSR of R/C.

The ILFSR, D={<[x], μ D[x], ν[x]>/[x] belongs to R/C} is called the Intuitionistic L-fuzzy Quotient Subring of B relative to C and denoted as B/C and is abbreviated as ILFQSR.

Notes

IV. Intuitionistic L-Fuzzy Ideal of an Intuitionistic L-Fuzzy Subring

Notes

In this section we define an ILFI of an ILFSR and some elementary results are obtained. Also we discuss the ILFI of an ILFSR under an epimorphism.

Definition 4.1. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFS.*
Let $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ *be an ILFSR with* $A \subseteq B$*. Then* A *is called an ILFI of* B *if for all* $x, y \in R$*,*

(i) $\mu_A(x - y) \geq \mu_A(x) \wedge \mu_A(y)$

(ii) $\mu_A(xy) \geq (\mu_B(x) \wedge \mu_A(y)) \vee (\mu_A(x) \wedge \mu_B(y))$

(iii) $\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y)$

(iv) $\nu_A(xy) \leq (\nu_B(x) \vee \nu_A(y)) \wedge (\nu_A(x) \vee \nu_B(y))$

Since R *is commutative,* $\nu_A(xy) \leq (\nu_B(x) \vee \nu_A(y)) \wedge (\nu_A(x) \vee \nu_B(y))$
for all $x, y \in R$ *if and only if* $\nu_A(xy) \leq \nu_B(x) \vee \nu_A(y)$*, for all* $x, y \in R$*.*

Definition 4.2. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFI.*
Then $A^* = \{x \in R / \mu_A(x) > 0, \nu_A(x) = 0\}$ *is an ideal of* R*, if* L *is regular.*

The proof of the following result is direct.

Theorem 4.3. *Let* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ *be an ILFSR and*
$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFI of* B *. If* L *is regular, then*
$A^* = \{x \in R / \mu_A(x) > 0, \nu_A(x) = 0\}$ *is an ideal of*
$B^* = \{x \in R / \mu_B(x) > 0, \nu_B(x) = 0\}$.

Theorem 4.4. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFI of an ILFSR* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$*. Then* A *is an ILFSR.*

Proof. For $x, y \in R$

$$\nu_A(x - y) \leq \nu_A(x) \vee \nu_A(y).$$

For $x, y \in R$

$$\begin{aligned} \nu_A(xy) &\leq (\nu_B(x) \vee \nu_A(y)) \wedge (\nu_A(x) \vee \nu_B(y)) \\ &\leq (\nu_A(x) \vee \nu_A(y)) \wedge (\nu_A(x) \vee \nu_A(y)) \\ &= \nu_A(x) \vee \nu_A(y). \end{aligned}$$

Hence A is an ILFSR.

Theorem 4.5. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFI of* R *and*
$B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ *be an ILFSR. Then* $A \cap B$ *is an ILFI of* B.

Proof. Clearly $A \cap B \subseteq B$ and $A \cap B$ is an ILFSR of R. For $x, y \in R$,

$$\begin{aligned}(\nu_A \cup \nu_B)(xy) &= \nu_A(xy) \vee \nu_B(xy)\\ &\leq [\nu_A(x) \wedge \nu_A(y)] \vee [\nu_B(x) \vee \nu_B(y)]\\ &= (\nu_A(x) \vee [\nu_B(x) \vee \nu_B(y)]) \wedge (\nu_A(y) \vee [\nu_B(x) \vee \nu_B(y)])\\ &= ([\nu_A(x) \vee \nu_B(x)] \vee \nu_B(y)) \wedge ([\nu_A(y) \vee \nu_B(y)] \vee \nu_B(x))\\ &\leq \nu_B(x) \vee [\nu_A(y) \vee \nu_B(y)]\\ &= \nu_B(x) \vee (\nu_A \cup \nu_B)(y).\end{aligned}$$

Therefore $A \cap B$ is an ILFI of B.

Theorem 4.6. *Let $C = \{\langle x, \mu_C(x), \nu_C(x)\rangle / x \in R\}$ be an ILFSR and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$, $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ be two ILFI of C. Then $A \cap B$ is an ILFI of C.*

Proof. Clearly $A \cap B \subseteq C$ and $A \cap B$ is an ILFSR. For $x, y \in R$,

$$\begin{aligned}(\nu_A \cup \nu_B)(xy) &= \nu_A(xy) \vee \nu_B(xy)\\ &\leq (\nu_C(x) \vee \nu_A(y)) \vee (\nu_C(x) \vee \nu_B(y))\\ &= \nu_C(x) \vee (\nu_A(y) \vee \nu_B(y))\\ &= \nu_C(x) \vee (\nu_A \cup \nu_B)(y).\end{aligned}$$

Therefore $A \cap B$ is an ILFI of C.

Theorem 4.7. *Let $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ be an ILFSR. Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ be an ILFI of B. Let $f : R \to S$ be an onto homomorphism. Then $f(A)$ is an ILFI of $f(B)$.*

Proof. Clearly $f(A)$ and $f(B)$ are ILFSR of S and $f(A) \subseteq f(B)$. Now for all $x, y \in S$,

$$\begin{aligned}f(\nu_A)(xy) &= \wedge\{\nu_A(w) : w \in R, f(w) = xy\}\\ &\leq \wedge\{\nu_A(uv) : u, v \in R, f(u) = x, f(v) = y\}\\ &\leq \wedge\{\nu_B(u) \vee \nu_A(v) / f(u) = x, f(v) = y, u, v \in R\}\\ &= (\wedge\{\nu_B(u) / u \in R, f(u) = x\}) \vee (\wedge\{\nu_A(v) / v \in R, f(v) = y\})\\ &= f(\nu_B)(x) \vee f(\nu_A)(y).\end{aligned}$$

Therefore $f(A)$ is an ILFI of $f(B)$.

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Theorem 4.8. *Let $f : R \to S$ be an onto homomorphism.*
Let $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in S\}$ be an ILFSR of S and $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in S\}$ be an ILFI of B. Then $f^{-1}(A)$ is an ILFI of $f^{-1}(B)$.

Proof. Clearly $f^{-1}(A)$ and $f^{-1}(B)$ are ILFSR of R and $f^{-1}(A) \subseteq f^{-1}(B)$. Now

$$\begin{aligned} f^{-1}(\nu_A)(xy) &= \nu_A(f(xy)) \\ &= \nu_A(f(x)f(y)) \\ &\leq \nu_B(f(x)) \vee \nu_A(f(y)) \\ &= f^{-1}(\nu_B)(x) \vee f^{-1}(\nu_A)(y). \end{aligned}$$

Therefore $f^{-1}(A)$ is an ILFI of $f^{-1}(B)$.

V. Isomorphism Theorems for ILFSR

Here we define homomorphism and isomorphism of an ILFSR. The fundamental theorem of ring homomorphism and the Third Isomorphism Theorem for rings are established for ILFSR.

Definition 5.1. *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ be an ILFSR of R and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in S\}$ be an ILFSR of S.*

(1) A weak homomorphism from A into B is an epimorphism f of R onto S such that $f(A) \subseteq B$. If f is a weak homomorphism of A into B, then A is said to be weakly homomorphic to B and written as $A \overset{f}{\sim} B$ or $A \sim B$.

(2) A weak isomorphism from A into B is a weak homomorphism f from A into B which is also an isomorphism of R onto S. If f is a weak isomorphism from A into B, then A is said to be weakly isomorphic to B and written as $A \overset{f}{\simeq} B$ or $A \simeq B$.

(3) A homomorphism from A onto B is a weak homomorphism f from A onto B such that $f(A) = B$. If f is a homomorphism of A onto B, then A is said to be homomorphic to B and written as $A \overset{f}{\approx} B$ or $A \approx B$.

(4) An isomorphism from A onto B is a weak isomorphism f from A into B such that $f(A) = B$. If f is an isomorphism from A onto B, then A is said to be isomorphic to B and written as $A \overset{f}{\cong} B$ or $A \cong B$.

Let $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ be an ILFSR of R.
Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ be an ILFI of B. Assume that L is regular. Then it is clear that A^* is an ideal of B^* and B/B^* is an ILFSR of B^*. Thus we can consider the ILFQSR of B/B^* relative to A^*. This ILFQSR is denoted as B/A.

Notes

Theorem 5.2. *Let* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ *be an ILFSR and* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *be an ILFI of* B. *Suppose that* L *is regular. Then* $B/B^* \approx B/A$.

Proof. Let f be the natural homomorphism from B^* onto B^*/A^*. Then

$$\begin{aligned} f(\nu_B/\nu_{B^*})([y]) &= \wedge\{(\nu_B/\nu_{B^*})(x)/x \in B^*, f(x) = [y]\} \\ &= \wedge\{\nu_B(z)/z \in [y]\} \\ &= (\nu_B/\nu_A)([y]) \end{aligned}$$

for all $y \in B^*$ where $[y] = y + A^*$.

Therefore $B/B^* \overset{f}{\approx} B/A$.

The result of the following theorems are proved for membership and non membership functions.

Theorem 5.3. *Let* $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ *and* $C = \{\langle x, \mu_C(x), \nu_C(x)\rangle / x \in S\}$ *be an ILFSR of the rings* R *and* S *such that* $B \approx C$. *Suppose that* L *is regular. Then there exists an ILFI* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ *of* B *such that* $B/A \cong C/C^*$.

Proof. Since $B \approx C$, there exists an epimorphism f of R onto S such that $f(B) = C$. Define an ILFS, $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$ as follows:

$$\mu_A(x) = \begin{cases} \mu_B(x), & x \in Ker\ f \\ 0, & \text{otherwise, for all } x \in R \end{cases}$$

and

$$\nu_A(x) = \begin{cases} 0 & x \in Ker\ f \\ \nu_B(x), & \text{otherwise, for all } x \in R \end{cases}$$

Clearly A is an ILFSR and $A \subseteq B$. If $x \in \mathrm{Ker} f$ then

$$\begin{aligned} \mu_A(xy) &= \mu_B(xy) \\ &\geq \mu_B(x) \wedge \mu_B(y) \\ &\geq \mu_B(x) \wedge \mu_A(y) \end{aligned}$$

for all $y \in R$. If $x \in R \setminus \mathrm{Ker} f$, then $\mu_A(x) = 0$. Hence

$$\mu_A(xy) \geq \mu_B(x) \wedge \mu_A(y)$$

Notes

for all $y \in R$. If $x \in \text{Ker} f$, then $\nu_A(x) = 0$ and so

$$\nu_A(xy) \leq \nu_B(x) \vee \nu_A(y)$$

for all $y \in R$. If $x \in R \backslash \text{Ker} f$, then

$$\begin{aligned} \nu_A(xy) &= \nu_B(xy) \\ &\leq \nu_B(x) \vee \nu_B(y) \\ &\leq \nu_B(x) \vee \nu_A(y) \end{aligned}$$

for all $y \in R$. Hence A is an ILFI of B.

Since $B \approx C$, $f(B^*) = C^*$. Let $g = f/B^*$. Then g is a homomorphism of B^* onto C^* and $\text{Ker} g = A^*$. Thus there exists an isomorphism h of B^*/A^* onto C^* such that $h([x]) = g(x)$ for all $x \in B^*$. For such an h, we have

$$\begin{aligned} h(\mu_B/\mu_A)(y) &= \vee\{(\mu_B/\mu_A)[x]/h([x]) = y, x \in B^*\} \\ &= \vee\{\vee\{\mu_B(z)/z \in [x]\}/g(x) = y, x \in B^*\} \\ &= \vee\{\mu_B(z)/z \in B^*, g(z) = y\} \\ &= \vee\{\mu_B(z)/z \in R, f(z) = y\} \\ &= f(\mu_B)(y) \\ &= \mu_C(y), \quad \text{for all } y \in C^*. \end{aligned}$$

and

$$\begin{aligned} h(\nu_B/\nu_A)(y) &= \wedge\{(\nu_B/\nu_A)[x]/h([x]) = y, x \in B^*\} \\ &= \wedge\{\wedge\{\nu_B(z)/z \in [x]\}/g(x) = y, x \in B^*\} \\ &= \wedge\{\nu_B(z)/z \in B^*, g(z) = y\} \\ &= \wedge\{\nu_B(z)/z \in R, f(z) = y\} \\ &= f(\nu_B)(y) \\ &= \nu_C(y), \text{ for all } y \in C^*. \end{aligned}$$

Therefore $B/A \overset{h}{\cong} C/C^*$.

Theorem 5.4. *Let* $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle / x \in R\}$, $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle / x \in R\}$ *and* $C = \{\langle x, \mu_C(x), \nu_C(x)\rangle / x \in R\}$ *be ILFSR. Let* A *be an ILFI of* B *and* A, B *be ILFI of* C*. Suppose that* L *is regular. Then*

$$(C/A)/(B/A) \cong C/B.$$

Notes

Proof. Clearly A^* is an ideal of B^* and A^*, B^* are ideals of C^*. By the Third Isomorphism Theorem for Rings,

$$(C^*/A^*)/(B^*/A^*) \overset{f}{\cong} C^*/B^*,$$

where f is given by

$$f(x + A^* + (B^*/A^*)) = x + B^* \text{ for all } x \in C^*.$$

Thus

$$\begin{aligned} f((\mu_C/\mu_A)/(\mu_B/\mu_A))(x + B^*) &= ((\mu_C/\mu_A)/(\mu_B/\mu_A))(x + A^* + (B^*/A^*)) \\ &= \vee\{(\mu_C/\mu_A)(y + A^*)/y \in C^*, y + A^* \in x + A^* + (B^*/A^*)\} \\ &= \vee\{\vee\{\mu_C(z)/z \in y + A^*\}/y \in C^*, y + A^* \in x + A^* + (B^*/A^*)\} \\ &= \vee\{(\mu_C(z)/z \in C^*, z + A^* \in x + A^* + (B^*/A^*)\} \\ &= \vee\{(\mu_C(z)/z \in x + A^* + (B^*/A^*)\} \\ &= \vee\{(\mu_C(z)/z \in C^*, f(z) \in x + B^*\} \\ &= (\mu_C/\mu_B)(x + B^*) \quad \text{for all } x \in C^*. \end{aligned}$$

and

$$\begin{aligned} f((\nu_C/\nu_A)/(\nu_B/\nu_A))(x + B^*) &= ((\nu_C/\nu_A)/(\nu_B/\nu_A))(x + A^* + (B^*/A^*)) \\ &= \wedge\{(\nu_C/\nu_A)(y + A^*)/y \in C^*, y + A^* \in x + A^* + (B^*/A^*)\} \\ &= \wedge\{\wedge\{\nu_C(z)/z \in y + A^*\}/y \in C^*, y + A^* \in x + A^* + (B^*/A^*)\} \\ &= \wedge\{(\nu_C(z)/z \in C^*, z + A^* \in x + A^* + (B^*/A^*)\} \\ &= \wedge\{(\nu_C(z)/z \in x + A^* + (B^*/A^*)\} \\ &= \wedge\{(\nu_C(z)/z \in C^*, f(z) \in x + B^*\} \\ &= (\nu_C/\nu_B)(x + B^*) \quad \text{for all } x \in C^*. \end{aligned}$$

Hence $(C/A)/(B/A) \overset{f}{\cong} C/B$.

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A Class of Improved Estimators for Estimating Population Mean Regarding Partial Information in Double Sampling

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Abstract - In this paper a class of improved estimators has been proposed for estimating population mean in two phase (double) sampling when only partial information is available on either of two auxiliary variables. Under simple random sampling (SRWOR), expressions of mean square error and bias have been derived to make comparison of suggested class with wide range of other estimators. Empirical study has also been given using five different natural populations. Empirical study confirmed that the suggested class of improved estimators is more efficient under percent relative efficiency (PRE) criterion.

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GJSFR-F Classification : *MSC 2010 : 62F12 , 06A06*

A CLASS OF IMPROVED ESTIMATORS FOR ESTIMATING POPULATION MEAN REGARDING PARTIAL INFORMATION IN DOUBLE SAMPLING

Strictly as per the compliance and regulations of :

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Hina Khan [α], Saleha Shouket [σ] & Aamir Sanaullah [ρ]

Abstract - In this paper a class of improved estimators has been proposed for estimating population mean in two phase (double) sampling when only partial information is available on either of two auxiliary variables. Under simple random sampling *(SRWOR)*, expressions of mean square error and bias have been derived to make comparison of suggested class with wide range of other estimators. Empirical study has also been given using five different natural populations. Empirical study confirmed that the suggested class of improved estimators is more efficient under percent relative efficiency (PRE) criterion.

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I. Introduction

The history of use of auxiliary information in survey sampling is as old as the history of survey sampling. Bowley (1926) and Neyman (1934, 38) provide foundation stones of modern sampling theory, dealing with stratified random sampling. Hansen and Hurwitz (1943) firstly use auxiliary information in selecting sample with varying probabilities. Snedecor and King (1942), Spurr (1952), Freese (1962), Unnikrishan and Kunte (1995), Armstrong and St-Jean (1994) provide applications of two phase (or double) sampling procedure.

The estimation of the population mean is an unrelenting issue in sampling theory and several efforts have been made to improve the precision of the estimators in the presence of multi-auxiliary variables. Mohanty (1967) suggested regression cum ratio estimator in double sampling using two auxiliary variables. Tripathi (1970) and Das (1988) describe the auxiliary information in four ways. Das and Tripathi (1978) initiate to use population variance of auxiliary variable for estimating the population variance. Srivastava and Jhajj (1980) also consider the use of population mean and variance of auxiliary variable for estimating population variance of the study variable. Several other authors have also used information on the parameters of auxiliary variable to find more precise estimates. Regarding the use of information on C_x, $\bar{Z}$, σ_z, $\beta_1(z)$, and $\beta_2(z)$ the researcher may be referred to Sear (1964), Singh et al. (1973), Sen(1978), Singh (2001), Uphadhyaya and Singh (2001), Singh et al. (2006), Singh et al (2007), and Singh et al. (2011).

Following Chand (1975) and Kiregyera (1980, 1984), Sahoo and Sahoo (1993) and Sahoo et al. (1994) have discussed a general frame work of estimation by using an

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additional auxiliary variable for double sampling when the population mean of the main auxiliary variable is unknown. Kiregyera (1984) developed two estimators, one is ratio-in-regression and other is regression-in-regression estimator. Mukerjee et al. (1987) developed three estimators following Kiregyera's (1984) technique. Sahoo's (1993) class of estimators covered a large number of estimators. Roy (2003) constructed a regression-type estimator of population mean of the main variable in the presence of available knowledge on second auxiliary variable, when the population mean of the first auxiliary variable was not known. Samiuddin and Hanif (2007) have reported three different methods of estimation in double sampling. These methods are proposed depending whether information of auxiliary variables is available or not at first phase of sampling. Singh et al. (2011) proposed chain ratio type estimator for population mean using some known values of population parameters of secondary auxiliary variable.

Notes

Let $U = (U_1, U_2, \ldots\ldots, U_n)$ be a finite population consisting of N units. Let y and (x, z) be the variate of interest and auxiliary characteristics respectively related to y assume real non-negative i^{th} value (y_i, x_i, z_i) $i = 1, 2, \ldots\ldots, N$ with population means $\bar{Y}$, $\bar{X}$, and $\bar{Z}$ respectively. Let a simple random sample without replacement *(SRSWOR)* is drawn in each phase, the two phase (or double) sampling scheme is as follows:

i. The first phase sample S_1 $(S_1 \subset U)$ of size n_1 is drawn to measure x and z say (x_1, z_1).
ii. The second phase sample S_2 $(S_2 \subset S_1)$ of size n_2 $(n_2$ ¡ $n_1)$ is drawn from the first phase sample S to measure y say y_2.

Let $$\bar{x}_1 = \frac{1}{n_1}\sum_{i\in s_1} x_i,\quad \bar{x}_2 = \frac{1}{n_2}\sum_{i\in s_2} x_i,\quad \bar{z}_1 = \frac{1}{n_1}\sum_{i\in s_1} z_i,\quad \bar{z}_2 = \frac{1}{n_2}\sum_{i\in s_2} z_i, \text{ and } \bar{y}_2 = \frac{1}{n_2}\sum_{i\in s_2} y_i$$

For a *SRSWOR*, we have some assumptions as following,

$$\left.\begin{array}{lll}
\bar{y}_1 = \bar{Y}(1+e_{\bar{y}_1}), & \bar{x}_1 = \bar{X}(1+e_{\bar{x}_1}), & \bar{z}_1 = \bar{Z}(1+e_{\bar{z}_1}) \\
\bar{y}_2 = \bar{Y}(1+e_{\bar{y}_2}), & \bar{x}_2 = \bar{X}(1+e_{\bar{x}_2}), \quad and & \bar{z}_2 = \bar{Z}(1+e_{\bar{z}_2}) \\
E(e_{\bar{y}_2}) = E(e_{\bar{z}_2}) = E(e_{\bar{z}_1}) = E(e_{\bar{x}_1}) = 0 & & \\
E(e_{\bar{y}_2})^2 = \theta_2 C^2{}_y & E(e_{\bar{z}_2})^2 = \theta_2 C_z^2 & \\
E(e_{\bar{z}_1})^2 = \theta_1 C_z^2, & E(e_{\bar{x}_1})^2 = \theta_1 C_x^2 & \\
E(e_{\bar{y}_2} e_{\bar{z}_2}) = \theta_2 C_y C_z \rho_{yz}, & E(e_{\bar{z}_2} e_{\bar{z}_1}) = \theta_1 C_z^2 & \\
E(e_{\bar{z}_1} e_{\bar{x}_1}) = \theta_1 C_z C_x \rho_{zx} & E(e_{\bar{y}_2} e_{\bar{x}_1}) = \theta_2 C_y C_x \rho_{yx} & \\
\theta_1 = \dfrac{1}{n_1} - \dfrac{1}{N} & \theta_2 = \dfrac{1}{n_2} - \dfrac{1}{N} &
\end{array}\right\} \quad (1.1)$$

In many practical situations even if $\bar{X}$ is unknown, information on a secondary auxiliary variable z, closely related to x but compared to x remotely related to y, is readily available on all units of population such that z_i denotes its value on i^{th} unit and $\bar{Z}$

as its known mean see Singh et al. (2004) and Singh et al.(2006). For instance, if the elements of population are hospitals, and y_i, x_i and z_i are respectively the number of deaths, number of patients admitted and number of available beds relating to the i^{th} hospital, then information on z_i's can be collected easily from the official records of the Health Department. This situation has also been discussed by chand (1975), Mukhergee et al. (1987), Sahoo and Sahoo (1993), Roy (2003) and among many others.

Notes

II. Some Available Estimators

In this section we reproduce some well known ratio type estimators for the population mean available for double sampling under *SRWOR* regarding only partial information are available.

1 The variance of the usual unbiased estimator $\bar{y}$ under SRSWOR scheme is as;

$$Var(T_1) = \theta \bar{Y}^2 C^2{}_y \tag{2.1}$$

2 Mohanty (1967) regression to ratio estimator

$$T_2 = [\bar{y}_2 + b_{yx}(\bar{x}_1 - \bar{x}_2)]\frac{\bar{Z}}{\bar{z}_2} \tag{2.2}$$

$$MSE(T_2) = \bar{Y}^2\left[\theta_2\left(C_y^2 + C_z^2 - C_y^2\rho_{xy}^2 - 2C_yC_z\rho_{yz} + 2C_yC_z\rho_{xy}\rho_{xz}\right) + \theta_1\left(C_y^2\rho_{xy}^2 - 2C_yC_z\rho_{xy}\rho_{xz}\right)\right] \tag{2.3}$$

3 Chand (1975) chain ratio estimator

$$T_3 = \bar{y}_2\frac{\bar{x}_2}{\bar{x}_1}\frac{\bar{z}_1}{\bar{Z}} \tag{2.4}$$

$$MSE(T_3) = \bar{Y}^2\left[\theta_2 C_y^2 + (\theta_2 - \theta_1)\left((C_x + \rho_{xy}C_y)^2 - C_y^2\rho_{xy}^2\right) + \theta_1\left((C_z + \rho_{yz}C_z)^2 - C_z^2\rho_{yz}^2\right)\right] \tag{2.5}$$

4 Kiregyera (1984) regression in regression estimator

$$T_4 = \bar{y}_2 + b_{yx}\left[(\bar{x}_1 - \bar{x}_2) - b_{xz}(\bar{z}_1 - \bar{Z})\right] \tag{2.6}$$

$$MSE(T_4) = \bar{Y}^2 C_y^2\left[\theta_2 - (\theta_2 - \theta_1)\rho_{xy}^2 - \theta_1\rho_{yz}^2 + \theta_1(\rho_{yz} - \rho_{xy}\rho_{xz})^2\right] \tag{2.7}$$

5 Bedi (1985) ratio estimator

$$T_5 = \bar{y}_2\left[\frac{\bar{z}_1}{\bar{z}_2}\right]^{\alpha} \tag{2.8}$$

$$MSE(T_5) = \bar{Y}^2 C_y^2\left[\theta_2 - (\theta_2 - \theta_1)\rho_{yz}^2\right] \tag{2.9}$$

6 Mukhergee et at (1987) regression estimator

$$T_6 = \bar{y}_2 + b_{yx}(\bar{x}_1 - \bar{x}_2) + b_{yx}b_{xz}(\bar{Z} - \bar{z}_1) + b_{yz}(\bar{Z} - \bar{z}_2) \qquad (2.10)$$

$$MSE(T_6) = \bar{Y}^2 C_y^2 \left[\theta_1(\rho_{yz} - \rho_{xy}\rho_{yz})^2 + \theta_2(1 - \rho_{yz}^2 - \rho_{xy}^2 + 2\rho_{xy}\rho_{yz}\rho_{zx})\right] \qquad (2.11)$$

7 Srivastava et al (1990) ratio estimator

$$T_7 = \bar{y}_2 \left[\frac{\bar{x}_1}{\bar{x}_2}\right]^{\alpha_1} \left[\frac{\bar{Z}}{\bar{z}_1}\right]^{\alpha_2} \qquad (2.12)$$

$$MSE(T_7) = \bar{Y}^2 C_y^2 \left[\theta_2 - (\theta_2 - \theta_1)\rho_{xy}^2 - \theta_1\rho_{yz}^2\right] \qquad (2.13)$$

8 Sahoo et al (1994a) regression in regression estimator

$$T_8 = \bar{y}_2 + b_{yx}(\bar{x}_1 - \bar{x}_2) + b_{yx}b_{xz}(\bar{z}_1 - \bar{z}_2) + b_{yx}b_{xz}(\bar{Z} - \bar{z}_1) \qquad (2.14)$$

$$MSE(T_8) = \bar{Y}^2 C_y^2 \left[\theta_2 + \theta_1\rho_{xy}^2\rho_{xz}^2 - (\theta_2 - \theta_1)\left(\rho_{xy}^2(1 - \rho_{xz}^2) - 2\rho_{xy}\rho_{xz}\rho_{yz}\right)\right] \qquad (2.15)$$

9 Singh (2001) chain ratio type estimator:

$$T_9 = \bar{y}_2 \left[\frac{\bar{x}_1}{\bar{x}_2}\right] \left[\frac{\alpha\bar{Z} + \sigma_z}{\alpha\bar{z}_1 + \sigma_z}\right]^g \qquad (2.16)$$

$$MSE(T_9) = \bar{Y}^2 \left[\theta_2 C_y^2 + (\theta_2 - \theta_1)\left(C_x^2 - 2C_yC_x\rho_{xy}\right) - \theta_1 C_y^2\rho_{yz}^2\right] \qquad (2.17)$$

10 Singh et al. (2004) generalized estimator:

$$T_{10} = \bar{y}_2 \left[\frac{\bar{x}_1}{\bar{x}_2}\right]^{\alpha_1} \left[\frac{a\bar{Z} + b}{a\bar{z}_1 + b}\right]^{\alpha_2} \left[\frac{a\bar{Z} + b}{a\bar{z}_2 + b}\right]^{\alpha_3} \qquad (2.18)$$

$$MSE(T_{10}) = \bar{Y}^2 C_y^2 \left[\theta_2 - \theta_1\rho_{yz}^2 - (\theta_2 - \theta_1)\rho_{y.xz}^2\right] \qquad (2.19)$$

11 Samiuddin and Hanif (2006) ratio cum regression estimator:

$$T_{11} = \left[\bar{y}_2 + b_{yz}(\bar{z}_1 - \bar{z}_2)\right]\left[\frac{\bar{X}}{\bar{x}_2}\right] \qquad (2.20)$$

$$MSE(T_{11}) = \bar{Y}^2 \left[\theta_2\left(C_y^2(1 - \rho_{yx}^2) + (C_x - C_y\rho_{xy})^2\right) + \theta_3\left(C_x^2\rho_{xz}^2 - (C_y\rho_{yz} - C_x\rho_{xz})^2\right)\right] \qquad (2.21)$$

12 Samiuddin and Hanif (2007) chain ratio estimator

$$T_{12} = \bar{y}_2 \left[\frac{\bar{x}_1}{\bar{x}_2}\right]^{\alpha_1} \left[\frac{\bar{z}_1}{\bar{z}_2}\right]^{\alpha_2} \left[\frac{\bar{Z}}{\bar{z}_2}\right]^{\alpha_3} \qquad (2.22)$$

$$MSE(T_{12}) = \bar{Y}^2 C_y^2 \left[\theta_2(1-\rho_{y.xz}^2) + \theta_1(1-\rho_{yz}^2)\rho_{yx.z}^2\right] \quad (2.23)$$

13 Singh et al. (2007) general family of ratio estimators

Notes

$$T_{13} = \bar{y}_2 \left[\frac{\bar{x}_1}{\bar{x}_2}\right]^{\alpha_1} \left[\frac{\bar{Z} + \rho_{xz}}{\bar{z}_1 + \rho_{xz}}\right]^{\alpha_2} \quad (2.24)$$

$$MSE(T_{13}) = \bar{Y}^2 C_y^2 \left[\theta_2 - \theta_1 \rho_{yz}^2 - \theta_3 \rho_{yx}^2\right] \quad (2.25)$$

14 H.P.Singh and N.Agnihortie (2008) ratio product estimator

$$T_{13} = \bar{y}_1 \left[\delta\left(\frac{a\bar{X} + b}{a\bar{x}_1 + b}\right) + (1-\delta)\left(\frac{a\bar{x}_1 + b}{a\bar{X} + b}\right)\right] \quad (2.26)$$

$$\min.MSE(T_{13}) = \theta_1 \bar{Y}^2 C_y^2 \left(1 - \rho_{yx}^2\right) \quad (2.27)$$

III. Proposed Estimators

In section-2 mentioned estimators have been widely used in the estimation of population mean in diverse situations regarding partial information. Now following estimators stated above, we have proposed two estimators in this section regarding the availability of partial information. One estimator (section-3.1) has been proposed regarding partial information on main auxiliary variable x and other estimator (section-3.2) have been proposed regarding partial information on secondary variable z for double sampling under *SRWOR*.

a) Proposed estimator in two phase (double) sampling

The new estimator $\hat{\bar{Y}}_1$ has been proposed for two phase sampling using two auxiliary variables regarding partial information on main auxiliary variable x. The estimator has been convinced by Srivastava (1971) and Singh (2001) ratio estimators.

$$\hat{\bar{Y}}_1 = \bar{y}_2 \left(\frac{\bar{x}_1}{\bar{x}_2}\right)^{\alpha} \left(\frac{a\bar{Z} + b}{a\bar{z}_1 + b}\right) \quad (3.1)$$

Where α, is an unknown constant whose values is to estimate. $a(\neq 0)$, and b are assumed to be known as either real numbers or (Linear or Non-linear) functions of some known parameters of auxiliary variable z such as standard deviation σ_z, coefficient of variation C_z, skewness $\beta_1(z)$, kurtosis $\beta_2(z)$.

Using the notations given in (1.1), $\hat{\bar{Y}}_1$ is expressed in the form of e's and up to the first degree of approximation as:

$$\hat{\bar{Y}}_1 \approx \bar{Y} + \bar{Y}\left[\alpha\left(e_{\bar{x}_1} - e_{\bar{x}_2}\right) - \omega e_{\bar{z}_1} + e_{\bar{y}_2}\right] \qquad \text{where} \quad \omega = \frac{a\bar{Z}}{a\bar{Z}+b},$$

$$\hat{\bar{Y}}_1 - \bar{Y} \approx \bar{Y}\left[\alpha\left(e_{\bar{x}_1} - e_{\bar{x}_2}\right) - \omega e_{\bar{z}_1} + e_{\bar{y}_2}\right] \tag{3.2}$$

Taking square and applying expectation, [given in (1.1)], the mean square error of (3.1) is obtained as:

$$MSE(\hat{\bar{Y}}_1) \approx \bar{Y}^2\begin{bmatrix}\theta_2 C_y^2 + \alpha^2(\theta_2-\theta_1)^2 C_x^2 + \theta_1\omega^2 C_z^2 - \\ 2\alpha(\theta_2-\theta_1)C_yC_x\rho_{xy} - 2\omega\theta_1 C_yC_z\rho_{yz}\end{bmatrix} \tag{3.3}$$

Differentiating (3.3) with respect to α and ω and setting equal zero. We have:

$$\alpha = \frac{C_y}{C_x}\rho_{yx} \qquad \textit{and} \qquad \omega = \frac{C_y}{C_z}\rho_{yz}$$

Taking the values of α and ω in equation (3.3), and simplifying. We get *min.MSE*($\hat{\bar{Y}}_1$):

$$\min.MSE(\hat{\bar{Y}}_1) = \bar{Y}^2 C_y^2\left[\theta_2 - \theta_3\rho_{xy}^2 - \theta_1\rho_{xy}^2\right] \qquad \text{where } \theta_3 = \theta_2 - \theta_1 \tag{3.4}$$

In order to derive bias of (3.1), we again use (3.2) upto the 2[nd] order of approximation as:

$$\hat{\bar{Y}}_1 - \bar{Y} \approx \begin{bmatrix}\alpha\left(e_{\bar{x}_1} - e_{\bar{x}_2}\right) - \alpha^2 e_{\bar{x}_1}e_{\bar{x}_2} + \frac{\alpha(\alpha-1)}{2!}e_{\bar{x}_1}^2 + \frac{\alpha(\alpha+1)}{2!}e_{\bar{x}_2}^2 - \omega e_{\bar{z}_1} - \omega\alpha\left(e_{\bar{x}_1} - e_{\bar{x}_2}\right)e_{\bar{z}_1} \\ + \omega^2 e_{\bar{z}_1}^2 + e_{\bar{y}_2} + \alpha e_{\bar{y}_2}\left(e_{\bar{x}_1} - e_{\bar{x}_2}\right) - \omega e_{\bar{y}_2}e_{\bar{z}_1}\end{bmatrix}$$

After applying expectation and simplifying, the optimum bias of (3.1) is:

$$Bias(\hat{\bar{Y}}_1) = \bar{Y}\left[-\frac{\theta_3}{2}C_y^2\rho_{yx}^2 + \frac{\theta_3}{2}C_x\left(C_y\rho_{xy} - \frac{C_x}{4}\right) - \frac{\theta_1}{4}C_y^2\rho_{yz}^2\right] \textit{where } \theta_3 = \theta_2 - \theta_1 \tag{3.5}$$

i. *Deduced Family of* $\hat{\bar{Y}}_1$

A large number of estimators have been deduced as a family of proposed estimator $\hat{\bar{Y}}_2$ under certain choices of the constants α, a, and b. These deduced estimators have been presented in the following table.

Notes

Deduced Estimator	α	a	b
$t_0 = \bar{y}_2\left(\frac{\bar{x}_1}{\bar{x}_2}\right)$ Shkhatme's (1962) ratio estimator	**1**	**0**	$b_0 \neq 0$
$t_1 = \bar{y}_2\left(\frac{\bar{x}_2}{\bar{x}_1}\right)$ Two phase product estimator	**-1**	**0**	$b_0 \neq 0$
$t_2 = \bar{y}_2\left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\bar{Z}}{\bar{z}_1}\right)$ Chand (1975) estimator	**1**	$a_0 \neq 0$	**0**
$t_3 = \bar{y}_2\left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\bar{Z}+C_z}{\bar{z}_1+C_z}\right)$ Sing and Upadhayaya's estimator (2001)	**1**	**1**	$\boldsymbol{C_z}$
$t_4 = \bar{y}_2\left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\bar{Z}C_z+\beta_2(z)}{\bar{z}_1C_z+\beta_2(z)}\right)$ Upadhyaya and Singh (2001)	**1**	$\boldsymbol{C_z}$	$\beta_2(z)$
$t_5 = \bar{y}_2\left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\bar{Z}+\sigma_z}{\bar{z}_1+\sigma_z}\right)$ Singh (2001) estimator	**1**	**1**	σ_z
$t_6 = \bar{y}_2\left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\beta_1(z)\bar{Z}+\sigma_z}{\beta_1(z)\bar{z}_1+\sigma_z}\right)$ Singh (2001) estimator	$\beta_1(z)$	**1**	σ_z
$t_7 = \bar{y}_2\left(\frac{\bar{x}_1}{\bar{x}_2}\right)\left(\frac{\bar{Z}+\rho_{xz}}{\bar{z}_1+\rho_{xz}}\right)$ Singh et al.(2007) estimator	**1**	**1**	ρ_{xz}

b) Another proposed estimator for two phase (double) sampling

In this section another estimator denoted by $\hat{\bar{Y}}_2$ has been suggested for double sampling using two auxiliary variables. The suggested estimator has been convinced by H.P. Singh and N. Agnihortie (2008) and Bedi (1985) ratio estimators for two phase sampling regarding partial information on secondary auxiliary variable.

$$\hat{\bar{Y}}_2 = \bar{y}_2\left[\frac{\bar{z}_1}{\bar{z}_2}\right]^{\alpha}\left[\delta\left(\frac{a\bar{X}+b}{a\bar{x}_1+b}\right)+(1-\delta)\left(\frac{a\bar{x}_1+b}{a\bar{X}+b}\right)\right] \quad (3.6)$$

Where α, and δ are the unknown constants whose values are to be estimated. $a(\neq 0)$, and b are assumed to be known as either real numbers or (Linear or Non-linear) functions of some known parameters of auxiliary variable x as (Section-3.1).

$$\hat{\bar{Y}}_2 = \bar{Y} + \bar{Y}\left[e_{\bar{y}_2} + \alpha\left(e_{\bar{z}_1} - e_{\bar{z}_2}\right) + \gamma\, e_{\bar{x}_1}(1-2\delta)\right] \quad \text{where} \quad \gamma = \frac{a\bar{X}}{a\bar{X}+b}$$

$$\hat{\hat{Y}}_2 - \bar{Y} \approx \bar{Y}\left[e_{\bar{y}_2} + \alpha\left(e_{\bar{z}_1} - e_{\bar{z}_2}\right) + \gamma\, e_{\bar{x}_1}(1-2\delta)\right] \tag{3.7}$$

Taking square and applying expectation, [given in (1.1)], the mean square error of (3.6) is obtained as:

$$MSE(\hat{\hat{Y}}_2) = \bar{Y}^2\begin{bmatrix} \theta_2 C_y^2 + \alpha^2(\theta_2-\theta_1)C_z^2 + \theta_1\gamma^2(1-2\delta)^2 C_x^2 + \\ 2\alpha(\theta_1-\theta_2)C_y C_z \rho_{yz} + 2\theta_1(1-2\delta)\gamma C_y C_x \rho_{xy} \end{bmatrix} \tag{3.8}$$

Notes

Differentiating (3.8) with respect to α and δ and setting equal zero. We have:

$$\alpha = \frac{C_y\rho_{yz}}{C_z} \qquad \text{and} \qquad \delta = \frac{1}{2}\left(C_x + \frac{C_y\rho_{yx}}{\gamma}\right)$$

Taking the values of α and δ in equation (3.8), and simplifying. We get *min.MSE*($\hat{\hat{Y}}_2$):

$$\min.MSE(\hat{\hat{Y}}_2) = \bar{Y}^2 C_y^2\left[\theta_2 - \theta_1\rho_{xy}^2 - \theta_3\rho_{yz}^2\right] \tag{3.9}$$

In order to derive bias of (3.6), we again use (3.7) upto the 2[nd] order of approximation as:

$$\hat{\hat{Y}}_2 - \bar{Y} \approx \bar{Y}\begin{bmatrix} e_{\bar{y}_2} + \alpha\left(e_{\bar{z}_1} - e_{\bar{z}_2}\right) + \frac{\alpha(\alpha-1)}{2}e_{\bar{z}_1}^2 + \frac{\alpha(\alpha+1)}{2}e_{\bar{z}_2}^2 - \alpha^2 e_{\bar{z}_1}e_{\bar{z}_2} + \gamma\, e_1 + \alpha\gamma\, e_1\left(e_{\bar{z}_1} - e_{\bar{z}_2}\right) - 2\delta\gamma\, e_{\bar{x}_1} \\ -2\delta\gamma\alpha e_{\bar{x}_2}\left(e_{\bar{z}_1} - e_{\bar{z}_2}\right) + \delta\gamma^2 e_{\bar{x}_1}^2 + \alpha e_{\bar{y}_2}\left(e_{\bar{z}_1} - e_{\bar{z}_2}\right) + \gamma e_{\bar{x}_1}e_{\bar{y}_2}(1-2\delta) \end{bmatrix}$$

After applying expectation and simplifying, the optimum bias of (3.6) is:

$$Bias(\hat{\hat{Y}}_2) = \bar{Y}\left[\frac{\theta_3}{2}\left(C_y\rho_{yz} - \frac{C_z}{2}\right)^2\right] \tag{3.10}$$

i. *Deduced Family of* $\hat{\hat{Y}}_2$

A large number of estimators have been deduced as a family of proposed estimator $\hat{\hat{Y}}_2$ under certain choices of the constants α, a, b and δ. These deduced estimators have been presented in the following table.

Deduced Estimator	α	a	b	δ
$t_0 = \bar{y}_2$ Usual mean per unit	**0**	**0**	**1**	δ_0
$t_1 = \bar{y}_2\dfrac{\bar{X}}{\bar{x}_1}$ Usual ratio type	**0**	**1**	**0**	**1**
$t_2 = \bar{y}_2\dfrac{\bar{x}_1}{\bar{X}}$ Usual product type	**0**	**1**	**0**	**0**

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$t_3 = \bar{y}_2 \dfrac{\bar{X} + C_x}{\bar{x}_1 + C_x}$ Sisodia and Diwivedi (1981) type estimator	**0**	**1**	**C_x**	**1**
$t_4 = \bar{y}_2 \dfrac{\bar{x}_1 + C_x}{\bar{X} + C_x}$ Pandey and Dubey (1988) type estimator	**0**	**1**	**C_x**	**0**
$t_5 = \bar{y}_2 \dfrac{\beta_2(x)\bar{x}_1 + C_x}{\beta_2(x)\bar{X} + C_x}$ Upadhyaya and Singh (1999) type estimator	0	$\beta_2(x)$	**C_x**	0
$t_6 = \bar{y}_2 \dfrac{C_x \bar{x}_1 + \beta_2(x)}{C_x \bar{X} + \beta_2(x)}$ Upadhyaya and Singh (1999) type estimator	0	**C_x**	$\beta_2(x)$	0
$t_7 = \bar{y}_2 \dfrac{\bar{x}_1 + \sigma_x}{\bar{X} + \sigma_x}$ G.N. Singh (2003) type estimator	0	1	σ_x	0
$t_8 = \bar{y}_2 \dfrac{\beta_1(x)\bar{x}_1 + \sigma_x}{\beta_1(x)\bar{X} + \sigma_x}$ G.N. Singh (2003) type estimator	0	$\beta_1(x)$	σ_x	0
$t_9 = \bar{y}_2 \dfrac{\beta_2(x)\bar{x}_1 + \sigma_x}{\beta_2(x)\bar{X} + \sigma_x}$ G.N. Singh (2003) type estimator	0	$\beta_2(x)$	σ_x	0
$t_{10} = \bar{y}_2 \dfrac{\bar{X} + \rho}{\bar{x}_1 + \rho}$ Singh and Tailor (2003) type estimator	0	1	ρ	1
$t_{11} = \bar{y}_2 \dfrac{\bar{x}_1 + \rho}{\bar{X} + \rho}$ Singh and Tailor (2003) type estimator	0	1	ρ	0
$t_{12} = \bar{y}_2 \dfrac{\bar{X} + \beta_2(x)}{\bar{x}_1 + \beta_2(x)}$ Singh et al. (2004) type estimator	0	1	$\beta_2(x)$	1
$t_{13} = \bar{y}_2 \dfrac{\bar{x}_1 + \beta_2(x)}{\bar{X} + \beta_2(x)}$ Singh et al. (2004) type estimator	0	1	$\beta_2(x)$	0
$t_{14} = \bar{y}_2 \dfrac{\bar{X}}{\bar{x}_1} \dfrac{\bar{z}_1}{\bar{z}_2}$ Chain ratio type estimator	1	1	0	1
$t_{15} = \bar{y}_2 \dfrac{\bar{x}_1}{\bar{X}} \dfrac{\bar{z}_1}{\bar{z}_2}$ Product to ratiotype estimator	1	1	0	0

In addition to these estimators a large number of estimators can also be deduced from the proposed family of estimators by putting values of α, a, b and δ. It is observed that the expression of the first order approximation of MSE of the given number of the family can be obtained by mere substituting the values of α, a, b and δ in (3.8).

IV. Empirical Illustration

To analyze the performance of proposed estimators in comparison to other estimators, five population data sets are being considered. In two phase sampling under *SRSWOR*, the comparison of proposed estimators $\hat{\bar{Y}}_1$ and $\hat{\bar{Y}}_2$ with respect to usual unbiased estimator, Mohanty (1967), Chand (1975), Mukhergee et at (1987), Srivastava et al (1990), Sahoo et al (1994a), Singh (2001), Singh et al. (2004), Samiuddin and Hanif (2006), Samiuddin and Hanif (2007), and Singh et al. (2007) have been made regarding the availability of partial information only. The descriptions of populations are given below.

Population-I.
Data used by Anderson (1958)
Y: Head length of second son **X:** Head length of first son **Z:** Head breadth of first son

N = 25, $\bar{Y}$ = 183.84, $\bar{X}$ =185.72, $\bar{Z}$ =151.12, C_y=0.0546, C_x=0.0526,
C_z=0.0488, ρ_{xy} =0.7108, ρ_{yz} =0.6932, ρ_{zx} =0.7346, n_1=10, n_2=7

Population-II.
(Source: Nachtshemim, Neter and Kutner. Advanced applied linear models, 2004)
Y: No of persons below poverty level **X:** No of unemployed persons
Z: Total population

N = 440, $\bar{Y}$ = 119.50, $\bar{X}$ = 906.79, $\bar{Z}$ = 159.17, C_y= 1.9955, C_x= 1.7501,
C_z= 1.5317, ρ_{xy} = 0.956, ρ_{yz} = 0.932, ρ_{zx} = 0.969, n_1= 88, n_2=18

Population-III.
(Source: Population census report of Okara district (1998), Pakistan)
Y: Population Matric and above **X:** Primary but below Matric **Z:** Population both sexes

N = 300, $\bar{Y}$ =41.5233, $\bar{X}$ = 141.58, $\bar{Z}$ = 1518.767, C_y= 1.2185, C_x= 1.088,
C_z= 0.9757, ρ_{xy} = 0.894, ρ_{yz} = 0.84, ρ_{zx} = 0.94, n_1= 60, n_2=12

Population-IV.
(Source: Population census report of Gujrat district (1998), Pakistan)
Y: Population Matric and above **X:** Primary but below Matric **Z:** Population both sexes

N = 300, $\bar{Y}$ = 131.5133, $\bar{X}$ = 356.8433, $\bar{Z}$ = 1407.407, C_y= 1.2532, C_x= 0.991,
C_z= 0.9545, ρ_{xy} = 0.927, ρ_{yz} = 0.893, ρ_{zx} = 0.972, n_1= 60, n_2=12

Population-V.
(Source: Nachtshemim, Neter and Kutner. Advanced applied linear models, 2004)
Y: Grade-point average following freshman year **Z:** ACT entrance examination score
X: High school class rank as percentile: lower percentile imply higher class rank

N = 705, $\bar{Y}$ = 2.9773, $\bar{X}$ = 76.95, $\bar{Z}$ = 24.54, C_y= 0.213123, C_x= 0.242157,
C_z= 0.16357, ρ_{xy} = 0.398, ρ_{yz} = 0.366, ρ_{zx} = 0.443, n_1= 141, n_2=28

Table 4.1 : PRE's of different proposed estimators of $\bar{Y}$ in double sampling w.r.t $\bar{y}$

Estimator	Population #:				
	1	2	3	4	5
Usual unbiased estimator $t_1 = \bar{y}$	100	100	100	100	100
Mohanty (1967) t_2	135.48	172.44	133.06	154.44	89.21

Notes

Chand (1975) t_3	32.58	30.15	30.47	33.19	33.32
Mukhergee et al. (1987) t_6	131.30	105.87	110.09	104.91	*118.38*
Srivastava et al. (1990) t_7	196.39	*1066.15*	*462.17*	*662.32*	*118.25*
Sahoo et al. (1994a) t_8	73.32	39.49	45.11	41.03	99.34
Singh (2001) t_9	124.37	415.59	306.62	344.23	75.64
Singh et al. (2004) t_{10}	170.97	631.94	285.64	399.93	106.0
Sammiudin & Hanif(2006) t_{11}	130.35	145.83	126.9	156.58	63.20
Sammiudin & Hanif (2007) t_{12}	137.36	582.78	265.37	370.30	103.07
Singh et al. (2007) t_{13}	196.39	*1066.15*	*462.17*	*662.32*	*118.25*
Proposed Estimator $\hat{\hat{\bar{Y}}}_1$	196.39	1066.15	462.17	662.32	118.25
Proposed Estimator $\hat{\hat{\bar{Y}}}_2$	197.99	808.77	358.69	520.19	116.01

Notes

V. Conclusion

We have suggested two improved estimators $\hat{\hat{\bar{Y}}}_1$ and $\hat{\hat{\bar{Y}}}_2$. From table 4.1, we conclude that the proposed estimators are better than usual unbiased estimator $\bar{y}$, Mohanty (1967), Chand (1975), Mukhergee et at (1987), Srivastava et al (1990), Sahoo et al (1994a), Singh (2001), Singh et al. (2004), Samiuddin and Hanif (2006), Samiuddin and Hanif (2007), and Singh et al. (2007). It is also observed that among the class of suggested estimators, $\hat{\hat{\bar{Y}}}_1$ performs more efficiently except in population-I and population-V, in comparison with proposed estimator $\hat{\hat{\bar{Y}}}_2$ and Mukhergee et al. (1987) respectively. It is further observed that $\hat{\hat{\bar{Y}}}_1$, Srivastava et al. (1990) and Singh et al. (2007) are performed equally. It is also observed that the performance of $\hat{\hat{\bar{Y}}}_2$ is also fairly good though it seems slightly less efficient in comparison with $\hat{\hat{\bar{Y}}}_1$. Hence proposed estimators are recommended for their practical use if only partial information are available.

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On Coefficient Estimates and Neighbourhood Problem for Generalized Sakaguchi Type Functions

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Abstract - In the present investigation, we introduce a new class $k\text{-}U^m(\rho,\beta,\lambda,\mu,\gamma,t)$ of analytic functions with negative coefficients. The various results obtained here for this function include coefficient estimate and inclusion relationships involving the neighbourhoods of the analytic function.

Keywords : Analytic function, uniformly starlike function, coefficient estimate, neighbourhood problem.

GJSFR-F Classification : MSC 2010: 11B65, 05A10

ON COEFFICIENT ESTIMATES AND NEIGHBOURHOOD PROBLEM FOR GENERALIZED SAKAGUCHI TYPE FUNCTIONS

Strictly as per the compliance and regulations of :

Notes

On Coefficient Estimates and Neighbourhood Problem for Generalized Sakaguchi Type Functions

B. Srutha Keerthi [α] & S. Chinthamani [σ]

Abstract - In the present investigation, we introduce a new class $k\text{-}U^m(\rho,\beta,\lambda,\mu,\gamma,t)$ of analytic functions with negative coefficients. The various results obtained here for this function include coefficient estimate and inclusion relationships involving the neighbourhoods of the analytic function.

Keywords and Phrases : Analytic function, uniformly starlike function, coefficient estimate, neighbourhood problem.

I. Introduction

Let A denote the family of functions of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n \qquad (1.1)$$

that are analytic in the open unit disk $\mathcal{U} = \{z : |z| < 1\}$. Denote by S the subclass of A of functions that are univalent in $\mathcal{U}$.

For $f \in A$ given by (1.1) and $g(z)$ given by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n \qquad (1.2)$$

their convolution (or Hadamard product), denoted by $(f * g)$, is defined as

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z) \quad (z \in \mathcal{U}) \qquad (1.3)$$

Note that $f * g \in A$.

A function $f \in A$ is said to be in $k\text{-}\mathcal{U}S(\gamma)$, the class of k-uniformly starlike functions of order γ, $0 \le \gamma < 1$, if satisfies the condition

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$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > k\left|\frac{zf'(z)}{f(z)} - 1\right| + \gamma \quad (k \geq 0) \tag{1.4}$$

and a function $f \in A$ is said to be in $k\text{-}\mathcal{U}C(\gamma)$, the class of k-uniformly convex functions of order γ, $0 \leq \gamma < 1$, if satisfies the condition

$$Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > k\left|\frac{zf''(z)}{f'(z)}\right| + \gamma \quad (k \geq 0) \tag{1.5}$$

Uniformly starlike and uniformly convex functions were first introduced by Goodman [8] and then studied by various authors. It is known that $f \in k\text{-}\mathcal{U}C(\gamma)$ or $f \in k\text{-}\mathcal{U}S(\gamma)$ if and only if $1+\frac{zf''(z)}{f'(z)}$ or $\frac{zf'(z)}{f(z)}$, respectively, takes all the values in the conic domain $\mathcal{R}_{k,\gamma}$ which is included in the right half plane given by

$$\mathcal{R}_{k,\gamma} = \{w = u + iv \in C : u > k\sqrt{(u-1)^2 + v^2} + \gamma, \beta \geq 0 \text{ and } \gamma \in [0,1)\}. \tag{1.6}$$

Denote by $\mathcal{P}(P_{k,\gamma})$, $(\beta \geq 0, 0 \leq \gamma < 1)$ the family of functions p, such that $p \in \mathcal{P}$, where $\mathcal{P}$ denotes well-known class of caratheodary functions. The function $P_{k,\gamma}$ maps the unit disk conformally onto the domain $\mathcal{R}_{k,\gamma}$ such that $1 \in \mathcal{R}_{k,\gamma}$ and $\partial\mathcal{R}_{k,\gamma}$ is a curve defined by the equality

$$\partial\mathcal{R}_{k,\gamma} = \{w = u+iv \in C : u^2 = (k\sqrt{(u-1)^2 + v^2}+\gamma)^2, \beta \geq 0 \text{ and } \gamma \in [0,1)\}. \tag{1.7}$$

where $0 \leq \alpha < 1$, $|t| \leq 1$, $t \neq 1$. Note that $S_S(0,-1) = S_s$ and $S_s(\alpha,-1) = S_s(\alpha)$ is called Sakaguchi function of order α.

Let us define the linear multiplier differential operator $D^m_{\lambda,\mu}f$ [11] which is shown as follows:

$$D^m_{\lambda,\mu}f(z) = z + \sum_{n=2}^{\infty} \phi^m(\lambda,\mu,n)a_n z^n \tag{1.8}$$

where

$$\phi^m(\lambda,\mu,n) = [1 + (\lambda\mu n + \lambda - \mu)(n-1)]^m, \tag{1.9}$$

$0 \leq \mu \leq 1$ and $m \in N_0 = N \cup \{0\}$.

It should be remarked that the operator $D^m_{\lambda,\mu}$ is a generalization of many other linear operators considered earlier. In particular, for $f \in A$ we have the following:

- $D^m_{1,0}f(z) \equiv D^m f(z)$ the operator investigated by Salagean (see [14]).
- $D^m_{\lambda,0}f(z) \equiv D^m_\lambda f(z)$ the operator studied by Al-Oboudi (see [1]).

Now, by making use of he differential operator $D^m_{\lambda,\mu}$, we define a new subclass of functions belonging to the class A.

Ref.

[8] Goodman, A. W. On uniformly starlike functions, J. Math. Anal. Appl. 155, 364-370, 1991.

Definition 1.1. *A function* $f(z) \in A$ *is said to be in the class* $k\text{-}\mathcal{U}^m(\rho,\beta,\lambda,\mu,\gamma,t)$ *if for all* $z \in \mathcal{U}$,

$$Re\left\{\frac{(1-t)[(\rho\beta z^3(D^m_{\lambda,\mu}f(z))''' + (2\rho\beta+\rho-\beta)z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))'}{\begin{array}{c}\{\rho\beta z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + (\rho-\beta)z[(D^m_{\lambda,\mu}f(z))' \\ -t(D^m_{\lambda,\mu}f(tz))'] + (1-\rho+\beta)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)]\}\end{array}}\right\}$$

$$\geq k\left|\frac{(1-t)[(\rho\beta z^3(D^m_{\lambda,\mu}f(z))''' + (2\rho\beta+\rho-\beta)z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))'}{\begin{array}{c}\{\rho\beta z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + (\rho-\beta)z[(D^m_{\lambda,\mu}f(z))' \\ -t(D^m_{\lambda,\mu}f(tz))'] + (1-\rho+\beta)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)]\}\end{array}} - 1\right| + \gamma$$

for $\lambda \geq \mu \geq 0,\ m,k \geq 0,\ |t| \leq 1,\ t \neq 1,\ 0 \leq \beta \leq \rho \leq 1.$

Furthermore, we say that a function $f(z) \in k\text{-}\mathcal{U}^m(\rho,\beta,\lambda,\mu,\gamma,t)$ is in the subclass $k\text{-}\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t)$ if $f(z)$ is of the following form:

$$f(z) = z - \sum_{n=2}^{\infty} a_n z^n \quad (a_n \geq 0, n \in N) \tag{1.10}$$

The aim of this paper is to study the coefficient bounds and certain neighbourhood results of the class $k\text{-}\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t)$.

This subclass was motivated by Murat Cagler and Halit Orhan See [17].

Ref.

[17] Murat Caglar and Halit Orhan, On Coefficient Estimates and Neighborhood Problem for Generalized Sakaguchi type Functions, arxiv: 1204.4546v1, [math.cv] 20 Apr. 2012.

Definition 1.2. *A function* $f(z) \in A$ *is said to be in the class* $k\text{-}\mathcal{U}^m(\rho,\lambda,\mu,\gamma,t)$ *if for all* $z \in \mathcal{U}$

$$Re\left\{\frac{(1-t)[\rho z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))']}{(1-\rho)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)] + \rho z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))']}\right\}$$

$$\geq k\left|\frac{(1-t)[\rho z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))']}{(1-\rho)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)] + \rho z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))']} - 1\right| + \gamma$$

for $\lambda \geq \mu \geq 0,\ m,k \geq 0,\ |t| \leq 1,\ t \neq 1,\ 0 \leq \rho \leq 1.$

Remark 1.1. *When* $\beta = 0$ *in the class* $k\text{-}\mathcal{U}^m(\rho,\beta,\lambda,\mu,\gamma,t)$, *we get the class* $k\text{-}\mathcal{U}^m(\rho,\lambda,\mu,\gamma,t)$ *as in Definition 1.2.*

Definition 1.3. *A function* $f(z) \in A$ *is said to be in the class* $k\text{-}\mathcal{U}C^m(\lambda,\mu,\gamma,t)$ *if for all* $z \in \mathcal{U}$,

$$Re\left\{\frac{(1-t)[z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))']}{z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))']}\right\}$$

$$\geq k\left|\frac{(1-t)[z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))']}{z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))']} - 1\right| + \gamma$$

for $\lambda \geq \mu \geq 0,\ m, k \geq 0,\ |t| \leq 1,\ t \neq 1.$

Definition 1.4. *A function* $f(z) \in A$ *is said to be in the class* k-$\mathcal{U}^m(\alpha, \lambda, \mu, \gamma, t)$ *if for all* $z \in \mathcal{U}$,

$$Re\left\{\frac{(1-t)[\alpha z^3(D^m_{\lambda,\mu}f(z))''' + (1+2\alpha)z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))']}{\alpha z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))']}\right\}$$

$$\geq k\left|\frac{(1-t)[\alpha z^3(D^m_{\lambda,\mu}f(z))''' + (1+2\alpha)z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))']}{\alpha z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))']} - 1\right| + \gamma$$

for $\lambda \geq \mu \geq 0,\ m, k \geq 0,\ |t| \leq 1,\ t \neq 1,\ 0 \leq \alpha \leq 1.$

Remark 1.2. *When* $\rho = 1$ *in the class* k-$\mathcal{U}^m(\rho, \lambda, \mu, \gamma, t)$ *and when* $\alpha = 0$ *in the class* k-$\mathcal{U}^m(\alpha, \lambda, \mu, \gamma, t)$, *we get the class* k-$\mathcal{UC}^m(\lambda, \mu, \gamma, t)$ *as in Definition 1.3.*

II. Coefficient Bounds of the Function Class

k-$\tilde{\mathcal{U}}^m(\rho, \beta, \lambda, \mu, \gamma, t)$

Firstly, we, shall need the following lemmas.

Lemma 2.1. *Let* $w = u + iv$. *Then*

$$Re\ w \geq \alpha \text{ if and only if } |w - (1+\alpha)| \leq |w + (1-\alpha)|.$$

Lemma 2.2. *Let* $w = u + iv$ *and* α, γ *are real numbers. Then*

$$Re\ w > \alpha|w-1| + \gamma \text{ if and only if } Re\{w(1+\alpha e^{i\theta}) - \alpha e^{i\theta}\} > \gamma.$$

Theorem 2.1. *The function* $f(z)$ *defined by (1.10) is in the class* k-$\tilde{\mathcal{U}}^m(\rho, \beta, \lambda, \mu, \gamma, t)$ *if and only if*

$$\Sigma\phi^m(\lambda, \mu, n)|n(k+1) - u_n(k+\gamma)||n(n-1)\rho\beta + (\rho-\beta)(n-1) + 1|a_n \leq 1-\gamma, \tag{2.1}$$

where $\lambda \geq \mu \geq 0,\ m, k \geq 0,\ |t| \leq 1,\ t \neq 1,\ 0 \leq \gamma < 1,\ 0 \leq \beta \leq \rho \leq 1,$ $u_n = 1 + t + \cdots + t^{n-1}$. *The result is sharp for the function* $f(z)$ *given by*

$$f(z) = z - \sum_{n=2}^{\infty}\frac{1-\gamma}{\phi^m(\lambda, \mu, n)|n(k+1) - u_n(k+\gamma)||n(n-1)\rho\beta + (\rho-\beta)(n-1) + 1|}z^n$$

Proof. By Definition 1.1 and by Lemma 2.2, we have,

$$Re\left\{\frac{(1-t)[\rho\beta z^3(D^m_{\lambda,\mu}f(z))''' + (2\rho\beta+\rho-\beta)z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))'](1+ke^{i\theta})}{\begin{array}{c}\{\rho\beta z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + (\rho-\beta)z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))'] \\ +(1-\rho+\beta)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)]\}\end{array}} - ke^{i\theta}\right\} \geq \gamma,$$

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where $-\pi < \theta < \pi$, or equivalently

$$Re\left\{\frac{F(z)}{E(z)}\right\} \geq \gamma \tag{2.2}$$

where

$$F(z) = (1-t)[\rho\beta z^3(D^m_{\lambda,\mu}f(z))''' + (2\rho\beta+\rho-\beta)z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))'](1+ke^{i\theta}) - ke^{i\theta}\{\rho\beta z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + (\rho-\beta)z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))'] + (1-\rho+\beta)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)]\}$$

and

$$E(z) = \rho\beta z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + (\rho-\beta)z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))'] + (1-\rho+\beta)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)] \tag{2.3}$$

By Lemma 2.1, (2.2) is equivalent to

$$|F(z) + (1-\gamma)E(z)| \geq |F(z) - (1+\gamma)E(z)| \quad \text{for} \quad 0 \leq \gamma < 1$$

But

$$|F(z) + (1-\gamma)E(z)| \geq |1-t|\left\{\begin{array}{l}(2-\gamma)|z| \\ -\Sigma\phi^m(\lambda,\mu,n)|n(n-1)\rho\beta + (\rho-\beta)(n-1)+1||n+u_n(1-\gamma)|a_n|z|^n \\ -k\Sigma\phi^m|n(n-1)\rho\beta + (\rho-\beta)(n-1)+1||n-u_n|a_n|z|^n\end{array}\right\}$$

Also,

$$|F(z) - (1+\gamma)E(z)| \leq |1-t|\left\{\begin{array}{l}\gamma|z| \\ +\Sigma\phi^m(\lambda,\mu,n)|n(n-1)\rho\beta + (\rho-\beta)(n-1)+1||n-u_n(1+\gamma)|a_n|z|^n \\ +k\Sigma\phi^m(\lambda,\mu,n)|n(n-1)\rho\beta + (\rho-\beta)(n-1)+1||n-u_n|a_n|z|^n\end{array}\right\}$$

and so

$$|F(z) + (1-\gamma)E(z)| - |F(z) - (1+\gamma)E(z)| \geq \left\{\begin{array}{l}2(1-\gamma)|z| \\ -\sum_{n=2}^{\infty} 2\phi^m(\lambda,\mu,n)|n(k+1) - u_n(k+\gamma)||n(n-1)\rho\beta + (\rho-\beta)(n-1)+1|a_n|z|^n\end{array}\right\} \geq 0$$

or

$$\Sigma\phi^m(\lambda,\mu,n)|n(k+1) - u_n(k+\gamma)||n(n-1)\rho\beta + (\rho-\beta)(n-1)+1|a_n \leq (1-\gamma)$$

Conversely, suppose that (2.1) holds, then we must show that (2.2) is true upon choosing the values of z on the positive real axis where $0 \leq z = r < 1$, the above inequality reduces to

$$Re\left\{\frac{(1-\gamma) - \Sigma\phi^m(\lambda,\mu,n)[n(n-1)\rho\beta + (\rho-\beta)(n-1)+1][n(k+1) - u_n(k+\gamma)]a_n z^{n-1}}{1 - \Sigma\phi^m(\lambda,\mu,n)[n(n-1)\rho\beta + (\rho-\beta)(n-1)+1]u_n a_n z^{n-1}}\right\} \geq 0$$

Since $Re(-e^{i\theta}) \geq -|e^{i\theta}| = -1$, the above inequality reduces to

$$Re\left\{\frac{(1-\gamma) - \sum_{n=2}^{\infty}\phi^m(\lambda,\mu,n)[n(n-1)\rho\beta + (\rho-\beta)(n-1)+1][n(k+1) - u_n(k+\gamma)]a_n r^{n-1}}{1 - \Sigma\phi^m(\lambda,\mu,n)[n(n-1)\rho\beta + (\rho-\beta)(n-1)+1]u_n a_n r^{n-1}}\right\} \geq 0$$

Letting $r \to 1^-$, we have desired concluison.

Corollary 2.1. *Let $\beta = 0$ in (2.1) then we have the result for the class defined in Definition 1.2 as*

$$\sum \phi^m(\lambda,\mu,n)|n(k+1) - u_n(k+\gamma)||\rho(n-1)+1|a_n \le (1-\gamma)$$

where $\lambda \ge \mu \ge 0$, $m,k \ge 0$, $|t| \le 1$, $t \ne 1$, $0 \le \gamma < 1$, $0 \le \rho \le 1$, $u_n = 1 + t + \cdots + t^{n-1}$.

Corollary 2.2. *Let $\rho = 1$, $\beta = 0$ in (2.1) then we have the result for the class defined in Definition 1.3 as*

$$\sum \phi^m(\lambda,\mu,n)|n(k+1) - u_n(k+\gamma)|na_n \le (1-\gamma)$$

where $\lambda \ge \mu \ge 0$, $m,k \ge 0$, $|t| \le 1$, $t \ne 1$, $0 \le \gamma < 1$, $u_n = 1+t+\cdots+t^{n-1}$.

Theorem 2.2. *The funciton $f(z)$ defined by (1.10) is in the class $k\text{-}\tilde{\mathcal{U}}^m(\alpha,\lambda,\mu,\gamma,t)$ if and only if*

$$\sum \phi^m(\lambda,\mu,n)|n(k+1) - u_n(k+\gamma)||\alpha(n-1)+1|a_n \le 1-\gamma$$

where $\lambda \ge \mu \ge 0$, $m,k \ge 0$, $|t| \le 1$, $t \ne 1$, $0 \le \gamma < 1$, $0 \le \alpha \le 1$, $u_n = 1 + t + \cdots + t^{n-1}$.
The result is sharp for the function $f(z)$ given by

$$f(z) = z - \sum_{n=2}^{\infty} \frac{1-\gamma}{\phi^m(\lambda,\mu,n)|n(k+1) - u_n(k+\gamma)||\alpha(n-1)+1|} z^n$$

Proof. The same procedure is followed as in Theorem 2.1 to prove this result.

Corollary 2.3. *Take $\alpha = 0$, then we get the result as in Corollary 2.2.*

III. Neighbourhood of the Function Class $k\text{-}\mathcal{U}^m(\rho,\beta,\lambda,\mu,\gamma,t)$

Following the earlier investigations (based upon the familiar concept of neighbourhoods of analytic functions) by Goodman [7], Ruscheweyh [12], Altintas et al. ([2, 3]) and others including Srivastava et al. ([15, 16]), Orhan ([9]), Deniz et al. [6], Catas [4].

Definition 3.1. *Let* $\lambda \ge \mu \ge 0$, $m,k \ge 0$, $|t| \le 1$, $t \ne 1$, $0 \le \gamma < 1$, $\alpha \ge 0$, $u_n = 1 + t + \cdots + t^{n-1}$ *we define the α-neighbourhood of a function $f \in A$ and denote by $N_\alpha(f)$ consisting of all functions* $g(z) = z - \sum_{n=2}^{\infty} b_n z^n \in S$ $(b_n \ge 0, n \in N)$ *satisfying*

$$\sum \frac{\phi^m(\lambda,\mu,n)|n(k+1) - u_n(k+\gamma)||n(n-1)\rho\beta + (\rho-\beta)(n-1)+1|}{1-\gamma}|a_n - b_n| \le \alpha$$

Ref.

[6] Deniz, E. and Orhan, H. Some properties of certain subclasses of analytic functions with negative coefficients by using generalized Ruscheweyh derivative operator, Czechoslovak Math. J., 60 (135), 699-713, 2010.

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Theorem 3.1. *Let* $f \in k\text{-}\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t)$ *and for all real* θ, *we have* $\gamma(e^{i\theta}-1)-2e^{i\theta} \neq 0$. *For any complex number* ϵ *with* $|\epsilon| < \alpha$ $(\alpha \geq 0)$, *if* f *satisfies the following condition:*

$$\frac{f(z)+\epsilon z}{1+\epsilon} \in k-\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t),$$

then $N_\alpha(f) \subset k\text{-}\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t)$.

Proof. It is obvious that $f \in k\text{-}\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t)$ if and only if

$$\left|\frac{u(z)(1+ke^{i\theta})-(ke^{i\theta}+1+\gamma)v(z)}{u(z)(1+ke^{i\theta})+(1-ke^{i\theta}-\gamma)v(z)}\right| < 1 \quad (-\pi < \theta < \pi)$$

where

$$u(z) = (1-t)[\rho\beta z^3(D^m_{\lambda,\mu}f(z))''' + (2\rho\beta+\rho-\beta)z^2(D^m_{\lambda,\mu}f(z))'' + z(D^m_{\lambda,\mu}f(z))']$$
$$v(z) = \rho\beta z^2[(D^m_{\lambda,\mu}f(z))'' - t^2(D^m_{\lambda,\mu}f(tz))''] + (\rho-\beta)z[(D^m_{\lambda,\mu}f(z))' - t(D^m_{\lambda,\mu}f(tz))'] + (1-\rho+\beta)[D^m_{\lambda,\mu}f(z) - D^m_{\lambda,\mu}f(tz)]$$

for any complex number S with $|S|=1$, we have

$$\frac{u(z)(1+ke^{i\theta})-(ke^{i\theta}+1+\gamma)v(z)}{u(z)(1+ke^{i\theta})+(1-ke^{i\theta}-\gamma)v(z)} \neq S$$

In other words, we must have

$$(1-S)u(z)(1+ke^{i\theta}) - (ke^{i\theta}+1+\gamma-S(ke^{i\theta}-1+\gamma))v(z) \neq 0$$

which is equivalent to

$$z - \frac{\begin{array}{c}\{\Sigma\phi^m(\lambda,\mu,n)(\rho\beta(n(n-1))+(\rho-\beta)(n-1)+1) \\ \times((n-u_n)(1+ke^{i\theta}-Ske^{i\theta})-S(n+u_n)-u_n\gamma(1-S))\}\end{array}}{\gamma(S-1)-2S} a_n z^n \neq 0$$

However, $f \in k\text{-}\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t)$ if and only if $\dfrac{(f*h)(z)}{z} \neq 0,\ z \in \mathcal{U}-\{\ 0\}$ where $h(z) = z - \sum_{n=2}^{\infty} c_n z^n$ and

$$c_n = \frac{\begin{array}{c}\{\sum\phi^m(\lambda,\mu,n)(\rho\beta(n(n-1))+(\rho-\beta)(n-1)+1) \\ \times((n-u_n)(1+ke^{i\theta}-Ske^{i\theta})-S(n+u_n)-u_n\gamma(1-S))\}\end{array}}{\gamma(S-1)-2S}$$

we note that

$$|c_n| \leq \frac{\sum\phi^m(\lambda,\mu,n)|\rho\beta(n(n-1))+(\rho-\beta)(n-1)+1||n(1+k)-u_n(k+\gamma)|}{1-\gamma}$$

Since $\dfrac{f(z)+\epsilon z}{1+\epsilon} \in k\text{-}\tilde{\mathcal{U}}^m(\rho,\beta,\lambda,\mu,\gamma,t)$, therefore

$z^{-1}\left(\dfrac{f(z)+\epsilon z}{1+\epsilon} * h(z)\right) \neq 0$ which is equivalent to

$$\frac{(f*h)(z)}{(1+\epsilon)z} + \frac{\epsilon}{1+\epsilon} \neq 0 \tag{3.1}$$

Now suppose that $\left|\dfrac{(f*h)(z)}{z}\right| < \alpha$. Then by (3.1), we must have

$$\left|\frac{(f*h)(z)}{(1+\epsilon)z} + \frac{\epsilon}{1+\epsilon}\right| \geq \frac{|\epsilon|}{|1+\epsilon|} - \frac{1}{|1+\epsilon|}\left|\frac{(f*h)(z)}{z}\right| > \frac{|\epsilon|-\alpha}{|1+\epsilon|} \geq 0$$

this is a contradiction by $|\epsilon| < \alpha$ and however, we have $\left|\dfrac{(f*h)(z)}{z}\right| \geq \alpha$. If $g(z) = z - \sum_{n=2}^{\infty} b_n z^n \in N_\alpha(f)$, then

$$\alpha - \left|\frac{(g*h)(z)}{z}\right| \leq \left|\frac{((f-g)*h)(z)}{z}\right| \leq \sum_{n=2}^{\infty} |a_n - b_n| c_n |z^n|$$

$$< \sum_{n=2}^{\infty} \frac{\phi^m(\lambda,\mu,n)|\rho\beta(n(n-1)) + (\rho-\beta)(n-1)+1||n(1+k) - u_n(k+\gamma)|}{1-\gamma}|a_n - b_n| \leq \alpha$$

Corollary 3.1. *When $\beta = 0$ in Theorem 3.1, we get the result for the class $k\text{-}\tilde{\mathcal{U}}^m(\rho,\lambda,\mu,\gamma,t)$.*

Corollary 3.2. *When $\rho = 1$, $\beta = 0$ in Theorem 3.1, we get the result for the class $k\text{-}\tilde{\mathcal{U}}\mathcal{C}^m(\lambda,\mu,\gamma,t)$.*

Remark 3.1. *Using the similar procedure, we can prove the result as in Theorem 3.1 for the class $k\text{-}\tilde{\mathcal{U}}^m(\alpha,\lambda,\mu,\gamma,t)$ in which $\alpha = 0$ implies the result for the class $k\text{-}\tilde{\mathcal{U}}\mathcal{C}^m(\lambda,\mu,\gamma,t)$.*

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Modeling and Analysis of an SEIR Epidemic Model with a Limited Resource for Treatment

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Abstract - In this paper an SEIR epidemic model with a limited resource for treatment is investigated. It is assumed that the treatment rate is proportional to the number of patients as long as this number is below a certain capacity and it becomes constant when that number of patients exceeds this capacity. Mathematical analysis is used to study the dynamic behavior of this model. Existence and stability of disease-free and endemic equilibria are investigated. It is shown that this kind of treatment rate leads to the existence of multiple endemic equilibria where the basic reproduction number plays a big role in determining their stability.

Keywords : SEIR epidemic model, global stability, basic reproduction number, tretment rate, Routh-Herwitz criterion, second additive compound matrix, Lyapunov function, Lasalle's invariance principle.

GJSFR-F Classification : MSC 2010: 40C05, 37B25

MODELING AND ANALYSIS OF AN SEIR EPIDEMIC MODEL WITH A LIMITED RESOURCE FOR TREATMENT

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Modeling and Analysis of an SEIR Epidemic Model with a Limited Resource for Treatment

Sarah A. Al-Sheikh

Abstract - In this paper an SEIR epidemic model with a limited resource for treatment is investigated. It is assumed that the treatment rate is proportional to the number of patients as long as this number is below a certain capacity and it becomes constant when that number of patients exceeds this capacity. Mathematical analysis is used to study the dynamic behavior of this model. Existence and stability of disease-free and endemic equilibria are investigated. It is shown that this kind of treatment rate leads to the existence of multiple endemic equilibria where the basic reproduction number plays a big role in determining their stability.

Keywords : SEIR epidemic model, global stability, basic reproduction number, tretment rate, Routh-Herwitz criterion, second additive compound matrix, Lyapunov function, Lasalle's invariance principle.

I. Introduction

There is a long and distinguished history of mathematical models in epidemiology, going back to the eighteenth century (Bernoulli 1760). Since that time, theoretical epidemiology has witnessed numerous developments. Some of these studies can be found in Baily (1975), Anderson and May (1991), and Hethcote (2000). A tremendous number of models have been formulated, analyzed and applied to a variety of infectious diseases qualitatively and quantitatively. Mathematical models have become important tools in analyzing the spread and control of infectious diseases. Furthermore, mathematical models now plays a key role in policy making, including health-economic aspects, emergency planning and risk assessment, control-program evaluation, and optimizing various detection. One of the fundamental results in mathematical epidemiology is that most mathematical epidemic models usually exhibit "threshold" behavior stated as follows: if the average number of secondary infections caused by an average infective, called the basic reproduction number, is less than one the disease will die out, while if it exceeds one there will be an endemic (see Driessche and Watmough, 2002, Brauer et all., 2008).

Most of the models in mathematical epidemiology are compartmental models, with the population being divided into compartments with the assumptions about the nature and time rate of transfer from one compartment to another. In this paper, an SEIR model is presented where there is an exposed period between being infected and becoming infective. Some of the research done on SEIR models can be found for example in (Zhang et all., 2006, Yi et all., 2009, Sun and Hsieh, 2010, Zhou and Cui, 2011, Shu et all. 2012). Treatment plays an

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important role in controlling or decreasing the spread of diseases such as measles, flue and tuberculosis (see Hyman and Li, 1998, Fang and Thieme, 1995, Wu and Feng ,2000). More recent work on the effect of treatment on the dynamic behavior can be found in (Wang, 2006, Zhang and Liu, 2008, Kar and Baeabyal, 2010, Zhou and Cui, 2011, Wang et all., 2012). In classical epidemic models, the treatment rate is assumed to be proportional to the number of infectives, which is almost impossible in reality. In this paper, the treatment rate is assumed to be proportional to the number of infectives when the capacity of treatment is not reached, and otherwise, takes the maximal capacity (See Wang, 2006, Kar and Baeabyal, 2010).

Notes

The organization of this paper is as follows: In the next section, the mathematical model is formed and the basic reproduction number is calculated. In section 3, Equilibria of the system are found and their existence conditions are presented. In section 4, stability of equilibria is investigated. Section 5, is devoted for the discussion of the results.

II. The Mathematical Model and the Basic Reproduction Number

To construct the SEIR model, we will divide the total population into four epidemiological classes which are succeptibles (S), exposed (E) infectious (I) and recovered (R). The model to be studied is of the following form:

$$\begin{aligned} \frac{dS}{dt} &= A - \beta SI - \mu S \\ \frac{dE}{dt} &= \beta SI - (\mu + \varepsilon) E \\ \frac{dI}{dt} &= \varepsilon E - (\mu + r + d) I - T(t) \\ \frac{dR}{dt} &= rI - \mu R + T(t) \end{aligned} \tag{1}$$

where A is the recruitment rate, β is the infection rate, μ is the natural death rate, ε is the progression rate to symptoms development(the rate at which an infected individual becomes infectious per unit time), r is the removal rate(the rate at which an infectious individual recovers per unit time), d is the disease-related death and $T(t)$ is the treatment rate function. In this paper the treatment function is defined by

$$T(I) = \begin{cases} cI & \text{if } 0 \le I \le I_o \\ k & \text{if } I > I_o \end{cases}$$

where $k = cI_o$. This means that the treatment rate is proportional to the number of infected people as long as the number of infectives is less than or equal to a fixed value I_o but after that the treatment rate becomes constant. This type of treatment is more realistic when patients have to be hospitalized and the number of beds is limited. This is also true for the case where the medications are not sufficient.(See Wang, 2006, Kar and Batabyal, 2010)

The variable R does not appear in the first three equations of (1), so it is enough to analyze the following reduced system

$$\frac{dS}{dt} = A - \beta SI - \mu S$$

Notes

$$\begin{aligned} \frac{dE}{dt} &= \beta SI - (\mu+\varepsilon)E \\ \frac{dI}{dt} &= \varepsilon E - (\mu+r+d)I - T(t) \end{aligned} \tag{2}$$

It follows from system (2) that $(S+E+I)' = A-\mu(S+E+I)-T(t) \leq A-\mu(S+E+I)$

Then $\lim_{n\to\infty}\sup(S+E+I) \leq \frac{A}{\mu}$. So the feasible region for system (2) is

$\Omega = \{(S,E,I) : S+E+I \leq \frac{A}{\mu}, S>0, E\geq 0, I \geq 0\}$

The region Ω is positively invariant with respect to system (2). Hence, system (2) is considered mathematically and epidemiologically well posed in Ω.

Now, the basic reproduction number R_o will be found by using the method of next generation matrix found in Driessche and Watmough, 2002.

System (2) always has the disease-free equilibrium $X_o = \left(\frac{A}{\mu},0,0\right)$. Near this disease free equilibrium I has to be less than I_o, so system (2) becomes

$$\begin{aligned} \frac{dS}{dt} &= A - \beta SI - \mu S \\ \frac{dE}{dt} &= \beta SI - (\mu+\varepsilon)E \\ \frac{dI}{dt} &= \varepsilon E - (\mu+r+d+c)I \end{aligned} \tag{3}$$

Let $X=(E,I,S)^T$. System (3) can be written as

$\frac{dX}{dt} = \mathcal{F}(X) - \mathcal{V}(X)$

where

$$\mathcal{F}(X) = \begin{pmatrix} \beta SI \\ 0 \\ 0 \end{pmatrix}, \mathcal{V}(X) = \begin{pmatrix} (\mu+\varepsilon)E \\ -\varepsilon E + (\mu+r+d+c)I \\ -A+\beta SI+\mu S \end{pmatrix}$$

The Jacobian matrices of $\mathcal{F}(X)$ and $\mathcal{V}(X)$ at the disease free equilibrium X_o are, respectively,

$$D\mathcal{F}(X_o) = \begin{pmatrix} F & 0 \\ 0 & 0 \end{pmatrix}, D\mathcal{V}(X_o) = \begin{pmatrix} V & 0 \\ J_1 & J_2 \end{pmatrix}$$

$$\text{where } F = \begin{pmatrix} 0 & \frac{\beta A}{\mu} \\ 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \mu+\varepsilon & 0 \\ -\varepsilon & \mu+r+d+c \end{pmatrix}$$

$FV^{-1} = \begin{pmatrix} \frac{\varepsilon\beta A}{\mu(\mu+\varepsilon)(\mu+r+d+c)} & \frac{\beta A}{\mu(\mu+r+d+c)} \\ 0 & 0 \end{pmatrix}$ is the next generation matrix of system (2).

The spectral radius of FV^{-1} is

$\rho\left(FV^{-1}\right) = \frac{\varepsilon\beta A}{\mu(\mu+\varepsilon)(\mu+r+d+c)}$

Hence, the basic reproduction number of system (2) is given by

$R_o = \frac{\varepsilon\beta A}{\mu(\mu+\varepsilon)(\mu+r+d+c)}$

III. Equilibria

In this section, equilibria of system (2) will be found and discussed.

First of all, the disease-free equilibria $X_o = \left(\frac{A}{\mu}, 0, 0\right)$ always exists when $I \leq I_o$.

An endemic equilibria of system (2) satisfies

$$\begin{aligned} A - \beta SI - \mu S &= 0 \\ \beta SI - (\mu+\varepsilon) E &= 0 \\ \varepsilon E - (\mu + r + d) I - T(I) &= 0 \end{aligned} \tag{4}$$

When $0 < I \leq I_o$, system (4) becomes

$$\begin{aligned} A - \beta SI - \mu S &= 0 \\ \beta SI - (\mu+\varepsilon) E &= 0 \\ \varepsilon E - (\mu + r + d + c) I &= 0 \end{aligned} \tag{5}$$

When $I > I_o$, system (4) becomes

$$\begin{aligned} A - \beta SI - \mu S &= 0 \\ \beta SI - (\mu+\varepsilon) E &= 0 \\ \varepsilon E - (\mu + r + d) I - k &= 0 \end{aligned} \tag{6}$$

If $R_o > 1$, system (5) admits a unique positive solution $X^* = (S^*, E^*, I^*)$ given by

$S^* = \frac{A}{\mu+\beta I^*} = \frac{A}{\mu R_o}$

$E^* = \frac{A}{\mu+\varepsilon} - \frac{\mu(\mu+r+d+c)}{\beta\varepsilon} = \frac{\mu(\mu+r+d+c)}{\beta\varepsilon}\left(R_o - 1\right)$

$I^* = \frac{\mu}{\beta}\left(R_o - 1\right)$

$I^* \leq I_o$if and only if $R_o \leq 1 + \frac{\beta I_o}{\mu} \triangleq P_o$

So, X^* is an endemic equilibrium of system (2) if and only if $1 < R_o \leq P_o$.

In order to obtain positive solutions of system (6), we solve S from the first equation of (6) to get $S = \frac{A}{\mu+\beta I}$. We also solve E from the thirds equation to get $E = \frac{\mu+r+d}{\varepsilon}I + \frac{k}{\varepsilon}$. Substitute into the second equation of (6), we have

$$aI^2 + bI + c = 0 \tag{7}$$

where

$a = \beta(\mu+\varepsilon)(\mu+r+d) > 0$

$b = (\mu+\varepsilon)(\mu(\mu+r+d)+\beta k) - \varepsilon\beta A$

$= (\mu+\varepsilon)(\mu(\mu+r+d)+\beta k - \mu(\mu+r+d+c)R_o)$

$c = \mu k(\mu+\varepsilon) > 0$

Notes

Let the discriminant of (7) be $\Delta = b^2 - 4ac$.

If $b \geq 0$, then (7) has no positive solution. Also if $\Delta < 0$, then (7) has no real solution. So we see that if $b < 0$ and $\Delta \geq 0$, than (7) has two positive solutions.

$\Delta \geq 0$ is equivalent to $[(\mu+\varepsilon)(\mu(\mu+r+d)+\beta k - \mu(\mu+r+d+c)R_o)]^2$

$\geq 4\mu\beta k(\mu+\varepsilon)^2(\mu+r+d)$

i.e., $R_o \leq 1 + \frac{\beta k - \mu c}{\mu(\mu+r+d+c)} - 2\frac{\sqrt{\mu\beta k(\mu+r+d)}}{\mu(\mu+r+d+c)}$

or $R_o \geq 1 + \frac{\beta k - \mu c}{\mu(\mu+r+d+c)} + 2\frac{\sqrt{\mu\beta k(\mu+r+d)}}{\mu(\mu+r+d+c)} \triangleq P_1$

Note that $b < 0$ is equivalent to $R_o > 1 + \frac{\beta k - \mu c}{\mu(\mu+r+d+c)}$

Therefore, (7) has two positive solutions I_1 and I_2 if $R_o \geq P_1$ where

$I_1 = \frac{-b-\sqrt{\Delta}}{2\beta(\mu+\varepsilon)(\mu+r+d)}$ and $I_2 = \frac{-b+\sqrt{\Delta}}{2\beta(\mu+\varepsilon)(\mu+r+d)}$

Set $S_1 = \frac{A}{\mu+\beta I_1}$ and $S_2 = \frac{A}{\mu+\beta I_2}$

$E_1 = E_2 = \frac{A}{\mu+\varepsilon} - \frac{\mu(\mu+r+d+c)}{\beta\varepsilon} = \frac{\mu(\mu+r+d+c)}{\beta\varepsilon}(R_o - 1)$

Then $X_i = (S_i, E_i, I_i)$, $i = 1, 2$ are endemic equilibria of (2) if $I_i > I_o$.

$I_1 > I_o$ if and only if $-b - \sqrt{\Delta} > 2\beta(\mu+\varepsilon)(\mu+r+d)I_o$

This implies that $b + 2\beta(\mu+\varepsilon)(\mu+r+d)I_o < 0$

It follows from the definition of b that

$R_o > 1 + \frac{\beta k - \mu c}{\mu(\mu+r+d+c)} + \frac{2\beta(\mu+r+d)I_o}{\mu(\mu+r+d+c)} \triangleq P_2$

By a similar argument we see that $I_2 < I_o$ if and only if $R_o < P_2$.

We summarize the above discussion in the following theorem

Theorem 1 *Let* $P_o = 1 + \frac{\beta I_o}{\mu}$, $P_1 = 1 + \frac{\beta k - \mu c}{\mu(\mu+r+d+c)} + 2\frac{\sqrt{\mu\beta k(\mu+r+d)}}{\mu(\mu+r+d+c)}$ *and* $P_2 = 1 + \frac{\beta k - \mu c}{\mu(\mu+r+d+c)} + \frac{2\beta(\mu+r+d)I_o}{\mu(\mu+r+d+c)}$.

1. *System (2) always have the disease-free equilibrium* $X_o = \left(\frac{A}{\mu}, 0, 0\right)$.
2. *The endemic equilibrium* $X^* = (S^*, E^*, I^*)$ *of system (2) exists if and only if* $1 < R_o \leq P_o$
3. *Two more endemic equilibria* $X_i = (S_i, E_i, I_i)$, $i = 1, 2$ *of system (2) exist if and only if* $R_o \geq P_1$ *and* $R_o > P_2$

Notes

IV. Stability of Equilibria

By analyzing the eigenvalues of the Jacobian matrices of system (2), we get results about the local stability of these equilibria.

a) Disease-free equilibrium X_o

The Jacobian matrix evaluated at X_o is

$$J(X_o) = \begin{pmatrix} -\mu & 0 & -\frac{\beta A}{\mu} \\ 0 & -(\mu+\varepsilon) & 0 \\ 0 & \varepsilon & -(\mu+r+d+c) \end{pmatrix}$$

and the eigenvalues are $-\mu$, $-(\mu+\varepsilon)$ and $-(\mu+r+d+c)$ which are all negative. So we have the following result

Lemma 2 *The disease-free equilibrium* X_o *is locally asymptotically stable.*

To investigate the global stability of X_o, consider the Lyapunov function $L = \varepsilon E + (\mu+\epsilon) I$

$$\frac{dL}{dt} = \varepsilon \frac{dE}{dt} + (\mu+\epsilon)\frac{dI}{dt} = (\epsilon\beta S - (\mu+\epsilon)(\mu+r+d+c)) I$$

$$\leq \left(\frac{\epsilon\beta A}{\mu} - (\mu+\epsilon)(\mu+r+d+c)\right) I = (\mu+\epsilon)(\mu+r+d+c)(R_o - 1) I \leq 0 \text{ if } R_o < 1.$$

The maximal compact invariant set in $\{(S, E, I) \in \Omega : \frac{dL}{dt} = 0\}$ is the singleton $\{X_o\}$. Using Lasalle's invariance principle (Edelstein-Kesher, 2005), we have the following theorem

Theorem 3 *If* $R_o < 1$, *the disease-free equilibrium* X_o *is globally asymptotically stable and the disease dies out. But if* $R_o > 1$, *then* X_o *is unstable.*

b) Endemic equilibrium X^*

The Jacobian matrix evaluated at X^* is

$$J(X^*) = \begin{pmatrix} -\beta I^* - \mu & 0 & -\beta S^* \\ \beta I^* & -(\mu+\varepsilon) & 0 \\ 0 & \varepsilon & -(\mu+r+d+c) \end{pmatrix}$$

$$= \begin{pmatrix} -\mu R_o & 0 & -\frac{\beta A}{\mu R_o} \\ \mu(R_o - 1) & -(\mu+\varepsilon) & 0 \\ 0 & \varepsilon & -(\mu+r+d+c) \end{pmatrix}$$

Notes

The characteristic polynomial of $J(X^*)$ is given by

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0$$

where

$$a_1 = 2\mu + r + d + c + \varepsilon + \mu R_o$$

$$a_2 = (\mu + \varepsilon + \mu R_o)(\mu + r + d + c) + \mu R_o(\mu + \varepsilon)$$

$$a_3 = \mu(\mu + \varepsilon)(\mu + r + d + c)R_o + \varepsilon\beta A\tfrac{(R_o - 1)}{R_o}$$

$$= 2\mu(\mu + \varepsilon)(\mu + r + d + c)R_o - \mu(\mu + \varepsilon)(\mu + r + d + c)$$

Clearly, $a_1 > 0$ and if $R_o > 1$ then $a_3 > 0$.

$a_1a_2 - a_3 = (2\mu + r + d + c + \varepsilon + \mu R_o)((\mu + \varepsilon + \mu R_o)(\mu + r + d + c) + \mu R_o(\mu + \varepsilon)) + \mu(\mu + \varepsilon)(\mu + r + d + c) - 2\mu(\mu + \varepsilon)(\mu + r + d + c)R_o > 0.$

Therefore, by Routh-Herwitz criteria, we conclude that the eigenvalues of $J(X^*)$ are all negative when $R_o > 1$. So, we have the following result

Lemma 4 *If $R_o > 1$, then the endemic equilibrium X^* is locally asymptotically stable.*

Now, we will investigate the global stability of X^*. To do so, we consider the following Lyapunov function

$$L = \left(S - S^* - S^*\ln\tfrac{S}{S^*}\right) + \left(E - E^* - E^*\ln\tfrac{E}{E^*}\right) + \tfrac{\mu+\varepsilon}{\varepsilon}\left(I - I^* - I^*\ln\tfrac{I}{I^*}\right)$$

Thus

$$\tfrac{dV}{dt} = \left(1 - \tfrac{S^*}{S}\right)\tfrac{dS}{dt} + \left(1 - \tfrac{E^*}{E}\right)\tfrac{dE}{dt} + \tfrac{\mu+\varepsilon}{\varepsilon}\left(1 - \tfrac{I^*}{I}\right)\tfrac{dI}{dt}$$

Substituting the expressions of the derivatives from system (2) and using the relation

$$A = \beta S^*I^* + \mu S^*$$

we get

$$\tfrac{dV}{dt} = \left(1 - \tfrac{S^*}{S}\right)\left[-\mu(S - S^*) + \beta S^*I^* - \beta SI\right] + \left(1 - \tfrac{E^*}{E}\right)\left[\beta SI - (\mu + \varepsilon)E\right]$$

$$+\tfrac{\mu+\varepsilon}{\varepsilon}\left(1 - \tfrac{I^*}{I}\right)\left[\varepsilon E - (\mu + r + d + c)I\right]$$

$$= -\mu\tfrac{(S-S^*)^2}{S} + \beta S^*I^* - \beta S^*I^*\tfrac{S^*}{S} + \beta S^*I - \beta SI\tfrac{E^*}{E} + (\mu + \varepsilon)E^* - (\mu + \varepsilon)E\tfrac{I^*}{I}$$

$$-\tfrac{\mu+\varepsilon}{\varepsilon}(\mu + r + d + c)I + \tfrac{\mu+\varepsilon}{\varepsilon}(\mu + r + d + c)I^*$$

Note that $\varepsilon E^* = (\mu + r + d + c)I^*$

This implies that

$$\beta S^*I - \tfrac{\mu+\varepsilon}{\varepsilon}(\mu + r + d + c)I = \beta S^*I - (\mu + \varepsilon)E^*\tfrac{I}{I^*} = \left[\beta S^*I^* - (\mu + \varepsilon)E^*\right]\tfrac{I}{I^*} = 0$$

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So

$$\frac{dV}{dt} = -\mu\frac{(S-S^*)^2}{S} + 3(\mu+\varepsilon)E^* - \beta S^* I^* \frac{S^*}{S} - \beta S I \frac{E^*}{E} - (\mu+\varepsilon)E\frac{I^*}{I}$$

$$= -\mu\frac{(S-S^*)^2}{S} + (\mu+\varepsilon)E^*\left(3 - \frac{S^*}{S} - \frac{SE^*I}{S^*EI^*} - \frac{EI^*}{E^*I}\right) \le 0$$

since the arithmetic mean is greater than or equal to the geometric mean of the quantities $\frac{S^*}{S}, \frac{SE^*I}{S^*EI^*}, \frac{EI^*}{E^*I}$. i.e., $\frac{S^*}{S} + \frac{SE^*I}{S^*EI^*} + \frac{EI^*}{E^*I} - 3 \ge 0$.Then $\frac{dV}{dt} = 0$ holds only when $S = S^*$, $E = E^*$ and $I = I^*$. So the maximal compact invariant set in $\{(S,E,I) \in \Omega : \frac{dL}{dt} = 0\}$ is the singleton $\{X^*\}$. Using Lasalle's invariance principle, we have the following theorem

Theorem 5 *If $R_o > 1$, the endemic equilibrium X^* is globally asymptotically stable*

Notes

c) Endemic equilibria X_1 and X_2

By analyzing the Jacobian matrix at these equilibria we find that

$$J(X_1) = \begin{pmatrix} -\beta I_1 - \mu & 0 & -\beta S_1 \\ \beta I_1 & -(\mu+\varepsilon) & 0 \\ 0 & \varepsilon & -(\mu+r+d) \end{pmatrix} = \begin{pmatrix} -\frac{A}{S_1} & 0 & -\beta S_1 \\ \beta I_1 & -\frac{\beta S_1 I_1}{E_1} & 0 \\ 0 & \varepsilon & \frac{k-\varepsilon E_1}{I_1} \end{pmatrix}$$

The second additive compound matrix of $J(X_1)$ is given by

$$J(X_1)^{[2]} = \begin{pmatrix} -\beta I_1 - \mu - (\mu+\varepsilon) & 0 & \beta S_1 \\ \varepsilon & -\beta I_1 - \mu - (\mu+r+d) & 0 \\ 0 & \beta I_1 & -(\mu+\varepsilon) - (\mu+r+d) \end{pmatrix}$$

For the local stability of X_1 we need the following lemma (See Arino et all., 2003, McCluskey and Driessche, 2004, Cai et all., 2008)

Lemma 6 *Let M be a 3×3 real matrix. If $tr(M)$, $\det(M)$ and $\det(M^{[2]})$ are all negative, then all of the eigenvalues of M have negative real parts.*

Now clearly $tr(J(X_1)) < 0$

$\det(J(X_1)) = -\frac{1}{E_1}\left(A\beta\varepsilon E_1 - Ak\beta + \beta^2\varepsilon E_1 I_1 S_1\right) < 0$ since $\varepsilon E_1 - k > 0$

$\det(J(X_1)^{[2]}) = [-\beta I_1 - \mu - (\mu+\varepsilon)][-\beta I_1 - \mu - (\mu+r+d)][-(\mu+\varepsilon) - (\mu+r+d)] + \varepsilon\beta^2 S_1 I_1$

We can see that $\det(J(X_1)^{[2]}) < 0$ if $\beta^2 I_1^2(\varepsilon + 2\mu + r + d) > \varepsilon\beta^2 S_1 I_1$

The same argument can be used for X_2 as well.

So, we have the following result

Theorem 7 *The endemic equilibria X_i $i = 1,2$ are locally asymptotically stable if* $\frac{S_i}{I_i} < 1 + \frac{2\mu+r+d}{\varepsilon}$

Notes

V. Discussion

In this paper an SEIR epidemic model is proposed to simulate the limited resources for the treatment of patients, which can occur as a consequence of lack of medications or limited beds in hospitals. This model was studied theoretically, and it was found that the dynamic behavior of the model can be determined by its basic reproduction number R_o. When $R_o < 1$, there exists no positve equilibrium and the disease-free equilibrium is globally asymptotically stable, that is the disease dies out. But when $R_o > 1$ the disease-free equilinrium becomes unstable and the disease persists. It was shown that this kind of treatment rate results in the existence of multiple endemic equilibria. An endemic equilibrium X^* exists when $1 < R_o \leq P_o$ in which case it will be globally asymptotically stable. Two more endemic equilibria X_1 and X_2 exist when $R_o \geq P_1$ and $R_o > P_2$. These equilibria are locally asymptotically stable if the ratio $\frac{S_i}{I_i}$ is less than the quantity $1 + \frac{2\mu+r+d}{\varepsilon}$.

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On $(LCS)_n$ -Manifolds Satisfying Certain Conditions on D-Conformal Curvature Tensor

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Abstract - In this paper we have characterized $(LCS)_n$ -manifolds with D -Conformal curvature tensor, concircular curvature tensor and projective curvature tensor.

Keywords : $(LCS)_n$ - Manifold, D - conformal curvature tensor, Projective curvature tensor, Concircular curvature tensor.

GJSFR-F Classification : MSC 2010: 30C20, 14P20

ON LCSN-MANIFOLDS SATISFYING CERTAIN CONDITIONS ON D-CONFORMAL CURVATURE TENSOR

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On $(LCS)_n$ - Manifolds Satisfying Certain Conditions on D-Conformal Curvature Tensor

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Abstract - In this paper we have characterized $(LCS)_n$ -manifolds with D -Conformal curvature tensor, concircular curvature tensor and projective curvature tensor.

Keywords and phrases : $(LCS)_n$ - Manifold, D - conformal curvature tensor, Projective curvature tensor, Concircular curvature tensor.

I. Introduction

An n -dimensional Lorentzian manifold M is smooth connected para contact Hausdorff manifold with Lorentzian metric g, i.e., M admits a smooth symmetric tensor field g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to \Re$ is a non degenerate inner product of signature $(-,+,\ldots.+)$ where T_pM denotes the tangent space of M at p and $\Re$ is the real number space. A non-zero vector $\upsilon \in (T_pM)$ is said to be time like (res., non-space like, null, space like) if it satisfies $g_p(\upsilon,\upsilon) < 0$ (resp., $\le 0, = 0, > 0$) (see [2]).

Definition1.1. In a Lorentzian manifold (M, g) a vector field P defined by

$$g(X, P) = A(X)$$

for any vector field $X \in \chi(M)$ is said to be concircular vector field if

$$(\nabla_X A)(Y) = \alpha\left[g(X,Y) + \omega(X)A(Y)\right]$$

where α is a non zero scalar function, A is a 1-form and ω is a closed 1-form.

Let M^n be a Lorentzian manifold admitting a unit time like concircular vector field ξ, called the characteristic vector field of the manifold. Then we have

(1.1) $$g(\xi,\xi) = -1$$

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Since ξ is the unit concircular vector field, there exist a non zero 1-form η such that

(1.2) $$g(X,\xi)=\eta(X)$$

the equation(1.2) of the following form holds

(1.3) $$(\nabla_X \eta)(Y)=\alpha[g(X,Y)+\eta(X)\eta(Y)] \quad (\alpha \neq 0)$$

for all vector field X,Y, where ∇ denotes the operator of covariant differentiation with respect to Lorentzian metric g and α is a non zero scalar function satisfying

(1.4) $$(\nabla_X \alpha)=(X\alpha)=\rho\eta(X),$$

where ρ being a scalar function. If we put

(1.5) $$\phi X=\frac{1}{\alpha}\nabla_X \xi,$$

then from (1.3) and (1.5), we have

(1.6) $$\phi^2 X=X+\eta(X)\xi,$$

from which it follows that ϕ is a symmetric $(1,1)$ -tensor. Thus the Lorentzian manifold M^n together with unit time like concircular vector field ξ, its associate 1-form η and $(1,1)$ –tensor field ϕ is said to be $(LCS)n$ -manifold. Especially, if we take $\alpha=1$, then the manifold becomes LP-Sasakian structure of Matsumoto (see [3]).

The D -conformal curvature tensor B (see [4]), projective curvature tensor P, concircular curvature tensor C (see [5]) on a Riemannian manifold (M^n,g), $(n>4)$ are defined as

(1.7)
$$B(X,Y)Z=R(X,Y)Z+\frac{1}{n-3)}\begin{bmatrix}S(X,Z)Y-S(Y,Z)X+g(X,Z)QY-g(Y,Z)QX-S(X,Z)\eta(Y)\xi\\+S(Y,Z)\eta(X)\xi-\eta(X)\eta(Z)QY+\eta(Y)\eta(Z)QX\end{bmatrix}$$
$$-\frac{(k-2)}{(n-3)}\{g(X,Z)Y-g(Y,Z)X\}+\frac{k}{(n-3)}\begin{Bmatrix}g(X,Z)\eta(Y)\xi-g(Y,Z)\eta(X)\xi\\+\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X\end{Bmatrix}$$

(1.8) $$P(X,Y)Z=R(X,Y)Z-\frac{1}{(n-1)}\{S(Y,Z)X-S(X,Z)Y\}$$

(1.9) $$C(X,Y)Z=R(X,Y)Z-\frac{r}{n(n-1)}\{g(Y,Z)X-g(X,Z)Y\}$$

respectively, where r is the scalar curvature, Q is the Ricci tensor and $k=\frac{(r+2)(n-1)}{(n-2)}$.

Ref.

[3] K.Matsumoto;On Lorentzian almost paracontact manifolds,Bull.Yamagata Univ.Natur.Soci.12(1989),151-156.

II. Preliminaries

A differentiable manifold M of dimension n is called $(LCS)n$ -manifold if it admits a $(1,1)$ – tensor ϕ, a contravarient vector field ξ, a covariant vector field η and a Lorentzian metric g which satisfy the following.

Notes

(2.1) $$\eta(\xi) = -1$$

(2.2) $$\phi^2 = I + \eta \otimes \xi$$

(2.3) $$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y)$$

(2.4) $$g(X,\xi) = \eta(X)$$

(2.5) $$\phi\xi = 0 , \quad \eta(\phi X) = 0$$

for all $X,Y \in TM.$ Also in a $(LCS)n$ –manifold the following relations are satisfied (see[4]).

(2.6) $$\eta(R(X,Y)Z) = (\alpha^2 - \rho)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]$$

(2.7) $$R(X,Y)\xi = (\alpha^2 - \rho)[\eta(Y)X - \eta(X)Y]$$

(2.8) $$R(\xi, X)Y = (\alpha^2 - \rho)[g(X,Y)\xi - \eta(Y)X]$$

(2.9) $$R(\xi, X)\xi = (\alpha^2 - \rho)[\eta(X)\xi + X]$$

(2.10) $$(\nabla_X \phi)(Y) = \alpha[g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X]$$

(2.11) $$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X)$$

(2.12) $$S(\phi X, \phi Y) = S(X,Y) + (n-1)(\alpha^2 - \rho)\eta(X)\eta(Y)$$

(2.13) $$(X\rho) = d\rho(X) = \beta\eta(X)$$

Definition.2.1. A Lorentzian concircular structure manifold is said to be η–Einstein if the Ricci operator Q satisfies

$$Q = aId + b\eta \otimes \xi ,$$

where a and b are smooth functions on the manifolds, In particular if $b = 0$, then M is an Einstein manifold.

III. Main Results

Theorem 3.1. There is no $(LCS)_n$ - manifold that satisfying $B(X,Y)Z = 0.$

Proof. Assume that in a $(LCS)_n$ -manifold

(3.1) $$B(X,Y)Z = 0.$$

Then it is follows from (1.7) and (3.1) that

(3.2)
$$R(X,Y)Z = -\frac{1}{(n-3)}\begin{bmatrix} S(X,Z)Y - S(Y,Z)X + g(X,Z)QY - g(Y,Z)QX \\ -S(X,Z)\eta(Y)\xi + S(Y,Z)\eta(X)\xi - \eta(X)\eta(Z)QY \\ +\eta(Y)\eta(Z)QX \end{bmatrix}$$
$$+\frac{(k-2)}{(n-3)}\{g(X,Z)Y - g(Y,Z)X\} - \frac{k}{(n-3)}\begin{Bmatrix} g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi \\ +\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \end{Bmatrix}$$

It can also written as

(3.3)
$$g(R(X,Y)Z,U) = -\frac{1}{(n-3)}\begin{bmatrix} S(X,Z)g(Y,U) - S(Y,Z)g(X,U) + g(X,Z)S(Y,U) \\ -g(Y,Z)S(X,U) - S(X,Z)\eta(Y)\eta(U) + S(Y,Z)\eta(X)\eta(U) \\ -\eta(X)\eta(Z)S(Y,U) + \eta(Y)\eta(Z)S(X,U) \end{bmatrix}$$
$$+\frac{(k-2)}{(n-3)}\{g(X,Z)g(Y,U) - g(Y,Z)g(X,U)\}$$
$$-\frac{k}{(n-3)}\begin{Bmatrix} g(X,Z)\eta(Y)\eta(U) - g(Y,Z)\eta(X)\eta(U) \\ +\eta(X)\eta(Z)g(Y,U) - \eta(Y)\eta(Z)g(X,U) \end{Bmatrix}$$

Taking $X = U = \xi$ in (3.3) and using (2.1) (2.4) and (2.11), it becomes

(3.4)
$$\left[\frac{(\rho-\alpha^2)(5n+3)+2(k-1)}{(n-3)}\right]\{g(Y,Z)+\eta(Y)\eta(Z)\} = 0$$

Then (3.4) implies that

(3.5)
$$g(Y,Z) + \eta(Y)\eta(Z) = 0.$$

From (3.5) and (2.3) it is seen that $g(\phi Y, \phi Z) = 0$, however, as this is not possible.

This proves the theorem3.1.

Theorem3.2. A Ricci D-conformal semi-symmetric $(LCS)_n$-manifold is an Einstein manifold with scalar curvature $r = 2n^2(\alpha^2 - \rho)$.

Proof. From (1.7) by virtue of (2.6) and (2.11), we obtain

(3.6)
$$\eta(B(X,Y)Z = \left[(\alpha^2 - \rho) + \frac{(k-2)}{(n-3)}\right]\{g(Y,Z)\ \eta(X) - g(X,Z)\ \eta(Y)\}$$

From (3.6), it follows that

(3.7)
$$\eta(R(X,Y)\xi) = 0.$$

Notes

and

(3.8) $$\eta(B(\xi,Y)Z = \left[(\alpha^2-\rho)+\frac{(k-2)}{(n-3)}\right]\{-g(Y,Z)-\eta(Y)\eta(Z)\}$$

Assume that M^n is a Lorentzian concircular manifold satisfies the condition

Notes

(3.9) $$B(X,Y)S(Z,W)=0.$$

From (3.9), it is obtained that

(3.10) $$S(B(X,Y)Z,W)+S(Z,B(X,Y)W)=0$$

Taking $X=W=\xi$ in (3.10) and using (3.6) (3.7) (3.8) and (2.11), we get

(3.11) $$S(Y,Z)=2n(\alpha^2-\rho)g(Y,Z)$$

This proves the theorem3.2.

Definition 3.1.A Riemannian manifold (M^n,g) is termed as Ricci D-conformal semi-symmetric if $B(X,Y)S=0.$

Theorem3.3. There is no $(LCS)_n$ -manifold that satisfying $R(X,Y)B=0.$

Proof. Assume that in a $(LCS)_n$ -manifold satisfies the conditions $R(\xi,Y)B=0.$, then it is expressed as

(3.12) $$R(X,Y)B(Z,V)W-B(R(X,Y)Z,V)W-B(Z,R(X,Y)V)W-B(Z,V)R(X,Y)W=0$$

for all vector field X,Y,Z,V and W on M^n.

For $X=\xi$, it is follows from (2.8) and (3.12) that

(3.13) $$(\alpha^2-\rho)\begin{bmatrix}B(Z,V,W,Y)-\eta(B(Z,V)W)\eta(Y)-g(Y,Z)\eta(B(\xi,V)W)+\eta(Z)\eta(B(Y,V)W)\\-g(Y,V)\eta(B(Z,\xi)W)+\eta(V)\eta(B(Z,Y)W)-g(Y,W)\eta(B(Z,V)\xi)+\eta(W)\eta(B(Z,V)Y\end{bmatrix}=0$$

In fact $Y=Z$ in (3.13) and by use of (3.6) (3.7) and (3.8) we have

(3.14) $$(\alpha^2-\rho)\left[B(Z,V,W,Y)-g(Z,Z)\eta(B(\xi,V)W)-g(Z,W)\eta(B(Z,V)\xi)W)+\eta(W)\eta(B(Z,V)Z\right]=0$$

From (3.14), by contracting we get

$$\left[\frac{-2(n-3)(\rho-\alpha^2)^2-2(\alpha^2-\rho)(k-1)}{(n-3)}\right]\{g(V,W)+\eta(V)\eta(W)\}=0$$

This implies that $g(V,W)=-\eta(V)\eta(W)$. Then from (2.3) we get $g(\phi V,\phi W)=0$, however, as this is not possible. This proves the theorem3.3.

Theorem3.4. A $(LCS)_n$ -manifold is projectively Ricci symmetric if and only if the manifold in an Einstein manifold.

Proof. Assume that in $(LCS)_n$ -manifold the condition $P(X,Y)\cdot S(Z,W)=0$ are satisfies, and then it can be expressed as

(3.15) $$S(P(X,Y)Z,W)+S(Z,P(X,Y)W=0$$

From (1.8) and (2.11) we get

(3.16) $$P(\xi,Y)Z=(\alpha^2-\rho)\left[g(Y,Z)\xi-\eta(Z)Y\right]-\frac{1}{(n-1)}\left[S(Y.Z)\xi-(n-1)(\alpha^2-\rho)\eta(Z)Y\right]$$

Taking $X=\xi$ in (3.15) by virtue of (2.11) and (3.16) we obtain

(3.17) $$S(Y,Z)=(n-1)(\alpha^2-\rho)\,g(Y,Z).$$

This proves the theorem3.4.

Theorem3.5. A $(LCS)_n$ -manifold is concircurly Ricci symmetric if and only if either scalar curvature $r=n(n-1)(\alpha^2-\rho)$ or the manifold in an Einstein manifold.

Proof. Assume that in $(LCS)_n$ -manifold satisfies the condition $C(X,Y)\cdot S(Z,W)=0$, and then it can be expressed as

(3.18) $$S(C(X,Y)Z,W)+S(Z,C(X,Y)W=0$$

From (1.9) and (2.11), we have

(3.19) $$C(\xi,Y)Z=\left[(\alpha^2-\rho)-\frac{r}{n(n-1)}\right]\{g(Y,Z)\xi-\eta(Z)Y\}$$

Taking $X=\xi$ in (3.18) by virtue of (3.19) and (2.11), we get

(3.20) $$\left[(\alpha^2-\rho)-\frac{r}{n(n-1)}\right]\{S(Y,Z)-2n(\alpha^2-\rho)g(Y,Z)\}=0$$

This implies that either $r=n(n-1)(\alpha^2-\rho)$ or $S(Y,Z)=2n(\alpha^2-\rho)g(Y,Z)$

This proves the theorem 3.5

Theorem3.6.A $(LCS)_n$ -manifold satisfies the condition $P(\xi,X)\cdot S=0$ if and only if the M^n is an Einstein manifold with scalar curvature $r=2n^2(\alpha^2-\rho)$.

Notes

Proof. The condition $P(\xi, X)\cdot S = 0$ implies

(3.21) $$S(P(\xi, X)Y, \xi) + S(Y, P(\xi, X)\xi = 0$$

By virtue of (2.8) and (2.11) ,equation (1.8) reduces that

(3.22) $$S(P(\xi, X)Y, \xi) = -2n(\alpha^2 - \rho)^2\{g(X,Y) + \eta(X)\eta(Y)\} + \frac{1}{(n-1)} 2n(\alpha^2 - \rho)\left\{S(X,Y) + 2n(\alpha^2 - \rho)\eta(X)\eta(Y)\right\}$$

and

(3.23) $$S(P(\xi, X)\xi, \xi) = (\alpha^2 - \rho)^2\left\{S(X,Y) + 2n(\alpha^2 - \rho)\eta(X)\eta(Y)\right\} - \frac{1}{(n-1)} 2n(\alpha^2 - \rho)\left\{S(X,Y) + 2n(\alpha^2 - \rho)\eta(X)\eta(Y)\right\}$$

Using (3.22)(3.23) in (3.21) , we get

$$S(X,Y) = 2n(\alpha^2 - \rho)g(X,Y)$$

This proves the theorem3.6

Corollary1. In $(LCS)_n$ -manifold the D-conformal curvature tensor B satisfies

(3.24) $$B(X,Y)\xi = \lambda\{\eta(Y)X - \eta(X)Y\}$$

where $$\lambda = \frac{(n+1)(\rho - \alpha^2) + (k-1)}{(n-3)}$$

Proof. Using (2.7) and (2.11) in (1.7) we get (3.24).

Definition3.2. The rotational motion (curl) of D-conformal curvature tensor B on a Riemannian manifold is given by

(3.25) $$Rot\, B = (\nabla_U B)(X,Y,Z) + (\nabla_X B)(U,Y,Z) + (\nabla_Y B)(U,X,Z) - (\nabla_Z B)(X,Y)U = 0$$

By virtue of second Bianchi identity

(3.26) $$(\nabla_U B)(X,Y,Z) + (\nabla_X B)(Y,U,Z) + (\nabla_Y B)(U,X,Z) = 0$$

Equation (3.25) reduces to

$$Rot\, B = -(\nabla_Z B)(X,Y)U$$

If the D-conformal curvature tensor is irrotational then curl B =0 and by (3.26),we have

$$(\nabla_Z B)(X,Y)U = 0$$

This implies that

(3.27) $$\nabla_Z\{B(X,Y)U\}=B(\nabla_Z X,Y)U+B(X,\nabla_Z Y)U+B(X,Y)\nabla_Z U$$

In view of (3.27) with $U=\xi$ it is seen that

(3.28) $$\nabla_Z\{B(X,Y)\xi\}=B(\nabla_Z X,Y)\xi+B(X,\nabla_Z Y)\xi+B(X,Y)\nabla_Z\xi$$

Theorem.3.7. If the *D*- conformal curvature tensor in *(LCS)*$_n$ -manifold is irrotational then the *D* - conformal curvature tensor *B* is given by (3.30)

Proof. Using (3.24) and (1.5) in (3.28), we get

(3.29) $$B(X,Y)\phi Z=\lambda\left[(\nabla_Z\eta)(Y)X-(\nabla_Z\eta)(X)Y\right]$$

Replacing Z by ϕZ in (3.29) by using (1.3) and (1.6) it is seen that

(3.30) $$B(X,Y)Z=\lambda\{g(\phi Z,Y)X-g(\phi Z,X)Y-\eta(Y)\eta(Z)X+\eta(X)\eta(Z)Y\}$$

This proves the theorem3.7.

Theorem3.8. If the D -conformal curvature tensor in $(LCS)_n$ -manifold is irrotational then the manifold is an η–Einstein manifold with scalar curvature

$$\tau=\left[\frac{n(n-3)+2n\{(n-1)(\alpha^2-\rho)-(k-2)\}-(n-1)(\alpha^2-\rho)}{(n-1)}\right]$$

Proof. Using (3.21) in (1.7) the curvature tensor of B in $(LCS)_n$ -manifold is given by

(3.31) $$R(X,Y)Z=\lambda\begin{bmatrix}g(Z,Y)X-g(Z,X)Y\\-\eta(Y)\eta(Z)X+\eta(X)\eta(Z)Y\end{bmatrix}-\frac{1}{(n-3)}\begin{bmatrix}S(X,Z)Y-S(Y,Z)X+g(X,Z)QY\\-g(Y,Z)QX-S(X,Z)\eta(Y)\xi\\+S(Y,Z)\eta(X)\xi-\eta(X)\eta(Z)QY\\+\eta(Y)\eta(Z)QX\end{bmatrix}$$
$$+\frac{(k-2)}{(n-3)}\{g(X,Z)Y-g(Y,Z)X\}-\frac{k}{(n-3)}\begin{Bmatrix}g(X,Z)\eta(Y)\xi-g(Y,Z)\eta(X)\xi\\+\eta(X)\eta(Z)Y-\eta(Y)\eta(Z)X\end{Bmatrix}$$

Let $X_i,\ i=1,2,3,\dots,n$ be an orthonornal basis of the tangent space at any point .Then the sum for $1\le i\le n$ of the relation (3.31) with $Y=D=X_i$, yields

(3.32) $$\sum R(X,X_i)X_i=\lambda[g(X_i,X_i)X-g(X,X_i)X_i]-\frac{1}{(n-3)}\begin{bmatrix}S(X,X_i)X_i-S(X_i,X_i)X+g(X,X_i)QX_i\\-g(X_i,X_i)QX+S(X_i,X_i)\eta(X)\xi\end{bmatrix}$$
$$+\frac{(k-2)}{(n-3)}\{g(X,X_i)X_i-g(X_i,X_i)X\}+\frac{k}{(n-3)}\{g(X_i,X_i)(X)\xi\}$$

Notes

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The Ricci tensor S is given by

(3.33) $$SX,Y) = \sum g(R(X,X_i)X_i,Y) + g(X,Y)$$

Taking inner product of (3.32) with Y and by virtue of (3.31) and (3.33), we get

(3.34) $$S(X,Y) = a\ g(X,Y) + b\eta(X)\eta(Y)$$

Notes

where $$a = \left[\frac{(n-3)+2(n-1)(\alpha^2-\rho)-2(k-2)}{(n-1)}\right],$$ and $b = (\alpha^2 - \rho)$

This implies that the manifold is an η–Einstein manifold.

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3. Submission of Manuscripts,

4. Manuscript’s Category,

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6. After Acceptance.

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The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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