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Mathematics and Decision Sciences

Discovering Thoughts and Inventing Future

Air Traffic Control
Sweden, Europe

Highlights

- Trimming Simple Hypergraphs
- Note on Semilattice Decompositions
- I-convergent Dierence Sequence
- Nonlocal Boundary Conditions

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A Survey of Mathematical Approaches to Two Phase Fluid Flow Techniques

By Dr. V.Venkataraman, Dr. K.Kannan & R.Dharmarajan
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Abstract - Two phase fluid systems are concerned with the motion of a liquid or gas containing immiscible inert identical solid particles. Common examples of two phase fluid systems include blood flow in arteries, flows in rocket tubes, dust in gas cooling systems to enhance heat transfer processes, movement of inert solid particles in atmosphere, and other suspended particles in seas and oceans. Naturally, studies of these systems are mathematically interesting and useful for modeling the physical phenomena to address complex issues related to fluid flow characteristics.

Keywords : Two phase fluid, unsteady flow, porous medium, transverse magnetic field, dissipative heat, Darcy's law.

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Strictly as per the compliance and regulations of :

A Survey of Mathematical Approaches to Two Phase Fluid Flow Techniques

Dr. V.Venkataraman [α], Dr. K.Kannan [σ] & R.Dharmarajan [ρ]

Abstract - Two phase fluid systems are concerned with the motion of a liquid or gas containing immiscible inert identical solid particles. Common examples of two phase fluid systems include blood flow in arteries, flows in rocket tubes, dust in gas cooling systems to enhance heat transfer processes, movement of inert solid particles in atmosphere, and other suspended particles in seas and oceans. Naturally, studies of these systems are mathematically interesting and useful for modeling the physical phenomena to address complex issues related to fluid flow characteristics.

Keywords : Two phase fluid, unsteady flow, porous medium, transverse magnetic field, dissipative heat, Darcy's law.

I. Importance of Two Phase Fluid Flows

For many years, engineers and scientists have been interested in gas – solid particle flows which arise in many industrial applications.

a) Heat Transfer [26]

In many engineering problems, we come across dusty fluid flows which involve body forces such as buoyant force. Practical application of these flows may be found in heat exchangers, utilizing liquid metal or liquid sodium coolants in the area of thermal instability in the study of heat transfer. A few applications include Gas cooling systems and hydro cyclones.

b) Cement Process Industry [37]

In cement factories, powdered fine cement is taken to silo by pneumatic conveying which means mixing of fine cement particles with air,. The cement dust air is transported through pipe lines to storage bins and the air is devoid of fine particles in a cyclone separator followed by electrostatic precipitators.

c) Steel Manufacturing Industry[40]

Finely ground iron ore is mixed with water to make a suspension, and then piped down to the desired location, separated at the port and shipped.

d) Fluidized Bed[37]

- Coal gasification for power generation.
- Reduction of gasoline from petroleum traction.
- Bed slag drier cement for treatment of special grade cement used for constructing bridges.

Ref.

37. Kannan.K(2000). "Study of some two phase fluid flow problems without and with heat conduction", **Ph.D. Thesis**, Alagappa University, Tamilnadu, India.

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e) *Magneto hydrodynamic generators (MHG)[26]*

Solid particles in form of ash or soot are suspended in the conducting fluid as a result of the corrosion during the combustion processes in MHD generators.

f) *Bio Fluids[40]*

Examples of dust – laden immiscible fluids separated by an interface occur in the locomotion of micro organisms, where the heating of long slender flagella or fields of hair like cilia propel the organisms through the surrounding fluid.

II. Basic Assumptions in two Phase Fluid Flows

In order to formulate the fundamental equations of motion of the two-phase fluid flows [40] in a reasonably simple manner and to bring out the essential features certain basic assumptions are made. They are as follows:

a) The fluid is an incompressible Newtonian fluid.
b) Dust particles are assumed to be spherical in shape, all having the same radius and mass, and are undeformable.
c) The bulk concentration (i.e., concentration by volume) of the dust is very small so that the net effect of the dust on the fluid particles is equivalent to an extra force KN $(\vec{q}_p - \vec{q})$ [1] per unit volume, where $\vec{q}(\vec{x}, t)$ is the velocity vector of the fluid, $\vec{q}_p(\vec{x}, t)$ is the velocity vector of the dust particles. N_o, the number density of the dust particles, i.e., the number of dust particles per unit volume of the mixture and K, the Stokes drag constant which is $6\pi\mu a$ for spherical particles of radius a, μ being the coefficient of viscosity of the clean fluid and it is also assumed that the Reynolds number of the relative motion of the dust and the fluid is small compared with unity.
d) The effect of the dust enters through the two parameters- f, the mass concentration parameter and τ, the relaxation time parameter of the dust particles. τ is a measure of time for the dust to adjust to changes in the fluid velocity. The former describes how much dust is present and the latter is determined by the size of the individual particles. Making the dust fine decreases τ, and making it coarse increases τ, in a manner proportional to the surface area of the particles.
e) The density of the material in the dust particles (ρ_1) is high compared with the fluid density (ρ) so that the mass concentration $\left(\frac{\rho}{\rho_1}\right)$ is small.
f) The buoyancy force on the particles is neglected since $\left(\frac{\rho}{\rho_1}\right)$ is small and so the fluid-phase supplies the entire pressure.
g) For sufficiently small particles, the velocity of sedimentation is small compared to the characteristic velocity and can be neglected.
h) Three dimensional parameters can be constructed out of the quantities defining the flow and the dust: the Reynolds number R, the mass concentration of dust f and another one involving the relaxation τ .
i) The distortion of the flow around the dust particles is neglected.
j) The particle will in general have a different temperature than that of the surrounding fluid, and therefore there will be temperature defects. Because of these temperature

Ref.

26. Ramamurthy, V. (1987). "Oscillating free conversion over two dimensional bodies in dusty fluid", **Ph.D. Thesis**, I.I.T., Khargpur, India.

defects, there will be heat transfer between the two phases. The heat transfer from a particle to the fluid has the form $\frac{\rho_p c_s (T_p - T)}{\tau_T}$, where $\rho_p = N_o m$, N_o is the number [40] density , m is the mass of each particle , ρ_p is the partial density of the dust particles i.e., mass of particle phase per unit volume of mixture of the two phases, c_s is the specific heat of the solid particles, c_p is the specific heat at constant pressure for the fluid, $\tau_T = \left(\frac{3}{2}\right) \Pr \left(\frac{c_s}{c_p}\right) \tau$ is the thermal equilibration time, $\Pr = \left(\frac{\mu c_p}{k}\right)$ = Prandtl number, k is the thermal conductivity of the fluid, T is the fluid temperature, T_p is the temperature of the dust and $\tau = \left(\frac{2}{9}\right)\left(\frac{\rho_p a^2}{\mu}\right)$. Consequently the heat transfer from the fluid to the dust will be negative of the heat transfer from the dust to the fluid.

III. Basic Governing Equations [36, 38, 39, 40]

Equation of continuity for the fluid

$$\operatorname{div} \vec{q} = 0 = 0 \tag{4.1}$$

Equation of conservation of momentum for the fluid:

$$\rho\left[\frac{\partial \vec{q}}{\partial t} + \left(\vec{q}\,.\mathrm{grad}\right)\vec{q}\right] = -\nabla p + \mu \nabla^2 \vec{q} + K N_0 \left(\vec{q}_p - \vec{q}\right) \tag{4.2}$$

The conservation of Energy equation for the fluid:

$$\rho c_p \left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right] = \frac{\partial}{\partial x}\left[K\frac{\partial T}{\partial x}\right] + \frac{\partial}{\partial y}\left[K\frac{\partial T}{\partial y}\right] + \frac{\partial}{\partial z}\left[K\frac{\partial T}{\partial z}\right] + \frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y} + w\frac{\partial p}{\partial z} + \phi + \frac{\rho_p c_p (T_p - T)}{\tau_T} \tag{4.3}$$

Where ϕ represents the viscous dissipation function given by

$$\phi = 2\left\{\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right\}$$

Ref.

40. Venkataraman.V (2007). " Study of a few two phase fluid flow problems in the presence and absence of magnetic fields ", **Ph.D. Thesis**, SASTRA University, Tamilnadu, India.

$$+\left\{\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right\}^2+\left\{\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}\right\}^2 \tag{4.4}$$

$$+\left\{\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right\}^2-\frac{2}{3}\left\{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right\}^2$$

The Continuity equation for the particle phase

$$\frac{\partial N}{\partial t}+\operatorname{div}\left[N\,\vec{q}_p\right]=0 \tag{4.5}$$

The conservation of momentum equation for the particle phase

$$N_o\, m\left[\frac{\partial \vec{q}_p}{\partial t}+(\vec{q}\,.\nabla)\vec{q}_p\right]=K\,N_o\,(\vec{q}-\vec{q}_p) \tag{4.6}$$

The conservation of energy for particle phase

$$\rho c_p\left[\frac{\partial T_p}{\partial t}+u_p\frac{\partial T_p}{\partial x}+v_p\frac{\partial T_p}{\partial y}+w_p\frac{\partial T_p}{\partial z}\right]=\frac{\rho_p c_p\left(T_p-T\right)}{\tau_T} \tag{4.7}$$

These independent equations are to be solved for unknowns

$$\vec{q}\equiv(u,v,w),\ \ \vec{q}_p\equiv(u_p,v_p,w_p),N,\ p,T,T_p\ .$$

When volume fraction of particle phase is taken into account, the equations of conservation of momentum for the fluids and dust take the form:

$$\rho(1-\phi)\frac{\partial \vec{q}}{\partial t}+\left(\vec{q}.\nabla\right)\vec{q}=(1-\phi)\left(-\nabla p+\mu\nabla^2\vec{q}\right)+K\,N_0\left(\vec{q}_p-\vec{q}\right) \tag{4.8}$$

$$N_o m\left[\frac{\partial \vec{q}_p}{\partial t}+(\vec{q}\,.\nabla)\vec{q}_p\right]=\phi(-\nabla p+\mu\nabla^2\vec{q}_p)+K\,N_o\,(\vec{q}-\vec{q}_p) \tag{4.9}$$

IV. Boundary Conditions[1 ,26, 37, 40]

The fundamental equations stated above are to be solved under appropriate boundary conditions to determine the flow fields of the fluid and dust particles. In general, the boundary conditions are as follows:

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a) The boundary conditions on the fluid phase

1. There will be no mass transfer at the solid boundary.
2. The fluid velocity vanishes at the solid boundary.
3. The fluid velocity must approach the free stream value as y approaches infinity. (For large distance)
4. The temperature of the fluid at the plate is that of the plate.
5. The temperature of the fluid must approach free stream value as y approaches infinity.

b) The boundary conditions on the dust particles

1. The dust particles may slip at the boundary and the boundary conditions are to be taken from ambient conditions.
2. The particle phase temperatures must approach their free stream values as y approaches infinity.

V Research Approaches

The importance of two phase particulate flows has led to the development of several multiphase theories. These theories are based mainly on either the Lagrangian approach or Eulerian approach. In Lagrangian approach the fluid treats as a continuum while the particle phase is governed by the kinetic theory. Motivated by the advantages of Eulerian approach establishing mathematical models for various types of flows augurs well and their utility in carrying out research studies, Picart et al[24] suggested Eulerian model as the tracking approach in evaluating the dispersion coefficients analytically from basic principles. Eulerian approach is thus and is also expected to provide valuable information for further advances of the tracking approach which is very far from being fully developed[32]. K-ε–lunate″ model is used for prediction of Turbulence fields. This model is supplemented with algebraic relations deduced from a second-order closure scheme. Then, the dispersion of discrete particles transported by the turbulence fields predicted above is computed. Modelling of the discrete particle dispersion is based on an Eulerian approach. The (monodispersed) particles are considered as a continuous field for which a transport equation is written. The transport equation contains a dispersion tensor which is computed in the framework of the (slightly extended) Tchen theory, assuming a two- parameter family of Lagrangian correlation functions for the fluid particles. Modifications can be included to account for crossing-trajectory effects.

VI. Mathematical Investigations

Saffman[1] carried out pioneering work on the stability of a laminar flow of a dusty gas by introducing relaxation parameter. By describing the motion of a gas carrying small dust particles, he derived the equations satisfied by small disturbances of a steady laminar flow. He also discussed the effect of the dust, and described by two parameters; the concentration of dust and a relaxation time τ which measures the rate at which the velocity of a dust particle adjusts to changes in the gas velocity and depends upon the size of the individual particles.

Marble[2] made a comparative study of experimental and theoretical results on gas and particle temperature in a rocket nozzle and pointed out the accuracy of predicted results of particle lag in rocket nozzles. Investigations into the dynamics of two phase flow received a strong impetus from concern with the losses of solid rocket motor performance associated with substantial mass fractions of solid particles in the exhaust.

Ref.

2. Marble, F.E. (1963). "Dynamics of a gas containing small solid particles", *Proceedings of Fifth AGARD Colloquium Combustion and Propulsion (1962)*, Pergamon Press, Oxford, pp.175-213.

Investigations on heat transfer by natural convection of two phase fluids in non-porous media have been made by Sukomel, Tsvetkov, and Kerimov[3]. Also the authors discussed the natural convection of a dusty fluid in an infinite rectangular channel with differentially heated vertical walls and adiabatic horizontal walls and solved by using the central and second upwind differencing methods. Its seems that the heat transfer rate decreases with an increase of mass concentration of dust particles, but it increases with an increase of the Rayleigh number.

Baral [4] studied the problem of the flow of a conducting dusty gas occupying a semi infinite space in the presence of a transverse magnetic field.

Rao [5] studied the unsteady flow of a dusty viscous liquid in a channel and a pipe under the influence of a pressure gradient varying in magnitude but not in direction also discussed the unsteady flow of a dusty viscous liquid through a circular cylinder.

Healy and Young [6] have investigated various aspects of hydrodynamic and hydro magnetic two phase flows in a non - rotating system. Pai[7] has discussed the fundamental equations of a mixture of a gas with solid particles and Rudinger [8] studied the effective drag coefficient for gas – particle flow in shock tubes.

Healy [9] investigated various aspects of hydro dynamic and hydro magnetic two phase flows in a non rotating system . Reddy [10] discussed the flow of a dusty viscous liquid through rectangular channel, and also investigated the effect of the presence of the dust particles on fluid particles and plotted it graphically.

Soundalgekar [11,12] addressed free convection effects in dissipative dusty medium. Dube and Sharma [13] discussed in their note on the flow of a dusty viscous liquid in a channel bounded by two parallel flat plates under the influence of a constant pressure gradient.

Datta and Jana [14] studied the effects of rotation in the dusty fluid flows in a channel. Gupta and Sharma [15] investigated the unsteady flow of a dusty viscous fluid through long confocal elliptical ducts when the axial pressure gradient is an arbitrary function of time in the presence of transverse magnetic field.

Soundalgekar [16] gave the closed form solution to the Stokes problem. He solved it by considering a vertical infinite plate, by taking into account the free-convection effects but neglecting viscous dissipative heat. The plate was assumed to be isothermal and the effects of heating or cooling of the plate on the flow were considered.

Soundalgekar, Bhat and Mohiuddin[17] discussed the analysis of free convection effects on Stoke's problem for a vertical plate in dissipative fluid using Finite difference method. Venkataraman and kannan [36] analyzed the flow pattern for dusty fluid too.The flow past an infinite vertical isothermal plate started impulsively in its own plane in a viscous incompressible fluid has been considered on taking into account the viscous dissipative heat. The coupled non-linear equations governing the flow are solved by finite-difference method. The velocity and temperature field have been shown graphically for G {greater-than or less-than} 0 (G, the Grashof number, $G > 0$, cooling of the plate by the free convection currents, $G < 0$, heating of the plate by the free convection currents) and the numerical values of the skin-friction and the rate of heat transfer are entered in tables. The effects of G and E (the Eckert number) on the flow field are also discussed.

The two-dimensional flow of a dusty fluid induced by sinusoidal wavy motion of an infinite wavy wall for low Reynolds numbers was discussed by Nag [18].

Ref.

11. Soundalgekar, V.M. (1973). "Free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction-I", *Proc. Roy. Soc. London, Ser.,* **A333**: 25-36.

Verma and Sarangi [19] discussed the unsteady flow of a dusty gas between two plates with roughness along their length by Fourier series method by considering the roughness to be small in comparison with the average distance between the plates.

Mitra.P [20] obtained an exact solution for the flow of an incompressible viscous dusty gas induced by two infinitely extended parallel plates when the lower plate is oscillating harmonically and the upper one is at rest in a rotating frame of reference. The velocity distributions for the dusty gas, dust particles and the clean gas for different values of the rotation parameter omega are represented graphically. It is found that with the increase in omega, the velocities of the dusty gas, dust particles, and the clean gas along the axis of x are decreased.

Palaniswamy and Purushotham [21] considered in the absence of dissipative processes the effect of fine dust on the internal gravity waves in the parallel shear flow of a Boussinesq fluid and have shown that the addition of dust particles results in a considerable reduction in the wave energy. The linear stability of plane parallel shear flows of a stably stratified incompressible fluid laden with uniformly distributed fine dust particles is studied. It is shown that the Miles criterion for stability, the Rayleigh–Synge criterion for instability, and Howard's semicircle theorem can be extended under the assumptions that the mass concentration is very small and that the relaxation time of the dust particles is very much less than the time scale characterizing the basic flow. The effect of fine dust is found to increase the region of instability. In addition, a semi ellipse criterion for instability has been given as an improvement over Howard's semicircle criterion.

The study reported by Jha [22] concerns the flow pattern set up in an electrically conducting dusty visco – elastic fluid of Maxwell type contained in the annular space between two concentric circular cylinders when the flow is induced by the transient axial motion of the cylinders under the combined influence of a constant pressure gradient and a uniform radial magnetic field strength fixed relative to the fluid.The response of the flow to the interaction between elastic, viscous and electromagnetic forces and to the interplay between the dust and fluid particles has been determined and the data are presented graphically. It is found that the presence of dust has negligible influence on skin friction drag at the inner cylinder, while the magnetic field increases it.

Debnath and Ghosh [23] investigated hydro magnetic Stokes flow in a rotating fluid with suspended small particles.

An initial investigation is made of the motion of an incompressible, viscous conducting fluid is made. The fluid is embedded with small spherical particles bounded by an infinite rigid non-conducting plate. Both the plate and the fluid are in a state of rotation with constant angular velocity about an axis normal to the plate. The flow is generated in the fluid-particle system is due to non-torsional oscillations of a given frequency superimposed on the plate in the presence of a transverse magnetic field. The operational method is used to derive exact solutions for the fluid and the particle velocities, and the wall shear stress. The small and the large time behaviour of the solutions is discussed. The ultimate steady-state solutions and the structure of the associated boundary layers are determined with physical implications. When the time is small, rotation and magnetic field affect the motion of the fluid relatively earlier than that of the particles. The motion for large time is through inertial oscillations of frequency equal to twice the angular velocity of rotation. The ultimate boundary layers are established through inertial oscillations. The shear stress at the plate is calculated for all values of the frequency parameter. The behaviour of the shear stress is discussed. The exact solutions for the velocity of fluid and the wall shear stress are evaluated numerically

Ref.

21. Palaniswamy, V.I. and Purushotham, C.M. (1984). "On the internal gravity waves in a shear flow of a dusty gas", *J. Math. Phy. Sci*, **18**(S): 89-99.

for the case of an impulsively moved plate. It is found that the drag and the lateral stress on the plate fluctuate during the non-equilibrium process of relaxation if the rotation is large. The present analysis is very general in the sense that many known results in various configurations are found to follow as special cases. The problem of unsteady laminar flow of a conducting dusty gas in a channel bounded by two parallel plates in the presence of a transverse magnetic field when the flow takes place due to sudden application of a pressure gradient which is any function of time has been considered by Rukmangadachari [25].

While studying oscillating free convection over two dimensional body in dusty fluid, Ramamurthy [26] showed that steady state heat transfer is always from wall to fluid and discussed the effect of Prandtl number on heat transfer of the dusty medium.

Sinclair and Jackson [27] investigated the Gas – Particle flow in a vertical pipe with particle – particle interactions.A theory for the fully-developed flow of gas and particles in a vertical pipe is presented. The relationship between gas pressure gradient and the flow rates of the two phases is predicted over the range of cocurrent and countercurrent flows. Velocity profiles and radial concentration profiles are also given. The model accounts for marked segregation of gas and particles in the radial direction.

Stoke's problem for free convection effects over two dimensional bodies in dusty fluid has been discussed by Ramamurthy [28].

Alichamkha[29], Ram and Takhar [30] studied to analyze fluid particle interaction mechanism in various types of flows. Dalal [31] investigated on heat transfer by natural convection of two phase fluids in non – porous medium.

Eulerian approach is thus and is also expected to provide valuable information for further advances of the tracking approach which is very far from being fully developed by Gouebet and Berlemont [32] .

Eulerian and Lagrangian methods are used for finding the behaviour of dust particles in turbulent flows

Debnath and Ghosh [33] also discussed the unsteady hydro magnetic flows of a dusty visco-elastic fluid between two moving plates. All the above investigations were carried out by the authors in a fluid system having non–rotational frame of reference. On the other hand the simultaneous influence of rotation and external magnetic field on electrically conducting two-phase fluid systems seems to be dynamically important and physically useful.

Kannan [34] analyzed the relative velocities of fluid and dust particles depending upon the concentration parameter while discussing unsteady flow of a dusty viscous fluid through circular cylinder.

Kannan and Ramamurthy [35] applied perturbation technique to obtain inner and outer solutions for the problem of induced dusty flow due to normal oscillation of the wavy wall and have shown an interested application of their result to mechanical engineering for the possibility of fluid transportation without external pressure.

Venkataraman and Kannan [36] discussed the the two phase fluid flow on an infinite vertical plate with viscous incompressible and dissipative heat. They discussed the effects of fluid flow for various values of Grashof and Eckert numbers. Also they observed that skin friction values increased in dusty fluid, heat transfer rate decreased for increase in mass concentration. For higher values of t, particles never allowed the reverse flow.

Ghosh.S and GhoshA.K [38] studied to analyze fluid – particle interaction mechanism in various types of flows.

Ref.

27. Sinclair, J.L. and Jackson, R. (1989). "Gas-particle flow in a vertical pipe with particle-particle interactions", *A. I. Che. J.*, **35**: 1473-1486.

Notes

An initial value investigation is made of the motion of an incompressible, viscous, conducting fluid with embedded small inert spherical particles bounded by an infinite rigid non-conducting plate. The unsteady flow is supposed to generate from rest in the fluid-particle system due to velocity tooth pulses being imposed on the plate in presence of a transverse magnetic field. It is assumed that no external electric field is acting on the system and the magnetic Reynolds number is very small. The operational method is used to obtain exact solutions for the fluid and the particle velocities and the shear stress at the plate. Quantitative analysis of the results is made to disclose the simultaneous effects of the magnetic field and the particles on the fluid velocity and the wall shear stress.

Kannan.K and Venkataraman.V [39] analyzed the viscous dusty fluid between two harmonically oscillating plates when a body force is applied at time t = 0 in the direction of the motion of the plates. It is inferred that growth of velocity of both the phases is rapid in the early stages but soon it tends to follow a steady pattern. When there is no body force, reverse type of flow occurs initially which shoots up rapidly to follow an oscillatory pattern.

Kannan.K and Venkataraman.V[41] analytically studied the heat transfer rate and free convection in an infinite and porous medium. They discussed velocity and temperature fields by using Rayleigh number. They extend the results up to second order mean in both fields. It is observed that, because of relaxation time of dust, the second order mean flow occurred.

Venkataraman.V and Kannan.K[42] has analyzed the two phase fluid flow in the existence of Magnetic field. They evaluated the numerical values of exact solution. Also found the reversal particle flow occurred only when the Eckert number is eight. Also in the presence of magnetic field, increase in mass concentration causes increase in fluid velocity and skin friction values decreases only when the rotation parameter values are decreased.

References Références Referencias

1. Saffman, P.G. (1961). "On the stability of laminar flow of a dusty gas", *Journal Fluid Mechanics*, **13**: 120-128.
2. Marble, F.E. (1963). "Dynamics of a gas containing small solid particles", *Proceedings of Fifth AGARD Colloquium Combustion and Propulsion (1962)*, Pergamon Press, Oxford, pp.175-213.
3. Sukomel, A.S., Tsvetkov, F.F. and Kerimov, R.V. (1967). "The study of local heat transfer from a tube wall to a turbulent flow of gas bearing suspended solid particles", *Teploenergetika*, **14**: 77.
4. Baral, M.C. (1968). "Plane parallel flow of a conducting dusty gas", *J. Phy. Soc. Japan*, **33**: 1732.
5. Rao, P.S.S. (1969). "Unsteady flow of a dusty viscous liquid through circular cylinder", *Def. Sci. J.*, **19**: 135-138.
6. Healy, J.V. and Young, H.T. (1970). "Oscillating two-phase channel flows", *Z. Angew. Math.Physics*, **21**: 454.
7. Pai, S.I. (1970). "A Review of Fundamental Equations of the mixture of a gas with solid particles", Institute for Fluid Dynamics and Applied Mathematics, University of Maryland, Tech. Note BN-668.
8. Rudinger, G. (1970). "Effective drag coefficient for gas-particle flow in shock tubes", *Trans. Am. Soc. Mech. Engg. J. Bas. Engg.*, **D92**: 165.

9. Healy, J.V. and Young, H.T. (1972). "The Stokes' problems for a suspension of particles", *Astronautica Acta*, **17**: 851.
10. Reddy, Y.B. (1972). "Flow of a dusty viscous liquid through rectangular channel", *Def. Sci. J.*, **22**: 149.
11. Soundalgekar, V.M. (1973). "Free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction-I", *Proc. Roy. Soc. London*, *Ser.*, **A333**: 25-36.
12. Soundalgekar, V.M. (1973). "Free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction-II", *Proc. Roy. Soc. London*, *Ser.*, **A333**: 37-50.
13. Dube, S.N. and Sharma, C.L. (1975). "A note on unsteady flow of a dusty viscous liquid in a channel bounded by two parallel flat plates", *J. Phy. Soc. Japan*, **38**: 297.
14. Datta, N. and Jana, R.N. (1976). "Oscillatory flow of a dusty gas in a rotating frame", *Bull. Math. de. la. Soc. Sci. Math. de. la. R.s.de Roumaine, Tome*, **20**(68): **34**: 391-395.
15. Gupta, M. and Sharma, H.S. (1978). "Unsteady flow of a dusty viscous fluid through confocal electrical ducts in presence of transverse magnetic field", *Proc. Twenty-Third ISTAM Congress,* pp.275-284.
16. Soundalgekar, V.M. (1973). "Free convection effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction-I, II", *Proc. Roy. Soc. London*, *Ser.*, **A333**: 25-50.
17. Soundalgekar, V.M., Bhat, J.P. and Mohiuddin, M. (1979). "Finite difference analysis of free convection effects on Stokes problem for a vertical plate in dissipative fluid", *Intl. J. Engg. Sci.*, **17**(12): 1283-1288.
18. Nag, S.K. (1980). "Flow of a dusty fluid past a wavy moving wall", *J. Phy. Soc. Japan*, **49**: 391-395.
19. Verma, P.D. and Sarangi, K.C. (1981). "Unsteady flow of a dusty viscous fluid between two wavy walls", *Def. Sci. J.*, **31**: 109-123.
20. Mitra.P (1981), "Osillatory flow of a dusty gas between two parallel plates in a rotating" *Revue roumaine des sciences techniques. Serie de mecanique appliquee* 26 (1), pp. 47-60
21. Palaniswamy, V.I. and Purushotham, C.M. (1984). "On the internal gravity waves in a shear flow of a dusty gas", *J. Math. Phy. Sci.*, **18**(S): 89-99.
22. Jha, P.K. (1985). "Transient flow of a conducting dusty Maxwell fluid in an annulus", *Indian J. Tech.*, **23**: 289-293.
23. Debnath, L. and Ghosh, A.K. (1986). "Hydromagnetic stokes flow in a rotating fluid with suspended small particles", *Appl. Sci.*, **43**: 165-182.
24. Picart, A., Berlemont, A. and Gouesbet, G. (1986). "Modelling and predicting turbulence fields and the dispersion of discrete particles transported by turbulent flows", *Intl. J. Multiphase Flow*, **12**(2): 237-261.
25. Rukmangadachari, E. (1986). "Flow of a conducting dusty gas in a channel bounded by two parallel flat plates", *Acta Ciencia*, **12**: 66-70.
26. Ramamurthy, V. (1987). "Oscillating free conversion over two dimensional bodies in dusty fluid", **Ph.D. Thesis**, I.I.T., Khargpur, India.

Notes

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Notes

27. Sinclair, J.L. and Jackson, R. (1989). “Gas-particle flow in a vertical pipe with particle-particle interactions”, *A. I. Che. J.*, **35**: 1473-1486.
28. Ramamurthy, V. (1990). “Free convection effects on the Stokes problem for an infinite vertical plate in a dusty fluid”, *J. Math. Phy. Sci.*, **24**: 297.
29. Ali Chamkha, J. (1994). “Unsteady flow of a dusty conducting fluid through a pipe”, *Mech. Res. Communications*, USA, **21**: 3281-3288.
30. Ram, P.C. and Takhar, H.S. (1994). “Unsteady MHD flow with a suspension of spherical particles through a squared channel”, *Intl. J. Non-Linear Mech.*, Great Britain, **29**(5): 775-780.
31. Dalal, D.C. (1994). Unsteady flow and heat transfer of a dusty fluid, **Ph.D. Thesis**, I.I.T., Kharagpur, India.
32. Gouesbet, G. and Berlemont, A. (1999). “Eulerian and Lagrangian approaches for predicting the behaviour of discrete particles in turbulent flows”, *Progress in Energy and Combustion Science*, **25**: 133-159.
33. Debnath, L., Ghoash, B.C. and Ghosh, N.C. (2000). “The Hydrodynamic flow of a dusty viscoelastic fluid between two infinite parallel plates”, *Computers and Mathematics with Applications*, **39**: 103-116.
34. Kannan, K. (2000). “Unsteady flow of a dusty viscous fluid through circular cylinder”, *Appl. Sci. Periodical*, INDIA, **2**(4): 195-201.
35. Kannan, K. and Ramamurthy, V. (2001). “Induced dusty flow due to normal oscillation of wavy wall”, *Intl. J. Math. Math. Sci.*, **26**(4): 199-233.
36. Venkataraman.V and Kannan.K (2004). ”Numerical solution of Stokes problem for free convection effects in dissipative dusty medium”, *Intl. J. Math. Math. Sci.*, **72:** 3975 - 3988.
37. Kannan.K(2000).“Study of some two phase fluid flow problems without and with heat conduction”, **Ph.D. Thesis,** Alagappa University, Tamilnadu, India.
38. Ghosh.S. and Ghosh, A.K. (2005). “On Hydromagnetic flow of a two-phase fluid near a pulsating plate”, *Indian Journal of Pure and Applied Mathematics*, **36**(10): 529-540.
39. Kannan.K and Venkataraman.V (2007).”FDM for the effects of dust particles in dusty fluid flow between two harmonically oscilating plates”, *International Journal of Computing and Applications,* Dec 2007 issue: pp.119 - 126
40. Venkataraman.V (2007). “ Study of a few two phase fluid flow problems in the presence and absence of magnetic fields ”, **Ph.D. Thesis**, SASTRA University, Tamilnadu, India.
41. Kannan.K and Venkataraman.V (2010). ”Free convection in an infinite porous dusty medium induced by pulsating point heat source”, *International Journal of Computational and Mathematical Sciences*”. Vol.4, No.6. pp 296 – 304.
42. Kannan.K and Venkataraman.V (2010). “Unsteady Flow of a dusty Viscous fluid in a rotating channel in the presence of transverse Magnetic field.

Notes

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Solution of Einstein's Field Equations for Mixed Potential of a Radiating Star

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Abstract - We have studied the behaviour of a radiating star when the interior expanding, shearing fluid particles are traveling in geodesic motion. A systematic approach enables us to write the junction condition as a Riccati equation. In this article we obtained two new solutions in terms of elementary functions with assuming a separation of variables and also have discussed the physical significance of these solutions.

Keywords : *Radiating star, Vaidya exterior equation, Einstein's field equations, Exact solutions, Variable separation.*

GJSFR-F Classification : *MSC 2010: 83C15*

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Solution of Einstein's Field Equations for Mixed Potential of a Radiating Star

M. A. Helal [α], M. K. H. Chowdury [σ], M. M. R. Chowdhury [ρ] & M. A. Islam [ω]

Abstract- We have studied the behaviour of a radiating star when the interior expanding, shearing fluid particles are traveling in geodesic motion. A systematic approach enables us to write the junction condition as a Riccati equation. In this article we obtained two new solutions in terms of elementary functions with assuming a separation of variables and also have discussed the physical significance of these solutions.

Keywords : Radiating star, Vaidya exterior equation, Einstein's field equations, Exact solutions, Variable separation.

I. Introduction

The interior space-time of the collapsing radiating star should match to the exterior space-time described by the Vaidya solution in 1951. To obtain realistic analytic solutions, different authors constructed different models. De Oliviera et al (1985) proposed a radiating model of an initial interior static configuration leading to slow gravitational collapse. Herrera et al (2004) proposed a relativistic radiating model with a vanishing Weyl-tensor, in a first order approximation, without solving the junction condition exactly. Then Maharaja and Govender (2005) & Herrera et al (2006) solved the relevant junction condition exactly, and generated classes of solutions in terms of elementary functions which contain the Friedmann dust solution as special case. The first exact solution, with nonzero shear was obtained by Naidu et al (2006) in 2006, considering geodesic motion of fluid particles; later in 2008, Rajah and Maharaja (2008) obtained two classes of nonsingular solutions by assuming that the gravitational function Y(r,t) is a separable function and solving a Riccati equation. Recently S.

Thirukanesh and Maharaj (2010) demonstrate and obtained exact solutions systematically without assuming separable forms and not fixing the temporal evolution of the model. We further extended it and obtained two new solutions by assuming that the gravitational potential $Y(t, r)$ and $B\,(r,\ t)$ is a separable function.

II. The Model

In general relativity, the form for the interior space time of a spherically symmetric collapsing star with nonzero shear when the fluid trajectories are geodesies is given by the line metric.

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$$ds^2 = -dt^2 + B^2 dr^2 + Y^2(d\theta^2 + sin^2\theta d\phi^2) \quad (1)$$

Here B and Y are functions of both the temporal coordinate t and radial coordinate *r*. The fluid four − velocity vector u is given by $u^a = \delta_0^a$ which is comoving. For the line element (1), the four acceleration $\dot{u}^a$, the expansion scalar Θ, and the magnitude of the shear scalar are given by

$$\dot{u}^a = 0 \quad (2a)$$

$$\Theta = \frac{\boldsymbol{B}}{B} + 2\,\frac{Y}{Y} \quad (2b)$$

$$\xi = \frac{1}{3}\left(\frac{\dot{y}}{y} - \frac{\dot{B}}{B}\right) \quad (2c)$$

respectively, and dots denote the differentiation with respect to *t*. The energy momentum tensor for the interior matter distribution is described by

$$T_{ab} = (p + \rho)\, u_a u_b + p g_{ab} + \pi_{ab} + q_a u_b + q_a u_a \quad (3)$$

where p is the isotropic pressure, p is the energy density of the fluid, π_{ab} is the stress tensor, and q_a is the heat flux vector. The stress tensor has the from

$$\pi_{ab} = (P_r - P_t)\,(n_a n_b - \frac{1}{3} h_{ab}) \quad (4)$$

Where P_r is the radial pressure, and P_t is the tangential pressure and n is a unit radial vector given by $n^a = (\frac{1}{3})\,\delta_1^a$. The isotropic pressure is given by

$$P = \frac{1}{3}\,(p_r + 2p_t) \quad (5)$$

In terms of the radial pressure and the tangential pressure, for the line element (1) and matter distribution (3) the Einstein field equations becomes

$$\rho = 2\frac{\dot{B}}{B}\frac{\dot{Y}}{Y} + \frac{1}{Y^2} + \frac{\dot{Y}^2}{Y^2} - \frac{1}{B^2}\left(2\frac{Y''}{Y} + \frac{Y'^2}{Y^2} - 2\frac{B'Y'}{BY}\right) \quad (6a)$$

$$P_r = -\frac{2\ddot{Y}}{Y} - \frac{\dot{Y}^2}{Y^2} - \frac{1}{Y^2} + \frac{1}{B^2}\frac{Y'^2}{Y^2} \quad (6b)$$

$$P_t = -\left(\frac{\ddot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{Y}}{Y} + \frac{\ddot{Y}}{Y}\right) + \frac{1}{B^2}\left(\frac{Y''}{Y} - \frac{B'Y'}{BY}\right) \quad (6c)$$

$$q = -\frac{2}{B^2}\left(\frac{\dot{B}Y'}{BY} - \frac{\dot{Y}'}{Y}\right) \quad (6d)$$

where the heat flux $q^a = (0, q, 0, 0)$ is radially directed and primes denote the differentiation with respect to *r*. These equation describe the gravitational interactions of a shearing matter distribution with heat flux and anisotropic pressure for particles travelling along geodesics from (6*a*) – (6*d*), we observe that if the gravitational potentials $B(t, r)$ and $y\,(t, r)$ are specified, then the expressions for the matter variables p, p_r, p_t and q follow by simple substitution.

The vaidya exterior space-time of radiating star is given by

$$ds^2 = -(1 - \frac{2m(v)}{R})\, dv^2 - 2dvdR + R^2\,(d\theta^2 + sin^2\theta\, d\,\phi^2) \quad (7)$$

where $m(v)$ denoted the mass of the fluid as measured by an observer at infinity. The matching of the interior space-time (1) with the exterior spacetime (7) generates the set if junction conditions on the hyper surface Σ given by

Notes

$$dt = (1 - \frac{2m}{R_\Sigma} + 2\frac{dR_\Sigma}{dv})^{1/2}\, dv \tag{8a}$$

$$Y(R_\Sigma, t) = R_\Sigma(v) \tag{8b}$$

$$m(v)_\Sigma = \left[\frac{Y}{2}\left(1 + \dot{Y}^2 - \frac{Y'^2}{B^2}\right)\right]_\Sigma \tag{8c}$$

$$(P_r)_\Sigma = (qB)_\Sigma \tag{8d}$$

Notes

The nonvanishing of the radial pressure at the boundary Σ is reflected in equation (8*d*). Equation (8*d*) is an additional constraint which has to be satisfied together with the system of equations (6*a*)- 6*d*). On substituting (6*b*) and (6*d*) in (8*d*) we obtain

$$2Y\ddot{Y} + \dot{Y}^2 - \frac{Y'^2}{B^2} + \frac{2}{B}Y\dot{Y}' - \frac{2\dot{B}}{B^2}YY' + 1 = 0 \tag{9}$$

which has to satisfied on Σ. Equation (9) governs the gravitational behaviour of the radiating anisotropic star with nonzero shear and no acceleration. As equation (9) is highly nonlinear, it is difficult to solve without some simplifying assumption. This equation comprises two unknown functions $B(t, r)$ and $Y(t, r)$.

III. Exact Solutions

For convenience rewrite equation (9) in the form of the Riccati equation in the gravitational potential B as follows

$$\dot{B} = \left[\frac{\ddot{Y}}{Y'} + \frac{\dot{Y}^2}{2YY'} + \frac{1}{2YY'}\right]B^2 + \frac{\dot{Y}'}{Y'}B - \frac{Y'}{2Y} \tag{10}$$

Equation (10) was analyzed by Nogueira and Chan (2004) who obtained approximate solutions using numerical techniques. To describe properly the physical features of a radiating relativistic star exact solutions are necessary, preferably written in terms of elementary functions. An exact solution was found by Naidu et al (2006), which was singular at the stellar centre.

The Riccati equation (10), which has to be satisfied on the stellar boundary Σ, is highly nonlinear and difficult to solve. In 2008, Rajah and Maharaj obtained solutions by assuming that the gravitational potential $Y(t, r)$ is a separable function and specifying the temporal evolution of the model. Later in 2010, Thirukanesh and Maharaj, demonstrate that it is possible to find another exact solutions systematically without assuming separable forms for $Y(t, r)$ and not fixing the temporal evolution of the model a priori by introduce the transformation

$$B = ZY' \tag{11}$$

Then equation (10) becomes

$$\dot{Z} = \frac{1}{2Y}[FZ^2 - 1] \tag{12}$$

where set $F = 2Y\ddot{Y} + \dot{Y}^2 + 1$

Observe that equation (12) becomes a separable equation in Z and t, and therefore integrable, let F be a constant or a function of r only. In other word, (12) is integrable as long as F is independent of t.

For F = 1 S. Thirukanesh and S. D. Maharaj obtained the solution

$$ds^2 = -dt^2 + \frac{4}{9}\left[\frac{1+f(r)\exp\left[3[R_1(r)t+R_2(r)]^{1/3}/R_1\right]}{1-f(r)\exp\left[3[R_1(r)t+R_2(r)]^{1/3}/R_1\right]}\right]^2 \frac{[R_1't+R_2']^2}{[R_1t+R_2]^{2/3}}\,dr^2 + [R_1t+R_2]^{4/3}(d\theta^2+\sin^2\theta\,d\phi^2) \tag{13}$$

For $R_1 = R^{3/2}$, $R_2 = aR^{3/2}$ the line element (13) reduces to

$$ds^2 = -dt^2 + (t+a)^{4/3}\left\{R'^2\left[\frac{1+f(r)\exp\left[3[R_1(r)t+R_2(r)]^{1/3}/R_1\right]}{1-f(r)\exp\left[3[R_1(r)t+R_2(r)]^{1/3}/R_1\right]}\right]^2 dr^2 + R^2(d\theta^2+\sin^2\theta\,d\phi^2)\right\} \tag{14}$$

Which is the first category of the Rajah and Maharaja (2008) models for an anisotropic radiating star with shear. They match the line element (14) with the Naidu et al solution

$$ds^2 = -dt^2 + t^{4/3}\left\{R'^2\left[\frac{1+f(r)\exp\left[3t^{1/3}/r\right]}{1-f(r)\exp\left[3t^{1/3}/r\right]}\right]^2 dr^2 + R^2(d\theta^2+\sin^2\theta\,d\phi^2)\right\} \tag{15}$$

Again for $F = 1 + R^2_1(r)$ Thirukanesh & Maharaj found the line element

$$ds^2 = -dt^2 + \frac{1}{\sqrt{R_1^2+1}}\left[\frac{1+g(r)(R_1t+R_2)^{\sqrt{R_1^2+1}/R_1}}{1-g(r)(R_1t+R_2)^{\sqrt{R_1^2+1}/R_1}}\right]^2 [R_1't+R_2']^2 + [R_1(r)t+R_2(r)]^2(d\theta^2+\sin^2\theta\,d\phi^2) \tag{16}$$

For $R_1 = R$, $R_2 = aR$ which is reduces to equation

$$ds^2 = -dt^2 + (t+a)^2\left\{\frac{R'^2}{R^2+1}\left[\frac{1+h(r)[t+a]^{\sqrt{R^2+1}/R}}{1-h(r)[t+a]^{\sqrt{R^2+1}/R}}\right]^2 dr^2 + R^2(d\theta^2+\sin^2\theta\,d\phi^2)\right\} \tag{17}$$

IV. New Solutions

Now we obtained solutions by assuming that the gravitational potential $Y(t, r)$ and $B(t, r)$ is a separable function and specifying the temporal evolution of the model Equation (9). Choose,

$Y=R(t)A(t)$ And $B=R(t)C(r)$

Now from equation (9) we have

Notes

$$2R(t)A(r).\ddot{R}(t)A(r)+\dot{R}^2(t)A^2(r)-\frac{R^2(t)A'^2(r)}{R^2(t)C^2(r)}+\frac{2R(t)A(r)\dot{R}(t)A'(r)}{R(t)C(r)}$$

$$-\frac{2\dot{R}(t)C(r)R(t)A(r)R(t)A'(r)}{R^2(t)C^2(r)}+1=0 \Rightarrow 2R(t)\ddot{R}(t)+\dot{R}^2(t)=\frac{A'^2(r)}{C^2(r)A^2(r)}-\frac{1}{A^2(r)}=\gamma^2(say)$$

Notes

Case-1

For $\gamma = 0$, then we have

$$\therefore \quad R(t)=(a+bt)^{2/3} \text{ and } A(r)=\pm\int_0^r C(r')\,dr'+d$$

Therefore we have,

$$B=(a+bt)^{2/3}C(r) \quad and \quad Y=(a+bt)^{2/3}A(r)$$

Then we can calculate the metric (1) is

$$ds^2=-dt^2+(a+bt)^{4/3}\left[C^2(r)dr^2+A^2(r)\left(d\theta^2+\sin^2\theta\, d\Phi^2\right)\right]$$

Where $A(r)=\int_0^r C(r')dr'+d$

Here the expansion scalar Θ is non zero and if time *(t)* is increase this will be decrease. Again pressure is zero but density is nonzero i.e. the universe is gaseous.
Now we draw a graph (using MATLAB) for the scale factor

$$R(t)=(a+bt)^{2/3}$$

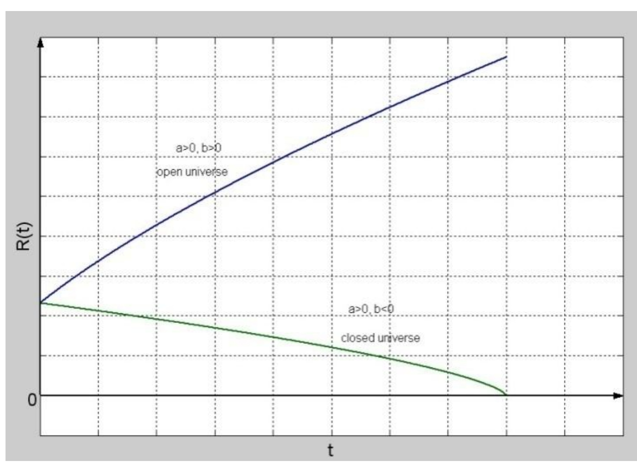

Figure 1 : This solution shows the universe is either open or closed.

Case-2

For $\gamma \neq 0$ we have

$$R(t)=\alpha+\gamma t \quad and \quad C(r)=\frac{A'(r)}{\sqrt{\gamma^2A^2+1}}$$

$$\therefore B=(\alpha+\gamma t)C(r) \quad and \quad Y=(\alpha+\gamma t)A(r)$$

Therefore the metric (1) can be written as

$$ds^2=-dt^2+(\alpha+\gamma t)^2\left[C^2(r)dr^2+A^2(r)\left(d\theta^2+\sin^2\theta\, d\Phi^2\right)\right]$$

Notes

Where A(r) is the function of r only.

And $q = 0$; $P = 0$; $\rho = 0$; $\Theta = \frac{3\gamma}{(1+\gamma t)}$

which is not a collapse solution.

Consider, $A = r$. Then, $C(r) = \frac{1}{\sqrt{\gamma^2 r^2+1}}$

Therefore the metric (1) becomes

$$ds^2 = -dt^2 + (1+\gamma t)^2 \left[\frac{dr^2}{(\gamma^2 r^2+1)} + r^2\left(d\theta^2 + \sin^2\theta\, d\Phi^2\right)\right]$$

For

$R = (\propto +\gamma t)$, $A(r) = r$ and $C(r) = \frac{1}{\sqrt{1+\gamma^2 r^2}}$ gives

$B = (\propto +\gamma t)\frac{1}{\sqrt{1+\gamma^2 r^2}}$ and $Y = (\propto +\gamma t) r$

Therefore we have,

$$\therefore q = 0$$

$$\Theta = 3\frac{\gamma}{(\propto+\gamma t)}$$

$$P = \frac{1}{(\alpha+\gamma t)^2}\left[-7\gamma^2 + 3\gamma^2 r^2 - \frac{6}{(\alpha+\gamma t)^2} + 3\right]$$

$$\rho = \frac{4\gamma^2}{(\alpha+\gamma t)^2}$$

Here the expansion scalar Θ and density are non zero and both will be decrease if time is increase. Pressure is also non zero.

V. Conclusion

For the first case, the above solutions indicates that the space is very diluted as $P = 0$. So we may consider the solution for dust, the density and the expansion scalar Θ decreases when t is increases and tends to zero when $t \to \infty$ The density decreases rapidly than the expansion scalar Θ. This solution shows the universe is either open or closed. So the solutions are physically realistic.

For case-2 we see that for $3(\gamma^2 r^2 + 1) > 7\gamma^2 + \frac{6}{(\alpha+\gamma t)^2}$ both pressure p and density p decreases when r and t increases. Also it is clear that the density p decreases rapidly than the expansion scalar Θ. The form of the solution is Robertson Walker type solution for open universe. The pressure decreases slowly when both r and t increases but does not tends to zero when both r and t tends to infinity. The solution is physically realistic.

References Références Referencias

1. De Oliviera, A. K. G., Santos, N. O., & Kolassis, C. A. (1985). Collapse of a Radiating Star, "Mon. Not. R. Astron. Soc. 216, 1001-1011"
2. Herrera, L., Le Denmat, G., Santos, N. O., & Wang, A. (2004). Shear-free Radiating Collapse & Conformal Flatness, "Int. J. Mod. Phys. D13, 583".
3. Herrera, L., Di Prisco, A., & Ospino, J. (2006). Some Analytical Models of Radiating Collapsing Spheres, "phys. Rev. D 74,044001".

Notes

4. Maharaj, S. D., & Govender, M. (2005). Radiating Collapse with Vanishing Weyl Stresses, "Int. J. Mod. Phys. D 14, 667".
5. Naidu, N. F., Govender, M. & Govender, K. S. (2006). Thermal Evolution of a Radiating Anisotropic Star with Shear, "Int. J. Mod. Phys. D 15, 1053".
6. Rajah, S. S., & Maharaj, S. D. (2008). A Riccati Equation in Radiative Stellar Collapse, "J. Math. Phys. 49, 012501".
7. Nogueira, P. C., & Chan, R. (2004). Radiating Gravitational Collapse with shear Viscosity & Bulk Viscosity, "Int. J. Mod. Phys. D 13, 1727".
8. Thirukkanesh, S. & Maharaj, S. D. (2010). Mixed potentials and radiative stellar collapse, "Journal of Mathematical Physics", 51, No.7, P-(2-7).

Notes

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On Trimming Simple Hypergraphs

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Abstract - We establish a necessary and sufficient condition for finding if the given hyperedge set in a simple hypergraph is a minimal hyperedge cover. Then we give a set theoretical proposition to find minimal hyperedge covers.

Keywords : Hypergraph, hyperedge, cover, degree.

AMS mathematics subject classification : 05C65

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[1] Stoll, R. R., 1963, Set Theory and logic, Dover Publications, New York, US.

On Trimming Simple Hypergraphs

D. Ramachandran [α] & R. Dharmarajan [σ]

Abstract - We establish a necessary and sufficient condition for finding if the given hyperedge set in a simple hypergraph is a minimal hyperedge cover. Then we give a set theoretical proposition to find minimal hyperedge covers.

Keywords : Hypergraph, hyperedge, cover, degree.

I. Introduction

Established terminologies, notations and theorems in set theory and propositional logic [1] are assumed. Let V be a finite nonempty set. Its cardinality (or, size) is denoted by $|V|$. By 2^V we mean the power set of V, or the set of all subsets (including the empty set φ) of V. By 2^{V*} we mean the set of all nonempty subsets of V; that is, $2^{V*} = 2^V - \{\varphi\}$.

Let E denote a family of nonempty subsets of V. If $\cup_{X \in E} X = V$, we say E *fills out* V. A *hypergraph* [2] on V is a pair (or, couple) H = (V, E) where V is a nonempty finite set and E is a family of nonempty subsets of V that fills out V. The set V is called the *vertex set* of H and each member of E is called a *hyperedge* of H. If no hyperedge in H equals all of V then we call H *non-trivial*. If the members of E are all distinct (that is, no two members coincide as subsets of V; or, $E \subsetneq 2^{V*}$) then H is *simple*. If no member of E is a subset (proper or otherwise) of another, then H is a *Sperner* hypergraph. Some authors (instances: [2] and [3]) take Sperner hypergraphs to be simple and vice versa but there is a distinction [4] between the two: Sperner hypergraphs are necessarily simple but simple hypergraphs need not be Sperner. See 1.1 that follows. All hypergraphs in this article are simple unless there are unambiguous indications to the contrary.

1.1: Example. Let H = (V, E) where V = {1, 2, 3, 4, 5}, E = $\{X_1, X_2, X_3\}$ with $X_1 = \{2, 3\}$, $X_2 = \{2\}$ and $X_3 = \{1, 3, 4, 5\}$. H is simple because X_1, X_2 and X_3 are all distinct as subsets of V. But H is not Sperner because the hyperedge X_2 is a subset of the hyperedge X_1.

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II. Trim Hypergraphs

Let H = (V, E). A hyperedge X in H is called *redundant* in H (or, *redundant* in E) if there exists $S \subseteq E - \{X\}$ such that S covers X; that is, $X \subseteq \cup_{Y \in S} Y$. For such an X, its *redundancy number* (rX) in H is defined as rX = $|S|$, provided no subset of E with cardinality less than rX covers X; that is, if $T \subset E - \{X\}$ and $|T| <$ rX then T does not cover X.

If H = (V, E) has no redundant hyperedges then we call E a *minimal hyperedge cover* for H and we call H a *trim hypergraph.*

If X is redundant in H with redundancy number rX then we write Xred(rX)H or Xred(rX)E or, if rX does not matter, just XredH or XredE. Evidently the redundancy number of a hyperedge depends on the hypergraph in which the hyperedge figures, and a redundant hyperedge in H has a unique redundancy number in H. The redundancy property of X in H could disappear if H is altered.

2.1: Example. V = {1, 2, 3, 4, 5, 6, 7, 8}, X_1 = {1, 3}, X_2 = {2, 3, 4}, X_3 = {4, 5, 6}, X_4 = {7, 8} and X_5 = {6, 8}. Let E = $\{X_j \mid j = 1$ through 5} and H = (V, E). Then X_5 is redundant in H and has rX_5 = 2 because:

(a) S = $\{X_3, X_4\} \subset E - \{X_5\}$ covers X_5 and

(b) no subset of E – $\{X_5\}$ with size less than 2 covers X_5.

2.2: Proposition. In H = (V, E), for r = 1, 2, . . ., we let Red(r)H = $\{X \in E$: Xred(r)H}. Then we have: H = (V, E) is Sperner if and only if Red(1)H = φ.

Proof. Suppose Red(1)H $\neq \varphi$. Then given $X \in$ Red(1)H, $X \subset Y$ for some $Y \in E - \{X\}$, whence H is not Sperner. Conversely, assume H is not Sperner. Then $X \subset Y$ for some X, $Y \in E$ such that $X \neq Y$. This means {Y} covers X, and so $X \in$ Red(1)H.

2.3: Proposition. A trim hypergraph is necessarily Sperner.

Proof. If H is not Sperner then Red(1)H $\neq \varphi$ by 2.2 and so H is not trim.

2.4: Example. The converse of 2.3 is not true. Let H = (V, E) where V = {1, 2, 3, 4, 5, 6, 7, 8}, X_1 = {1, 3}, X_2 = {2, 3, 4}, X_3 = {4, 5, 6}, X_4 = {7, 8} and X_5 = {6, 8} and E = $\{X_j \mid j = 1$ through 5}. H is Sperner, for no hyperedge in H contains another. But H has a redundant edge – namely X_5, because E – $\{X_5\}$ covers X_5.

Notes

Let H = (V, E). For a vertex $x \in V$, the number of hyperedges that contain x is defined to be the *degree of x in H*, and this number is denoted by dx(H) or dx. Evidently $dx(H) \geq 1$ for each $x \in V$.

2.5: Proposition. A hyperedge X is redundant in H if and only if no vertex of X is of degree 1.

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Proof. **(**$\Longrightarrow$) Let H = (V, E). Suppose some $y \in X$ has dy = 1. Then no subset of E – {X} can cover X because X is the only hyperedge containing y. Hence X is not redundant in H.

($\Longleftarrow$) Let $X \in E$ be given, say $X = \{x_1, \ldots, x_r\}$, and suppose no vertex in X is of degree 1. By hypothesis, to each $x_i \in X$ there exists $Y_i \in E$ with $Y_i \neq X$ and $x_i \in Y_i$. So $X \subseteq \cup_{i=1 \text{ to } r} Y_i$, whence X is redundant in H.

We call H = (V, E) a *partitioned hypergraph* (PHG) if its hyperedges form a partition of V – meaning, $V = X_1 \cup \ldots \cup X_k$ and $X_i \cap X_j = \varphi$ for $i \neq j$. A PHG is necessarily Sperner, though not conversely. The following proposition is straightforward.

2.6: Proposition. If H = (V, E) is a PHG, then H is trim.

Let H = (V, E). The omission of at least one hyperedge from while retaining the vertex set V is called a *rarefaction* of H. Suppose T is a nonempty subset of E such that:

(a) T has no redundant hyperedges and

(b) T fills out V.

Let T(H) = (V, T). Then we call T(H) *a trim form of H.*

2.7: Proposition. Let H = (V, E). Then there exists a trim form of H by rarefaction of H.

Proof. Let $E = \{X_1, \ldots, X_k\}$ so that $|E| = k$. The following is a rarefaction process leading to a trim form of H – if H is not trim in the first place.

Step 1: Begin by taking R (V) = φ and T = E.

Input X_1. Let $Y_1 = \cup_{i \neq 1} X_i$ and $A_1 = X_1 - Y_1$. Then $A_1 = \varphi \Longleftrightarrow E_1\text{red}(H)$.

(For, $A_1 = \varphi \Longrightarrow X_1 \subseteq Y_1$ which means $E - \{X_1\}$ covers X_1. Conversely, X_1redH means $E - \{X_1\}$ covers X_1.)

Update R (V) and T as follows:

If $A_1 = \varphi$, then let $R(V) \leftarrow R(V) \cup \{X_1\}$ and $T \leftarrow E - R(V)$.

Step j $(2 \leq j \leq k)$: Input X_j. Let $Y_j = \cup_{i \neq j} X_i$ and $A_j = X_j - Y_j$.

If $A_j = \varphi$, (which means X_j is redundant in T) then R (V) ← R (V) ∪ $\{X_j\}$ and T ← E − R (V).

Repeat the above procedure till all the k hyperedges have been input and tested for redundancy in the rarefied form of H at the point of input. At the end of the kth iteration, T(H) = (V, T) is a trim form of H. This is because even if k−1edges are redundant, the last input then cannot be so.

2.8: Example. V = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, X_1 = {1, 2}, X_2 = {2, 3, 4}, X_3 = {3, 5, 6}, X_4 = {4, 7, 8}, X_5 = {6, 8}, X_6 = {3} and X_7 = {9, 10}. E = {X_i : i = 1 through 7} and H = (V, E). We aim to obtain a trim form of H.

Initial: R (V) = φ & T = E.

Step 1: Input: X_1. $Y_1 = \cup_{i \neq 1} X_i$ = {2, 3, 4, 5, 6, 7, 8, 9, 10} and $A_1 = X_1 - Y_1$ = {1} ≠ φ.

So X_1 is not redundant in H. *Update:* R (V) = φ & T_1 = E = {X_1, . . . , X_7}. H_1 = H.

Step 2: Input: X_2. $Y_2 = \cup_{i \neq 2} X_i$ = V and $A_2 = X_2 - Y_2$ = φ. So X_2 is redundant in H.

Update: R (V) = {X_2} & T_2 = {X_1, X_3, X_4, X_5, X_6, X_7}. H_2 = (V, T_2).

Step 3: Input: X_3. $Y_3 = \cup_{i \neq 2, 3} X_i$ = {1, 2, 3, 4, 6, 7, 8, 9, 10} and $A_3 = X_3 - Y_3$ = {5} ≠ φ.

Update: R (V) = {X_2} &T_3 = {X_1, X_3, X_4, X_5, X_6, X_7}. H_3 = (V, T_3).

Step 4: Input: X_4. $Y_4 = \cup_{i \neq 2, 4} X_i$ = {1, 2, 3, 5, 6, 8, 9, 10} and $A_4 = X_4 - Y_4$ = {4, 7} ≠ φ.

Update: R (V) = {X_2} & T_4 = { X_1, X_3, X_4, X_5, X_6, X_7}. H_4 = (V, T_4).

Step 5: Input: X_5. $Y_5 = \cup_{i \neq 2, 5} X_i$ = V and $A_5 = X_5 - Y_5$ = φ.

Update: R (V) = {X_2, X_5} & T_5 = {X_1, X_3, X_4, X_6, X_7}. H_5 = (V, T_5).

Step 6: Input: X_6. $Y_6 = \cup_{i \neq 2, 5, 6} X_i$ = V and $A_6 = X_6 - Y_6$ = φ.

Update: R (V) = {X_2, X_5, X_6} & T_6 = {X_1, X_3, X_4, X_7}. H_6 = (V, T_6).

Step 7: Input: X_7. $Y_7 = \cup_{i \neq 2, 5, 6, 7} X_i$ = {1, 2, 3, 4, 5, 6, 7, 8} and $A_7 = X_7 - Y_7$ = {9, 10}≠φ.

Update: R (V) = {X_2, X_5, X_6} & T_7 = {X_1, X_3, X_4, X_7}. H_7 = (V, T_7).

Result: T(H) = (V, T), where T = T_7 ={X_1, X_3, X_4, X_7}, is a trim form of H = (V, E).

2.9: Example. A given HG can have two or more trim forms. A trim form obtained by the above rarefaction process is the consequence of the order in which the hyperedges are input. In this example, two different input orders lead to two different trim forms of the same HG.

Given: H = (V, E), with V = {1, 2, 3, 4} and E = {X_1, X_2, X_3, X_4}, where X_1 = {1, 2}, X_2 = {2, 3}, X_3 = {3, 4} and X_4 = {1, 2, 4}.

Notes

Table 1: Trimming process 1

Initial: R (V) = φ & T = E.			
Input order	T updated	Status of the input	R (V) updated
E_1	$T = T_1 = \{E_1, E_2, E_3, E_4\}$	E_1 red T_1 (E_1 has no remote vertex in T_1)	$\{E_1\}$
E_2	$T = T_2 = \{E_2, E_3, E_4\}$	E_2 red T_2 (E_2 has no remote vertex in T_2)	$\{E_1, E_2\}$
E_3	$T = T_3 = \{E_3, E_4\}$	E_3 not red T_3 ($3 \in E_3$ is remote in T_3)	$\{E_1, E_2\}$
E_4	$T = T_4 = \{E_3, E_4\}$	E_4 not red T_4 ($1 \in E_4$ is remote in T_4)	$\{E_1, E_2\}$
Output: $T = T_4 = \{E_3, E_4\}$ is a trim form of H.			

Table 2: Trimming process 2:

Initial R (V) = φ & S = E.			
Input order	S updated	Status of the input	R (V) updated
E_2	$S = S_1 = \{E_1, E_2, E_3, E_4\}$	E_2 red S_1 (E_2 has no remote vertex in S_1)	$\{E_2\}$
E_4	$S = S_2 = \{E_1, E_3, E_4\}$	E_4 red S_2 (E_4 has no remote vertex in S_2)	$\{E_2, E_4\}$
E_1	$S = S_3 = \{E_1, E_3\}$	E_1 not red in S_3 ($1 \in E_1$ is remote in S_3)	$\{E_2, E_4\}$
E_3	$S = S_4 = \{E_1, E_3\}$	E_3 not red in S_4 ($3 \in E_3$ is remote in S_4)	$\{E_2, E_4\}$
Output: $S = S_4 = \{E_1, E_3\}$ is a trim form of H.			

III. Concluding Remarks

Trimming a hypergraph is a special case of the set-cover problem [3] which is a classic problem in computer science. The algorithm we have presented (proposition 2.7) uses basic set theory and hence works for all simple hypergraphs, irrespective of the differences in the cardinalities of the hyperedges. The output is an edge cover (which can also be termed set cover) from the set of hyperedges. Among the several (finitely many) such outputs, at least one is bound to be a minimal edge cover for the vertex set.

Ref.

[3] Eiter, T. and Gottlob, G., 1995, "Identifying the minimal transversals of a hypergraph and related problems", SIAM J. on Computing, 24, pp 1278-1034.

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References Références Referencias

[1] Stoll, R. R., 1963, Set Theory and logic, Dover Publications, New York, US.

[2] Berge, C., 1989, Hyergraphs - Combinatorics on finite sets, North-Holland Mathematical Library, Amsterdam, The Netherlands, Chap. 1.

[3] Eiter, T. and Gottlob, G., 1995, "Identifying the minimal transversals of a hypergraph and related problems", SIAM J. on Computing, 24, pp 1278-1034.

[4] Rosen, K.H. (Editor-in-chief), 2000, Handbook of Discrete and Combinatorial Mathematics, CRC Press LLC, Florida, US.

Notes

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A Note on Semilattice Decompositions of Epigroups

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Abstract - The purpose of this paper is to study epigroups admitting a decomposition into a semilattice of σ_n –simple semigroups. We give further remark on semilattice decompositions of epigroups and characterize them by using some relations, ideals on/of S and certain special elements in S.

Keywords : epigroup; semilattice decompositions; σ_n *-simple semigroups.*

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Strictly as per the compliance and regulations of :

Ref.

1. Putcha M S. Semilattice decompositions of semigroups[J]. Semigroup Forum, 1973, 6: 12-34.

A Note on Semilattice Decompositions of Epigroups

Liu Jing-Guo

Abstract - The purpose of this paper is to study epigroups admitting a decomposition into a semilattice of σ_n-simple semigroups. We give further remark on semilattice decompositions of epigroups and characterize them by using some relations, ideals on/of S and certain special elements in S.

Keywords : epigroup; semilattice decompositions; σ_n -simple semigroups

I. Introduction and Preliminaries

The relation $\longrightarrow$ introduced by M. S. Putcha in [1] and T. Tamura in [2], plays a crucial role in semilattice decompositions of semigroups. General properties of the graphs that correspond to these relations were studied by M. S. Putcha in [3] and the structure of semigroups in which the minimal paths in the graph corresponding to $\longrightarrow$ are bounded was described by M. Cirić and S. Bogdanović in [4]. The latter semigroups have also been studied by S. Bogdanović, M. Ćirić and Ž. Popović in [5]. Further, semilattice decompositions are especially interesting when they are considered for epigroups. A characterization of the least semilattice congruence on such semigroups was given by M. S. Putcha in [6], and by L. N. Shevrin (See the survey paper [7]). In [8], Ž. Popović, S. Bogdanović and M. Ćirić study epigroups admitting a decomposition into a semilattice of σ_n-simple semigroups and described them in terms of properties of their idempotents. In this paper we will give a note on semilattice decompositions of epigroups by using some relations, ideals on/of S and certain special elements in S.

Now we give precise definitions of the notions used above and the ones that will be used in the further text. $\mathbb{N}$ will be used in the sequel to denote the set of all positive integers. Let S be a semigroup. For a subset A of S, we define

$$\sqrt{A} = \{x \in S \,|\, (\exists\, n \in \mathbb{N})\, x^n \in A\}.$$

A subset A of S is completely semiprime if for any $x \in S, x^2 \in A$ implies $x \in A$. If A is an ideal of S, then it is completely semiprime if and only if $\sqrt{A} \subseteq A$. The division relation $|$ and the relation $\longrightarrow$ on S are defined by

$$a|b \Leftrightarrow (\exists\, x, y \in S^1)\; b = xay, \quad a \longrightarrow b \Leftrightarrow (\exists\, k \in \mathbb{N})\; a|b^k.$$

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For $n \in \mathbb{N}$, $n \geqslant 2$, the relation $\longrightarrow^n$ on S is defined by

$$a \longrightarrow^n b \Leftrightarrow (\exists x \in S) a \longrightarrow^{n-1} x \longrightarrow b,$$

and for $n = 1$, $\longrightarrow^n = \longrightarrow$. In other words, $\longrightarrow^n$ is the n-th power of $\longrightarrow$ in the semigroup of binary relations on S. The transitive closure of $\longrightarrow$ is denoted by $\longrightarrow^\infty$. For $n \in \mathbb{N}$ and $a \in S$, the sets $\Sigma_n(a)$ and $\Sigma(a)$ are defined by

$$\Sigma_n(a) = \{x \in S \mid a \longrightarrow^n x\}; \qquad \Sigma(a) = \{x \in S \mid a \longrightarrow^\infty x\},$$

and the equivalence relations σ_n and σ on S are defined by

$$(a, b) \in \sigma_n \Leftrightarrow \Sigma_n(a) = \Sigma_n(b); \qquad (a, b) \in \sigma \Leftrightarrow \Sigma(a) = \Sigma(b).$$

In other words,

$$\Sigma_1(a) = \sqrt{SaS},\ \Sigma_{n+1}(a) = \sqrt{S\Sigma_n(a)S} \supseteq \Sigma_n(a);\ \text{and}\ \Sigma(a) = \bigcup_{n \in \mathbb{N}} \Sigma_n(a).$$

As it was proved by M. Ćirić and S. Bogdanović in [4], σ is the least semilattice congruence on S and $\Sigma(a)$ is the least completely semiprime ideal of S containing a, called the principal radical of S generated by a. The set $\Sigma_n(a)$ is called the n-radical generated by a. Let A be a nonempty subset of a semigroup S. Then

$$\Sigma(A) \overset{def}{=} \bigcup_{a \in A} \Sigma(a)$$

is the least completely semiprime ideal of S containing A. A semigroup S is σ_n-simple if σ_n coincides with the universal relation on S, and σ_1-simple semigroups are also called archimedean semigroups. The set of all idempotents of a semigroup S is denoted by $E(S)$. If $e \in E(S)$, then

$$G_e = \{x \in S | x \in eS \cap Se,\ e \in xS \cap Sx\}$$

is the largest subgroup of S having e as its identity, called the maximal subgroup of S determined by e, and the set K_e is defined by $K_e = \sqrt{G_e}$. An element a of S is group-bound if at least one of its powers lies in some subgroup of S. There is exactly one such subgroup, and its identity is denoted by a^ω. A semigroup S is called an epigroup if every element a of S is group-bound. Any epigroup S is partitioned into the subsets K_e called unipotency classes. The idempotent of the unipotency class to which an element a belongs will be denoted by e_a(here $e_a = a^\omega$). The element $\overline{a} = (ae_a)^{-1}$ is the inverse of ae_a in the group G_{e_a}. This element is called the pseudo-inverse of a. The following equalities hold:

$$\overline{a}a = a\overline{a} = e_a, e_a\overline{a} = \overline{a}, a^m e_a = a^m \text{ for some } m \in \mathbb{N}.$$

We will denote by $\mathcal{K}$ the equivalence relation on an epigroup S corresponding to the partition of the given epigroup S into its unipotency classes and $\mathcal{H}$, $\mathcal{D}$ and $\mathcal{J}$ are the well known Green relations.

For undefined notions and notations we refer to the book [10].

Ref.
4. Ciric M, Bogdanovic S. Semilattice decompositions of semigroups[J]. Semigroup Forum, 1996, 52:119-132.

II. The Main Result

We start this section by recalling some results obtained from the paper [4] by M. Ćirić and S. Bogdanović, the paper [9] by Ž. Popović, S. Bogdanović and M. Ćirić.

Theorem 2.1[4] *Let $n \in \mathbb{N}$. Then the following conditions on a semigroup S are equivalent:*

(i) S is a semilattice of σ_n-simple semigroups;
(ii) every σ_n-class of S is a subsemigroup;
(iii) for every $a \in S$, $\Sigma_n(a)$ is an ideal of S;
(iv) $(\forall a, b \in S)\ \Sigma_n(ab) = \Sigma_n(a) \cap \Sigma_n(b)$;
(v) $(\forall a, b, c \in S)\ a \longrightarrow^n b \ \&\ b \longrightarrow^n c \Longrightarrow a \longrightarrow^n c$;
(vi) $\sigma_n = \longrightarrow^n \cap (\longrightarrow^n)^{-1}$ on S.

Theorem 2.2[9] *Let S be an epigroup and $n \in \mathbb{N}$. Then S is a semilattice of σ_n-simple semigroups if and only if for every a of $S, a\sigma_n a^\omega$.*

Next we prove some auxiliary lemmas.

Lemma 2.1 *Let a be a group-bound element of a semigroup S. Then for every $b \in S$ and every $n \in \mathbb{N}$, $\overline{a} \longrightarrow^n b$ implies $a \longrightarrow^n b$. In other words, for every $n \in \mathbb{N}$,*

$$\Sigma_n(\overline{a}) \subseteq \Sigma_n(a).$$

Proof Since $\overline{a} = \overline{a}a\overline{a} \in SaS$, we have $S\overline{a}S \subseteq SaS$. It follows that

$$\Sigma_1(\overline{a}) = \sqrt{S\overline{a}S} \subseteq \sqrt{SaS} \subseteq \Sigma_1(a).$$

Now, by induction we easily verify that $\Sigma_n(\overline{a}) \subseteq \Sigma_n(a)$, for every $n \in \mathbb{N}$.

Lemma 2.2 *Let a be some element of an epigroup S. Then for every $b \in S$ and every $n \in \mathbb{N}$,*

$$\overline{a} \longrightarrow^n b \ \text{ if and only if } \ a^\omega \longrightarrow^n b.$$

In other words, for every $n \in \mathbb{N}$,

$$\Sigma_n(\overline{a}) = \Sigma_n(a^\omega).$$

Proof Since $\overline{a}\mathcal{H}a^\omega$, then $\overline{a}\mathcal{D}a^\omega$. This together with Lemma 5 in [4], and the known fact $\mathcal{D} = \mathcal{J}$ for any epigroup, $\mathcal{D} \subseteq \sigma_1 \subseteq \sigma_2 \subseteq \cdots \subseteq \sigma_n \subseteq \cdots$, we have $\overline{a}\sigma_n a^\omega$.

Lemma 2.3 *Let b be a group-bound element of a semigroup S. Then for every $a \in S$ and every $n \in \mathbb{N}$,*

$$a \longrightarrow^n b \ \text{ if and only if } \ a \longrightarrow^n \overline{b}.$$

Proof Let $m \in \mathbb{N}$ such that $b^m \in G_{e_b}$. Consider an arbitrary $a \in S$. Suppose that $a \longrightarrow b$. Then $b^k = uav$, for some $u, v \in S$, $k \in \mathbb{N}$, and thus

$$\overline{b}^k = (\overline{b}b\overline{b})^k = \overline{b}^k b^k \overline{b}^k = \overline{b}^k uav\overline{b}^k \in SaS.$$

Hence we obtain $a \longrightarrow \overline{b}$.

Ref.

9. Howie J M. Fundamentals of Semigroup Theory[M]. London: Clarendon Press, Oxford, 1995.

Conversely suppose that $a \longrightarrow \bar{b}$. Then $\bar{b}^k = uav$, for some $u, v \in S$, $k \in \mathbb{N}$, and hence for some $m \in \mathbb{N}$

$$b^{mk} = b^{mk}b^{\omega} = b^{mk}(b^{\omega})^k = b^{mk}(\bar{b}b)^k = b^{mk}\bar{b}^k b^k = b^{mk}uavb^k \in SaS.$$

Thus we get $a \longrightarrow b$. Therefore, we have proved that our assertion holds for $n = 1$. By induction we easily verify that this assertion holds for every $n \in \mathbb{N}$.

Lemma 2.4 *For any epigroup, we have* $\mathcal{K} \vee \mathcal{D} = (\longrightarrow \cap \longrightarrow^{-1})^{\infty}$.

Proof It is easy to verify that $\mathcal{K} \subseteq (\longrightarrow \cap \longrightarrow^{-1})$, $\mathcal{D} \subseteq (\longrightarrow \cap \longrightarrow^{-1})$. Since the join $\mathcal{K} \vee \mathcal{D}$ is the smallest equivalence containing $\mathcal{K}$ and $\mathcal{D}$ and $(\longrightarrow \cap \longrightarrow^{-1})^{\infty}$ is an equivalence, it follows that $\mathcal{K} \vee \mathcal{D} \subseteq (\longrightarrow \cap \longrightarrow^{-1})^{\infty}$.

Conversely, by virtue of Corollary 3 in [6], $(\longrightarrow \cap \longrightarrow^{-1})^{\infty}$ is the transitive closure of $\sim \circ \mathcal{D}$ on S, where $\sim$ is the Schwartz's equivalence ($a \sim b$ if and only if $a^i = b^j$ for some $i, j \in \mathbb{N}$). But then as $\sim \subseteq \mathcal{K}$, we have $(\longrightarrow \cap \longrightarrow^{-1})^{\infty} \subseteq \mathcal{K} \vee \mathcal{D}$.

For any ideal I of S, we set

$$Q_{I_1} = \sqrt{I},\ Q_{I_{n+1}} = \sqrt{SQ_{I_n}S} \supseteq Q_{I_n},\ n \in \mathbb{N}.$$

Now we are prepared for the main result of the paper.

Theorem 2.3 *Let S be an epigroup and $n \in \mathbb{N}$. Then the following conditions are equivalent:*

(i) S is a semilattice of σ_n-simple semigroups;

(ii) $(\forall a \in S)\ a\sigma_n\bar{a}$;

(iii) Every σ_n-class of S is a subepigoup;

(iv) $\sqrt{\sigma_n} \subseteq \sigma_n$;

(v) $\sqrt{\mathcal{D}} \subseteq \sigma_n$;

(vi) $\mathcal{K} \subseteq \sigma_n$;

(vii) $\mathcal{K} \vee \mathcal{D} \subseteq \sigma_n$;

(viii) For any ideal I of S, the set Q_{I_n} is and ideal.

Proof (i)$\Longrightarrow$(ii) For any element a of S, $a^i \in G_{e_a}$ and $a^i = a^i\bar{a}a$, for some $i \in \mathbb{N}$. Then $\bar{a} \longrightarrow a$ and $a|\bar{a}$ and if (i) holds, then by (vi) of Theorem 1 it follows $a\sigma_n\bar{a}$.

(ii)$\Longrightarrow$(iii) By Lemma 2.3, for every $a \in S$, $a\sigma_n\bar{a}\sigma_n a^{\omega}$. Notice that σ_n is an equivalence relation on S. Again by Theorem 2.1, S is a semilattice of σ_n-simple semigroups and Every σ_n-class of S is a subsemigroup. This together with the assumption of (ii), every σ_n-class of S is a subepigoup, since a subsemgiroup of an epigroup that is closed under pseudo-inversion is a subepigroup.

(iii)$\Longrightarrow$(iv) Let $a\sqrt{\sigma_n}b$. Then $a^m\sigma b^n$ for some $m, n \in \mathbb{N}$. By hypothesis and Theorem 2.1 we have $a\sigma_n a^m\sigma b^n\sigma b$. Thus $a\sigma_n b$, which was to be proved.

(iv)$\Longrightarrow$(v) By Lemma 5 in [4] (see the proof of Lemma 2.2) we have $\mathcal{D} \subseteq \sigma_n$ and thus $\sqrt{\mathcal{D}} \subseteq \sqrt{\sigma_n}$. Therefor (v) holds.

(v)$\Longrightarrow$(vi) It is known that in epigroup $\mathcal{K} \subseteq \sqrt{\mathcal{H}} \subseteq \sqrt{\mathcal{D}}$. So (vi) holds.

(vi)$\Longrightarrow$(vii) By (vi) we have $\mathcal{K} \vee \mathcal{D} \subseteq \sigma_n$, since $\mathcal{D} \subseteq \sigma_n$ always holds and these relation are all equivalence relations on S.

Ref.

6. Putcha M S. Semigroups in which a power of each element lies in a subgroup[J]. Semigroup Forum, 1973, 5: 354-361.

(vii)$\Longrightarrow$(viii) Notice that $a(\mathcal{K}\vee\mathcal{D})a^\omega$ holds such that $a\sigma_n a^\omega$ by assumption, hence S is a semilattice of σ_n-simple semigroups by Theorem 2.2 and thus, $\longrightarrow^n=\longrightarrow^\infty$, which implies $\Sigma_n(a)=\Sigma(a)$. Hence for any nonempty subset A of S,

$$\Sigma_n(A)=\bigcup_{a\in A}\Sigma_n(a)=\bigcup_{a\in A}\Sigma(a)=\Sigma(A)$$

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is the smallest completely semiprime ideal of S containing A. Let I be an ideal of S and $a\in I$. Then

$$SaS\subseteq I,\Sigma_1(a)\subseteq Q_{I_1},\ldots,\Sigma_n(a)\subseteq Q_{I_n}$$

and thus $\Sigma_n(I)=\Sigma(I)\subseteq Q_{I_n}$. On the other hand, for any $b\in Q_{I_n}$, that is,

$$a\longrightarrow x_1\longrightarrow\ldots\longrightarrow x_{n-1}\longrightarrow b,$$

where $a\in I, x_i\in Q_{I_n}, 1\leq i<n, i\in\mathbb{N}$. It follows that $b\in\Sigma_n(a)$ and thus $Q_{I_n}\subseteq\Sigma(I)$.

(viii)$\Longrightarrow$(i) For any $a\in S$, Let $I=S^1aS^1$. Obviously I is an ideal of S. Then by the hypothesis of (viii), together with Lemma 1 in [4] and Theorem 2.1, $\Sigma_n(a)$ is an ideal. Again by Theorem 2.1, S is a semilattice of σ_n-simple semigroups.

Remark Notice that in the proof ((vii)$\Longrightarrow$(viii)) of Theorem 2.3, $a(\mathcal{K}\vee\mathcal{D})a^\omega$ always holds such that $a\sigma_n a^\omega$ by assumption and hence σ_n is a semilattice congruence on S. Therefor by Lemma 2.4, $\mathcal{K}\vee\mathcal{D}=(\longrightarrow\cap\longrightarrow^{-1})^\infty\subseteq\sigma_n$ prevails, since $(\longrightarrow\cap\longrightarrow^{-1})^\infty$ is the smallest semilattice congruence on S.

References Références Referencias

1. Putcha M S. Semilattice decompositions of semigroups. Semigroup Forum, 1973, 6: 12-34.
2. Tamura T. On Putcha's theorem concerning semilattice of Archimedean semigroups. Semigroup Forum, 1972, 5: 83-86.
3. Putcha M S. Minimal sequences in semigroups. Transactions of the American Mathematical Society,1974, 189: 93-106.
4. Ciric M, Bogdanovic S. Semilattice decompositions of semigroups. Semigroup Forum, 1996, 52:119-132.
5. Bogdanovic S, Ciric M and Popovic, Z. Semilattice decompositions of semigroups revisited. Semi-group Forum, 2000, 61: 263-276.
6. Putcha M S. Semi-groups in which a power of each element lies in a subgroup. Semigroup Forum, 1973, 5: 354-361.
7. Shevrin L N. Epigroups. In: Kudravtsev V B, Rosenberg I G (eds.) Structural Theory of Automata, Semigroups, and Universal Algebra. Berlin: Springer, 2005: 331-380.
8. Popovic, Z, Bogdanovic S and Ciric M. A note on semilattice decompositions of completely regular semigroups. Novi Sad Journal of Mathematics, 2004, 34: 167-174.
9. Howie J M. Fundamentals of Semigroup Theory. London: Clarendon Press, Oxford, 1995.

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Generalized I-convergent Difference Sequence Spaces defined by a Moduli Sequence

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Abstract - In this article we introduce the sequence space $c_0^I(F,\Delta^n)$ and $\ell_\infty^I(F,\Delta^n$ for the sequence of $\mathrm{F} = (\mathrm{fk})$ and given some inclusion relations.

Keywords : Ideal, filter, sequence of moduli, difference sequence space, I-convergent sequence space.

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10. H.Kizmaz, on certain sequence spaces, Canadian Math.Bull.,24(1981), 169-176.

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Generalized I-convergent Difference Sequence Spaces Defined by a Moduli Sequence

M. Aiyub

Abstract - In this article we introduce the sequence space $c_0^I(F,\Delta^n)$ and $\ell^I_\infty(F,\Delta^n$ for the sequence of F = (fk) and given some inclusion relations.

Keywords and phrases : Ideal,filter, sequence of moduli,difference sequence space, I-convergent sequence space

I. Introduction and Preliminaries

Let ω, ℓ_∞, c_0 be the set of all sequences of complex numbers, the linear spaces of bounded, convergent and null sequences $x = (x_k)$ with complex terms, respectively, normed by

$$\|x\|_\infty = \sup_k |x_k|, \text{where} K \in \mathbb{N} = 1, 2, 3......$$

The idea of difference sequence spaces was introduced by H. Kizmaz [10]. In 1981 , Kizmaz defined the sequence spaces as follow;

$$\ell_\infty(\Delta) = \{x = (x_k) \in \omega : (\Delta x_k) \in \ell_\infty\},$$

$$c(\Delta) = \{x = (x_k) \in \omega : (\Delta x_k) \in c\},$$

$$c_0(\Delta) = \{x = (x_k) \in \omega : (\Delta x_k) \in c_0\},$$

where

$$\Delta x = (x_k - x_{k+1}) \text{ and } \Delta^0 x = (x_k),$$

These are Banach space with the norm

$$\|x\|_\Delta = |x_1| + \|\Delta x\|_\infty.$$

Later Colak and Et [2] defined the sequence spaces:

$$\ell_\infty(\Delta^n) = \{x = (x_k) \in \omega : (\Delta^n x_k) \in \ell_\infty\},$$

$$c(\Delta^n) = \{x = (x_k) \in \omega : (\Delta^n x_k) \in c\},$$

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$$c_0(\Delta^n) = \{x = (x_k) \in \omega : (\Delta^n x_k) \in c_0\},$$

where $n \in \mathbf{N}, \Delta^0 x = (x_k), \Delta x = (x_k - x_{k+1}), \quad \Delta^n x = (\Delta^n x_k) = (\Delta^{n-1} x_k - \Delta^{n-1} x_{k+1})$ and so that

$$\Delta^n x_k = \sum_{v=0}^{n} (-1)^v \begin{bmatrix} n \\ v \end{bmatrix} x_{k+v},$$

and so that these are Banach space with the norm

$$\|x\|_\Delta = \sum_{i=1}^{n} |x_i| + \|\Delta^n x\|_\infty.$$

The idea of modulus was defined by Nakano [15] in 1953. A function $f : [0, \infty) \to [0, \infty)$ is called a modulus if

(*i*) $f(t) = 0$ if and only if $t = 0$,

(*ii*) $f(t + u) \leq f(t) + f(u)$, for all $t, u \geq 0$,

(*iii*) f is increasing and

(*iv*) f is continuous from the right at 0.

Let X be a sequence spaces.Then the sequence spaces $X(f)$ is defined as

$$X(f) = \{x = (x_k) : (f(|x_k|)) \in X\}$$

for a modulus f. Maddox and Ruckle[14,16]

Kolak[11,12] gave an extension of $X(f)$ by considering a sequence of moduli $F = (f_k)$,that is

$$X(F) = \{x = (x_k) : (f_k(|x_k|)) \in X\}.$$

After then Gaur and Mursaleen[9] defined the following sequence spaces

$$\ell_\infty(F, \Delta) = \{x = (x_k) : (\Delta x_k) \in \ell_\infty(F)\},$$

$$c_0(F, \Delta) = \{x = (x_k) : (\Delta x_k) \in c_0(F)\},$$

for a sequence of moduli $F = (f_k)$.

we defined the following sequence spaces:

$$\ell_\infty(F, \Delta^n) = \{x = (x_k) : (\Delta^n x_k) \in \ell_\infty(F)\},$$

$$c_0(F, \Delta^n) = \{x = (x_k) : (\Delta^n x_k) \in c_0(F)\},$$

for a sequence of moduli $F = (f_k)$. We will give the necessary and sufficient conditions for the inclusion relations between $X(\Delta^n)$ and $Y(F, \Delta^n)$, where $X, Y = \ell_\infty$ or c_0. Sequence of moduli have been studied by C.A.Bektas and R. Colak[1] and many other authours.

The notion of statical convergence was introduced by H.Fast[6]. Later on it was studied by J.A.Fridy [7,8] from the sequence space point view and linked with the summability theory.

The notion of I-convergence is a generalization of the statical convergence. It was studied at initial stage by Kostyrko, Salat and Wilezynski [13]. Later on it was studied by Salat [19],Salat, Tripathy and Ziman [20], Demric[3]

Ref.

15. H.Nakano, Concave modulars, J.Math Soc. Japan., 5(1953),29-49.

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Let $\mathbb{N}$ be a non empty set. Then a family of sets $I \subseteq 2^N$ (power set of $\mathbb{N}$) is said to be an ideal if I is additive i.e $(A, B) \in I \Rightarrow (A \cup B) \in I$ and i.e $A \in I, B \subseteq A \Rightarrow B \in I$. A non empty family of sets $\pounds(I) \subseteq 2^N$ is said to be filter on N if and only if $\Phi \notin \pounds(I)$ for $A, B \in \pounds(I)$ we have $(A \cap B) \in \pounds(I)$ and for each $A \in \pounds(I)$ and $A \subseteq B$ implies $B \in \pounds(I)$.

An ideal $I \subseteq 2^N$ is called non trivial if $I \neq 2^N$. A non trivial ideal $I \subseteq 2^N$ is called admissible if $\{(x) : x \in \mathbb{N}\} \subseteq I$. A non trivial ideal is maximal if there cannot exist any non-trivial ideal $J \neq I$ containing I as a subset. For each ideal I, there exist a filter $\pounds(I)$ corresponding to I, i.e $\pounds(I) = \{K \subseteq \mathbb{N} : K^c \in I\}$,where $K^c = N - K$.

Definition 1.1. A sequence $(x_k) \in \omega$ is said to be I-convergent to a number L if for every $\epsilon > 0$. $\{k \in \mathbb{N} : |x_k - L| \geq \epsilon\} \in I$.In this case we write $I - \lim x_k = L$.

Definition 1.2. A sequence $(x_k) \in \omega$ is said to be I-null if L=0. In this case we write $I - \lim x_k = 0$.

Definition 1.3. A sequence $(x_k) \in \omega$ is said to be I-cauchy if for every $\epsilon > 0$, there exist a number $m = m(\epsilon)$ such that $\{k \in \mathbb{N} : |x_k - x_m| \geq \epsilon\} \in I$.

Definition 1.4. A sequence $(x_k) \in \omega$ is said to be I-bounded if there exist $M > 0$ such that $\{K \in \mathbb{N} : |x_k| \geq M\}$.

We need the following Lemmas.

Lemma 1.5. The condition $\sup_k f_k(t) < \infty, t > 0$ hold if and only if there is a point $t_0 > 0$ such that $\sup_k f_k(t_0) < \infty$ (see [1,9]).

Lemma 1.6. The condition $\inf_k f_k(t) > 0$ hold if and only if there exist is a point $t_0 > 0$ such that $\inf_k f_k(t_0) > 0$ (see [1,9]).

Lemma 1.7. Let $K \in \pounds(I)$ and $M \subseteq N$. If $M \neq I$ then $M \cap K \neq I$ (see [20]).

Lemma 1.8. If $I \subseteq 2^N$ and $M \subseteq N$. If $M \neq I$ then $M \cap K \neq I$ (see [13]).

II. Main Results

In this article we introduce the following classes of sequence spaces.

$$c_0^I(F, \Delta^n) = \{(x_k) \in \omega : I - \lim f_k(|\Delta^n x_k|) = 0\} \in I,$$

$$\ell_\infty^I(F, \Delta^n) = \{(x_k) \in \omega : I - \sup_k f_k(|\Delta^n x_k|) < \infty\} \in I$$

Theorem 2.1. For a sequence $F = f_k$ of moduli, the following statements are equivalent:

(a) $\ell_\infty^I(\Delta^n) \subseteq \ell_\infty^I(F, \Delta^n)$,

(b) $c_0^I(\Delta^n) \subseteq c_0^I(F, \Delta^n)$,

(c) $\sup_k f_k(t) < \infty, (t > 0)$.

Proof. (a) implies (b) is obvious .

(b) implies (c). Let $c_0^I(\Delta^n) \subseteq c_0^I(F, \Delta^n)$. Suppose that (c) is not true. Then by Lemma (1.5)

$$\sup_k f_k(t) = \infty, \text{ for all } t > 0,$$

Ref.

1. C.A.Bektas, R.Colak, Generalized difference sequence spaces defined by a sequence of moduli, Soochow. J.Math., 29(2)(2003), 215-220 164.

and therefore there is a sequence (k_i) of positive integers such that

$$f_{k_i}(\frac{1}{i}) > i, \quad \text{for each} \quad i = 1, 2, 3...... \tag{1}$$

Define $x = (x_k)$ as follow

$$x_k = \begin{cases} \frac{1}{i} \text{ if } \ k = k_i, i = 1, 2, 3.....; \\ 0 \ \ \text{otherwise.} \end{cases}$$

Then $x \in c_0^I(\Delta^n)$ but by (1), $x \notin \ell_\infty^I(F, \Delta^n)$ which contradicts (b). Hence (c) must hold.
(c) implies (a). Let (c) be satisfied and $x \in \ell_\infty^I(F, \Delta^n)$. If we suppose that $x \notin \ell_\infty^I(F, \Delta^n)$ then

$$\sup_k f_k(|\Delta^n x_k|) = \infty \text{ for } \ \Delta^n x \in \ell_\infty^I$$

If we take $t = |\Delta^n x|$ then $\sup_k f_k(t) = \infty$ which contradicts (c).
Hence $\ell_\infty^I(\Delta^n) \subseteq \ell_\infty^I(F, \Delta^n)$.

Theorem 2.2. For a sequence $F = f_k$ is a sequence of moduli, the following statements are equivalent:

(*a*) $c_0^I(F, \Delta^n) \subseteq c_0^I(\Delta^n)$,

(*b*) $c_0^I(F, \Delta^n) \subseteq \ell_\infty^I(\Delta^n)$,

(*c*) $\inf_k f_k(t) > 0, (t > 0)$.

Proof. (a) implies (b) is obvious.

(b) implies (c). Let $c_0^I(F, \Delta^n) \subseteq \ell_\infty^I(\Delta^n)$. Suppose that (c) is not true. Then by Lemma (1.6)

$$\inf_k f_k(t) = 0, \quad (t > 0)$$

and therefore there is a sequence (k_i) of positive integers such that

$$f_{k_i}(i^2) < \frac{1}{i} \ \text{ for each } \quad i = 1, 2, 3...... \tag{2}$$

Define $x = (x_k)$ as follow

$$x_k = \begin{cases} i^2, \text{ if } \ k = k_i \ \ i = 1, 2, 3.....; \\ 0 \ \ \text{otherwise.} \end{cases}$$

By (2) $x \in c_0^I(F, \Delta^n)$ but $x \notin \ell_\infty^I(\Delta^n)$ which contradicts (b). Hence (c) must hold.
(c) implies (a). Let (c) be satisfied and $x \in c_0^I(F, \Delta^n)$ that is

$$I - \lim_k f_k(|\Delta^n x_k|) = 0.$$

Suppose that $x \notin c_0^I(\Delta^n)$. Then for some number $\epsilon_0 > 0$ and positive integer k_0 we have $|\Delta^n x_k| \leq \epsilon_0$ for $k > k_0$. Therefore $f_k(\epsilon_0) \geq f_k(|\Delta^n x_k|)$ for $k > k_0$ and hence $\lim_k f_k(\epsilon_0) > 0$, which contradicts our assumption that $x \notin c_0^I(\Delta^n)$.

Thus $c_0^I(F, \Delta^n) \subseteq c_0^I(\Delta^n)$.

Theorem 2.3. The inclusion $\ell_\infty^I(F, \Delta^n) \subseteq c_0^I(\Delta^n)$ holds if and only if

$$\lim_k f_k(t) = \infty \ \text{ for } \ t > 0. \tag{3}$$

Notes

Proof. Let $\ell^I_\infty(F,\Delta^n) \subseteq c^I_0(\Delta^n)$ such that $\lim_k f_k(t) = \infty$ for $t > 0$ doesn't hold. Then there is a number $t_0 > 0$ and a sequence (k_i) of positive integer such that

$$f_{k_i}(t_0) \leq M < \infty. \tag{4}$$

define the sequence $x = (x_k)$ by

$$x_k = \begin{cases} t_0, & \text{if } \ k = k_i \ \ i = 1,2,3.....; \\ 0 & \text{otherwise.} \end{cases}$$

Notes

Thus $x \in \ell^I_\infty(F,\Delta^n)$ by (4). But $x \notin c^I_0(\Delta^n)$, so that (3) must hold.If $\ell^I_\infty(F,\Delta^n) \subseteq c^I_0(\Delta^n)$. Conversely, let (3) hold. If $x \in \ell^I_\infty(F,\Delta^n)$,then $f_k(|\Delta^n x_k|) \leq M < \infty$, for $k = 1,2,3$.......Suppose that $x \notin c^I_0(\Delta^n)$. Then for some number $\epsilon_0 > 0$ and positive integer k_0 we have $|\Delta^n x_k| < \epsilon_0$ for $k \geq k_0$. Therefore $f_k(\epsilon_0) \geq f_k(|\Delta^n x_k|) \leq M$ for $k \geq k_0$, which contradicts(3). Hence $x \in c^I_0(\Delta^n)$.

Theorem 2.4. The inclusion $\ell^I_\infty(\Delta^n) \subseteq c^I_0(F,\Delta^n)$ holds if and only if

$$\lim_k f_k(t) = 0, \quad \text{for } t > 0. \tag{5}$$

Proof. Suppose that $\ell^I_\infty(\Delta^n) \subseteq c^I_0(F,\Delta^n)$ but (5) doesn't hold,

Then

$$\lim_k f_k(t_0) = l \neq 0, \quad \text{for some } t_0 > 0 \qquad (6).$$

Define the sequence $x = (x_k)$ by

$$x_k = t_0 \sum_{v=0}^{k-n} (-1)^n \begin{bmatrix} n+k-v-1 \\ k-v \end{bmatrix}$$

for $k = 1,2,3,....$ Then $x \notin c^I_0(F,\Delta^n)$ by (6). Hence (5) must hold.
conversely, let $x \in \ell^I_\infty(\Delta^n)$ and suppose that (5) holds.
Then$|\Delta^n x_k| \leq M < \infty$ for $K = 1,2,3....$ There for $f_k(|\Delta^n x_k|) \leq f_k(M)$ for $k = 1,2,3.....$ and $\lim_k f_k(|\Delta^n x_k|) \leq \lim_k f_k(M) = 0$ by (5). Hence $x \in c^I_0(F,\Delta^n)$

References Références Referencias

1. C.A.Bektas, R.Colak, Generalized difference sequence spaces defined by a sequence of moduli, Soochow. J.Math., 29(2)(2003), 215-220 164.
2. R.Colak and M.ve Et, on some generalized difference sequence spaces and related matrix transformations, Hokkaido Math.J.,26(3)(1997),483-492.
3. K.Demerici, I-limit superior and limit inferior, Math.Commun., 6(2001),165-172.
4. K.Dems, On I-Cauchy sequences, Real Analysis Exchange., 30(2005), 123-128.
5. A.Esi and M.Isik, Some generalized difference sequence spaces, Thai J.Math.,3(2)(2005),241-247.
6. H.Fast, Sur Ia convergence statistique, Colloq.Math., 2(1951), 241-244.
7. J.A.Fridy, On statical convergence, Analysis., 5(1985),301-313.
8. J.A.Fridy, Statistical limit points, Proc. Amer.Math.Soc., 11(1993), 1187-1192.
9. A.K. Gaur and M.Mursaleen, Di˙erence sequence spaces defined by a sequence of moduli, Demonstratio Math., 31(1998), 275-278.

10. H.Kizmaz, on certain sequence spaces, Canadian Math.Bull.,24(1981), 169-176.
11. E.Kolak, On strong boundedness and summability with respect to a sequence of moduli, Acta Comment.Univ.Tartu.,960(1993),41-50.
12. E.Kolak, Inclusion theorems for some sequence spaces defined by a sequence of modulii, Acta Comment.Univ. Tartu., 970(1994), 65-72.
13. P.kostyrko.T.Salat and W.Wilczynski, I-Convergence, Real Analysis Exchange., 26 (2)(2000), 669-686.
14. I.J.Maddox, Sequence spaces defined by a modulus, Math.Camb.Phil.Soc., 100(1986),161-166.
15. H.Nakano, Concave modulars, J.Math Soc. Japan., 5(1953),29-49.
16. W.H.Ruckle, On perfect Symmetric BK-spaces,Math.Ann.,175(1968),121-126.
17. W.H.Ruckle, Symmetric coordinate space and symmetric bases,Canad,J.Math., 19(1967),828-838.
18. W.H.Ruckle, FK-spaces in which the sequence of coordinate vectors is bounded. Canad.J.Math.,25(5)(1973),973-975.
19. T.salat, On statitical convergent sequences of real numbers, Math. Solvaca., 30(1980), 139-150.
20. T.salat, B.C.Tripathy and M.Ziman, On some properties of I-convergence, Tatra Mt. Math.Publl., 28(2004),279-286.

Notes

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A Priori Estimates and Continuous Dependence of Solutions of Thermophysical Problems as Nonlocal Boundary Conditions Pass into Local One

By N. I. Yurchuk & T. S. Shlapakova
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Abstract - A priori estimates of the differences of solutions of non-local and local problems for heat equations are established. Using them prove continuous dependence of solutions of nonlocal problems as nonlocal boundary conditions pass into local ones.

Keywords : a priori estimates, continuous dependence, nonlocal boundary conditions pass into local conditions.

GJSFR-F Classification : MSC 2010: 35B45

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A Priori Estimates and Continuous Dependence of Solutions of Thermophysical Problems as Nonlocal Boundary Conditions Pass into Local One

N. I. Yurchuk [α] & T. S. Shlapakova [σ]

Abstract - A priori estimates of the differences of solutions of non-local and local problems for heat equations are established. Using them prove continuous dependence of solutions of nonlocal problems as nonlocal boundary conditions pass into local ones.

Keywords : a priori estimates, continuous dependence, nonlocal boundary conditions pass into local conditions.

I. Introduction

Let a rod of length l is placed to a continuum with $h(t)$ temperature (Fig. 1) and it enters into a continuum in α point. The temperature $u_\alpha(x,t)$ into the rod is satisfied the following equation

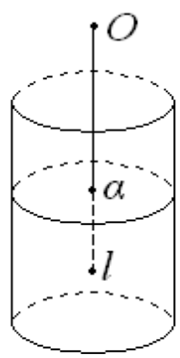

Fig 1

$$\frac{\partial u_\alpha}{\partial t} - a(x,t)\frac{\partial^2 u_\alpha}{\partial x^2} = f(x,t), \qquad (1)$$

an initial condition

$$u_\alpha(x,0) = \varphi_\alpha(x), \qquad (2)$$

one of the boundary condition in the point $x = 0$

$$u_\alpha(0,t) = 0 \qquad \text{or} \qquad \frac{\partial u_\alpha(0,t)}{\partial x} = 0 \qquad (3)$$

and one of the non-local condition on the segment (α, l)

$$\frac{1}{l-\alpha}\int_\alpha^l u_\alpha(\xi,t)\,d\xi = h(t) \quad \text{or} \quad \frac{u_\alpha(l,t) - u_\alpha(\alpha,t)}{l-\alpha} = h(t). \qquad (4)$$

Each problem (1) – (4) for each $0 \le \alpha < l$ is well-posed according to Hadamard, i.e. it has a solution for each $0 \le \alpha < l$, the solution is unique, the solution depends continuously on f, φ_α and h. Such problems were investigated in works of Kamynin [7],

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Ref.
7. N. I. Kamynin; A boundary value problem in the theory of the heat condition with nonclassical boundary condition, Th., Vychisl., Mat., Fiz., 4:6, (1964), p. 1006 – 1024.

Ionkin-Moiseyev [6], Kartynnik [8], Benouar-Yurchuk [1], Benouar-Bouziani [2], Gasimov [4], Yurchuk [11] ect. (see [12] and referred to the literature).

When a rod has been removing from the continuum, i. e. when $\alpha \to l$, problems (1) – (4) take the form of

$$\frac{\partial u_l}{\partial t} - a(x,t)\frac{\partial^2 u_l}{\partial x^2} = f(x,t), \quad (5)$$

$$u_l(x,0) = \varphi_l(x), \quad (6)$$

$$u_l(0,t) = 0 \text{ or } \frac{\partial u_l(0,t)}{\partial x} = 0, \quad (7)$$

$$u_l(l,t) = h(t) \text{ or } \frac{\partial u_l(l,t)}{\partial x} = h(t). \quad (8)$$

These problems are also well-posed according to Hadamard. They are easier than problems (1) – (4) and they have been researched by the number of authors. The solutions of such problems describe a temperature into a rod when the end of the rod contacts a continuum with $h(t)$ temperature. The importance of the investigation of problems (1) – (4) has been pointed out by Samarskii [9] since these problems are encountered in plasma physic. In fact, the solutions $u_l(x,t)$ of problems (1) – (4) may be described by the temperature in graphite rod which is placed in continuum of atomic reactor. When atomic reactor has been stopping, graphite rod is removed from continuum of the reactor. To know a behavior of a temperature $u_\alpha(x,t)$ when $\alpha \to l$ is important for fire safety.

In this paper a priori estimates for the difference $u_\alpha(x,t) - u_l(x,t)$ are established. Using these estimates prove that

if $\alpha \to l$, $\varphi_\alpha \to \varphi_l$ then $u_\alpha \to u_l$. Therefore, there establish a new important property that a solution of mixed problems (1)-(4) with non-local conditions continuously change as non-local conditions pass into local ones. As mention, this property has an applied importance.

II. Problems with Integral Boundary Conditions

In this section we give a priori estimates and show that $u_\alpha \to u_l$ when u_α is satisfied integral condition from (4) and $u_l(l,t) = h(t)$.

Theorem 1. Let the coefficient $a(x,t)$ in (1) and (5) is a continuous differentiable function on $G = [0,l] \times [0,T]$ $\quad 0 < a_0 \le a(x,t) \le a_1$,

$f \in L_2(G)$, $h \in W_2^1(0,T)$, $\varphi_\alpha, \varphi_l \in W_2^1(0,l)$, $\varphi_\alpha(0) = \varphi_l(0) = 0$ or $\varphi'_\alpha(0) = \varphi'_l(0)$ depending on conditions from (3), $\varphi_l(0) = h(0)$, $\frac{1}{l-\alpha}\int_\alpha^l \varphi_\alpha(x)dx = h(0)$.

Then there exist constant $c > 0$ independent of u_α, u_l and such l that

$$\int_0^l \int_0^T (1-x)\left[\left|\frac{\partial u_\alpha}{\partial t} - \frac{\partial u_l}{\partial t}\right|^2 + \left|\frac{\partial^2 u_\alpha}{\partial x^2} - \frac{\partial^2 u_l}{\partial x^2}\right|^2\right]dxdt +$$

Ref.

6. N. I. Ionkin and E. I. Moiseev; Solutions of boundary value problem in heat condition theory with nonlocal conditions, Diffentsial'ye Uravneniya, 13 (1977), p. 294 – 304.

$$+\sup_{0\le t\le T}\int_0^l\left[|u_\alpha-u_l|^2+(l-x)\left|\frac{\partial u_\alpha}{\partial x}-\frac{\partial u_l}{\partial x}\right|^2\right]dx\le$$

$$\le C\left\{\int_0^l\left|\varphi'_\alpha(x)-\varphi'_l(x)\right|^2dx+\left|h(0)-\frac{1}{l-\alpha}\int_\alpha^l u_l(x,0)dx\right|^2+\right. \tag{9}$$

$$\left.+\int_0^T\left[\left|h'(t)-\frac{1}{l-\alpha}\int_\alpha^l\frac{\partial u_l(x,t)}{\partial t}dx\right|^2+\left|h(t)-\frac{1}{l-\alpha}\int_\alpha^l u_l(x,t)dx\right|^2dt\right]\right\}$$

The proof of inequality (9) is given in [1] when $u_\alpha(0,t)=0$ and in [2] when $\frac{\partial u_\alpha(0,t)}{\partial x}=0.$ From inequality (9) follows

Theorem 2. Let the conditions of theorem 1 be satisfied. If

$$\lim_{\alpha\to l}\int_0^l\left|\varphi'_l(x)-\varphi'_\alpha(x)\right|^2dx=0, \tag{10}$$

then

$$\lim_{\alpha\to l}\left\{\int_0^l\int_0^T(l-x)\left[\left|\frac{\partial u_\alpha}{\partial t}-\frac{\partial u_l}{\partial t}\right|^2+\left|\frac{\partial^2 u_\alpha}{\partial x^2}-\frac{\partial^2 u_l}{\partial x^2}\right|^2\right]dxdt+\right. \tag{11}$$

$$\left.+\sup_{0\le t\le T}\int_0^l\left[(l-x)\left|\frac{\partial u_\alpha}{\partial x}-\frac{\partial u_l}{\partial x}\right|^2+|u_\alpha-u_l|^2\right]dx\right\}=0.$$

Proof. Since

$$\lim_{\alpha\to l}\left|\frac{1}{l-\alpha}\int_\alpha^l u_l(x,t)dx-h(t)\right|=\lim_{\alpha\to l}\left|\frac{1}{l-\alpha}\int_\alpha^l u_l(x,t)dx-u_l(l,t)\right|=0,$$

$$\lim_{\alpha\to l}\left|\frac{1}{l-\alpha}\int_\alpha^l\frac{\partial u_l(x,t)}{\partial x}dx-h'(t)\right|=\lim_{\alpha\to l}\left|\frac{1}{l-\alpha}\int_\alpha^l\frac{\partial u_l(x,t)}{\partial t})dx-\frac{\partial u_l(l,t)}{\partial t})\right|=0,$$

$$\lim_{\alpha\to l}\left|\frac{1}{l-\alpha}\int_\alpha^l u_l(x,0)dx-h(0)\right|=\lim_{\alpha\to l}\left|\frac{1}{l-\alpha}\int_\alpha^l u_l(x,0)dx-u_l(l,0)\right|=0$$

then from (9) using (10) follows (11). To complete the proof show that for each function $\varphi_l\in W_2^1(0,l)$ satisfying the conditions $\varphi_l(0)=0$ or $\varphi'_l(0)=0$ and $\varphi_l(l)=h(0)$ there exist the functions $\varphi_\alpha\in W_2^1(0,l)$ such that there hold the relations

$$\varphi'_\alpha(0)=0 \text{ or } \varphi'_\alpha(0)=0 \text{ and } \frac{1}{l-\alpha}\int_\alpha^l\varphi_\alpha(x)dx=h(0). \tag{12}$$

If $\varphi_\alpha(0)=0$ that these functions should be the following

$$\varphi_\alpha(x)=\varphi_l(x)+\frac{2x}{l+\alpha}\left(h(0)-\frac{1}{l-\alpha}\int_\alpha^l\varphi_l(x)dx\right),$$

and if $\varphi'_\alpha(0)=0$ such functions should be

$$\varphi_\alpha(x)=\varphi_l(x)+\frac{3x^2}{l^2+l\alpha+\alpha^2}\left(h(0)-\frac{1}{l-\alpha}\int_\alpha^l\varphi_l(x)dx\right).$$

Ref.

2. N. E. Benouar and A. Bouziani; Problème mixte avec conditions intègrales pour une classe d'équation parabolique, C. R. A. S, Paris, 321, Série I, (1995), p. 1177 – 1182.

III. Problems with Second Nonlocal Condition

In the rectangle $G=(0,l)\times(0,T)$ consider the set of mixed problems with nonlocal conditions:

$$\frac{1}{a(x,t)}\frac{\partial u_\alpha}{\partial t}-\frac{\partial^2 u_\alpha}{\partial x^2}=f(x,t),\quad 0<\alpha<l, \tag{13}$$

$$u_\alpha(x,0)=\varphi_\alpha(x),\quad u_\alpha(0,t)=0,\quad \frac{u_\alpha(l,t)-u_\alpha(\alpha,t)}{l-\alpha}=h(t), \tag{14}$$

and two mixed problems with local conditions:

$$\frac{1}{a(x,t)}\frac{\partial u_0}{\partial t}-\frac{\partial^2 u_0}{\partial x^2}=f(x,t), \tag{15}$$

$$u_0(x,0)=\varphi_0(x),\quad u_0(0,t)=0,\quad u_0(l,t)=h(t), \tag{16}$$

$$\frac{1}{a(x,t)}\frac{\partial u_l}{\partial t}-\frac{\partial^2 u_l}{\partial x^2}=f(x,t), \tag{17}$$

$$u_l(x,0)=\varphi_l(x),\quad u_l(0,t)=0,\quad \frac{\partial u_l(l,t)}{\partial x}=h(t). \tag{18}$$

Suppose that the coefficient $a(x,t)$ in equations (13), (15), (17) is a continuous and continuous differentiable function and

$$a_1\geq a(x,t)\geq a_0>0,\quad f\in L_2(G),\quad h\in W_2^1(0,T),\quad \varphi_\alpha,\ \varphi_0,\varphi_l\in W_2^1(0,l),\quad \varphi_\alpha(0)=\varphi_0(0)=\varphi_l(0)=0,$$

$$\varphi_0(l)=h(0),\quad \varphi_l'(l)=h(0),\quad \frac{\varphi_\alpha(l)-\varphi_\alpha(\alpha)}{l-\alpha}=h(0).$$

It's known that under these assumptions there exist unique solutions of local equations to (15), (16) and (17), (18) and these solutions have almost everywhere on G first-order derivative with respect to x and second-order derivative with respect to t. When smoothness of data of the problem increases the smoothness of the solution will increase.

In works [3], [4] the solution existence and uniqueness of set of problems (13), (14) are proved for each $0<\alpha<l$. It's obtained as follows. Initially in problems (13), (14) replace the unknown functions by the formula

$$u_\alpha(x,t)=v_\alpha(x,t)+xh(t), \tag{19}$$

For new functions $v_\alpha(x,t)$ there obtain for each $0<\alpha<l$ the problem

$$\frac{1}{a(x,t)}\frac{\partial v_\alpha}{\partial t}-\frac{\partial^2 v_\alpha}{\partial x^2}=f(x,t)\equiv f(x,t)-\frac{x}{a}h'(t), \tag{20}$$

$$v_\alpha(x,0)=\tilde{\varphi}_\alpha(x)\equiv\varphi_\alpha(x)-xh(0),\quad v_\alpha(0,t)=0,\quad \frac{v_\alpha(l,t)-v_\alpha(\alpha,t)}{l-\alpha}=0. \tag{21}$$

and there establish a priori estimate

$$\sup_{0\leq t\leq T}\int_0^l \psi_\alpha(x)v_\alpha^2(x,t)dx+\int_0^T\int_0^l \psi_\alpha(x)\left(\frac{\partial v_\alpha(x,t)}{\partial x}\right)^2 dxdt\leq$$

Ref.

3. N. E. Benouar and A. Bouziane; Mixed problem with integral conditions for third order parabolic equation, Kobe J. Math., 15 (1998), p. 47 – 58.

$$\le C\left(\int_0^l \psi_\alpha(x)\tilde{\varphi}_\alpha^2(x)dx + \int_0^T\int_0^l \psi_\alpha(x) f^2(x,t)dxdt\right), \qquad (22)$$

where constant C is independent of v_α and

$$\psi_\alpha = \begin{cases} 1, & 0 \le x \le \alpha \\ \dfrac{l-x}{l-\alpha}, & \alpha \le x \le l. \end{cases}$$

Notes

Using a priori estimates (22) prove the existence and uniqueness of problem (20), (21) and therefore of problems (13), (14) for each $0 < \alpha < l$.

Then establish a priori estimates of the differences $u_\alpha - u_0$ and $u_\alpha - u_l$. Denote by $u(x,t)$ one of the solutions u_0 of problems (15), (16) or by u_l of problems (17), (18). A new function $w(x,t)$ is introduced as follows

$$w_\alpha = u - u_\alpha + x\left[h(t) - \frac{u(l,t)-u(\alpha,t)}{l-\alpha}\right]. \qquad (23)$$

This function is the solution of the problem

$$\frac{1}{a}\frac{\partial w_\alpha}{\partial t} - \frac{\partial^2 w_\alpha}{\partial x^2} = \frac{x}{a}\left[h'(t) - \frac{\dfrac{\partial u(l,t)}{\partial t} - \dfrac{\partial u(\alpha,t)}{\partial t}}{l-\alpha}\right], \qquad (24)$$

$$w_\alpha(x,0) = \varphi - \varphi_\alpha - x\left[\frac{u(l,0)-u(\alpha,0)}{l-\alpha} - h(0)\right], \qquad (25)$$

$$w_\alpha(0,t) = 0, \qquad \frac{w_\alpha(l,t)-w_\alpha(\alpha,t)}{l-\alpha} = 0, \qquad (26)$$

where $\varphi(x)$ – one of the function $\varphi_0(x)$ or $\varphi_l(x)$ depending on designation of the function u. Therefore there holds a similar (22) a priori estimate for w_α

$$\sup_{0\le t\le T}\int_0^l \psi_\alpha(x) w_\alpha^2(x,t)dx + \int_0^T\int_0^l \psi_\alpha(x)\left(\frac{\partial w_\alpha(x,t)}{\partial x}\right)^2 dxdt \le$$

$$\le 2C\left(\int_0^l \psi_\alpha(x)\left|\varphi(x)-\varphi_\alpha(x)\right|^2 dx + \int_0^l \psi_\alpha(x)x^2\left|\frac{u(l,0)-u(\alpha,0)}{l-\alpha} - h(0\right|^2) dx + \right. \qquad (27)$$

$$\left. + \int_0^T\int_0^l \psi_\alpha \frac{x^2}{a^2(x,t)}\left|h'(t) - \frac{\dfrac{\partial u(l,t)}{\partial t} - \dfrac{\partial u(\alpha,t)}{\partial t}}{l-\alpha}\right|^2 dxdt\right).$$

This inequality should be valid if the inequality $\psi_\alpha(x) \ge l - x$ is applied for left-hand side and for the inequality $\psi_\alpha(x) \le 1$ is applied for right-hand side. One of the solution u_0 or u_l of problems (15), (16) or (17), (18) accordingly is substituted in resulting inequality after those estimates instead of $u(x,t)$. As a result there obtain a priori estimates

$$\sup_{0\le t\le T}\int_0^l (l-x)\left|u_0-u_\alpha\right|^2dx+\int_0^T\int_0^l (l-x)\left|\frac{\partial u_0}{\partial x}-\frac{\partial u_\alpha}{\partial x}\right|^2 dxdt\le$$

$$\le C\left\{\int_0^l\left|\varphi_0(x)-\varphi_\alpha(x)\right|^2dx+\int_0^T\left[\left|\frac{\alpha h(t)-u_0(\alpha,t)}{l-\alpha}\right|^2+\right.\right. \qquad (28)$$

$$\left.\left.+\left|\frac{\alpha h'(t)-\dfrac{\partial u_0(\alpha,t)}{\partial t}}{l-\alpha}\right|^2\right]dt+\left|\frac{\alpha h(0)-\varphi_0(\alpha)}{l-\alpha}\right|^2\right\},$$

$$\sup_{0\le t\le T}\int_0^l (l-x)\left|u_l-u_\alpha\right|^2dx+\int_0^T\int_0^l (l-x)\left|\frac{\partial u_l}{\partial x}-\frac{\partial u_\alpha}{\partial x}\right|^2 dxdt\le$$

$$\le C\left\{\int_0^l\left|\varphi_l(x)-\varphi_\alpha(x)\right|^2dx+\int_0^T\left[\left|\frac{u_l(l,t)-u_l(\alpha,t)}{l-\alpha}-h(t)\right|^2+\right.\right. \qquad (29)$$

$$\left.\left.+\left|\frac{\dfrac{\partial u_l(1,t)}{\partial t}-\dfrac{\partial u_l(\alpha,t)}{\partial t}}{l-\alpha}-h'(t)\right|^2\right]dt+\left|\frac{\varphi_l(l)-\varphi_l(\alpha)}{l-\alpha}-h(0)\right|^2\right\}.$$

From these priori estimates follows the next continuous dependence of the solutions of problem (13), (14) on parameter α.

Theorem 3. If

$$\lim_{\alpha\to 0}\int_0^l\left|\varphi_\alpha(x)-\varphi_0(x)\right|^2dx=0, \qquad (30)$$

then

$$\lim_{\alpha\to 0}\left[\sup_{0\le t\le T}\int_0^l (l-x)\left|u_\alpha-u_0\right|^2dx+\int_0^T\int_0^l (l-x)\left|\frac{\partial u_\alpha}{\partial x}-\frac{\partial u_0}{\partial x}\right|^2 dxdt\right]=0. \qquad (31)$$

Proof. Since for the solutions of problem (15), (16) there hold the relations $\lim_{\alpha\to 0}\int_0^T u_0(\alpha,t)dt=0$, $\lim_{\alpha\to 0}\int_0^T\left|\frac{\partial u_0(\alpha,t)}{\partial t}\right|^2dx=0$ and $\lim_{\alpha\to 0}\varphi_0(\alpha)=0$, then from (30) using (28) follows (31). To complete the proof show that for any function $\varphi_0\in W_2^1(0,T)$ satisfying the conditions $\varphi_0(0)=0$ and $\varphi_0(l)=h(0)$ there exist the functions $\varphi_\alpha\in W_2^1(0,T)$ such that $\varphi_\alpha(0)=0$, $\frac{\varphi_\alpha(l)-\varphi_\alpha(\alpha)}{l-\alpha}=h(0)$ and (30) is valid. It's clear that it's valid for $\varphi_\alpha(x)=\varphi_0(x)-x\left(\frac{\alpha h(0)-\varphi_0(\alpha)}{l-\alpha}\right)$. Thus theorem 3 is proved.

Theorem 4. If

$$\lim_{\alpha\to l}\int_0^l\left|\varphi_\alpha(x)-\varphi_l(x)\right|^2dx=0, \qquad (32)$$

then

$$\lim_{\alpha\to l}\left[\sup_{0\le t\le T}\int_0^l (1-x)\left|u_\alpha-u_l\right|^2dx+\int_0^T\int_0^l (l-x)\left|\frac{\partial u_\alpha}{\partial x}-\frac{\partial u_l}{\partial x}\right|^2 dxdt\right]=0. \qquad (33)$$

Notes

Notes

Proof. Since for solutions of problems (17), (18) there hold the relations

$$\lim_{\alpha\to l}\int_0^T\left[\left|\frac{u_l(l,t)-u_l(\alpha,t)}{l-\alpha}-h(t)\right|^2+\left|\frac{\frac{\partial u_l(l,t)}{\partial t}-\frac{\partial u_l(\alpha,t)}{\partial t}}{l-\alpha}-h'(t)\right|^2\right]dt=0$$

and $\lim_{\alpha\to l}\frac{\varphi_l(x)-\varphi_l(\alpha)}{l-\alpha}=h(0)$ that from (32) using (29) follows (33). To complete the investigation show that for any function $\varphi_l\in W_2^1(0,T)$ satisfying the conditions $\varphi_l(0)=0$, $\varphi_l'(l)=h(0)$ there exist the functions $\varphi_\alpha\in W_2^1(0,T)$ such that $\varphi_\alpha(0)=0,$ $\frac{\varphi_\alpha(l)-\varphi_\alpha(\alpha)}{l-\alpha}=h(0)$ and (32) is valid. These functions can be the following

$$\varphi_\alpha(x)=\varphi_l(x)-x\left(\frac{\varphi_l(l)-\varphi_l(\alpha)}{l-\alpha}-h(0\right).$$

Theorem4 is proved.

References Références Referencias

1. N.E. Benouar and N. I. Yurchuk; Mixed problem with an integral condition for parabolique equation with the Bessel operator, Differentsial'nye Uravneniya, 27 (1991), p. 2094 – 2098.
2. N. E. Benouar and A. Bouziani; Problème mixte avec conditions intègrales pour une classe d'équation parabolique, C. R. A. S, Paris, 321, Série I, (1995), p. 1177 – 1182.
3. N. E. Benouar and A. Bouziane; Mixed problem with integral conditions for third order parabolic equation, Kobe J. Math., 15 (1998), p. 47 – 58.
4. E. A. Gasymov; Mixed problems on the conjugaison of parabolic system of different order with nonlocal boundary conditions, Differentsial'nye Uravneniya, 26, № 8, (1991), p. 1003 – 1012.
5. N. I. Ionkin; Solutions of boundary value problems in heat condition theory with nonlocal boundary conditions, Differentsial'nye Uravneniya, 13:2 (1977), p. 294 – 304.
6. N. I. Ionkin and E. I. Moiseev; Solutions of boundary value problem in heat condition theory with nonlocal conditions, Diffentsial'ye Uravneniya, 13 (1977), p. 294 – 304.
7. N. I. Kamynin; A boundary value problem in the theory of the heat condition with nonclassical boundary condition, Th., Vychisl., Mat., Fiz., 4:6, (1964), p. 1006 – 1024.
8. Kartynnik; Problème mixte à trois points avec conditions intégrales suivant les variables spaciales pour les equations paraboliques du second ordre' Equations différentielles, 26:9 (1990), p. 1568 – 1775.
9. Samarskii; Some problems in differential equation theory, Differentsial'niye Uravneniya, 16:11 (1980), p. 1925 – 1935.
10. N. I. Yurchuk; Mixed problem with an integral condition for certain parabolic equations, Differentsial'nye Uravneniya, 22:12, (1986), p. 2117 – 2126.
11. N. I. Yurchuk; Estimation a priori at la dependance continue de la solution du problème mixte pour l'équation parabolique de la forme des conditions aux limites, Méthodes analytiques de l'analyse et équations différentielles, AMADE, 2003, Cottenhan, Cambrige scientifigue, 2006, p. 247 – 254.

12. Yurchuk N. I.; A priori estimate and a continuous dependence of a solution of mixed problems for parabolic equations on the form of boundary conditions, Analytic Method of Analysis and Differential Equations: AMADE. 2003. Cambridge Scientific Publishers. 2005. pp. 243 – 250.
13. N. I. Yurchuk, Tcharie Kokou; Moussa Zakari Djibibe, Continuous dependence of solutions to mixed boundary value problems for a parabolic equations, Electronic Journal of Differential Equations. Vol. 2008 (2008). № 17. pp. 1 – 10.
14. N. I. Yurchuk, Charie Koku; A priory estimates and continuous dependence of solutions of mixed problems for parabolic equations as nonlocal boundary conditions pass into local ones, Differential Equations. 2008. Vol. 44. № 33. pp. 434 – 441.

Notes

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Numerical Simulation of the Effect of Distant Tsunami along the Coast of Peninsular Malaysia and Southern Thailand through an Open Boundary Condition in a Linear Model

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Abstract - The 26 December 2004 Indonesian tsunami was the third known global tsunami and reached every distant corner of the globe. An effort has been made here to evaluate the effect of this distant tsunami in a limited area model domain. The effect of distant tsunami has been simulated through an open boundary condition in a Cartesian coordinate shallow water linear model. The open boundary condition is applied to simulate the tsunami propagation when it is assumed that the tsunami is generated far away from the region of interest. The computational domain covers the region so that the 26 December 2004 Indonesian tsunami source is well within the model domain. First, the initial disturbance of the tsunami source are examined along the western open boundary of the model domain and then the boundary condition is formulated and adjusted in such a manner that the effects of tsunami due to the source along the coasts are same as the effects due to the formulated boundary condition. The response of the open boundary condition is investigated along the coastal region of Peninsular Malaysia and southern Thailand. The results are compared with the data available in the website and a very reasonable agreement is observed.

Keywords : Shallow water equations; Open boundary condition; Distant tsunami; Tsunami source; Damping amplitude, Indonesian tsunami 2004.

GJSFR-F Classification : MSC 2010: 76F65

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I. Introduction

A tsunami is a natural coastal hazard generated in the deep ocean by vertical displacement of ocean water column and propagates across the ocean from the point of generation to the coast. It is usually a shallow water wave. A wave is characterized as a shallow water wave when the ratio between the water depth and its wavelength gets very small ($h \,/\, L < 0.05$). Tsunami has a very large wavelength and the speed is directly proportional to the depth of water. So it propagates in deep ocean at a very high speed with a limited loss of energy since the rate at which a tsunami wave loses its energy is inversely proportional to its wavelength. Thus the effect of a tsunami source along a particular region far away from the source position may be significant if the waves move through deep ocean.

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The response of the 2004 Indonesian tsunami wave reached every distant corner of the globe (Kowalik et al., 2005). The first known global tsunami that associated with the Krakatau Volcano explosion of 27 August 1883 (Murty, 1977) was generated in the same region where the 2004 earthquake occurred. The second known global tsunami was the Chilean tsunami of May 22, 1960 and tsunami waves observed at many far-field sites were very strong (Berkman and Symons, 1964). The 2004 Indonesian tsunami, resulting from a strong under sea earthquake that occurred off the coast of Sumatra of Indonesia, was the third known global tsunami and it was clearly recorded by a large number of tide gauges throughout the world ocean, including tide gauges located in the North Pacific and North Atlantic (Rabinovich et al., 2006). The distant effect of this tsunami was noticed as far as Struisbaai in South Africa, some 8,500 km away from the source zone, where a 1.5 m high surge was recorded about 16 hours after the earthquake. The tsunami also reached Antarctica where oscillations of up to 1 m were recorded with disturbances lasting a couple of days (Indian Ocean Tsunami at Syowa Station, Antarctica, 2007). Some of the tsunami's energy escaped into the Pacific Ocean, where it produced small but measurable tsunamis along the western coasts of North and South America, typically around 20 to 40 cm (NOAA, 2005). Mid-ocean ridges played a major role as wave guides that transferred the tsunami energy to distant regions outside the source area in the Indian Ocean (Kowalik et al., 2005; Titov et al., 2005).

The effect of a tsunami source along a particular region far away from the source position may be significant if the waves move through deep ocean. Since the response of the 2004 Indonesian tsunami reached every distant corner of the globe, it is necessary to estimate the response along a particular region of interest due to a source located far away from that region. This may be done through a global model that contains both the source and the region of interest. However a global model is very expensive in terms of both computer storage and CPU time and is not suitable for real time simulation. In hydrodynamical computations problem arises when the theoretical model applies to an infinite or semi-infinite region. In this case, where the original domain of the problem under investigation is infinite or very large, open boundaries may be used. An open boundary is an artificial boundary of a computational domain through which propagation of waves or flow should pass in order to leave the computational domain without giving rise to spurious reflection (Joolen et. al. 2003). The main purpose of using the open boundaries is to allow waves and disturbances originating within the model domain to leave the domain without affecting the interior solution.

Imamura et al. (1988) developed a shallow water model to simulate far field or distant tsunami where they used the finite difference method with the leap-frog scheme. Cho and Yoon (1998) improved the model of Imamura et al. (1988). The limitation of the models developed by Imamura et al. (1988) and Cho and Yoon (1998) is that the models should be used to the case of constant water depth with a uniform finite difference grid. Yoon (2002) again improved the model of Imamura et al. (1988) over a slowly varying topography. But as the model of Yoon (2002) has a large number of hidden grids the model is not suitable to calculate the tsunami propagation in deep sea (Cho et al., 2007).

Tidal oscillation in a limited area model domain may be generated through an open boundary (Johns et al. 1985, Roy 1995). A sinusoidal wave is allowed to propagate towards the model domain through an open boundary by using appropriate amplitude, phase and time period of the wave. The response of this type of boundary condition at every grid point is also sinusoidal. By adjusting the amplitude and phase it is possible to generate a representative tidal oscillation of specific time period in the model domain. Similarly, in computing the effect of distant tsunami source in a limited area model

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surrounding the region of interest, a boundary condition may be incorporated along the open boundary, which is facing the tsunami source. From our literature survey it is found that not many works have been carried out on formulating appropriate open boundary conditions that may be imposed to compute tsunami response due to distant source in a limited area model domain.

Roy et al. (2006) developed a Cartesian co-ordinate non-linear shallow water model to formulate an open boundary condition to investigate the effect of distant tsunami along the coastal belt of Peninsular Malaysia and southern Thailand associated with Indonesian tsunami of 2004. The convective terms in the shallow water equations are insignificant and their effect is negligible in tsunami propagation in deep sea. However, the convective terms are weakly significant for the wave height near the coast. Thus a linear model can be applied for tsunami propagation in the deep sea, on the other hand, non-linear model should be applied to compute run-up or water level near the coast. The advantage of a linear model is that it needs less computer memory and computation time since the convective terms are excluded in the model; see for instance Zahibo et al. (2006). From the above discussion it should be conclude that it is better to use linear model instead of non-linear model for any hydrodynamical computations in deep sea. Since the west open boundary of our model domain is in the deep sea we are more interested to develop a linear model.

In this study, we describe the formulation of an open boundary condition for computing the effect of a distant tsunami in a Cartesian co-ordinate shallow water linear model. A linear Cartesian coordinate shallow water model has been developed to compute tsunami along the west coast of Peninsular Malaysia and Thailand associated with Indonesian tsunami of 2004. The analysis area of this model is a rectangular region approximately between 2° N, 14° N and 101.5° E, 91° E. First, the response of the tsunami source associated with the 2004 Indonesian tsunami is investigated along the west open boundary of the model domain. The 2004 tsunami source is incorporated as an initial condition during the computation. The linear shallow water equation with boundary conditions is applied to compute the maximum amplitudes and time series of water levels along the western open boundary. On the basis of the time series and amplitude, the open boundary condition is formulated for the western open boundary by a proper choice of the values for the amplitude, phase, period and the scale factor and at the same time the tsunami source near Sumatra is removed. The formulated boundary condition is imposed as an effect of tsunami source to compute the distant tsunami in absence of any tsunami source in the model domain.

II. Governing Equations and Boundary Conditions

The shallow water equations, which describe the inviscid flow of a thin layer of fluid in two dimensions, are a commonly accepted governing approximation for tsunami propagation in the deep ocean as well as in near-shore regions including inundation (Aizinger and Dawson, 2002). The depth averaged shallow water equations in Cartesian co-ordinates and the boundary conditions of this model are as follows:

A system of rectangular Cartesian coordinates is used in which the origin, O, is in the undisturbed sea surface (MSL), x-axis directed towards the west and y-axis directed towards the north and z-axis is directed vertically upwards. We consider the displaced position of the free surface as $z=\zeta(x,y,t)$ and the sea floor as $z = -h(x, y)$ so that the total depth of the fluid layer is $\zeta+h$. Taking into account that the characteristic wavelength exceeds the water depth, neglecting the convective terms and using the parameterization of the bottom stress via the depth averaged velocity components, due to Karim (2006), the linear shallow water equations are:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left[(\zeta + h)u\right] + \frac{\partial}{\partial y}\left[(\zeta + h)v\right] = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} - f\,v = -g\frac{\partial \zeta}{\partial x} - \frac{C_f u(u^2+v^2)^{1/2}}{\zeta + h} \qquad (2)$$

$$\frac{\partial v}{\partial t} + f\,u = -g\frac{\partial \zeta}{\partial y} - \frac{C_f v(u^2+v^2)^{1/2}}{\zeta + h} \qquad (3)$$

For numerical treatment it is convenient to express the Eqs. (2) & (3) in the flux form by using the Eq. (1).

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \qquad (4)$$

$$\frac{\partial \tilde{u}}{\partial t} - f\,\tilde{v} = -g\,(\zeta + h)\frac{\partial \zeta}{\partial x} - \frac{C_f\,\tilde{u}\left(u^2+v^2\right)^{1/2}}{\zeta + h} \qquad (5)$$

$$\frac{\partial \tilde{v}}{\partial t} + f\,\tilde{u} = -g\,(\zeta + h)\,\frac{\partial \zeta}{\partial y} - \frac{C_f\,\tilde{v}\left(u^2+v^2\right)^{1/2}}{\zeta + h} \qquad (6)$$

where, $(\tilde{u}, \tilde{v}) = (\zeta + h)\,(u, v)$

In the bottom stress terms of (5) & (6), u and v have been replaced by $\tilde{u}$ and $\tilde{v}$ in order to solve the equations in a semi-implicit manner.

In addition to the fulfillment of the surface and bottom conditions, appropriate conditions have to be satisfied along the boundaries of the model area for all time. Theoretically the only boundary condition needed in the vertically integrated system is that the normal component of the vertically integrated velocity vanishes at the coast and this may be expressed as $u\cos\alpha + v\sin\alpha = 0$, for all $t \ge 0$, where α denotes the inclination of the outward directed normal to x-axis. It follows that $u = 0$ along y-directed boundaries and $v = 0$ along the x-directed boundaries.

At the open-sea boundaries the waves and disturbance are allowed, generated within the model domain, to leave the domain without affecting the interior solution. Thus the normal component of velocity cannot vanish and so a radiation type of boundary is generally used. Following Heaps (1973), the following radiation type of condition may be used in our model:

$u\cos\alpha + v\sin\alpha = -\left(g/h\right)^{1/2}\zeta$, for all $t \ge 0$. Note that the velocity structure in a shallow water wave is described by $w = \dfrac{g\zeta}{\sqrt{gh}}$, where w is the horizontal particle velocity.

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Other than the coastal boundary of the domain the boundaries are considered as straight lines along the open sea. Thus there lie three open sea boundaries (Fig. 1). Commonly used radiation type of boundary conditions for the open sea boundaries, due to Johns et al. (1981), are:

$$u-(g/h)^{1/2}\zeta=0 \quad \text{at the west open boundary} \quad (7)$$

$$v+(g/h)^{1/2}\zeta=0 \quad \text{at the south open boundary} \quad (8)$$

$$v-(g/h)^{1/2}\zeta=0 \quad \text{at the north open boundary} \quad (9)$$

The coastal belts of the main land and islands are the closed boundaries where the normal components of the current are taken as zero.

III. Numerical Discretisation

The governing shallow water equations and the boundary conditions are discretised by finite difference (forward in time and central in space) and are solved by a conditionally stable semi-implicit method using a staggered grid system which is similar to Arakawa C system (Arakawa and Lamb, 1977). Let there are M gridlines parallel to y-axis and N gridlines parallel to the x-axis so that the total number of grid points are $M \times N$. We define the grid points (x_i, y_j) in the domain by

$$x_i = (i-1)\Delta x, \qquad i = 1, 2, 3, \ldots, M \quad (10)$$

$$y_j = (j-1)\Delta y, \qquad j = 1, 2, 3, \ldots, N \quad (11)$$

The sequence of discrete time instants is given by

$$t_k = k\,\Delta t, \qquad k = 1, 2, 3, \ldots \quad (12)$$

Discretisation in time can be performed with either explicit or implicit schemes. An explicit scheme uses numerical values from current time steps only in the advanced time step computations. In an implicit scheme values from current time steps and from the advanced time step are used in advanced time step computations. Here we use the semi-implicit scheme to descretise the equation. The indexing of the horizontal coordinates is (i, j) and the time steps are indexed by the superscript k. For the purpose of discretisation the following notations are used.

For any dependent variable $\chi(x, y, t)$, let us consider:

$$\chi(x_i, y_j, t_k) = \chi_{i\,j}^k$$

$$\frac{1}{2}(\chi_{i+1\,j}^k + \chi_{i-1\,j}^k) = \overline{\chi_{i\,j}^k}^{x}$$

$$\frac{1}{2}(\chi_{i\,j+1}^k + \chi_{i\,j-1}^k) = \overline{\chi_{i\,j}^k}^{y}$$

$$\frac{1}{4}(\chi_{i+1\,j}^k + \chi_{i-1\,j}^k + \chi_{i\,j+1}^k + \chi_{i\,j-1}^k) = \overline{\chi_{i\,j}^k}^{xy}$$

The discretised form of continuity equation (4) is

$$\frac{\zeta_{ij}^{k+1}-\zeta_{ij}^{k}}{\Delta t}+\frac{\left(\zeta_{i+1j}^{k}+h_{i+1j}\right)u_{i+1j}^{k}-\left(\zeta_{i-1j}^{k}+h_{i-1j}\right)u_{i-1j}^{k}}{2\Delta x}+\frac{\left(\zeta_{ij+1}^{k}+h_{ij+1}\right)v_{ij+1}^{k}-\left(\zeta_{ij-1}^{k}+h_{ij-1}\right)v_{ij-1}^{k}}{2\Delta y}=0 \quad (13)$$

from which we compute ζ_{ij}^{k+1} for $i=2,4,6,\ldots,M-2$ and $j=3,5,7,\ldots,N-2$

The boundary condition (7) is discretised as

$$u_{M-1j}^{k}-(g/h_{M-1j})^{1/2}\frac{1}{2}\left(\zeta_{M-2j}^{k+1}+\zeta_{Mj}^{k+1}\right)=0 \quad (14)$$

from which we compute ζ_{Mj}^{k+1} for $j=1,3,5,7,\ldots,N$

The boundary condition (8) is discretised as

$$v_{i2}^{k}+(g/h_{i2})^{1/2}\frac{1}{2}\left(\zeta_{i1}^{k+1}+\zeta_{i3}^{k+1}\right)=0 \quad (15)$$

from which we compute ζ_{i1}^{k+1} for $i=2,4,6,8,\ldots,M-2$

The boundary condition (9) is discretised as

$$v_{iN-1}^{k}-(g/h_{iN-1})^{1/2}\frac{1}{2}\left(\zeta_{iN-2}^{k+1}+\zeta_{iN}^{k+1}\right)=0 \quad (16)$$

from which we compute ζ_{iN}^{k+1} for $i=2,4,6,8,\ldots,M-2$

The discretised form of linear *x*-momentum equation (5) is

$$\frac{u_{ij}^{k+1}-u_{ij}^{k}}{\Delta t}-f\,\overline{v_{ij}}^{xy}=-g\frac{\zeta_{i+1j}^{k+1}-\zeta_{i-1j}^{k+1}}{2\Delta x}-\frac{C_f\,u_{ij}^{k+1}\left(\left(u_{ij}^{k}\right)^2+\left(\overline{v_{ij}^{k}}^{xy}\right)^2\right)^{1/2}}{\overline{\zeta_{ij}^{k+1}}^{x}+h_{ij}} \quad (17)$$

from which we compute u_{ij}^{k+1} for $i=3,5,7,\ldots,M-1$ and $j=3,5,7,\ldots,N-2$. Note that in the last term u_{ij}^{k+1} is in advanced time level and this ensures a semi-implicit nature of the numerical method.

Similarly, the discretised form of linear *y*-momentum equation (6) is

$$\frac{v_{ij}^{k+1}-v_{ij}^{k}}{\Delta t}+f\,\overline{u_{ij}}^{xy}=-g\frac{\zeta_{ij+1}^{k+1}-\zeta_{ij-1}^{k+1}}{2\Delta y}-\frac{C_f\,v_{ij}^{k+1}\left(\left(\overline{u_{ij}^{k}}^{xy}\right)^2+\left(v_{ij}^{k}\right)^2\right)^{1/2}}{\overline{\zeta_{ij}^{k+1}}^{y}+h_{ij}} \quad (18)$$

from which we compute v_{ij}^{k+1} for $i=2,4,6,\ldots,M-2$ and $j=2,4,6,\ldots,N-1$. As before, in the last term v_{ij}^{k+1} is in advanced time level and this ensures a semi-implicit nature of the numerical method. Here $\zeta_{ij}^{k+1},u_{ij}^{k+1},v_{ij}^{k+1}$ are the water elevations, velocity components in the *x* and *y* directions respectively at the advanced time level.

The model domain is a rectangular region approximately between 2^0 N, 14^O N and

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91^0 E, 101.5^0 E, which includes the tsunami source region associated with 2004 Indonesian tsunami (Fig.1). The origin of the Cartesian coordinate system is at O (3.125° N, 101.5° E), the x-axis is directed towards west at an angle 15° (anticlockwise) with the latitude line through O and the y-axis is directed towards north inclined at an angle 15° (anticlockwise) with the longitude line through O. The grid size of the rectangular mesh is given by $\Delta x = \Delta y = 4$ km and number of grids in x-direction and y-direction are respectively $M = 230$ and $N = 319$ so that there are 73370 grid points in the computational domain. The time step Δt is taken as 10 seconds and this satisfies the CFL criterion and thus ensures the stability of the numerical scheme. Following Kowalik et al. (2005), the value of the friction coefficient C_f is taken as 0.0033 through out the model area. The bathymetries for the model area are collected from the Admiralty bathymetric charts.

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IV. Tsunami Source Generation and Initial Condition

Accurate initial conditions are required to obtain reasonable results from numerical simulation of tsunami. The generation of an earthquake tsunami source depends essentially on the pattern and dynamics of motions in the earthquake source zone and on the initial seafloor movements. The magnitude of the earthquake gives a relationship among the three parameters – length, width and dislocation. The generation mechanism of the 2004 Indonesian tsunami was mainly a static sea floor uplift caused by an abrupt slip at the India/Burma plate interface. A detailed description of the estimation of the extent of earthquake rupture along with the maximum uplift and subsidence of the seabed has been reported in Kowalik et al. (2005) and this estimation was based on Okada (1985). From the deformation contour, it is seen that the estimated uplift and subsidence zone is between 92° E to 97°E and 2°N to 10°N with a maximum uplift of 507 cm at the west and maximum subsidence of 474 cm at the east. Following Kowalik et al. (2005) the disturbance in the form of rise and fall of sea surface is assigned as the initial condition in the model with a maximum rise of 5 m to maximum fall of 4.75 m to generate the response along the western open boundary. The initial sea surface elevations are taken as zero everywhere except in the part of the source zone which is activated at the initial time. Also the initial x and y components of velocity are taken as zero throughout the model area.

V. Open Boundary Condition

The wave propagation from the tsunami source has been investigated along the western open boundary of the model domain. The amplitudes of tsunami wave along the western open boundary have been computed to estimate the amplitude of the boundary condition to be formulated. For generating tidal oscillation in a limited area model through a boundary, the radiation type of boundary condition along with a sinusoidal term, containing amplitude, period and phase is needed (Johns et al. 1985, Roy 1995). The amplitude of this sinusoidal term remains constant with respect to time. But in case of tsunami propagation through a particular point in the sea, the time series is also oscillatory but with damping amplitude. On the basis of time series data and amplitude of the tsunami wave due to the source, the formulated open boundary condition (due to Roy et al., 2006) that represents the effect of distant tsunami is given by

$$u - (g/h)^{1/2}\zeta = -2(g/h)^{1/2} e^{(-st)} a \sin(2\pi t/T + \varphi) \quad \text{at the west open boundary} \qquad (19)$$

where a is the amplitude, T is the period, φ is the phase of the wave and s is the scale factor used for damping the amplitude of the wave with respect to time. In Eq. (19), the following conditions are imposed:

$s = 0$ for $t \leq T$

and $s > 0$ for $t > T$.

Through this condition we are allowing one wave, with full amplitude, to enter into the domain through the open boundary before damping of the amplitude begins.

Based on the amplitudes obtained through the source of Indonesian tsunami 2004, the assigned amplitudes (a) in (19) are adjusted so that the response in model domain is similar to that associated with the source of Indonesian tsunami 2004. By trial and error method, the values of phase (φ), period (T) and the scale factor in Eq. (19) have also been adjusted and these are $\varphi = 0$, $T = 0.5$ hr and $s = 0.01$. The formulation of the open boundary condition is such that in absence of the source its response in the domain is similar to that of the original source of the Indonesian tsunami of 2004.

Figure 2 shows the time series of sea surface fluctuation at a particular grid point at the western open boundary of the model domain, where solid line indicates the computed amplitudes due to the tsunami source and dotted line indicates the amplitudes that are imposed as the boundary condition. Both the time series are found to be almost identical, which means that the boundary condition (19) is capable of generating time series which is similar to that generated by the source.

VI. Distant Tsunami Computation Through the Open Boundary Condition

We first simulate the 2004 Indonesian tsunami propagation along the west open boundary of the model domain. The simulated data are then applied to formulate the boundary condition. The effects of the formulated open boundary condition are then investigated along the west coast of Thailand and Peninsular Malaysia in absence of the tsunami source. Wave propagation from the boundary is computed and the water levels along the coastal belts of the west coast of Thailand and Penang Island are estimated.

The propagation of tsunami wave due to the imposed boundary condition from the open boundary and the arrival time at the coast have been studied. Tsunami travel time is an important parameter in the tsunami prediction and warning. We consider the 0.1 m sea level rise as the arrival of tsunami. Figure 3 shows the contour plot of time, in minutes, for attaining +0.1 m sea level rise at each grid point in the model domain. It is seen that after imposition of the boundary condition at the west open boundary of the model domain, the disturbance propagates gradually towards the coast (Fig. 3). The arrival time of the wave at Phuket is approximately 110 min and the same at Penang is approximately 240 min. If we use the tsunami source, within the model domain, in our linear model the arrival time of tsunami from the source at Phuket and Penang islands are 95 min and 230 min respectively (Fig. 4). In the present study we compute the response of the open boundary condition imposed at the west boundary, which is away from the source zone of the Indonesian tsunami 2004. This is why the computed arrival time due to the boundary condition is delayed by up to 10 to 15 min. This time difference can be estimated by measuring the total distance of the open boundary from the coast and its travel time. Thus the corrected time related to the tsunami source at Sumatra should be 10 to 15 min earlier than the present computed time.

Figure 5 depicts the maximum water level contours, along the coast from Penang Island to Phuket. The surge amplitude is increasing from south to north; the maximum

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water level at Penang Island is from 1.5 m to 3.5 m, whereas at Phuket region it is 3.5 m to 7.5 m. The surge amplitude is increasing very fast near the shoreline everywhere. The computed water levels indicate that the north coast of Penang Island is vulnerable for stronger surges. Similarly the north-west part of Phuket is at risk of highest surge due to the source at Sumatra. The times of attaining maximum elevations along these regions due to the formulated boundary condition are also computed and it is found that this time at each location is approximately 10 to 15 min later than the time of attaining +0.1 m.

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The computed time histories of water surface fluctuations at different locations of the coastal belt of Phuket and Penang Island are stored at an interval of 30 seconds and are shown in Figures 6 and 7 respectively. Figure 6 depicts the time series of water levels for the Phuket region in south west Thailand. At the east coast of Phuket, the maximum water level is 4.4 m and the minimum is - 4.0 m and the water level continues to oscillate for a long time (Fig. 6a). It is important to note that at approximately 1.6 hrs after imposition of boundary condition, instead of increasing, the water level starts decreasing as the response to the boundary condition and reaches a minimum level of – 4.0 m. Then the water level increases continuously to reach a level of 4.4 m (1st crest) at 2.2 hrs before going down again. At the south-west coast of Phuket the time series begins with a depression of – 6.0 m and the maximum water level reaches up to 7.4 m and the oscillation continues with low amplitudes (Fig. 6b).

Figure 7 shows the time series of water levels at two locations at the north and south coasts of Penang Island in Malaysia. At north coast the maximum elevation is approximately 4.5 m (Fig. 7a). At approximately 3.75 hrs after imposition of boundary condition, the water level starts decreasing as the response to the boundary condition and reaches a minimum level of –1.6 m. Then the water level increases continuously to reach a level of 3.6 m (1st crest) at 4 hrs 30 min before going down again. The water level oscillates and this oscillation continues for several hours. At the south coast the maximum elevation is approximately 2 m and the minimum is – 2.5 m (Fig. 7b).

We have seen in previous section that the time series of sea surface fluctuation at each coastal location begins with the sea level fall (withdrawal of water from the coast) instead of rise of water as the response of imposed boundary condition (Figs. 6, 7). The sinusoidal boundary condition is imposed at the boundary so that the phase is from trough to crest. Thus from the boundary the first wave propagates so that the trough is in front of the crest. To investigate this initial withdrawal we change the phase of the imposed boundary condition. Relative to the west coast of Malaysia and Thailand, the phase of the imposed boundary condition on open boundary is in the form of trough to crest. To identify whether this phase of the boundary condition is responsible for initial withdrawal of water from the coastal belt, an investigation is undertaken by a boundary condition of same intensity, but with the reverse phase, that is, from crest to trough. Figure 8 depicts the time series of water levels at the same locations as in Fig. 6b of Phuket and Fig. 7b of Penang Island associated with the reversed boundary condition. This figure shows that, at each location, the tsunami surge is not preceded by withdrawal of water from the coast. Similar results are found along the other coastal belts of Phuket and Penang (not shown). Thus, the initial withdrawal of water from the coasts depends upon the nature of the phase of the imposed boundary condition.

The computed arrival time of wave generated by imposed boundary condition is compared with the data available in USGS website (Tab. 1). According to USGS report the tsunami waves reached at Phuket within two hours time after the earthquake and the arrival time of tsunami at Penang is between 3 hr 30 min and 4 hours. It is mentioned that the formulated boundary condition is imposed on the open boundary of the model which is far from the tsunami source. Thus the corrected arrival time related to the tsunami source at Sumatra is obtained by shifting 10 to 15 min earlier than the computed arrival times of wave of imposed boundary condition. Therefore the computed time of attaining maximum surge due to the original tsunami source is 100 min and 230 min for Phuket and Penang respectively. Thus the computed time is almost identical with the website data.

On the other hand maximum water level surrounding Penang is 1.5 - 3.5 m and the same for Phuket is 3.5 – 7.0 m computed by the formulated boundary condition. In USGS website it is reported that wave height reached 7 to 11 m surrounding Phuket and due to Roy et al. (2006) the wave height reached 2.0 – 3.5 m surrounding Penang. From table 1 and the above discussion it is clear that computed tsunami arrival time and maximum water levels along the island boundaries of Penang and Phuket agree well with the observed data or data available in the USGS website.

VII. Conclusion

A two-dimensional linear model has been developed to formulate an appropriate boundary condition. The response of the 26 December 2004 Indonesian tsunami is computed at the west open boundary of the model domain. Then by the amplitudes of tsunami wave with the adjusted values of phase, period and scale factor of the boundary condition the appropriate boundary condition is formulated to compute the distant tsunami along the coastal boundary of west coast Peninsular Malaysia and southern Thailand. It is observed that the response of the formulated boundary condition in absence of the source is similar to that of the original source of the Indonesian tsunami of 2004. The computed water levels along the coastal belts of Penang and Phuket are found to be quite reasonable and consistent with the water level data. The arrival time of tsunami due to the boundary condition is little bit later than the arrival time of tsunami due to the source because the boundary condition is away from the source zone of the Indonesian tsunami 2004. It is found that the initial withdrawal of water from a coastal belt depends upon the phase of the boundary condition. Thus the computed results obtained by the formulated boundary condition are in good agreement with the observed data. Since this model is computationally efficient in comparison with a nonlinear model, authors believe that in real time operational distant tsunami forecast programs in a particular coastal region this open boundary technique is practically applicable.

Table 1 : Computed and observed / USGS tsunami propagation time and water levels for Penang and Phuket

	Location	Computed water level (m)	Observed/ USGS
Propagation time (min)	Penang	230	< 240
	Phuket	100	< 120
Max. water level (m)	surrounding Penang	1.5 - 3.5	2.0 – 3.5
	surrounding Phuket	3.5 – 7.0	7 - 11

Notes

References Références Referencias

1. Aizinger, V. & Dawson, C., 2002. A discontinuous Galerkin method for two-dimensional flow and transport in shallow water. *Advances in Water Resources*, 25(1), 67-84.
2. Arakawa, A. & Lamb, V., 1977. Computational design of the basic dynamical processes of the UCLA general circulation model. In J. Chang (Ed.), *Methods in Computational Physics* (7, 173-265). New York: Academic Press.
3. Berkman, S. C. & Symons, J.M., 1964. The tsunami of May 22, 1960 as recorded at tide gauge stations. Washington, D.C.: U.S. Department of Commerce, Coast and Geodetic Survey.
4. Cho, Y.-S., Sohn, D.H. & Lee, S.O., 2007. Practical modified scheme of linear shallow-water equations for distant propagation of tsunamis. *Ocean Engineering*, 34, 1769–1777.
5. Cho, Y.-S. & Yoon, S.B., 1998. A modified leap-frog scheme for linear shallow-water equations. *Coastal Engineering Journal*, 40 (2), 191–205.
6. Heaps, N.S., 1973. A three dimensional numerical model of the Irish Sea. *Geophysics. J. Astron. Soc*, 35, 99 – 120.
7. Imamura, F., Shuto, N. & Goto, C., 1988. Numerical simulations of the transoceanic propagation of tsunamis. In: Proceedings of *the Sixth Congress Asian and Pacific Regional Division*, IAHR, Japan, pp. 265–272.
8. Indian Ocean Tsunami at Syowa Station Antarctica, 2007. Retrieved 10 April, 2007, from http://www1.kaiho.mlit.go.jp/KANKYO/KAIYO/KOUHOU/iotunami/iotunami˙eng.html
9. Joolen, V., Givoli, D. & Neta, B., 2003. High-order Non-reflecting Boundary conditions for Dispersive Waves in Cartesian, Cylindrical and Spherical Coordinate Systems. *International Journal of Computational Fluid Dynamics*, 17 (4), 263-274.
10. Johns, B., Dube, S.K., Mohanti, U.C. & Sinha, P.C., 1981. Numerical Simulation of surge generated by the 1977 Andhra cyclone. *Quart. J. Roy. Soc. London*, 107, 919 – 934.
11. Johns, B., Rao, A. D., Dube, S. K. & Sinha P. C., 1985. Numerical modeling of tide surges interaction in the Bay of Bengal. *Phil. Trans. R. Sco. of London*, A313, 507-535.
12. Karim, M.F., Roy, G.D., Ismail, A.I.M. & Meah, M.A., 2006. A Linear Cartesian Coordinate Shallow Water Model for Tsunami Computation along the West coast of Thailand and Malaysia. *International Journal of Ecology & Development*, 4 (S06), 1 - 14.
13. Kowalik, Z., Knight, W. & Whitmore, P. M., 2005. Numerical Modeling of the Global Tsunami: Indonesian Tsunami of 26 December 2004. *Sc. Tsunami Hazards*, 23(1), 40 – 56.
14. Murty, T. S., 1977. Seismic Sea Waves – Tsunamis. Ottawa: *Bull. Fish. Res. Board Canada*.
15. NOAA, 2005. West Coast/Alaska Tsunami Warning Center. Retrieved 10 April, 2007, from http://wcatwc.arh.noaa.gov/IndianOSite/IndianO12-26-04.htm
16. Okada, Y., 1985. Surface Deformation due to Shear and Tensile Faults in a Half Space. *Bull, Seism. Soc. Am.*, 75, 1135 – 1154.

17. Rabinovich, A. B., Thomson, R.E. & Stephenson, F.E., 2006. The Sumatra tsunami of 26 December 2004 as observed in the North Pacific and North Atlantic oceans. *Surveys in Geophysics*, 27, 647 – 677.
18. Roy, G. D., 1998. Mathematical Modeling of Tide, Surge and their Interaction along the Coast of Bangladesh. Paper presented at the Mini-Workshop on Appl. Math., SUST, Sylhet, Bangladesh.
19. Roy, G. D., Karim, M. F. & Ismail, A. M., 2006. Numerical Computation of Some Aspects of 26 December 2004 Tsunami along the West Coast of Thailand and Peninsular Malaysia Using a Cartesian Coordinate Shallow Water Model. *Far East J. Appl. Math.*, 25(1), 57-71.
20. Titov, V. V., Rabinovich, A.B., Mofjeld, H.O., Thomson, R.E. & Gonzalez, F.I., 2005. The Global Reach of the 26 December 2004 Sumatra Tsunami. *Science*, 309, 2045 - 2048.
21. Yoon, S.B., 2002. Propagation of tsunamis over slowly varying topography. *Journal of Geophysical Research*, 107 (C10), 4(1) – 4(11).
22. Zahibo, N., Pelinovsky, E., Talipova. T., Kozelkov, A. & Kurkin, A., 2006. Analytical and numerical study of nonlinear effects at tsunami modeling. *Applied Mathematics and Computation*, 174(2), 795 - 809.

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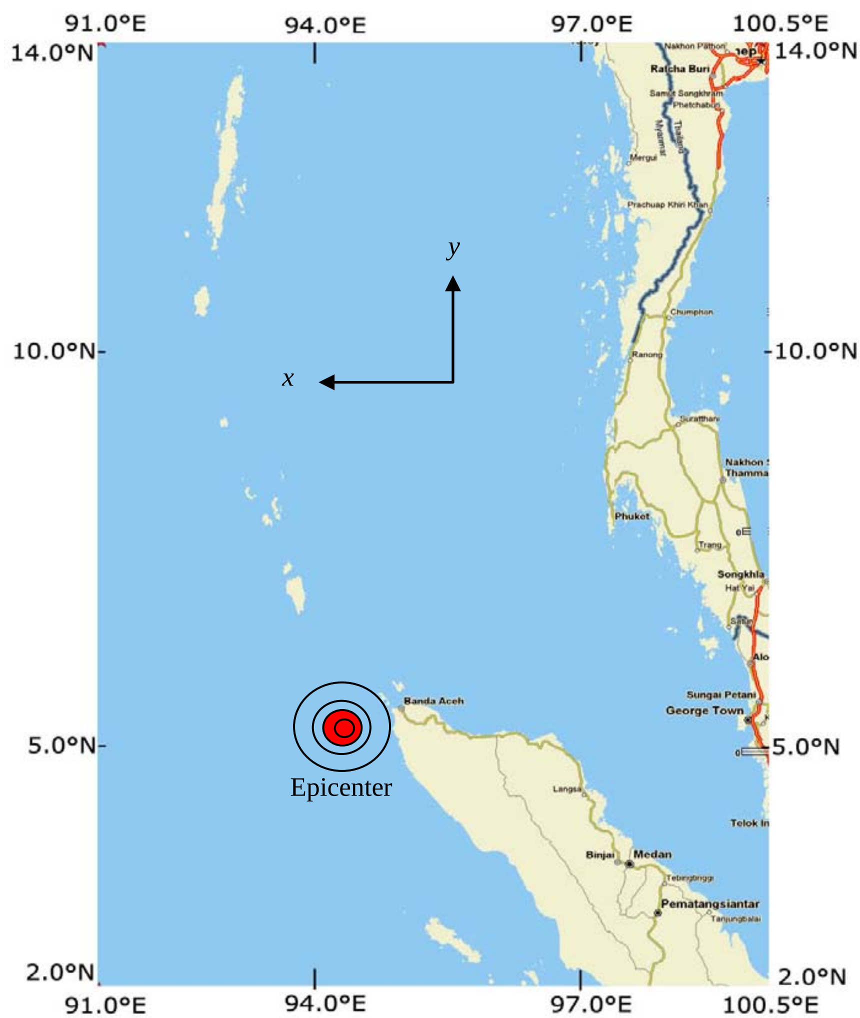

Figure 1 : Model Domain including the coastal geometry and the epicenter of the 2004 earthquake (Courtesy: Roy et al., 2006).

Notes

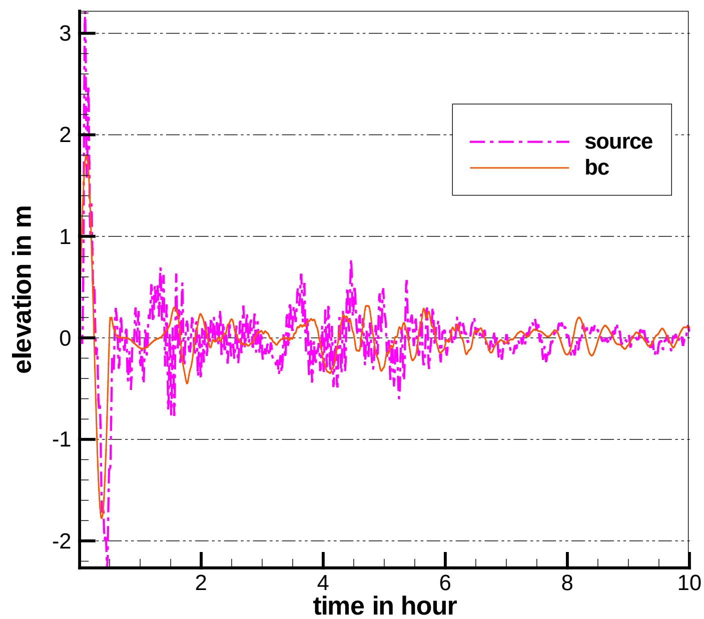

Figure 2 : Time series of sea surface fluctuation at the western open boundary

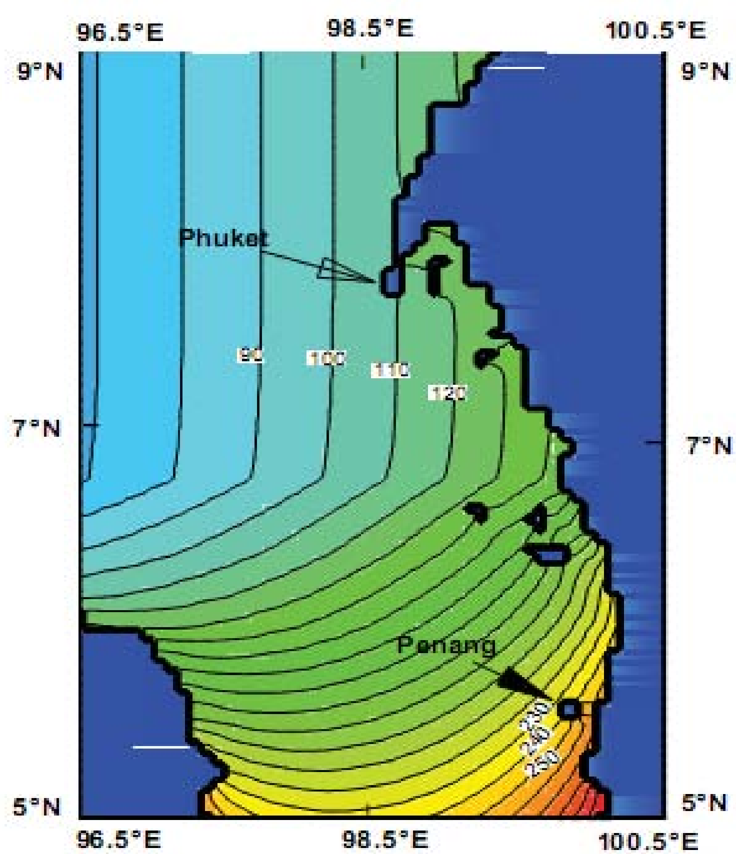

Figure 3 : Tsunami propagation time in minutes towards Phuket and Penang due to the formulated open boundary condition

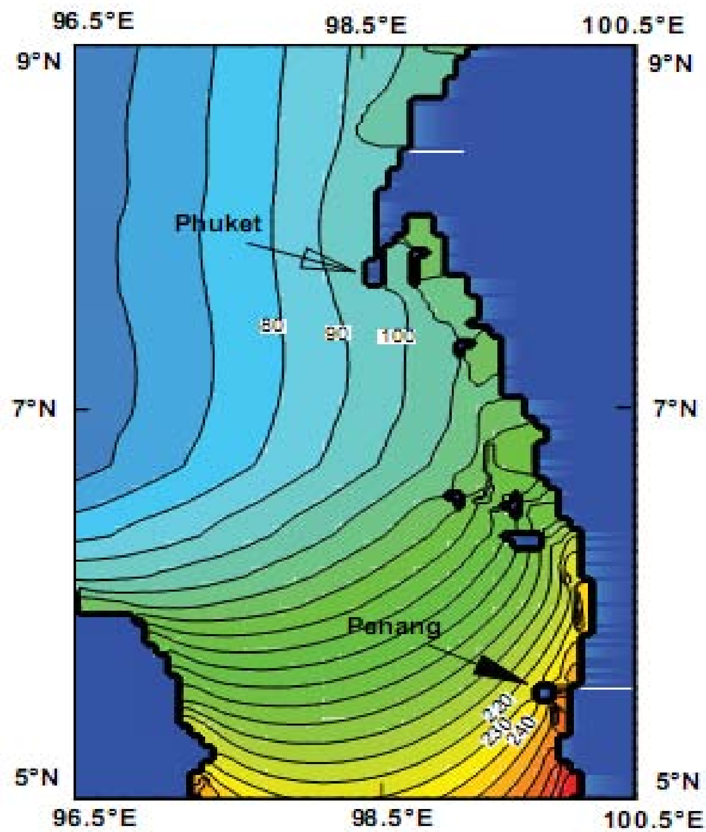

Figure 4 : Tsunami propagation time in minutes towards Phuket and Penang due to the 2004 Indonesian tsunami source

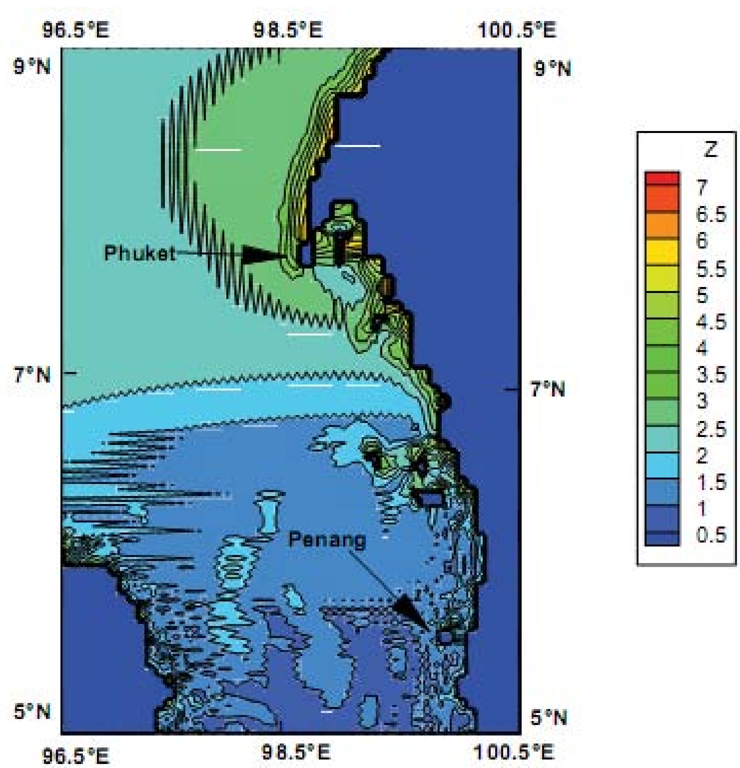

Figure 5 : Contour of maximum elevation associated with the formulated boundary condition around the west coast of Thailand and Malaysia

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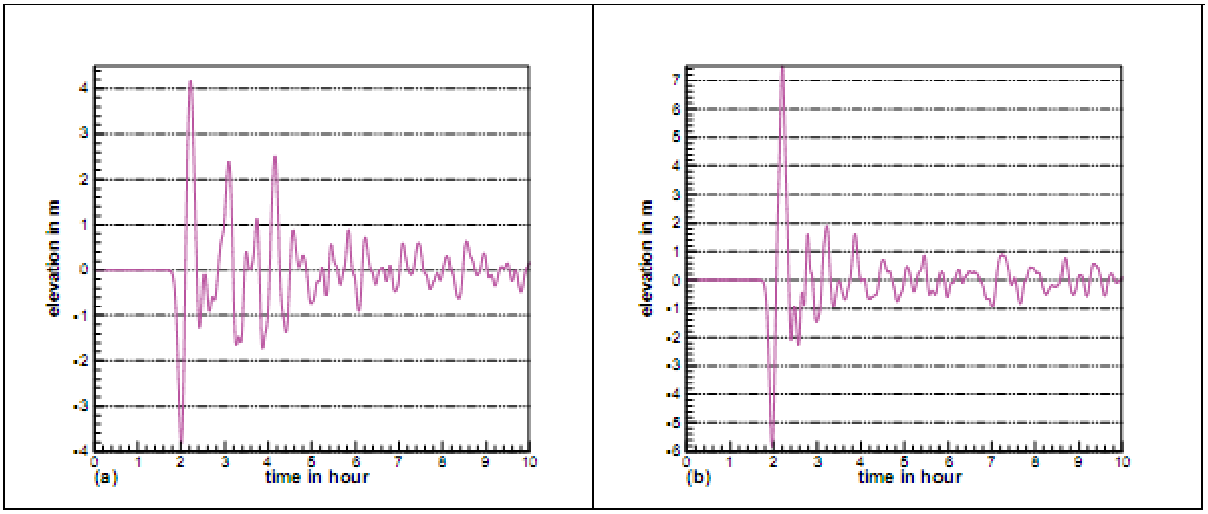

Figure 6 : Time series of computed elevation at two coastal locations of Phuket Island associated with the formulated boundary condition: (a) East Phuket, (b) South-west Phuket

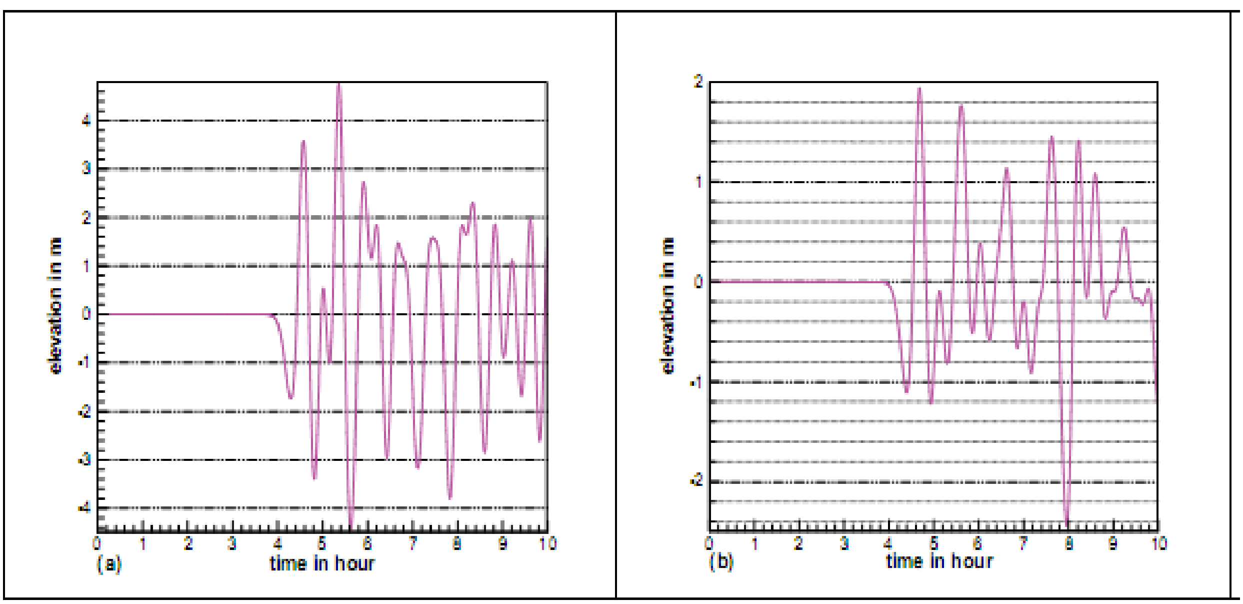

Figure 7 : Time series of computed elevation at two coastal locations of Penang Island associated with the formulated boundary condition: (a) Batu Ferringi, (b) South coast

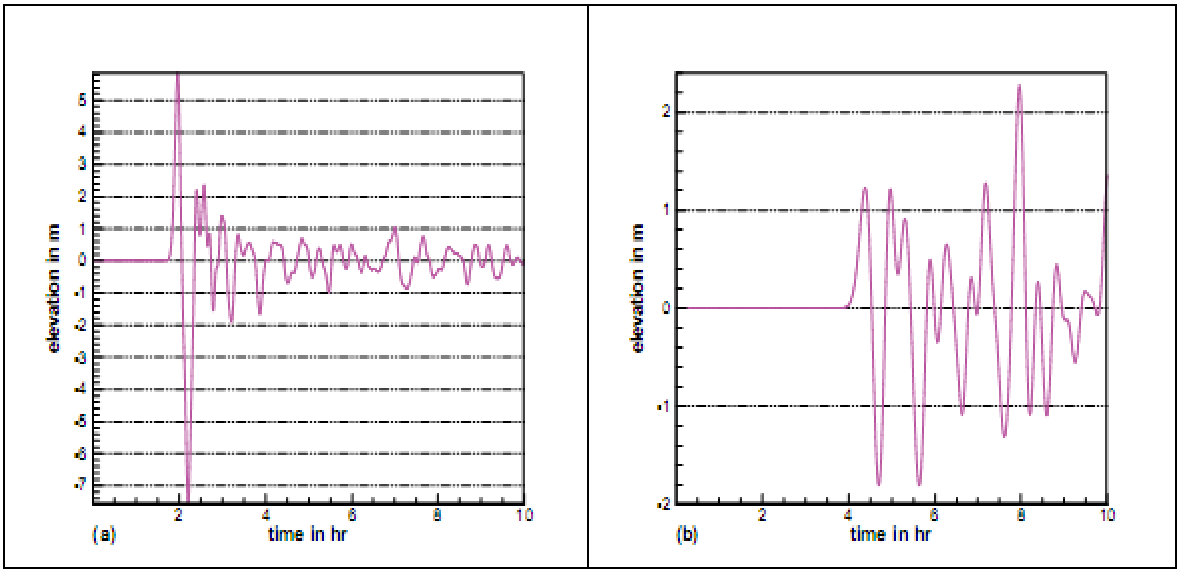

Figure 8 : Same as Figure 6b and 7b, except that the phase of the boundary condition has been reversed.

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On ∅-Recurrent Generalized Sasakian-Space-Forms

By Venkatesha, Sumangala B. & C. S. Bagewadi
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Abstract - The object of this paper is to study ∅-recurrent generalized Sasakian-space-forms. It is proved that a ∅- recurrent generalized Sasakian-space-forms is an η - Einstein manifold, provided $f_1 - f_3 \neq 0$ and ∅-recurrent generalized Sasakian-space-form having a non-zero constant sectional curvature is locally ∅-symmetric.

Keywords : generalized Sasakian-space-forms, ∅-recurrent, locally ∅-recurrent, $\eta\eta$-Einstein manifold.

AMS Subject Classification (2000) : 53C15, 53C25

Strictly as per the compliance and regulations of :

On ∅-Recurrent Generalized Sasakian-Space-Forms

Venkatesha [α], Sumangala B. [σ] & C. S. Bagewadi [ρ]

Abstract - The object of this paper is to study ∅-recurrent generalized Sasakian-space-forms. It is proved that a ∅-recurrent generalized Sasakian-space-forms is an η-Einstein manifold, provided $f_1 - f_3 \neq 0$ and ∅-recurrent generalized Sasakian-space-form having a non-zero constant sectional curvature is locally ∅-symmetric.

Keywords : generalized Sasakian-space-forms, ∅-recurrent, locally ∅-recurrent, η-Einstein manifold.

I. Introduction

A Riemannian manifold with constant sectional curvature C is known as real-space-form and its curvature tensor is given by

$$R(X,Y)Z = C\{g(Y,Z)X - g(X,Z)Y\}.$$

A Sasakian manifold (M, ϕ, ξ, η, g) is said to be a Sasakian-space-form [1], if all the ϕ-sectional curvatures $K(X \wedge \phi X)$ are equal to a constant C, where $K(X \wedge \phi X)$ denotes the sectional curvature of the section spanned by the unit vector field X orthogonal to ξ and ϕX. In such a case, the Riemannian curvature tensor of M is given by,

$$\begin{aligned} R(X,Y)Z &= (C+3)/4\{g(Y,Z)X - g(X,Z)Y\} \\ &+ (C-1)/4\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ (C-1)/4\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}. \end{aligned} \tag{1.1}$$

As a natural generalization of these manifolds, P. Alegre, D. E. Blair and A. Carriazo [1] [2] introduced the notion of generalized Sasakian-space-form . It is defined as almost contact metric manifold with Riemannian curvature tensor satisfying an equation similar to (1.1), in which the constant quantities $(C+3)/4$ and $(C-1)/4$ are replaced by differentiable functions, i.e.,

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$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}, \end{aligned}$$

for any vector fields X, Y, Z on M.

Generalized Sasakian-space-forms and Sasakian-space-forms have been studied by several authors, viz., [1], [3], [4], [6], [10]. The notion of local symmetry of a Riemannian manifold has been weakened by many authors in several ways to a different extent. As a weaker version of local symmetric, T.Takahashi [11] introduced the notion of locally ϕ-symmetry on a Sasakian manifold. Generalizing the notion of ϕ-symmetry, U.C.De and co-authors [9] introduced the notion of ϕ-recurrent Sasakian manifold. Further ϕ-recurrent Kenmotsu manifold, ϕ-recurrent LP-Sasakian manifold, concircular ϕ-recurrent LP-Sasakian manifold, pseudo-projectively ϕ-recurrent Kenmotsu manifold were studied by U.C. De and co-authors and Venkatesha and C.S. Bagewadi in their papers [7], [12], [13].

In the present paper we have studied ϕ-recurrent generalized Sasakian-space-form and proved that a ϕ-recurrent generalized Sasakian-space-form is an η-Einstein manifold and a locally ϕ-recurrent generalized Sasakian-space-form is a manifold of constant curvature. Further it is shown that if a ϕ-recurrent generalized Sasakian-space-form has a non zero constant sectional curvature, then it reduces to a locally ϕ-symmertic manifold.

II. Preliminaries

An odd-dimensional Riemannian manifold (M, g) is called an almost contact manifold if there exists on M, a $(1,1)$ tensor field ϕ, a vector field ξ (called the structure vector field) and a 1-form η such that

$$\phi^2(X) = -X + \eta(X)\xi, \qquad \eta(\xi) = 1, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.2}$$

for any vector fields X, Y on M.

In particular, in an almost contact metric manifold we also have

$$\phi\xi = 0, \qquad \eta \circ \phi = 0. \tag{2.3}$$

Ref.

3. P. Alegre and A. Cariazo, Structures on generalized Sasakian-space-form, Differential Geom. and its application 26 (2008), 656-666.

$$(\nabla_X \xi) = X - \eta(X)\xi, \tag{2.4}$$

$$(\nabla_X \eta)(Y) = g(X,Y) - \eta(X)\eta(Y). \tag{2.5}$$

Given an almost contact metric manifold (M, ϕ, ξ, η, g), we say that M is an generalized Sasakian-space-form, if there exists three functions f_1, f_2 and f_3 on M such that

$$\begin{aligned} R(X,Y)Z &= f_1\{g(Y,Z)X - g(X,Z)Y\} \\ &+ f_2\{g(X,\phi Z)\phi Y - g(Y,\phi Z)\phi X + 2g(X,\phi Y)\phi Z\} \\ &+ f_3\{\eta(X)\eta(Z)Y - \eta(Y)\eta(Z)X \\ &+ g(X,Z)\eta(Y)\xi - g(Y,Z)\eta(X)\xi\}, \end{aligned} \tag{2.6}$$

for any vector fields X, Y, Z on M, where R denotes the curvature tensor of M. This kind of manifold appears as a natural generalization of the well-known Sasakian-space-forms $M(C)$, which can be obtained as particular cases of generalized Sasakian-space-forms by taking $f_1 = (C+3)/4$ and $f_2 = f_3 = (C-1)/4$. Further in a $(2n+1)$-dimensional generalized Sasakian-space-form, we have [2]

$$QX = (2nf_1 + 3f_2 - f_3)X - (3f_2 + (2n-1)f_3)\eta(X)\xi, \tag{2.7}$$

$$S(X,Y) = (2nf_1 + 3f_2 - f_3)g(X,Y) - (3f_2 + (2n-1)f_3)\eta(X)\eta(Y), \tag{2.8}$$

$$r = 2n(2n+1)f_1 + 6nf_2 - 4nf_3, \tag{2.9}$$

$$R(X,Y)\xi = (f_1 - f_3)[\eta(Y)X - \eta(X)Y], \tag{2.10}$$

$$R(\xi,X)Y = (f_1 - f_3)[g(X,Y)\xi - \eta(Y)X], \tag{2.11}$$

$$\eta(R(X,Y)Z) = (f_1 - f_3)(g(Y,Z)\eta(X) - g(X,Z)\eta(Y)), \tag{2.12}$$

$$S(X,\xi) = 2n(f_1 - f_3)\eta(X), \tag{2.13}$$

$$\begin{aligned} (\nabla_W R)(X,Y)\xi &= (df_1(W) - df_3(W))\{\eta(Y)X - \eta(X)Y\} \\ &+ (f_1 - f_2)\{g(Y,W)X - g(X,W)Y\} - R(X,Y)W. \end{aligned} \tag{2.14}$$

Definition 2.1. *A generalized Sasakian - space - form is said to be locally ϕ-symmetric if*

$$\phi^2((\nabla_W R)(X,Y)Z) = 0. \tag{2.15}$$

Definition 2.2. *A generalized Sasakian-space-form is said to be ϕ-recurrent if there exist a non zero 1-form A such that*

$$\phi^2((\nabla_W R)(X,Y)Z) = A(W)R(X,Y)Z, \tag{2.16}$$

for any arbitrary vector field X, Y, Z and W.

III. ∅-Recurrent Generalized Sasakian - Space - Forms

Two vector fields X and Y are said to be co-directional, if $X = fY$ where f is a non-zero scalar. i.e.,

$$g(X,Z) = fg(Y,Z), \tag{3.1}$$

for all X.

Now from (2.1) and (2.16), we have

$$(\nabla_W R)(X,Y)Z = \eta((\nabla_W R)(X,Y)Z)\xi - A(W)R(X,Y)Z. \tag{3.2}$$

From (3.2) and the Bianchi identity we get

$$A(W)\eta(R(X,Y)Z) + A(X)\eta(R(Y,W)Z) + A(Y)\eta(R(W,X)Z) = 0. \tag{3.3}$$

By virtue of (2.12) we obtain from (3.3) that

$$\begin{aligned}(f_1 - f_3)[A(W)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\} + A(X)\{g(W,Z)\eta(Y)\\ -g(Y,Z)\eta(W)\} + A(Y)\{g(X,Z)\eta(W) - g(W,Z)\eta(X)\}] = 0.\end{aligned} \tag{3.4}$$

Putting $Y = Z = e_i$ in (3.4) and taking summation over i, $1 \leq i \leq 2n+1$, we get

$$2n(f_1 - f_3)[A(W)\eta(X) - A(X)\eta(W)] = 0.$$

If $(f_1 - f_3) \neq 0$, then

$$A(W)\eta(X) = A(X)\eta(W), \tag{3.5}$$

for all vector fields X,W.

Replacing X by ξ in (3.5), we get

(3.6) $$A(W) = \eta(\rho)\eta(W),$$

where $A(X) = g(X,\rho)$ and ρ is the vector field associated to the 1-form A. From (3.1) and (3.6) we can state the following:

Theorem 3.1. *In a ϕ-recurrent generalized Sasakian-space-form, the characteristic vector field ξ and the vector field ρ associated to the 1-form A are co-directional and the 1-form A is given by (3.6), provided $f_1 - f_3 \neq 0$.*

Notes

Let us consider a ϕ-recurrent generalized Sasakian space form. Then from (3.2) we have,

$$(\nabla_W R)(X,Y)Z = \eta((\nabla_W R)(X,Y)Z)\xi - A(W)R(X,Y)Z,$$

from which it follows that

$$-g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\eta(U) = A(W)g(R(X,Y)Z,U). \tag{3.7}$$

Notes

Let $\{e_i\}, i = 1,2,...,2n+1$ be an orthonormal basis of the tangent space at any point of the space form. Then putting $X = U = e_i$, in (3.7) and taking summation over i, $1 \leq i \leq 2n+1$, we get

(3.8) $$-(\nabla_W S)(Y,Z) = A(W)S(Y,Z).$$

Replacing Z by ξ in (3.8) we get,

$$(\nabla_W S)(Y,\xi) = -A(W)S(Y,\xi). \tag{3.9}$$

Now we have

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi). \tag{3.10}$$

Using (2.4), (2.5) and (2.13) in (3.10), we get

$$(\nabla_W S)(Y,\xi) = 2n(f_1 - f_3)g(Y,W) - S(Y,W). \tag{3.11}$$

Now using (3.11) in (3.10), we obtain

$$S(Y,W) = 2n(f_1 - f_3)g(Y,W) + A(W)S(Y,\xi). \tag{3.12}$$

Using (2.13) and (3.6) in (3.12), we get

$$S(Y,W) = 2n(f_1 - f_3)g(Y,W) + 2n(f_1 - f_3)\eta(\rho)\eta(W)\eta(Y). \tag{3.13}$$

This leads to the following theorem:

Theorem 3.2. *A ϕ-recurrent generalized Sasakian-space-form is an η-Einstein manifold provided $f_1 - f_3 \neq 0$.*

From (2.14), we have

$$\begin{aligned}(\nabla_W R)(X,Y)\xi &= (df_1(W) - df_3(W))\{\eta(Y)X - \eta(X)Y\} \\ &+ (f_1 - f_3)\{g(Y,W)X - g(X,W)Y\} - R(X,Y)W.\end{aligned}$$

By virtue of (2.12), it follows from (2.14) that

$$\eta(\nabla_W R)(X,Y)\xi = 0. \tag{3.14}$$

In view of (3.14) and (3.2), we obtain

$$A(W)R(X,Y)\xi \;=\; -(\nabla_W R)(X,Y)\xi. \tag{3.15}$$

Notes

By using (2.14) in (3.15), we get

$$\begin{aligned} &-(df_1(W)-df_3(W))\{\eta(Y)X-\eta(X)Y\} \\ -(f_1-f_3)\{g(Y,W)X-g(X,W)Y\} &+ R(X,Y)W = A(W)R(X,Y)\xi. \end{aligned} \tag{3.16}$$

Hence if X and Y are orthogonal to ξ, then we get from (2.10) that

$$R(X,Y)\xi = 0.$$

Thus we obtain

$$R(X,Y)W \;=\; (f_1-f_3)\{g(Y,W)X-g(X,W)Y\}, \tag{3.17}$$

for all X, Y, W.

This leads to the following theorem:

Theorem 3.3. *A locally ϕ-recurrent generalized Sasakian-space-form is a manifold of constant curvature.*

Again let us suppose that a generalized Sasakain-space-form is ϕ-recurrent. Then from (3.2) and (2.14), it follows that

$$\begin{aligned} (\nabla_W R)(X,Y)Z &= [-(df_1(W)-df_3(W))\{g(X,Z)\eta(Y)-g(Y,Z)\eta(X)\} \\ &- (f_1-f_3)\{g(Y,W)g(X,Z)-g(X,W)g(Y,Z)\} \\ &- g(R(X,Y)W,Z)]\xi - A(W)R(X,Y)Z. \end{aligned} \tag{3.18}$$

From (3.18) it follows that

$$\phi^2(\nabla_W R)(X,Y)Z = A(W)R(X,Y)Z - A(W)(f_1-f_3)\{g(Y,Z)\eta(X)-g(X,Z)\eta(Y)\}, \tag{3.19}$$

which yields

$$\phi^2(\nabla_W R)(X,Y)Z \;=\; A(W)R(X,Y)Z.$$

Notes

Hence we state the following:

Theorem 3.4. *A generalized Sasakian-space-form satisfying the relation (3.18) is ϕ-recurrent provided that X and Y are orthogonal to ξ.*

Next, we suppose that in a ϕ-recurrent generalized *Sasakian-space-form* the sectional curvature of a plane $\pi \subset T_pM$ defined by

$$K_p(\pi) \quad = \quad g(R(X,Y)Y,X),$$

is a non zero constant K, where X, Y is any orthonormal basis of π. Then we have

$$g((\nabla_Z R)(X,Y)Y,X) = 0. \tag{3.20}$$

By virtue of (3.20) and (3.2) we obtain

$$g((\nabla_Z R)(X,Y)Y,\xi)\eta(X) \quad = \quad A(Z)g(R(X,Y)Y,X). \tag{3.21}$$

Since in a ϕ-recurrent generalized Sasakian-space-form, the relation (3.18) holds good. Using (3.18) in (3.21) we get

$$\begin{aligned}&[-(df_1(Z) - df_3(Z))\{g(X,Y)\eta(Y) - g(Y,Y)\eta(X)\}\\ &-A(Z)\eta(R(X,Y)Y)]\eta(X) = A(Z)g(R(X,Y)Y,X).\end{aligned} \tag{3.22}$$

If X and Y are orthogonal to ξ,

$$-A(Z)\eta(X)\eta(R(X,Y)Y) = KA(Z). \tag{3.23}$$

Putting $Z = \xi$ in (3.23) we obtain

$$\eta(\rho)[\eta(X)\eta(R(X,Y)Y) + K] = 0.$$

Which implies that

$$\eta(\rho) = 0. \tag{3.24}$$

Hence by (3.6) we obtain from (2.16) that

$$\phi^2((\nabla_W R)(X,Y)Z) = 0. \tag{3.25}$$

This leads to the following theorem:

Theorem 3.5. *If a ϕ-recurrent generalized Sasakian-space-form has a non-zero constant sectional curvature, then it reduces to a locally ϕ-symmetric manifold provided that X and Y are orthogonal to ξ.*

References Références Referencias

1. Alfonso Carriazo, David. E. Blair and Pablo Alegre, proceedings of the Ninth International Workshop on Differential Geometry, 9 (2005), 31-39.
2. P. Alegre, D. Blair and A. Carriazo, Generalized Sasakian-space-forms, Israel J. Math. 14 (2004), 157-183.
3. P. Alegre and A. Cariazo, Structures on generalized Sasakian-space-form, Differential Geom. and its application 26 (2008), 656-666.
4. Belkhelfa. M., Deszcz. R., Verstraelen. L., Symmetric properties of Sasakian-space-forms, Soochow J.math., 31(2005), 611-616.
5. D.E. Blair, Contact manifolds in Riemannian geometry, Lecture Notes in Mathematics, 509 Springer-Verlag, Berlin, 1976.
6. U. C. De and A. Sarkar, Some result on Generalized Sasakian-space-forms, Thai J. Math, 8(1)(2010), 1-10.
7. U. C. De, Ahmet Yildiz, A. Funda Yaliniz, On ∅-recurrent Kenmotsu manifolds, Turk J. Math, 33 (2009),17-25.
8. U. C. De and A. Sarkar, On the Projective Curvature tensor of Generalized Sasakian-space-forms, Quaestiones Mathematicae, 33 (2010), 245-252.
9. U. C. De, A. A. Shaikh, Sudipta Biswas, On ∅-recurrent Sasakian manifold, Novi. Sad. J. Math., 32(2)(2003), 43-48.
10. Sarkar and U. C. De, Some Curvature properties of generalized Sasakian-space-forms,Lobachevskii Journal of Mathematics, 33(1)(2012), 22-27.
11. T. Takahashi, Sasakian ∅-symmetric space, Tohoku Math. J., 29(1977), 91-113.
12. Venkatesha and C. S. Bagewadi, On concircular ∅-recurrent LP-Sasakian manifolds, Differential Geometry- Dynamical Systems, 10 (2008), 312-319.
13. Venkatesha and C. S. Bagewadi, On Pseudo projective ∅-recurrent Kenmotsu manifolds, Soochow journal of mathematics, 32(3)(2006), 433-439.

Notes

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Some Summation Theorems Involving Bailey Theorem

By Salahuddin, M.P. Chaudhary & Ashish Arora
Amity University, India

Abstract - Authors obtain five new summations theorems involving Gamma functions, Bailey theorem and recurrence relation of Gamma functions, which are not available in the literature of special functions.

Keywords : *Bailey theorem, Gaussian hypergeometric function.*

MSC : *33C20; 33C80; 39A10*

SOME SUMMATION THEOREMS INVOLVING BAILEYTHEOREM

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Keywords : Bailey theorem, Gaussian hypergeometric function.

I. Introduction

Generalized Gaussian hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{matrix} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{matrix} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{matrix}\right] \equiv {}_AF_B\left[\begin{matrix} (a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relation is defined as follows: [E. D. p.51(10), Andrews p.363(9.16), H.T. F. I p.103(32)]

$$ (a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; & \\ c\ \ ; & z \end{matrix}\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ ; & \\ c\ \ ; & z \end{matrix}\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1\ ; & \\ c\ \ ; & z \end{matrix}\right] \tag{2} $$

Recurrence relation of gamma function is defined as follows:

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{3} $$

Legendre duplication formula is defined,as

$$ \sqrt{\pi}\ \Gamma(2z) = 2^{(2z-1)}\ \Gamma(z)\ \Gamma\left(z+\frac{1}{2}\right) \tag{4} $$

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$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{5}$$

$$= \frac{2^{(a-1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{6}$$

Bailey summation theorem [Prud, p.491(7.3.7.8)]is as follows:

$${}_2F_1\left[\begin{array}{ccc} a, 1-a & ; & \frac{1}{2} \\ c & ; & 2 \end{array}\right] = \frac{\Gamma(\frac{c}{2})\ \Gamma(\frac{c+1}{2})}{\Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} = \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c-1}\ \Gamma(\frac{c+a}{2})\ \Gamma(\frac{c+1-a}{2})} \tag{7}$$

II. Main Summation Theorems

Theorem-1:

$${}_2F_1\left[\begin{array}{ccc} a\ ,\ -a-11 & ; & \frac{1}{2} \\ c & ; & 2 \end{array}\right]$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}} \times \left[\frac{-122760a + 35546a^2 + 7161a^3 + 23a^4 - 33a^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})} + \right.$$

$$+ \frac{-a^6 + 122880c - 175560ac + 5820a^2c + 3960a^3c + 180a^4c + 140288c^2 - 70356ac^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})} +$$

$$+ \frac{-4218a^2c^2 + 396a^3c^2 + 18a^4c^2 + 57600c^3 - 10560ac^3 - 960a^2c^3 + 10880c^4 - 528ac^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})} +$$

$$+ \frac{-48a^2c^4 + 960c^5 + 32c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})} +$$

$$+ \frac{4(15120 - 15510a + 405a^2 + 330a^3 + 15a^4 + 27024c - 11902ac - 719a^2c + 66a^3c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})} +$$

$$\left. + \frac{4(3a^4c + 15200c^2 - 2640ac^2 - 240a^2c^2 + 3680c^3 - 176ac^3 - 16a^2c^3 + 400c^4 + 16c^5)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}\right] \tag{8}$$

Theorem-2:

$${}_2F_1\left[\begin{array}{ccc} a\ ,\ -a-12 & ; & \frac{1}{2} \\ c & ; & 2 \end{array}\right]$$

$$= \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+12}} \times \left[\frac{-245640a + 96002a^2 + 8301a^3 - 769a^4 - 69a^5}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})} + \right.$$

$$+ \frac{-a^6 + 245760c - 376304ac + 40100a^2c + 7246a^3c + 84a^4c - 6a^5c + 280576c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})} +$$

$$+ \frac{-163376ac^2 - 376a^2c^2 + 1104a^3c^2 + 24a^4c^2 + 115200c^3 - 27776ac^3 - 1184a^2c^3 + 32a^3c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})} +$$

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$$+\frac{21760c^4-1840ac^4-80a^2c^4+1920c^5-32ac^5+64c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{665280-690744a+14030a^2+19587a^3+1211a^4-3a^5-a^6+1249536c-588880ac}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{-44500a^2c+5426a^3c+444a^4c+6a^5c+776896c^2-156688ac^2-19384a^2c^2+48a^3c^2}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{24a^4c^2+222720c^3-14464ac^3-2336a^2c^3-32a^3c^3+32320c^4-80ac^4-80a^2c^4}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}+$$

$$\left.+\frac{2304c^5+32ac^5+64c^6}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}\right] \tag{9}$$

Theorem-3:

$${}_2F_1\left[\begin{matrix} a & , & -a-13 & ; & \frac{1}{2} \\ & c & & ; & \end{matrix}\right]$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+13}}\times\left[\frac{-2948400a+1178604a^2+123942a^3-12978a^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})}+\right.$$

$$+\frac{-1638a^5-42a^6+2949120c-4789512ac+547218a^2c+125489a^3c+1869a^4c-273a^5c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{-7a^6c+3612672c^2-2332512ac^2-9072a^2c^2+26208a^3c^2+1008a^4c^2+1662976c^3}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{-479024ac^3-27384a^2c^3+1456a^3c^3+56a^4c^3+376320c^4-43680ac^4-3360a^2c^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{44800c^5-1456ac^5-112a^2c^5+2688c^6+64c^7}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+14}{2})}+$$

$$+\frac{-2(-665280+752856a-74246a^2-18135a^3-275a^4+39a^5+a^6-1249536c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{-2(691392ac+4512a^2c-7488a^3c-288a^4c-776896c^2+207376ac^2+11896a^2c^2)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}+$$

$$+\frac{-2(-624a^3c^2-24a^4c^2-222720c^3+24960ac^3+1920a^2c^3-32320c^4+1040ac^4)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}+$$

$$\left.+\frac{-2(80a^2c^4-2304c^5-64c^6)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+13}{2})}\right] \tag{10}$$

Theorem-4:

$${}_2F_1\left[\begin{matrix} a & , & -a-14 & ; & \frac{1}{2} \\ & c & & ; & \end{matrix}\right]$$

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$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+14}} \times \left[\frac{-5897520a + 2856228a^2 + 68104a^3 - 39225a^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})} + \right.$$

$$+ \frac{-2225a^5 - 3a^6 + a^7 + 5898240c - 10079808ac + 1788240a^2c + 186120a^3c - 6280a^4c}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{-648a^5c - 8a^6c + 7225344c^2 - 5180672ac^2 + 245040a^2c^2 + 53080a^3c^2 + 720a^4c^2 - 24a^5c^2}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{3325952c^3 - 1143840ac^3 - 16400a^2c^3 + 4320a^3c^3 + 80a^4c^3 + 752640c^4 - 117600ac^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{-4560a^2c^4 + 80a^3c^4 + 89600c^5 - 5184ac^5 - 192a^2c^5 + 5376c^6 - 64ac^6 + 128c^7}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+14}{2})} +$$

$$+ \frac{17297280 - 19716432a + 1898236a^2 + 587096a^3 + 11665a^4 - 2143a^5 - 101a^6 - a^7}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})} +$$

$$+ \frac{33818496c - 19565024ac - 242880a^2c + 293240a^3c + 15560a^4c - 24a^5c - 8a^6c}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})} +$$

$$+ \frac{22698368c^2 - 6656608ac^2 - 479040a^2c^2 + 34280a^3c^2 + 2400a^4c^2 + 24a^5c^2 + 7344512c^3}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})} +$$

$$+ \frac{-977440ac^3 - 103760a^2c^3 + 160a^3c^3 + 80a^4c^3 + 1285760c^4 - 57120ac^4 - 7920a^2c^4}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})} +$$

$$\left. + \frac{-80a^3c^4 + 124544c^5 - 192ac^5 - 192a^2c^5 + 6272c^6 + 64ac^6 + 128c^7}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+15}{2})} \right] \tag{11}$$

Theorem-5:

$${}_2F_1\left[\begin{matrix} a & , & -a-15 & ; & \frac{1}{2} \\ & c & & ; & 2 \end{matrix}\right]$$

$$= \frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+15}} \times \left[\frac{-82570320a + 40531212a^2 + 1372620a^3}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+16}{2})} + \right.$$

$$+ \frac{-697871a^4 - 50040a^5 - 62a^6 + 60a^7 + a^8 + 82575360c - 147571200ac + 27457920a^2c}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+16}{2})} +$$

$$+ \frac{3460800a^3c - 136640a^4c - 20160a^5c - 448a^6c + 107053056c^2 - 82606080ac^2}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+16}{2})} +$$

$$+ \frac{4212928a^2c^2 + 1188000a^3c^2 + 21600a^4c^2 - 1440a^5c^2 - 32a^6c^2 + 53788672c^3}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+16}{2})} +$$

$$+ \frac{-20832000ac^3 - 380800a^2c^3 + 134400a^3c^3 + 4480a^4c^3 + 13862912c^4 - 2625600ac^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+16}{2})} +$$

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$$+\frac{-139040a^2c^4+4800a^3c^4+160a^4c^4+2007040c^5-161280ac^5-10752a^2c^5+164864c^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+16}{2})}+$$

$$+\frac{-3840ac^6-256a^2c^6+7168c^7+128c^8}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+16}{2})}+$$

$$+\frac{-16(-2162160+2633820a-422912a^2-56175a^3+2065a^4+315a^5+7a^6-4227312c)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})}+$$

$$+\frac{-16(2745900ac-122940a^2c-37425a^3c-685a^4c+45a^5c+a^6c-2837296c^2)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})}+$$

$$+\frac{-16(1001700ac^2+19530a^2c^2-6300a^3c^2-210a^4c^2-918064c^3+165300ac^3)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})}+$$

$$+\frac{-16(8770a^2c^3-300a^3c^3-10a^4c^3-160720c^4+12600ac^4+840a^2c^4-15568c^5)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})}+$$

$$\left.+\frac{-16(360ac^5+24a^2c^5-784c^6-16c^7)}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+15}{2})}\right] \qquad (12)$$

III. Derivations of The Summation Theorems(1) to (5)

Proof of theorem-1:

putting $b=-a-11, z=\frac{1}{2}$ in known result (2),we get

$$(2a+11)\ {}_2F_1\left[\begin{matrix} a\ ,\ -a-11 & ; \frac{1}{2} \\ c & ; \end{matrix}\right]$$

$$= a\ {}_2F_1\left[\begin{matrix} a+1\ ,\ -a-11 & ; \frac{1}{2} \\ c & ; \end{matrix}\right] + (a+6)\ {}_2F_1\left[\begin{matrix} a\ ,\ -a-10 & ; \frac{1}{2} \\ c & ; \end{matrix}\right]$$

Now using Bailey theorem, we get

$$\text{L.H.S} = \text{a}\ \frac{\sqrt{\pi}\ \Gamma(\text{c})}{2^{\text{c}+10}} \times \left[\frac{-8304-3790\text{a}+5067\text{a}^2+551\text{a}^3-3\text{a}^4-\text{a}^5-3152\text{c}-14484\text{ac}}{\Gamma(\frac{\text{c+a+11}}{2})\ \Gamma(\frac{\text{c-a}}{2})}+\right.$$

$$+\frac{1202a^2c+252a^3c+6a^4c+7080c^2-5948ac^2-216a^2c^2+12a^3c^2+3840c^3-672ac^3}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-32a^2c^2+624c^4-16ac^4+32c^5}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-8304a-3790a^2+5067a^3+551a^4-3a^5-a^6-3152ac-14484a^2c+1202a^3c}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

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$$+\frac{252a^4c+6a^5c+7080ac^2-5948a^2c^2-216a^3c^2+12a^4c^2+3840ac^3-672a^2c^3}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{-32a^3c^3+624ac^4-16a^2c^4+32ac^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]+$$

$$(a+11)\ \frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+10}}\times\left[\frac{-1226a+3406a^2+553a^3+2a^4-a^5+12288c-16156ac+482a^2c}{\Gamma(\frac{c+a+10}{2})\ \Gamma(\frac{c-a+1}{2})}+\right.$$

$$+\frac{228a^3c+6a^4c+12800c^2-5480ac^2-252a^2c^2+12a^3c^2+4480c^3-608ac^3-32a^2c^3}{\Gamma(\frac{c+a+10}{2})\ \Gamma(\frac{c-a+1}{2})}+$$

$$+\frac{640c^4-16ac^4+32c^5}{\Gamma(\frac{c+a+10}{2})\ \Gamma(\frac{c-a+1}{2})}+$$

$$+\frac{30240-27516a-1984a^2+527a^3+a^5+54048c-18604ac-2758a^2c+12a^3c+6a^4c}{\Gamma(\frac{c-a}{2})\ \Gamma(\frac{c+a+11}{2})}+$$

$$\left.+\frac{30400c^2-612a^2c^2-12a^3c^2+7860c^3-32ac^3-32a^2c^3+800c^4+16ac^4+32c^5}{\Gamma(\frac{c-a-1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]$$

$$=\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}}\times\left[\frac{-16608a-7580a^2+10134a^3+1102a^4-6a^5-2a^6-6304ac-28968a^2c}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+\right.$$

$$+\frac{2404a^3c+504a^4c+12a^5c+14160ac^2-11896a^2c^2-432a^3c^2+24a^4c^2+7680ac^3}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1344a^2c^3-64a^3c^3+1248ac^4-32a^2c^4+64ac^5}{\Gamma(\frac{c+a+11}{2})\ \Gamma(\frac{c-a}{2})}+$$

$$+\frac{-1320a+28370a^2+29771a^3-664a^4-802a^5-58a^6-a^7-31384ac-38334a^2c}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{26665a^3c+3395a^4c+15a^5c-5a^5c-5a^6c+6080ac^2-46256a^2c^2+2382a^3c^2}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{696a^4c^2+180a^5c^2+19320ac^3-11620a^2c^3-520a^3c^3+20a^4c^3+6480ac^4}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{-928a^2c^4-48a^3c^4+784ac^5-16a^2c^5+32ac^6}{\Gamma(\frac{c-a+1}{2})\ \Gamma(\frac{c+a+12}{2})}\right]+$$

$$+\frac{\sqrt{\pi}\ \Gamma(c)}{2^{c+11}}\times\left[\frac{-1349040a+117116a^2+120092a^3+15239a^4+485a^5-196a^6-a^7}{\Gamma(\frac{c+a+12}{2})\ \Gamma(\frac{c-a+1}{2})}+\right.$$

$$+\frac{1351680c-1654016ac-248766a^2c+28535a^3c+6505a^4c+345a^5c+5a^6c+1543168c^2}{\Gamma(\frac{c+a+12}{2})\ \Gamma(\frac{c-a+1}{2})}+$$

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$$+\frac{-499428ac^2-140854a^2c^2-6462a^3c^2+294a^4c^2+18a^5c^2+633600c^3-20280ac^3}{\Gamma(\frac{c+a+12}{2})\,\Gamma(\frac{c-a+1}{2})}+$$

$$+\frac{-20060a^2c^3-1400a^3c^3-20a^4c^3+119680c^4+9472ac^4-656a^2c^4-48a^3c^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{665280-544872a-98680a^2+7626a^3+2198a^4+126a^5+2a^6+1189056c-301192ac}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{-97884a^2c-5252a^3c+156a^4c+12a^5c+668800c^2-8720ac^2-19784a^2c^2-1488a^3c^2}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-24a^4c^2+161920c^3+14016ac^3-768a^2c^3-64a^3c^3+17600c^4+1952ac^4+32a^2c^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{704c^5+64ac^5}{\Gamma(\frac{c-a-1}{2})\,\Gamma(\frac{c+a+12}{2})}\right]$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+11}}\times\left[\frac{665280-561480a-106260a^2+17760a^3+3300a^4+120a^5+1189056c}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}+\right.$$

$$+\frac{-307496ac-126852a^2c-2848a^3c+660a^4c+24a^5c-668800c^2+5440ac^2-31680a^2c^2}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{-1920a^3c^2+161920c^3+21696ac^3-2112a^2c^3-128a^3c^3+17600c^4+3200ac^4}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{+704c^5+128ac^5}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}+$$

$$+\frac{-1350360a+145486a^2+149863a^3+14575a^4-317a^5-77a^6-2a^7+1351680c}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-1685400ac-287100a^2c+55200a^3c+9900a^4c+360a^5c+1543168c^2-493340ac^2}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-187110a^2c^2-4080a^3c^2+990a64c^2+36a^5c^2+633600c^3-960ac^3-31680a^2c^3-1920a^3c^3}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$\left.+\frac{+119680c^4+15952ac^4-1584a^2c64-96a^3c^4+10560c^5+1920ac^5+352c66+64ac^6}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}\right]$$

On simplification , we get

$${}_2F_1\left[\begin{matrix} a\ , & -a-11 & ; & \frac{1}{2} \\ & c & ; & 2\end{matrix}\right]$$

$$=\frac{\sqrt{\pi}\,\Gamma(c)}{2^{c+11}}\times\left[\frac{-122760a+35546a^2+7161a^3+23a^4-33a^5}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+\right.$$

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$$+\frac{-a^6+122880c-175560ac+5820a^2c+3960a^3c+180a^4c+140288c^2-70356ac^2}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-4218a^2c^2+396a^3c^2+18a^4c^2+57600c^3-10560ac^3-960a^2c^3+10880c^4-528ac^4}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{-48a^2c^4+960c^5+32c^6}{\Gamma(\frac{c-a+1}{2})\,\Gamma(\frac{c+a+12}{2})}+$$

$$+\frac{4(15120-15510a+405a^2+330a^3+15a^4+27024c-11902ac-719a^2c+66a^3c+3a^4c)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}$$

$$\left.+\ \frac{4(15200c^2-2640ac^2-240a^2c^2+3680c^3-176ac^3-16a^2c^3+400c^4+16c^5)}{\Gamma(\frac{c-a}{2})\,\Gamma(\frac{c+a+11}{2})}\right]$$

which proves the theorem -1.

Proof of theorems 2-5: On the similar lines of proof of theorem-1, we can prove other theorems 2-5.

References Références Referencias

1. Arora, Asish, Singh, Rahul , Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems *Journal of Rajasthan Academy of Physical Sciences.*, 7(2008), 335-342.
2. Choi, J., Harsh, H. and Rathie, A. K.; Some summation formulae for the Apple's function F_1, *East Asian Math. Journal*, 17(2001), 233-237.
3. Erdélyi, A., Magnus, W., Okerhettinger, F. and Tricomi, F. G.; *Higher transcendental functions* Vol.1 (Bateman Manuscript Project) McGraw-Hill book P. Inc. New York, Toronto and London, 1953.
4. Krupnikov, E. D., Kölbig, K. S.; Some special cases of the generalized hypergeometric function ${}_{q+1}F_q$, *Journal of computational and Applied Math.*, 78(1997), 79-95.
5. Lavoie, J. L.; Notes on a paper by J. B. Miller, *J. Austral. Math. Soc. Ser. B*, 29(1987), 216-220.
6. Lavoie, J. L.; Some summation formulae for the series ${}_3F_2$, *Math. Comput.*, 49(1987), 269-274.
7. Lavoie, J. L., Grondin, F. and Rathie, A.K.; Generalizations of Watson's theorem on the sum of a ${}_3F_2$, *Indian J. Math.*, 34(1992), 23-32.
8. Lavoie, J. L., Grondin, F. and Rathie, A.K.; Generalizations of Whipple's theorem on the sum of a ${}_3F_2$, *J. Comput. Appl. Math.*, 72(1996), 293-300.
9. Lavoie, J. L., Grondin, F. Rathie, A. K. and Arora, K.; Generalizations of Dixon's theorem on the sum of a ${}_3F_2$, *Math. Comput.*, 62, 267-276.
10. Mitra, C. S.; *J. Indian Math. Soc.* (N.S.), 7(1943), 102-109.
11. Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; *Integrals and Series Vol. 3: More Special Functions.* Nauka, Moscow, 1986. Translated from the Russian

Notes

by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne, 1990.

12. Rainville, E. D.; The contiguous function relations for ${}_pF_q$ with applications to Bateman's $J_n^{u,v}$ and Rice's $H_n\ (\zeta, p, \nu)$, *Bull. Amer. Math. Soc.*, 51(1945), 714-723.

13. Shashikant, Sharma, S. and Rathie, A. K.; Some summation formulae for the Apple's function F_1, *Proc. of the fourth Int. Conf. SSFA*, 4(2003), 81-84.

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Pathway Fractional Integral Operator Associated with Certain Special Functions

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Abstract - The aim; of the present paper is to study a pathway fractional integral operator concerning with the pathway model and pathway probability density of some product of special functions. The results derived here are quite general in nature, and hence encompass several cases of interest.

Keywords : *Pathway fractional integral operator, Fox H-function, M-series.*

GJSFR-F Classification : *MSC 2010: 45P05*

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7. Mathai, A.M., A pathway to matrix variate gamma and normal densities, Linear Algebra and its Applications,396 (2005), 317-328.

Pathway Fractional Integral Operator Associated with Certain Special Functions

V.B.L.Chaurasia [α] & Jaswant Singh [σ]

Abstract - The aim; of the present paper is to study a pathway fractional integral operator concerning with the pathway model and pathway probability density of some product of special functions. The results derived here are quite general in nature, and hence encompass several cases of interest.

Keywords : Pathway fractional integral operator, Fox H-function, M-series.

I. Introduction

Nair [11] introduced the Pathway fractional integral operator which is defined in the following manner

$$\left(P_{0+}^{(\eta,\alpha)} f\right)(x) = x^{\eta} \int_{0}^{\left[\frac{x}{a(1-\alpha)}\right]} \left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)\,dt, \tag{1.1}$$

where $f(x) \in L(a,b)$, $\eta \in C$, $Re(\eta) > 0$, $a > 0$ and pathway parameter $\alpha < 1$.

Mathai [7] introduced the pathway model and further studied by Mathai and Haubold ([8], [9]). For real scalar α, the pathway model for scalar random variables is denoted by following probability density function (p.d.f.)W.

$$f(x) = c\,|x|^{\gamma-1}\left[1-a(1-\alpha)|x|^{\delta}\right]^{\frac{\beta}{1-\alpha}}, \tag{1.2}$$

where $\gamma > 0, \delta > 0, \beta \geq 0, \{1-a(1-\alpha)|x|^{\delta}\} > 0, \gamma > 0, -\infty < x < \infty$, c is the normalizing constant and α is known as pathway parameter. The normalizing constant, for real α, is as follows:

$$c = \frac{1}{2}\,\frac{\delta\left[a(1-\alpha)\right]^{\frac{\gamma}{\delta}}\,\Gamma\left(\frac{\gamma}{\delta}+\frac{\beta}{1-\alpha}+1\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{1-\alpha}+1\right)},\ \alpha < 1 \tag{1.3}$$

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$$=\frac{1}{2}\frac{\delta[a(1-\alpha)]^{\frac{\gamma}{\delta}}\Gamma\left(\frac{\beta}{1-\alpha}\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\Gamma\left(\frac{\beta}{1-\alpha}-\frac{\gamma}{\delta}\right)}, \text{for } \frac{1}{\alpha-1}-\frac{\gamma}{\delta}>0, \alpha>1, \tag{1.4}$$

Notes

$$=\frac{1}{2}\frac{\delta(a\beta)^{\frac{\gamma}{\delta}}}{\Gamma\left(\frac{\gamma}{\delta}\right)} \text{ for } \alpha\to 1. \tag{1.5}$$

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It is a finite range density with $\{1-a(1-\alpha)\,|\,x\,|^{\delta} > 0$, for $\alpha < 1$, and (1.2) remains in the extended generalized type – 1 beta family. For $\alpha < 1$, the pathway density in (1.2) includes the extended type – 1 beta density, the triangular density, the uniform density and many other p.d.f.

We have, for $\alpha > 1$,

$$f(x)=c\,|\,x\,|^{\gamma-1}[1+a\,(\alpha-1)\,|\,x\,|^{\delta}]^{\frac{\beta}{\alpha-1}}, \tag{1.6}$$

where $\alpha > 1$, $\delta > 0$, $\beta \geq 0$, $-\infty < x < \infty$, which is extended generalized type – 2 beta model for real x. It includes the type – 2 beta density, the F-density, the student –t density, the Cauchy density and many more. The pathway parameter $\alpha < 1$ has only been considered here. For $\alpha \to 1$, (1.2) and (1.6) take the exponential form, since

$$\lim_{\alpha\to 1} c\,|x|^{\gamma-1}\,[1-a(1-\alpha)|\,x\,|^{\delta}]^{\frac{\eta}{1-\alpha}}$$

$$=\lim_{\alpha\to 1} c\,|\,x\,|^{\gamma-1}[1+a(\alpha-1)|\,x\,|^{\delta}]^{\frac{\eta}{\alpha-1}}$$

$$=c\,|\,x|^{\gamma-1}\,e^{-a\eta|x|^{\delta}}. \tag{1.7}$$

This includes generalized Gamma-, the Weibull-, the Chi-square, the Laplace-, the Maxwell-Boltzmann and other related density.

For $\alpha \to 1_-$, $\left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{1-\alpha}} \to e^{-\frac{a\,\eta}{x}t}$ U, the operator (1.1) reduces to the well known Laplace integral transform of f with parameter $\frac{a\eta}{x}$

$$(P_{0+}^{(\eta,1)}f)x=x^{\eta}\int_0^{\infty} e^{-\frac{a\eta}{x}t} f(t)\,dt$$

$$= x^{\eta} L_r\left(\frac{a^{\eta}}{x}\right) \tag{1.8}$$

For $\alpha = 0$, $a = 1$, then replacing η by $\eta-1$ in (1.1) the integral operator reduces to Riemann-Liouville fraction integral operator.
Sharma [13] introduced the generalized M-series as follows

$$ {}_{\rho}^{\alpha',\beta'}M_{\sigma}(z) = \sum_{k=0}^{\infty} \frac{(a_1')_k \ldots (a_\rho')_k}{(b_1')_k \ldots (b_\sigma')_k} \frac{z^k}{\Gamma(\alpha' k + \beta')},$$

$$= \psi_1(k) \tag{1.9}$$

where $z, \alpha', \beta' \in C$, $\mathrm{Re}(\alpha') > 0$, $\forall z$ if $\rho \le \sigma$, $|z| < \alpha'^{\alpha'}$, for other details see [13].

The following series representation of H-function [12] will be required

$$H_{P,Q}^{M,N}\left[z \left| \begin{matrix} (e_P, E_P) \\ (f_Q, F_Q) \end{matrix} \right. \right] = \sum_{h=1}^{N} \sum_{v=0}^{\infty} \frac{(-1)^v \chi(\xi)}{v!\, E_h} \left(\frac{1}{z}\right)^{\xi}, \tag{1.10}$$

where $\xi = \dfrac{e_h - 1 - v}{E_h}$ and $(h = 1, \ldots, N)$

and

$$\chi(\xi) = \frac{\prod_{j=1}^{M} \Gamma(f_j + F_j \xi) \prod_{\substack{j=1 \\ j \ne h}}^{N} \Gamma(1 - e_j - E_j \xi)}{\prod_{j=M+1}^{Q} \Gamma(1 - f_j - F_j \xi) \prod_{j=N+1}^{P} \Gamma(e_j + \xi E_j)},$$

$$= \psi_2(\xi) \tag{1.11}$$

for convergence condition and other details see ([4] and [13]).
For the sake of brevity

$$T_1 = \sum_{i=1}^{N} E_i - \sum_{i=N+1}^{P} E_i + \sum_{i=1}^{M} F_i - \sum_{i=M+1}^{Q} F_i \tag{1.12}$$

$$T_2 = \sum_{i=1}^{n} \alpha_i - \sum_{i=n+1}^{q} \alpha_i + \sum_{i=1}^{m} \beta_i - \sum_{i=m+1}^{q} \beta_i \tag{1.13}$$

Ref.

13. Sharma, Manoj and Jain, Renu, A note on a generalized M-series as a special function of fractional calculus, Fract. Cal. Appl. Anal., 12(4) (2009), 449-452.

II. Main Results

Theorem 1. Let $\eta,\omega\in C, \mathrm{Re}(\beta)>0, \mathrm{Re}(\delta)>0, \mathrm{Re}\left(1+\frac{h}{1-\alpha}\right)>0, \mathrm{Re}(\rho)>0, \alpha<1, b\in R,$

$$c\in R, \mathrm{Re}\left(\omega+\delta\frac{f_j}{F_j}\right)>0, \mathrm{Re}\left(\omega+\beta\frac{b'_j}{\beta'_j}\right)>0, |\arg c|<\frac{1}{2}T_1\pi, |\arg b|<\frac{1}{2}T_2\pi, \beta^*>0,$$

$$T_1, T_2>0, \rho\le\sigma, |d|<\alpha'^{\alpha'}, \beta^*>0, j=1,\dots,Q;\ j'=1,\dots,q.$$

Then

$$P_{0+}^{(\eta,\alpha)}\left\{t^{\omega-1}\ {}_{\rho}\overset{\alpha',\beta'}{M}_{\sigma}\,[dt^{-\beta^*}]H_{P,Q}^{M,N}\left[c\,t^{\delta}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]H_{p,q}^{m,n}\left[bt^{\beta}\left|\begin{matrix}(a_p,\alpha_p)\\(b_q,\beta_q)\end{matrix}\right.\right]\right\}$$

$$=\psi_1(k)\frac{d^k\,x^{\eta+\omega-\beta^*k}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\omega-\beta^*k}}H_{P,Q}^{M,N}\left[\frac{c\,x^{\delta}}{[a(1-\alpha)]^{\delta}}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$

$$.\ H_{p+1,q+1}^{m,n+1}\left[\frac{b\,x^{\beta}}{a(1-\alpha)^{\beta}}\left|\begin{matrix}(1-\omega+\delta\xi+\beta^*k,\beta),(a_p,\alpha_p)\\(b_q,\beta_q),\left(-\omega-\delta\xi+\beta^*k-\frac{\eta}{1-\alpha},\beta\right)\end{matrix}\right.\right], \qquad (2.1)$$

Proof. Making use of (1.9), (1.10) and (1.1) and appealing to a known result [11], we arrive at the desired result (2.1).

Theorem 2. Let $\eta,\gamma,\delta,\beta,T_1,T_2>0, \mathrm{Re}(\eta)>0, \mathrm{Re}(\gamma)>0, \mathrm{Re}(\omega)>0, \mathrm{Re}\left(1+\frac{\eta}{1-\alpha}\right)>\max.$

$$[0,-\mathrm{Re}(\omega)], b,c\in R, \alpha<1, \mathrm{Re}\left(\omega+\delta\frac{f_j}{F_j}\right)>0, |\arg c|<\frac{1}{2}T_1\pi, \rho\le\sigma \text{ and } |d|<, \beta^*>0,$$

$j=1,\dots,Q$.

Then

$$P_{0+}^{(\eta,\alpha)}\left\{t^{\omega-1}\ {}_{\rho}\overset{\alpha',\beta'}{M}_{\sigma}\,[dt^{-\beta^*}]H_{P,Q}^{M,N}\left[c\,t^{\delta}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]E_{\beta,\rho}^{\gamma}(b\,t^{\beta})\right\}$$

Ref.

11. Nair, Seema S., Pathway fractional integration operator, Fract. Cal. Appl. Anal., 12(3) (2009), 237-259.

$$=\psi_1(k)\frac{d^k\, x^{\eta+\omega-\beta*k}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{\Gamma(\gamma)\,\Gamma[a(1-\alpha)]^{\omega-\beta*k}}\;{}_2\psi_2\left[\frac{b\,x^{\beta}}{a(1-\alpha)^{\beta}}\left|\begin{matrix}(\omega-\delta\xi+\beta*k,\beta),(\gamma,1)\\(\omega,\beta),\left(1+\omega+\frac{\eta}{1-\alpha}-\delta\xi-\beta*k,,\beta\right)\end{matrix}\right.\right],$$

$$.\;H^{M,N}_{P,Q}\left[\frac{c\,x^{\delta}}{[a(1-\alpha)]^{\delta}}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right], \tag{2.2}$$

where $E^{\gamma}_{\beta,\omega}(b)$ is the generalized Mittag-Leffler function (see [14],[15]).

Proof. The result in (2.2) can be derived from Theorem 1 by taking m =1 = n, p = 1, q = 2, $b_1=0,\beta_1=0,b_2=1-\omega,\beta_2=\beta,\alpha_1=1-\gamma$ and $\alpha_1=1$. We have the required result.

Theorem 3. Let $\eta,\gamma,v\in C,\delta>0,\alpha<1,\rho\le\sigma,|d|<\alpha'^{\alpha'},\mathrm{Re}(\eta)>0,c\in R,\mathrm{Re}(\gamma+v)>0,$

$$\mathrm{Re}\left(1+\frac{\eta}{1-\alpha}\right)>0,\mathrm{Re}\left(\gamma+\delta\frac{f_j}{F_j}\right)>0,|\arg c|<\frac{1}{2}T_1\pi,T_1>0,\beta^*>0,\ j=1,\ldots,Q.$$

Then

$$P^{(\eta,\alpha)}_{0+}\left\{\left(\frac{t}{2}\right)^{\gamma-1}{}_{\rho}M^{\alpha',\beta'}_{\sigma}\left[d\left(\frac{t}{2}\right)^{-\beta*}\right]J_v(t)\,H^{M,N}_{P,Q}\left[c\left(\frac{t}{2}\right)^{\delta}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]\right\}$$

$$=\psi_1(k)\frac{d^k\,x^{\eta+v+\gamma-\beta*k}\,\Gamma\left(1+\frac{\eta}{1-\alpha}\right)}{[a(1-\alpha)]^{\gamma+v-\beta*k}\,2^{\gamma+v+\eta-\beta*k}}\;H^{M,N}_{P,Q}\left[\frac{c\,x^{\delta}}{[2a(1-\alpha)]^{\delta}}\left|\begin{matrix}(e_P,E_P)\\(f_Q,F_Q)\end{matrix}\right.\right]$$

$$.\;{}_1\psi_2\left[-\frac{x^2}{4a^2(1-\alpha)^2}\left|\begin{matrix}(\gamma+v-\delta\xi-\beta*k,2)\\(v+1,1),\left(1+v+\gamma-\delta\xi-\beta*k+\frac{\eta}{1-\alpha},2\right)\end{matrix}\right.\right]. \tag{2.3}$$

Here ${}_p\psi_q$ denotes the generalized Wright hypergeometric function ([14],[15]).

Proof. The result in (2.3) can be established by taking m =1, n = 0, p = 0, q = -2, $b_1=0$, $\beta_1=0,b_2=-v,\beta_2=1,\omega=\gamma+v,b'=1,\beta=2$ and replacing t by $\frac{t}{2}$ after a little simplification, we have the desired result.

14. Wright, E.M., The asymptotic expansion of the generalized Bessel function, Proc. London Math. Soc., 38 (1934), 257-270.

Ref.

III. Special Cases

1. Letting $\beta^* \to 0$ in the result (2.1), we get the result recently obtained by Chaurasia and Ghiya [1] for ρ, ρ_1 and $\rho_2 \to 0$.
2. Making $\beta^*, \delta \to 0$ in the results (2.1) through (2.3), we have the results recently derived by Chaurasia and Gill in [2].
3. Giving suitable values to the parameters in the results (2.1) through (2.3), we get the results recently obtained by Nair in [11].

A large number of simpler corresponding results pertaining to simpler functions can be obtained easily merely by specializing the parameters in them.

Notes

References Références Referencias

1. Chaurasia, V.B.L. and Ghiya, Neeti, Pathway fractional integral operator pertaining to special functions, Global J.Sci., Front. Res., 10(6)(Ver.1.0), (2010), 79-83.
2. Chaurasia, V.B.L. and Gill,Vinod, Pathway fractional integral operator involving H-functions (Communicated).
3. Erdélyi, et al., Higher transcendental functions, Vol.II, McGraw-Hill, New York, 1953.
4. Fox, C., The G and H-functions as symmetrical Fourier kernels, Trans. Amer. Math.Soc., 98 (1961), 395-429.
5. Kilbas, A.A., Saigo, M.and Saxena, R.K., Generalized Mittag-Leffler function and generalized fractional calculus operators, Integral Transform Special Functions, 15 (2004), 31-49.
6. Kilbas, A.A., Srivastava, H.M. and Trugille, J.J., Theory and applications of fractional differential equations, Elsevier, Amsterdams, 2006.
7. Mathai, A.M., A pathway to matrix variate gamma and normal densities, Linear Algebra and its Applications,396 (2005), 317-328.
8. Mathai, A.M. and Haubold, H.J., On generalized distribution and pathways, Phy. Letters, 372(2008), 2109-2113.
9. Mathai, A.M. and Haubold, H.J., Pathway models,superstatisties, trellis statistics and a generalized measure of entropy, Physica, A375 (2007), 110-122.
10. Mathai, A.M. and Saxena, R.K., The H-function with applications in Statistics and other Disciplines, Wiley, New York, 1978.
11. Nair, Seema S., Pathway fractional integration operator, Fract. Cal. Appl. Anal., 12(3) (2009), 237-259.
12. Skibiński, P., Some expansion Theorems for the H-function, Ann. Polon. Math., 23 (1970), 125-138.
13. Sharma, Manoj and Jain, Renu, A note on a generalized M-series as a special function of fractional calculus, Fract. Cal. Appl. Anal., 12(4) (2009), 449-452.
14. Wright, E.M., The asymptotic expansion of the generalized Bessel function, Proc. London Math. Soc., 38 (1934), 257-270.
15. Wright, E.M., The generalized Bessel function of order greater than one, Quart. J. Math. Oxford Ser., 11 (1940), 36-48.

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- Paper Title should be of Font Size 24 with one Column section.
- Author Name in Font Size of 11 with one column as of Title.
- Abstract Font size of 9 Bold, “Abstract” word in Italic Bold.
- Main Text: Font size 10 with justified two columns section
- Two Column with Equal Column with of 3.38 and Gaping of .2
- First Character must be three lines Drop capped.
- Paragraph before Spacing of 1 pt and After of 0 pt.
- Line Spacing of 1 pt
- Large Images must be in One Column
- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
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You can use your own standard format also.
Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

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Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)Title should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

(d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

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References

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22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

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Key points to remember:

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A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

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· Use present tense to report well accepted

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· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

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- Significant conclusions or questions that track from the research(es)

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- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

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- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
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Approach:

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- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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