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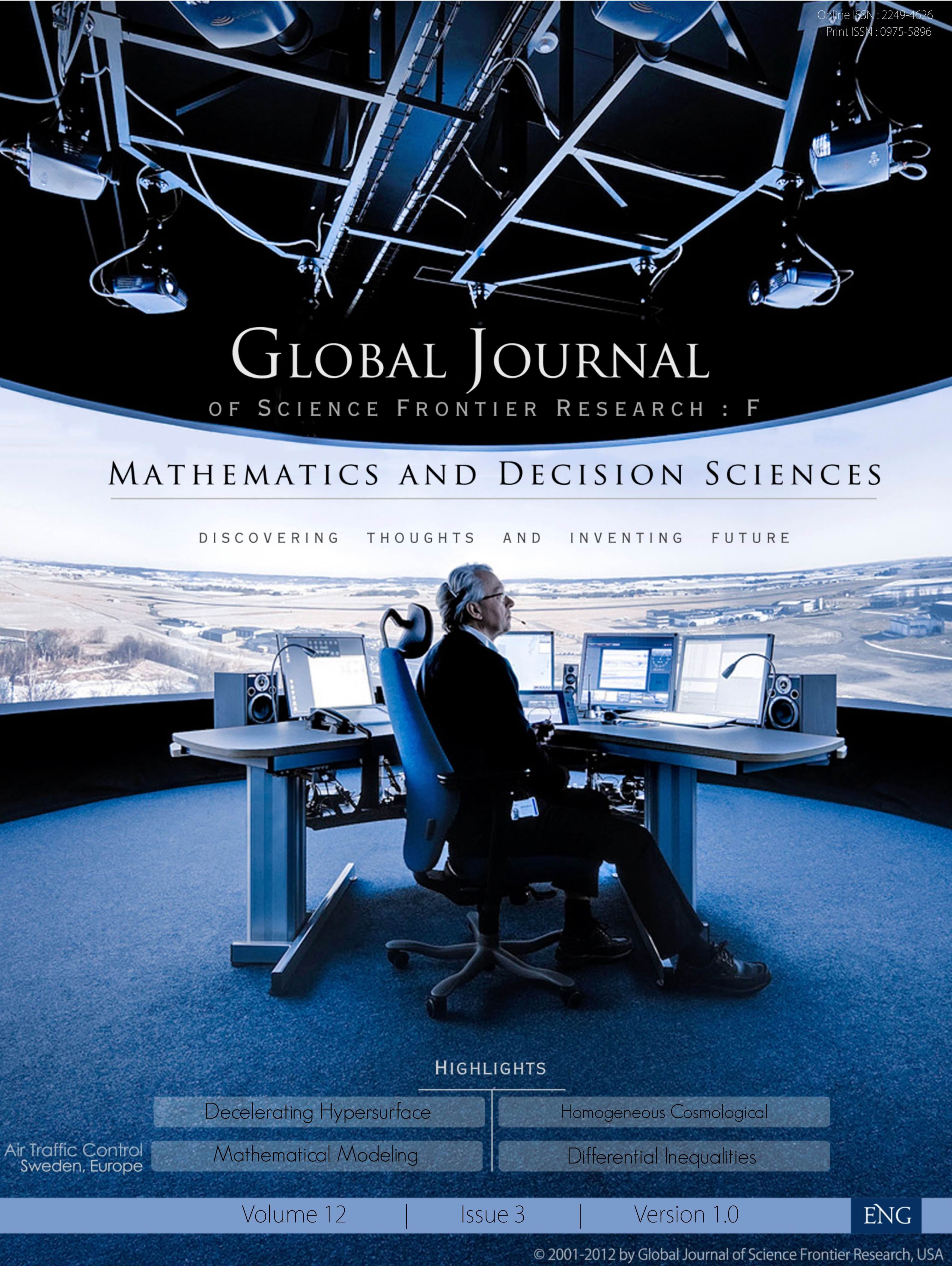

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Mathematics and Decision Sciences

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On The Response of a Non-Uniform Beam Transvered by Mobile Distributed Loads

By Ogunyebi S. N & Sunday J
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Abstract – The problem being investigated in this paper is that of the response of non-uniform beam under tensile stress and resting on an elastic foundation. The fourth order partial differential equation governing the problem is solved when the beam is transverse by mobile distributed loads. The elastic properties of the beam, the flexible rigidity, and the mass per unit length are expressed as functions of the spatial variable using Struble's method. It is observed that the deflection of non-uniform beam under the action of moving masses is higher than the deflection of moving force when only the force effects of the moving load are considered. From the analysis, the response amplitudes of both moving force and moving mass problems decrease with increasing foundation constant.

Keywords : *Distributed Load, Non-uniform, Elastic Foundation, moving Mass.*

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ON THE RESPONSE OF A NON-UNIFORM BEAM TRANSVERED BY MOBILE DISTRIBUTED LOADS

Strictly as per the compliance and regulations of :

On the Response of a Non-Uniform Beam Transvered by Mobile Distributed Loads

Ogunyebi S. N [α] & Sunday J [σ]

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I. Introduction

Structural engineers usually encountered problem that arises especially when a beam is being transverse by a moving load. The theory of vibration of structures has treated some of these problem i.e vibrations of turbines, hulls of shills and bridge girders of variable dept etc. Beam on elastic foundation subjected to moving masses have received extensive attention in the literature.

Kolousek et al [3] used normal mode analysis to address the problem of flexible vibration of non-uniform beam. This was followed by Sadiku and Leipholz [6] who only studied the dynamics of a uniform beam by considering the inertia effect of a moving mass and later developed the Green's function of the associated differential problem thereby obtained a closed form solution.

In a later development, Oni [10] presented the problem of dynamic analysis of a non uniform beam to several moving masses under concentrated load. The beam considered is under tensile stress and by the method of Galarkin, the result is obtained for the first mode response of the beam. Chau and Seng [8] worked on the static response of beams on non-linear elastic foundation where the deformed shape of the structure was represented by a Fourier series, and thereafter, the giving equation is reduced to a set of second order simultaneous equations using Galarkin's method. In all the aforementioned works, the practical cases where the elastic systems are of variable cross section and of distributed moving loads use not considered.

The paper therefore presents the problem of dynamic response of a non-uniform beam to moving masses on elastic foundation traversed by mobile distributed load.

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Ref.
3. Kolousek V. et al (1967): Civil engineering Structures Subjected to Dynamic Loads (in slovak) SVTL, Bratislava.
10. Oni, S.T (1996): Response of a non- uniform beam resting on an elastic foundation to several moving masses. Abacus Journal of Mathematical Association of Nigeria. Vol.12,Pp 103-120.

II. Derivation and Assembly of the Governing Equation

Consider a moving load $\Delta(x,t)$ of mass M acting on a Bernoulli-Euler beam (Non-uniform) uniformly loaded and move at a constant velocity c as shown below:

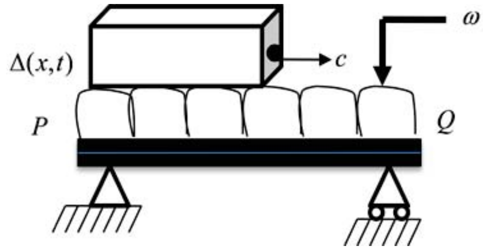

Figure 1: Uniformly distributed load on simply supported beam.

In the structure above, the displacement is governed by the equation

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2\overline{U}(x,t)}{\partial x^2}\right]+\alpha^m(x)\frac{\partial^2\overline{U}(x,t)}{\partial t^2}-N\frac{\partial^2\overline{U}(x,t)}{\partial x^2}+k(x)\overline{U}(x,t)=\Delta_f(x,t)\overline{U}(x,t)\left[1-\frac{\Delta^*}{g}\left(\overline{U}(x,t)\right)\right]$$

where $U(x,t)$ is transverse displacement, E is the Young modulus, $I(x)$ is variable moment of inertia, $EI(x)$ is flexible rigidity, α^m, Δ_f is the substantive acceleration operator, g is the acceleration due to gravity.

For the non-uniform beam such as above, its properties such as moment of inertia I and the mass per unit length of the beam α_{m} vary along the span of L of the beam.

The structure under consideration is simply supported and carrying an arbitrary number of masses M moving with constant velocities.

The Operator Δ^* is defined as

$$\Delta^*=\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t}+c^2\frac{\partial^2}{\partial x\partial t}+c^2\frac{\partial^2}{\partial x^2} \tag{2.2}$$

and the load $\Delta(x,t)$ is given as

$$\Delta_f(x,t)=MH(x-ct)\left[g-\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t}+c^2\frac{\partial^2}{\partial x^2}\right] \tag{2.3}$$

where $H(x-ct)$ is the Heaviside function.

Furthermore, the boundary condition for the dynamical system is taken to be arbitrary and the initial condition of the motion is

$$\overline{U}(x,t)=0=\frac{\partial}{\partial t}\overline{U}(x,t) \tag{2.4}$$

Substituting equations (2.2), (2.3), into (2.1), the governing of motion takes the form

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\overline{U}(x,t)\right]+\alpha^m(x)\frac{\partial^2}{\partial t^2}\overline{U}(x,t)-N\frac{\partial^2}{\partial x^2}\overline{U}(x,t)+K(x)\overline{U}(x,t)$$

$$+MH(x-ct)\left[\frac{\partial^2}{\partial t^2}+2c\frac{\partial^2}{\partial x\partial t}+c^2\frac{\partial^2}{\partial x^2}\right]\overline{U}(x,t)=MgH(x-ct) \tag{2.5}$$

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Equation 2.5 can be further be simplified to give further simplification yields;

$$N_1\left(1+Sin\frac{\pi x}{L}\right)^3\frac{\partial^4}{\partial x^4}\overline{U}(x,t)+N_2\left(1+Sin\frac{\pi x}{L}\right)^2Cos^2\frac{\pi x}{L}\frac{\partial^3}{\partial x^3}\overline{U}(x,t)$$

$$+\left[N_3\left(1+Sin\frac{\pi x}{L}\right)Cos^2\frac{\pi x}{L}-N_4(1+\sin\frac{\pi x}{L})^2Sin\frac{\pi x}{L}-N_5\right]\frac{\partial^2\overline{U}(x,t)}{\partial x^2}$$

$$+\left(1+Sin\frac{\pi x}{L}\right)\frac{\partial^2\overline{U}(x,t)}{\partial t^2}+N_6\overline{U}(x,t)+\frac{M}{\alpha^m{}_o}H(x-ct)\left[\frac{\partial^2}{\partial t^2}\overline{U}(x-ct)+2c\frac{\partial^2\overline{U}(x,t)}{\partial x\partial t}+c^2\frac{\partial^2\overline{U}(x,t)}{\partial x^2}\right]$$

$$=\frac{mg}{\alpha^m{}_o}H(x-ct) \qquad (2.6)$$

where,

$$N_1=\frac{EI_o\pi}{\alpha^m{}_o},N_2=\frac{6\pi EI_o}{\alpha^m{}_oL},N_3=\frac{6\pi^2EI_o}{\alpha^m{}_oL^2},N_4=\frac{3\pi^2EI_o}{\alpha^m{}_oL}N_5=\frac{N}{\alpha^m{}_o},N_6=\frac{K_o}{\alpha^m{}_o} \qquad (2.7)$$

Equation (2.6) is a non-homogenous partial differential equation with variable coefficients. Clearly, it is seen that the closed from solution does not exists.

III. Solution Procedure

To solve equation (2.6), an approximate solution is sought. One of the approximate methods best suited to solve diverse problems in dynamics of structures is the Galarkin's method [7]. This method requires that the solution of equation (2.6) be of the form

$$\overline{U}_n=\sum_{m=1}^{n}Y_m(t)X_m(x) \qquad (3.1)$$

where $X_m(x)$ is chosen such that all the boundary conditions are satisfied. Equation (3.1) when substituted into equation (2.6) yields;

$$\sum_{m=1}^{n}\left[N_1\left(1+Sin\frac{\pi x}{L}\right)^3Y_m(t)X_m{}^{IV}(x)+N_2\left(1+Sin\frac{\pi x}{L}\right)^2Cos^2\frac{\pi x}{L}Y_m(t)X_m{}^{III}(x)\right.$$

$$+\left[N_3\left(1+Sin\frac{\pi x}{L}\right)Cos^2\frac{\pi x}{L}-N_4(1+\sin\frac{\pi x}{L})^2Sin\frac{\pi x}{L}-N_5\right]Y_m(t)X_m{}^{II}(x)$$

$$\left.+\left(1+Sin\frac{\pi x}{L}\right)\ddot{Y}_m(t)X_m(x)+N_6Y_m(t)X_m(x)+\frac{M}{\alpha^m{}_o}H(x-ct)\left[\ddot{Y}_m(t)X_m(x)+2c\dot{Y}_m(t)X_m{}^{I}(x)+c^2Y_m(t)X_m{}^{II}(x)\right]\right]\Bigg\}$$

$$-\frac{Mg}{\alpha^m{}_o}H(x-ct)=0 \qquad (3.2)$$

Ref.

7. Wh, J.S et al (1987):The Dynamic analysis of a flat plate under a moving load by a definite element method. International Journal of Numerical methods in Engineering 24,743-762.

In order to determine $Y_m(t)$, it is required that the expression on the left hand side of equation (3.2) be orthogonal to function $X_m(x)$. Hence,

$$\int_0^L \left\{ \sum_{m=1}^{n} \left[N_1 \left(1+Sin\frac{\pi x}{L}\right)^3 Y_m(t) X_m{}^{IV}(x) + N_2\left(1+Sin\frac{\pi x}{L}\right)^2 Cos^2\frac{\pi x}{L} Y_m(t) X_m{}^{III}(x) \right.\right.$$

$$+\left[N_3\left(1+Sin\frac{\pi x}{L}\right)Cos^2\frac{\pi x}{L} - N_4(1+\sin\frac{\pi x}{L})^2 Sin\frac{\pi x}{L} - N_5\right] Y_m(t) X_m{}^{II}(x)$$

$$+\left(1+Sin\frac{\pi x}{L}\right)\ddot{Y}_m(t)X_m(x) + N_6 Y_m(t) X_m(x) + \frac{M}{\alpha^m{}_o} H(x-ct)\left[\ddot{Y}_m(t)X_m(x) + 2c\dot{Y}_m(t)X_m{}^{I}(x) + c^2 Y_m(t) X_m{}^{II}(x)\right]\Bigg]$$

$$\left. -\frac{Mg}{\alpha^m{}_o} H(x-ct) \right\} X_k(x)dx = 0$$

Since our dynamical system has simple supports at the edges $x=0$ and $x=L$, we choose;

$$X_m(x) = Sin\frac{m\pi x}{L} \tag{3.3}$$

Consequently, using (3.4) in (3.3) gives

$$\sum_{m=1}^{n}\left\{ Ha\ddot{Y}_m(t) + H_b Y_m(t) + \frac{M}{\alpha^m{}_o}\left[H_c(t)\ddot{Y}_m(t) + 2cH_d(t)\dot{Y}_m(t) + c^2 H_e(t) Y_m(t)\right]\right\} = \frac{Mg}{\alpha^m{}_o} H_f(t) \tag{3.5}$$

where

$$H_a = \int_0^L \left(1+Sin\frac{\pi x}{L}\right) Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L} dx\,, \qquad H_b = Q_1 + Q_2 + Q_3 - Q_4 - Q_5 + Q_6$$

$$H_c(t) = \int_0^L H(x-ct) Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \qquad H_d(t) = \frac{m\pi}{L}\int_0^L H(x-ct) Cos\frac{m\pi x}{L} Sin\frac{k\pi x}{L} dx$$

$$H_e(t) = \frac{m^2\pi^2}{L^2}\int_0^L H(x-ct) Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \quad H_f(t) = \int_0^L H(x-ct) Sin\frac{k\pi x}{L} dx \tag{3.6}$$

and

$$Q_1 = \frac{m^4\pi^4}{L^4} N_1 \int_0^L \left(1+\sin\frac{\pi x}{L}\right)^3 Sin\frac{\pi x}{L} Sin\frac{k\pi x}{L}dx, \qquad Q_2 = \frac{m^3\pi^3}{L^3} N_2\int_0^L\left(1+\sin\frac{\pi x}{L}\right)^2 Cos\frac{\pi x}{L} Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx$$

$$Q_3 = \frac{m^2\pi^2}{L^2} N_3 \int_0^L \left(1+\sin\frac{\pi x}{L}\right) Cos^2\frac{\pi x}{L} Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \quad Q_4 = \frac{m^2\pi^2}{L^2} N_4\int_0^L\left(1+\sin\frac{\pi x}{L}\right)^2 Sin\frac{\pi x}{L} Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx$$

$$Q_5 = N_5\int_0^L Sin\frac{m\pi x}{L} Sin\frac{k\pi x}{L}dx, \qquad Q_6 = N_6\int_0^L \sin\frac{m\pi x}{L}\sin k\frac{\pi x}{L}dx \tag{3.7}$$

When the integrals (3.6) and (3.7) are evaluated, the result is a series of coupled differential equations called Galarkin's equations for n-degree of freedom system governing

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the coefficients of all lower and higher modes of the beam. Thus, restricting ourselves to the analysis of the first mode response, we set $m=1$ and $n=1$ in equation (3.5) for analytical approximation.

Following the method of [9] where Heaviside function is expresses as Fourier cosine series. Thus, equation (3.5) leads to

$$\sum_{m=1}^{n} H_a \ddot{Y}_m(t) + H_b Y_m(t) + \Gamma_1 \left[\left(\frac{1}{L} I_1 + \frac{2}{n\pi L} \sum_{n=1}^{\infty} Cos \frac{n\pi ct}{L} I_2 + C' I_3 \right) \ddot{Y}_m(t) \right.$$

$$\left(\frac{2Cm\pi}{L^2} I_4 + \frac{4Cm\pi}{nL^2} \sum_{n=1}^{\infty} Cos \frac{n\pi ct}{L} I_5 + \frac{2CC' m\pi}{L^2} I_6 \right) \dot{Y}_m(t)$$

$$\left. + \left(\frac{C^2 m^2 \pi^2}{L^3} I_7 + \frac{2C^2 m^2 \pi}{nL^3} \sum_{n=1}^{\infty} Cos \frac{n\pi ct}{L} I_8 + \frac{C^2 m^2 \pi^2 C'}{L^2} I_9 \right) Y_m(t) = P_m \left[Cos \frac{\lambda_m ct}{L} - Cos\, \lambda_m \right] \quad (3.8)$$

$$\text{where } \Gamma_1 = \frac{M}{\alpha^m{}_0} \text{ and } P_m = \frac{MgL}{\alpha^m{}_0 \lambda_m} \quad (3.9)$$

which is the transformed equation of the dynamical system.

IV. Analytical Approximate Solution

a) Simply Supported Traversed By Moving Force

An approximate model of the system, when the inertia effect of the moving mass is neglected, is the moving force problem associated with the system. Setting $\Gamma_1 = 0$, we have

$$\ddot{Y}_m(t) + \beta_{mf}^2 Y_m(t) = P_m \left[Cos \frac{\lambda_m ct}{L} - Cos\, \lambda_m \right] \quad (4.1)$$

$$\text{where } \beta_{mf} = \frac{H_b}{H_a} \quad (4.2)$$

Subjecting equation (4.2) to Laplace transform defined by

$$(\tilde{\cdot}) = \int_0^{\infty} e^{-st}\, dt \quad (4.3)$$

where S is a Laplace transform. It yields,

$$\overline{U}_n(x,t) = \sum_{m=1}^{n} P_m \left[\frac{Cos Z_k t - Cos \beta_{mf} t}{\beta_{mf}^2 - Z_k^2} - E(m) \frac{\left(1 - Cos \beta_{mf} t\right)}{\beta_{mf}} \right] \times Sin \frac{n\pi x}{L} \quad (4.4)$$

where $Z_k = \dfrac{\lambda_m ct}{L}$ and $E(m) = Cos \lambda_m$

which is the response to moving force solution of the elastic system at constant velocity.

b) Simply Supported Traversed By Moving Mass

For the moving mass solution, we set $\Gamma_1 \neq 0$, in this case, the entire solution to the problem is sought. To this end, a modification of the asymptotic method of Struble[6] often used for treating weakly homogeneous and non-homogenous non-linear system is employed. Further arrangement of equation (3.8) yields

Ref.

6. Sadiku S Leiophtz H.H.E (1987): On the dynamics of elastic systems with moving concentrated masses. Ing. Archiv 57, 223-242.
9. Smith, J.W (1988): ibration of structures. Application in Civil Engineering Design. Chapman and hall Ltd London.

$$\left[H_a+\Gamma_1\left(\frac{1}{L}I_1+\frac{2}{n\pi L}\sum_{n=1}^{\infty}Cos\frac{n\pi ct}{L}I_2+C^{\square}I_3\right)\right]\ddot{Y}_m(t)$$

$$+\left[\Gamma_1\left(\frac{2Cm\pi}{L^2}I_4+\frac{4Cm\pi}{nL^2}\sum_{n=1}^{\infty}Cos\frac{n\pi ct}{L}I_5+\frac{2CC^{\square}m\pi}{L^2}I_6\right)\right]\dot{Y}_m(t)$$

$$+\left[H_b+\Gamma_1\left(\frac{C^2m^2\pi^2}{L^3}I_7+\frac{2C^2m^2\pi}{nL^3}\sum_{n=1}^{\infty}Cos\frac{n\pi ct}{L}I_8+\frac{C^2m^2\pi^2C^{\square}}{L^2}I_9\right)\right]Y_m(t)=P_m\left[Cos\frac{\lambda_m ct}{L}-C\text{os}\,\lambda_m\right] \tag{4.5}$$

At this juncture, we seek the modified frequency corresponding to the frequency of the free system due to the presence of moving mass [8].To this end, the solution to equation (4.5) can be written as

$$Y_m(t)=N(m,t)\left[\beta_{mf}t-\varphi(m,t)\right] \tag{4.6}$$

where $\beta_{mf}t$ and $\varphi(m,t)$ are constants.

Therefore when the mass of the particle is considered, the first approximation to the homogeneous system is given as

$$Y_m(t)=D^{\square}(m,t)\left[\beta_{jj}t-\varphi(m,t)\right] \tag{4.7}$$

where

$$\beta_{jj}=\left\{1-\frac{\Gamma_j}{H_a}\left[\beta_{mf}\left(\frac{1}{L}I_1+C^{\square}I_3\right)-\frac{C^2m^2\pi^2}{L^2}\left(2I_7-I_9\right)\right]\right\} \tag{4.8}$$

Equation (4.8) is called the modified frequency corresponding to the frequency of the free system due to the presence of the moving mass. Thus, the entire equation (4.5) takes the form

$$\frac{d^2}{dt^2}Y_m(t)+\beta_{mf}^2Y_m(t)=\frac{P_m\Gamma_j}{H_a}\left[Cos\frac{\lambda_m ct}{L}-C\text{os}\,\lambda_m\right] \tag{4.9}$$

which is a prototype of equation (4.1)and when inverted we have

$$\overline{U}_n(x,t)=\sum_{m=1}^{n}\frac{P_m\Gamma_j}{H_a}\left[\frac{CosZ_kt-Cos\beta_{jj}t}{\beta_{jj}^2-Z_k^2}-E(m)\frac{\left(1-Cos\beta_{jj}t\right)}{\beta_{jj}}\right]\times Sin\frac{n\pi x}{L} \tag{4.10}$$

Equation (4.10) is the transverse displacement response to moving mass solution for simply supported beam on elastic foundation.

V. Discussion of Results

Resonance condition

It is desirable to inspect closely the response amplitude of the dynamical system. Following [12], the moving force in equation (4.8) attains a resonance whenever

$$\beta_{mf}=\frac{m\pi c}{L} \tag{5.1}$$

while when

$$\beta_{jj}=\frac{m\pi c}{L} \tag{5.2}$$

Ref.

8. Chau, F.W and Seng, O.L (198): Beams on non-linear elastic foundation. Journal of Applied Mechanics. Vol. 2, Pp 587-600.
12. Oni S.T and Ogunyebi S.N (2008): Dynamical analysis of a prestressed elastic beam with general boundry conditions under the action of uniform distributed masses.

gives for the moving mass problem. Re-written equation (4.8) in the form

$$\beta_{jj} = \beta_{mf}\left\{\frac{1}{\beta_{mf}} - \frac{\Gamma_j}{H_a}\left[\left(\frac{1}{L}I_1 + C\square I_3\right) - \frac{C^2 m^2 \pi^2}{\beta_{mf} L^2}\left(2I_7 - I_9\right)\right]\right\} \tag{5.3}$$

which implies

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$$\beta_{mf} = \frac{L/m\pi c}{\beta_{mf}\left\{\frac{1}{\beta_{mf}} - \frac{\Gamma_j}{H_a}\left[\left(\frac{1}{L}I_1 + C\square I_3\right) - \frac{C^2 m^2 \pi^2}{\beta_{mf} L^2}\left(2I_7 - I_9\right)\right]\right\}} \tag{5.4}$$

VI. Conclusion

In view of the condition for resonance established above, it is deduced that for the same natural frequency, the critical speed for the moving force simply supported beam is greater than that of the moving mass problem. Thus for the same natural frequency, resonance is reached earlier in the moving mass system than in the moving force system.

For practical purposes, a one dimensional structures (Beam) are used as mathematical models in the buildings and bridges construction. Hence appropriate precaution may now be taken by the structural engineers to forestall the occurrence of resonance in the structure by integrating the necessary vibration absorber into the model.

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New Results on Q-Product Identities Based on Ramanujan's Findings

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Abstract - In this paper author has established four q-product identities by using elementary method. These identities are new and not available in the literature of special functions.

Keywords : Generating functions, triple product identities.

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New Results on q-Product Identities Based on Ramanujan's Findings

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Keywords : Generating functions, triple product identities.

I. Introduction

For $|q| < 1$,

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1 - aq^n) \tag{1.1}$$

$$(a;q)_\infty = \prod_{n=1}^{\infty}(1 - aq^{(n-1)}) \tag{1.2}$$

$$(a_1, a_2, a_3, ..., a_k; q)_\infty = (a_1;q)_\infty (a_2;q)_\infty (a_3;q)_\infty ... (a_k;q)_\infty \tag{1.3}$$

Ramanujan [2, p.1(1.2)]has defined general theta function, as

$$f(a,b) = \sum_{-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} \quad ; \quad |ab| < 1, \tag{1.4}$$

Jacobi's triple product identity [3,p.35] is given, as

$$f(a,b) = (-a;ab)_\infty (-b;ab)_\infty (ab;ab)_\infty \tag{1.5}$$

Special cases of Jacobi's triple products identity are given, as

$$\phi(q) = f(q,q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)_\infty^2 (q^2;q^2)_\infty \tag{1.6}$$

$$(q) = f(q,q^3) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.7}$$

$$f(-q) = f(-q,-q^2) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.8}$$

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Ref.

2. B.C. Berndt; *What is a q-series?*, preprint.
3. B.C. Berndt; *Ramanujan's notebook Part III*, Springer-Verlag, New York, 1991.

Equation (1.8) is known as Euler's pentagonal number theorem. Euler's another well known identity is as

$$(q;q^2)_\infty^{-1} = (-q;q)_\infty \tag{1.9}$$

Throughout this paper we use the following representations

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty = (q^a,q^b,q^c\cdots q^t;q^n)_\infty \tag{1.10}$$

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty = (q^a,q^b,q^c\cdots q^t;q^n)_\infty \tag{1.11}$$

$$(-q^a;q^n)_\infty(-q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty = (-q^a,-q^b,q^c\cdots q^t;q^n)_\infty \tag{1.12}$$

Notes

Now we can have following q-products identities, as

$$(q^2;q^2)_\infty = \prod_{n=0}^{\infty}(1-q^{2n+2})$$

$$= \prod_{n=0}^{\infty}(1-q^{2(4n)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+1)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+2)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+3)+2})$$

$$= \prod_{n=0}^{\infty}(1-q^{8n+2}) \times \prod_{n=0}^{\infty}(1-q^{8n+4}) \times \prod_{n=0}^{\infty}(1-q^{8n+6}) \times \prod_{n=0}^{\infty}(1-q^{8n+8})$$

or,

$$(q^2;q^2)_\infty = (q^2;q^8)_\infty(q^4;q^8)_\infty(q^6;q^8)_\infty(q^8;q^8)_\infty = (q^2,q^4,q^6,q^8;q^8)_\infty \tag{1.13}$$

$$(q^4;q^4)_\infty = \prod_{n=0}^{\infty}(1-q^{4n+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{4(3n)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+1)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+2)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{12n+4}) \times \prod_{n=0}^{\infty}(1-q^{12n+8}) \times \prod_{n=0}^{\infty}(1-q^{12n+12})$$

or,

$$(q^4;q^4)_\infty = (q^4;q^{12})_\infty(q^8;q^{12})_\infty(q^{12};q^{12})_\infty = (q^4,q^8,q^{12};q^{12})_\infty \tag{1.14}$$

$$(q^4;q^{12})_\infty = \prod_{n=0}^{\infty}(1-q^{12n+4}) = \prod_{n=0}^{\infty}(1-q^{12(5n)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+1)+4}) \times$$

$$\times \prod_{n=0}^{\infty}(1-q^{12(5n+2)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+3)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+4)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{60n+4}) \times \prod_{n=0}^{\infty}(1-q^{60n+16}) \times \prod_{n=0}^{\infty}(1-q^{60n+28}) \times \prod_{n=0}^{\infty}(1-q^{60n+40}) \times \prod_{n=0}^{\infty}(1-q^{60n+52})$$

or,

$$(q^4;q^{12})_\infty = (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$

$$= (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \tag{1.15}$$

Similarly we can compute following as

$$(q^5;q^5)_\infty = (q^5;q^{15})_\infty (q^{10};q^{15})_\infty (q^{15};q^{15})_\infty \tag{1.16}$$

$$(q^6;q^6)_\infty = (q^6;q^{24})_\infty (q^{12};q^{24})_\infty (q^{18};q^{24})_\infty (q^{24};q^{24})_\infty = (q^6,q^{12},q^{18},q^{24};q^{24})_\infty \tag{1.17}$$

$$\begin{aligned}(q^6;q^{12})_\infty &= (q^6;q^{60})_\infty (q^{18};q^{60})_\infty (q^{30};q^{60})_\infty (q^{42};q^{60})_\infty (q^{54};q^{60})_\infty \\ &= (q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty \end{aligned}\tag{1.18}$$

The outline of this paper is as follows. In sections 2, some recent results obtained by the author [1], and also some well known results are recorded in [6;7], those are useful to the rest of the paper. In section 3, we state and prove four q-product identities, which are new and not recorded in the literature of special functions.

II. Preliminaries

In [1], following identities are being established

$$(q;q^2)_\infty = (q,q^3,q^5;q^6)_\infty \tag{2.1}$$

$$\left[\frac{(-q;q^2)^8_\infty - (q;q^2)^8_\infty}{q}\right]^{\frac{1}{4}} = \frac{2}{\left[(q^2;q^4)_\infty\right]^2} \tag{2.2}$$

$$\frac{(q^2;q^2)_\infty}{(q^4;q^4)_\infty} = (q,-q;q^2)_\infty \tag{2.3}$$

$$(q^2;q^2)_\infty = (q^2;q^4)_\infty (q^4;q^4)_\infty \tag{2.4}$$

In Ramanujan's notebook [7, p.107], Chapter IX, Entry $7(iii)$ is recorded as

$$\phi(q) + \phi(-q) = \frac{1}{4}\phi(q^2) \tag{2.5}$$

In Ramanujan's notebook [7, p.198], Chapter XVI, following entries are recorded as

Entry $24(i)$:

$$\frac{f(q)}{f(-q)} = \frac{\psi(q)}{\psi(-q)} = \frac{\chi(q)}{\chi(-q)} = \sqrt{\frac{\phi(q)}{\phi(-q)}} \tag{2.6}$$

where $\chi(q)$ is given in [7, p.197], Chapter XVI, Entry $22(iv)$, as

$$\chi(q) = \prod(q,q^2) = (1+q)(1+q^3)(1+q^5)(1+q^7) \; and\ constant \tag{2.7}$$

Entry $24(ii)$:

$$f^3(-q) = \phi^2(-q)\psi(q) = 1 - 3q + 5q^3 - 7q^6 + 9q^{10} - \; and\ constant \tag{2.8}$$

Entry $24(iii)$:

$$\chi(q) = \frac{f(q)}{f(-q^2)} = \sqrt[3]{\frac{\phi(q)}{\psi(-q)}} = \frac{\phi(q)}{f(q)} = \frac{f(-q^2)}{\psi(-q)} \tag{2.9}$$

where $\chi(q)$ is given by (2.7)

Ref.

1. M.P. Chaudhary; *Development on q-product identities*, preprint.
7. S. Ramanujan; *Notebooks (Volume II)*, Tata Institute of Fundamental Research, Bombay, 1957.

Entry $24(iv)$:

$$f^3(-q^2) = \phi(-q)\psi^2(x) \tag{2.10}$$

and

$$\chi(q)\chi(-q) = \chi(-q^2) \tag{2.11}$$

where $\chi(q)$ is given by (2.7)

III. Main Results

In this section, we established following new results with the help of $\psi(.)$ and $\phi(.)$ functions or in more general language we can say that by using the properties of Jacobi's triple product identity as $\psi(.)$ and $\phi(.)$ functions are its special cases. These results are not recorded in the literature of special functions

$$(-q^2;q^4)_\infty = 2(-q,-q;q^2)_\infty^{\frac{1}{2}}[(-q;q^2)_\infty^2 + (q;q^2)_\infty^2]^{\frac{1}{2}} \tag{3.1}$$

$$(-q;q^2)_\infty(q;q)_\infty = (q;q^2)_\infty(-q;-q)_\infty \tag{3.2}$$

$$(q;q)_\infty = (q;q^2)_\infty(q^2;q^2)_\infty = (q,q^2;q^2)_\infty \tag{3.3}$$

$$(-q;-q)_\infty = (-q;q^2)_\infty(q^2;q^2)_\infty \tag{3.4}$$

Proof of (3.1): By substituting, $q=-q$ and $q=q^2$ respectively in (1.6), we have

$$\phi(-q) = (q;q^2)_\infty^2(q^2;q^2)_\infty \;;\; \phi(q^2) = (-q^2;q^4)_\infty^2(q^4;q^4)_\infty$$

by substituting the values $\phi(-q)$, $\phi(q^2)$, and employing (1.6) in (2.5), we get

$$(q^2;q^2)_\infty[(-q;q^2)_\infty^2 + (q;q^2)_\infty^2] = \frac{1}{4}(-q^2;q^4)_\infty^2(q^4;q^4)_\infty$$

further using (2.3), and after simplification, we get

$$(-q^2;q^4)_\infty = 2(-q,-q;q^2)_\infty^{\frac{1}{2}}[(-q;q^2)_\infty^2 + (q;q^2)_\infty^2]^{\frac{1}{2}}$$

which established (3.1)

Proof of (3.2): By substituting, $q=-q$ in (1.7) and (1.8) respectively, we have

$$\psi(-q) = \frac{(q^2;q^2)_\infty}{(-q;q^2)_\infty} \;;\; f(q) = (-q;-q)_\infty$$

by substituting the values of $f(q)$ and $\psi(-q)$, and employing (1.7) and (1.8), in first and second part of (2.6), after little simplification, we get

$$\frac{(-q;-q)_\infty}{(q;q)_\infty} = \frac{(-q;q^2)_\infty}{(q;q^2)_\infty}$$

which can also be written as

$$(-q;q^2)_\infty(q;q)_\infty = (q;q^2)_\infty(-q;-q)_\infty$$

which established (3.2)

Notes

Notes

Note: We verified that the result (3.2), can also be proved by taking any other two parts of (2.6).

Proof of (3.3): By (1.6) and (1.8) respectively, we have

$$\phi^2(-q) = (q;q^2)^4_\infty (q^2;q^2)^2_\infty \ ; \ f^3(-q) = (q;q)^3_\infty$$

by substituting the values of $\phi^2(-q)$ and $f^3(-q)$, and employing (1.7), in first and second part of (2.8), after little simplification, we get

$$(q;q)_\infty = (q;q^2)_\infty (q^2;q^2)_\infty = (q,q^2;q^2)_\infty$$

which established (3.3)

Note: If we put $q = q^2$ in (3.3), then we find (2.4) a result already proved by the author in [1].

Proof of (3.4): By (1.7) and (1.8) respectively, we have

$$(-q) = \frac{(q^2;q^2)_\infty}{(-q;q^2)_\infty} \ ; f^3(q) = (-q;-q)^3_\infty \ ; \ f^3(-q^2) = (q^2;q^2)^3_\infty$$

by substituting the values of $(-q)$, $f^3(q)$, $f^3(-q^2)$ and employing (1.6), in second and third part of (2.9), after little simplification, we get

$$(-q;-q)_\infty = (-q;q^2)_\infty (q^2;q^2)_\infty$$

which established (3.4)

Note: We verified that the result (3.4), can also be proved by taking any other two parts of (2.9).

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New Theorems Involving the Generalized Mellin-Barnes Type of Contour Integrals and General Class of Polynomials

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Abstract - In the present investigation, First we establish three new theorems, which involves generalized Mellin-Barnes type of contour integrals and general class of polynomials. Next, we obtain certain new integrals and expansion formulas by the application of our theorems. By giving suitable values to the parameters, main integral reduces to Fox's H-function and generalized wright hypergeometric function, etc. Our Main findings provide interesting unification and extensions of a number of new results.

Keywords : $\overline{H}$-function, general class of polynomials, generalized wright hypergeometric function.

GJSFR-F Classication : MSC 2000: 33C45, 33C60

NEW THEOREMS INVOLVING THE GENERALIZED MELLIN-BARNES TYPE OF CONTOUR INTEGRALS AND GENERAL CLASS OF POLYNOMIALS

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Ref.

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New Theorems Involving the Generalized Mellin-Barnes Type of Contour Integrals and General Class of Polynomials

Praveen Agarwal [α] & Mehar Chand [σ]

Abstract- In the present investigation, First we establish three new theorems, which involves generalized Mellin-Barnes type of contour integrals and general class of polynomials. Next, we obtain certain new integrals and expansion formulas by the application of our theorems. By giving suitable values to the parameters, main integral reduces to Fox's H-function and generalized wright hypergeometric function, etc. Our Main findings provide interesting unification and extensions of a number of new results.

Keywords : *$\bar{H}$ -function, general class of polynomials, generalized wright hypergeometric function.*

I. Introduction

In 1987, Inayat-Hussain [1, 2] introduced generalization form of Fox's H-function, which is popularly known as $\bar{H}$-function. Now $\bar{H}$-function stands on fairly firm footing through the research contributions of various authors [1, 2, 3, 9, 10, 14, 15, and 16]. $\bar{H}$-function is defined and represented in the following manner [10].

$$\bar{H}^{m,n}_{p,q}[z] = \bar{H}^{m,n}_{p,q}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L z^{\xi}\bar{\phi}(\xi)d\xi \quad (z\neq 0) \tag{1.1}$$

where

$$\bar{\phi}(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

It may be noted that the $\bar{\phi}(\xi)$ contains fractional powers of some of the gamma function and m,n,p,q are integers such that $1\le m\le q, 1\le n\le p$ $(\alpha_j)_{1,p},(\beta_j)_{1,q}$ are positive real numbers and $(A_j)_{1,n},(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(\alpha_j)_{1,p}$ and $(\beta_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\bar{H}$-function can be seen in the papers [9, 10]

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The behavior of the $\overline{\mathrm{H}}$-function for small values of $|z|$ follows easily from a result given by Rathie [3]:

$$\overline{\mathrm{H}}^{m,n}_{p,q}[z] = o\left(|z|^{\alpha}\right);\ \text{Where}$$

$$\alpha = \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{\alpha_j}\right), |z| \to 0 \tag{1.3}$$

$$\Omega = \sum_{j=1}^{m} |B_j| + \sum_{j=m+1}^{q} |b_j B_j| - \sum_{j=1}^{n} |a_j A_j| - \sum_{j=n+1}^{q} |A_j| > 0,\ 0 < |z| < \infty \tag{1.4}$$

The following function which follows as special cases of the $\overline{\mathrm{H}}$-function will be required in the sequel [10]

$${}_p\overline{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};z\right] = \overline{H}^{1,p}_{p,q+1}\left[-z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right] \tag{1.5}$$

The general class of polynomials $S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}[x]$ will be defined and represented as follows [6, p.185, eqn. (7)]:

$$S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}[x] = \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} x^{l_i} \tag{1.6}$$

where $n_1,\ldots,n_r = 0,1,2,\ldots;m_1,\ldots m_r$ are arbitrary positive integers, the coefficients $A_{n_i,l_i}(n_i,l_i \ge 0)$ are arbitrary constants, real or complex. $S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}[x]$ yields a number of known polynomials as its special cases. These includes, among other, the Jacobi polynomials, the Bessel Polynomials, the Lagurre Polynomials, the Brafman Polynomials and several others [8, p. 158-161].

The following formulas [12, p.77, Ens. (3.1), (3.2) & (3.3)] will be required in our investigation.

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad (a>0;b\ge 0;c+4ab>0;\mathrm{Re}(p)+1/2>0) \tag{1.7}$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad (a\ge 0;b>0;c+4ab>0;\mathrm{Re}(p)+1/2>0) \tag{1.8}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad (a>0;b>0;c+4ab>0;\mathrm{Re}(p)+1/2>0) \tag{1.9}$$

II. Main Theorems

In our investigation following result [11, p. 75] is also required.

$$\text{If}\ (1-y)^{\alpha+\beta-\gamma}\ {}_2F_1(2\alpha,2\beta;2\gamma;y) = \sum_{r=0}^{\infty} a_r y^r \tag{2.1}$$

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then

$$ {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)=\sum_{r=0}^{\infty}\frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r}a_r X^r \tag{2.2}$$

Let X stands for $\left(ax+\frac{b}{x}\right)^2+c$

Notes

First Theorem:

$$\int_0^\infty X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\middle|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right)\end{matrix}\right] \tag{2.3}$$

The above result will be converge under the following conditions

i. $a>0;b\geq 0;c+4ab>0$ and $\mu_i>0,\delta\geq 0$.

ii. $\operatorname{Re}\left[\lambda+\delta\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z|<\frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

iv. $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Second Theorem:

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\middle|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right)\end{matrix}\right] \tag{2.4}$$

The above result will be converge under the following conditions

i. $a\geq 0;b>0;c+4ab>0$ and $\mu_i>0,\delta\geq 0$.

ii. $\operatorname{Re}\left[\lambda+\delta\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z| < \frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

iv. $-\frac{1}{2} < (\alpha-\beta-\gamma) < \frac{1}{2}$

Third Theorem:

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] \overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{2.5}$$

The above result will be converge under the following conditions

i. $a>0;b>0;c+4ab>0$ and $\mu_i>0,\delta\geq 0$.

ii. $\text{Re}\left[\lambda+\delta\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z| < \frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

iv. $-\frac{1}{2} < (\alpha-\beta-\gamma) < \frac{1}{2}$

Proof :

To prove the first theorem, using the result given by equation (2.2) and express $\overline{H}$ -function occurring on the L.H.S. of equation (2.3) in terms of Mellin-Barnes type of contour integral given by equation (1.1) and the general class of polynomials $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ in series form with the help of equation (1.6) and then interchanging the order of integration and summation we get:

$$\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\frac{1}{2\pi i}\int_L\overline{\phi}(\xi)z^{\xi}\left[\int_0^\infty\left[\left(ax+\frac{b}{x}\right)+c\right]^{-\lambda+r-\sum_{i=1}^{k}\mu_i l_i-\delta\xi-1}dx\right]d\xi \tag{2.6}$$

Further using the result (1.7) the above integral becomes

$$\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\frac{1}{2\pi i}\int_L\overline{\phi}(\xi)z^{\xi}\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1/2}}\frac{\Gamma\left(\lambda-r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1/2\right)}{\Gamma\left(\lambda-r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1\right)}d\xi \tag{2.7}$$

Notes

Then interpreting with the help of (1.1) and (2.7) provides first integral.

Proceeding on the same parallel lines, theorems second and third given by (2.4) and (2.5) can be obtained by using the results (1.8) and (1.9) respectively.

Special Cases :

(3.1) If we put $A_j = B_j = 1$, $\bar{H}$-function reduces to Fox's H-function [7, p. 10, Eqn. (2.1.1)], then the equation (2.3), (2.4) and (2.5) takes the following form.

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\,\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\,\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.1.1}$$

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\,\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\,\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.1.2}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\,\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\,\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.1.3}$$

The Conditions of validity of (3.1.1), (3.1.2) and (3.1.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.2) By applying the our results given in (2.3), (2.4) and (2.5) to the case of Hermite polynomials [4, 5] by setting $S_n^2(x)\to x^{n/2}H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $m_1,\dots,m_k = 2; n_1,\dots,n_k = n; k=1; v_i = v, y_i = y, A_{n_i,l_i} = (-1)^l$, we have the following interesting results.

Ref.

7. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publishers, New Delhi, Madras (1982).

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2}H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.2.1}$$

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2}H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.2.2}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2}H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \tag{3.2.3}$$

The Conditions of validity of (3.2.1), (3.2.2) and (3.2.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.3) By applying the our results given in (2.3), (2.4) and (2.5) to the case of Lagurre polynomials [4, 5] by setting $S_n^2(x)\to L_n^{(\alpha')}[x]$ in which $m_1,\ldots,m_k=1; n_1,\ldots,n_k=n; k=1, v_i=v, y_i=y, A_{n_i,l_i}=\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}$, we have the following interesting results.

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\, a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

Ref.

4. C. Szego, Orthogonal polynomials, Amer. Math. Soc. Colloq. Publ. 23 Fourth edition, Amer. Math. Soc. Providence, Rhode Island (1975).
5. E.M. Wright, The asymptotic expansion of the generalized Bessel Function. Proc. London Math. Soc. (Ser.2), 38(1935), 257-260.

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \quad (3.3.1)$$

$$\int_0^{\infty}\frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \quad (3.3.2)$$

$$\int_0^{\infty}\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]\overline{H}^{m,n}_{p,q}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$\overline{H}^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-\lambda+r-\mu l,\delta;1)\end{matrix}\right.\right] \quad (3.3.3)$$

The Conditions of validity of (3.3.1), (3.3.2) and (3.3.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.4) If we put $n=p, m=1, q=q+1, b_1=0, \beta_1=1, a_j=1-a_j, b_j=1-b_j$, then the $\overline{\mathrm{H}}$-function reduces to generalized wright hypergeometric function [17] i.e. $\overline{H}^{1,p}_{p,q+1}\left[z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right]={}_p\overline{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\right]$, the equations (2.3), (2.4) and (2.5) takes the following form.

$$\int_0^{\infty}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\ldots,m_k}_{n_1,\ldots,n_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\overline{\psi}_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$${}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \quad (3.4.1)$$

$$\int_0^{\infty}\frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S^{m_1,\ldots,m_k}_{n_1,\ldots,n_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\overline{\psi}_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

Ref.

17. Wright, E.M., The asymptotic expansion of the generalized hypergeometric function. J. London Math. Soc. 10 (1935a), 286-293.

$$ {}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i;\delta,1\right),\left(a_j,\alpha_j;A_j\right)_{1,p} \\ \left(b_j,\beta_j;B_j\right)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \tag{3.4.2}$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\overline{\psi}_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$ {}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),\left(a_j,\alpha_j;A_j\right)_{1,p} \\ \left(b_j,\beta_j;B_j\right)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \tag{3.4.3}$$

The Conditions of validity of (3.4.1), (3.4.2) and (3.4.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.5) If we put $\alpha=\gamma$, in the main theorem, the value of a_r in (2.1) comes out to be equal to $\frac{\beta_r}{r!}$ and the result (2.3), (2.4) and (2.5) gives the following interesting integral.

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.5.1}$$

$$\int_0^\infty\frac{1}{X^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]\overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right),\left(a_j,\alpha_j;A_j\right)_{1,n},\left(a_j,\alpha_j\right)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},\left(b_j,\beta_j;B_j\right)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.5.2}$$

Notes

$$\int_0^\infty \left(a+\frac{b}{X^2}\right) X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] \overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.5.3)$$

Notes

The conditions of validity of (3.5.1), (3.5.2) and (3.5.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

(3.6) If we put $\beta=\alpha+\frac{1}{2}$ and $\alpha=-f$ (f is non-negative integer) in (3.5.1), (3.5.2) and (3.5.3), we have:

$$\int_0^\infty X^{-\lambda-1}(1-X)^f S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] \overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{f}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.6.1)$$

$$\int_0^\infty \frac{1}{X^2} X^{-\lambda-1}(1-X)^f S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] \overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.6.2)$$

$$\int_0^\infty \left(a+\frac{b}{X^2}\right) X^{-\lambda-1}(1-X)^f S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^k y_i X^{-\mu_i}\right] \overline{H}_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},\left(-\lambda+r-\sum_{i=1}^k \mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \quad (3.6.3)$$

The conditions of validity of (3.6.1), (3.6.2) and (3.6.3) easily follow from those given in (2.3), (2.4) and (2.5) respectively.

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Notes

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An Efficient Class of Dual to Product-Cum- Dual to Ratio Estimators of Finite Population Mean In Sample Surveys

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Abstract - This paper considers a class of dual to product-cum-dual to ratio estimators for estimating finite population mean of the study variate using auxiliary variate. The bias and mean square error of the proposed estimator have been obtained. The asymptotically optimum estimator (AOE) in the class has also been identified along with its approximate bias and mean square error. Theoretical and empirical studies have been done to demonstrate the superiority of the proposed estimators over the other estimators.

Keywords : *Finite population mean; Auxiliary variate; Dual to product-cum-dual to ratio estimator; Mean square error; Efficiency.*

GJSFR-F Classication : *MSC 2010: 62D05.*

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Strictly as per the compliance and regulations of :

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Sanjib Choudhury [α] & B. K. Singh [σ]

Abstract - This paper considers a class of dual to product-cum-dual to ratio estimators for estimating finite population mean of the study variate using auxiliary variate. The bias and mean square error of the proposed estimator have been obtained. The asymptotically optimum estimator (AOE) in the class has also been identified along with its approximate bias and mean square error. Theoretical and empirical studies have been done to demonstrate the superiority of the proposed estimators over the other estimators.

Keywords : Finite population mean; Auxiliary variate; Dual to product-cum-dual to ratio estimator; Mean square error; Efficiency.

I. Introduction

It is well established in sample surveys that auxiliary information is often used to improve the precision of estimators of population parameters. The use of auxiliary information at the estimation stage appears to have started with the work of Cochran (1940). He developed the ratio estimator to estimate the population mean or total of the study variate y by using supplementary information on an auxiliary variate x, positively correlated with y. The ratio estimator is most effective when the relationship between study variate y and auxiliary variate x is linear through the origin and the mean square error of y is proportional to x. When the auxiliary variate x is negatively correlated with the study variate y, Robson (1957) proposed the product estimator of the population mean or total. In fact, for the better utilization of a given auxiliary information on an auxiliary variate x, Murthy (1964) has suggested the use of

- ratio estimator $\bar{y}_R$ if, $\rho C_y / C_x > 1/2$,
- product estimator $\bar{y}_P$ if, $\rho C_y / C_x < -1/2$,
- unbiased estimator $\bar{y}$ if, $-1/2 \le \rho C_y / C_x \le 1/2$,

where C_y, C_x and ρ are coefficient of variation of y, coefficient of variation of x and correlation coefficient between y and x respectively.

Consider a finite population $U = (u_1, u_2, \ldots, u_N)$ of size N units. Let y and x denote the study and auxiliary variates respectively. A sample of size n $(n < N)$ is drawn using simple random sampling without replacement (SRSWOR) scheme to estimate the

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population mean $\bar{Y}=(1/N)\sum_{i=1}^{N}y_i$ of the study variate y. Let the sample mean $(\bar{x},\bar{y})$ be the unbiased estimator of $\bar{X},\bar{Y}$ based on n observations.

The usual ratio and product estimators for $\bar{Y}$ are $\bar{y}_R=\bar{y}\left(\bar{X}/\bar{x}\right)$ and $\bar{y}_P=\bar{y}\left(\bar{x}/\bar{X}\right)$ respectively,

where $\bar{y}=(1/n)\sum_{i=1}^{n}y_i$ and $\bar{x}=(1/n)\sum_{i=1}^{n}x_i$.

Consider a transformation $x_i^*=(1+g)\bar{X}-gx_i$, $i=1,2,\ldots,N$, where $g=n/(N-n)$. Then $x_i^*=(1+g)\bar{X}-gx_i$ is an unbiased estimator for $\bar{X}$ and the correlation of $\left(\bar{y},\bar{x}^*\right)$ is negative.

Using the transformation of x_i^*, Srivenkataramana (1980) obtained dual to ratio estimator as $\bar{y}_R^*=\bar{y}\left(\bar{x}^*/\bar{X}\right)$ and Bandyopadhyay (1980) obtained dual to product estimator as $\bar{y}_P^*=\bar{y}\left(\bar{X}/\bar{x}^*\right)$.

In this paper, we have proposed a class of dual to product-cum-dual to ratio estimator for estimating population mean $\bar{Y}$. Numerical illustrations are given in support of the present study.

II. The Proposed Class of Estimator

For estimating population mean $\bar{Y}$, we have proposed a class of dual to product-cum-dual to ratio estimator as

$$\bar{y}_{PR}^*=\bar{y}\left[\alpha\left(\frac{\bar{X}}{\bar{x}^*}\right)+(1-\alpha)\left(\frac{\bar{x}^*}{\bar{X}}\right)\right] \tag{1}$$

where α is a suitably chosen scalar.

To obtain the bias and mean square error (MSE) of $\bar{y}_{PR}$ to the first degree of approximation, we write

$e_0=\left(\bar{y}-\bar{Y}\right)/\bar{Y}$ and $e_1=\left(\bar{x}-\bar{X}\right)/\bar{X}$,

such that

$$\left.\begin{aligned}&E(e_0)=E(e_1)=0,\quad E(e_0^2)=f_1C_y^2,\\&E(e_1^2)=f_1C_x^2,\quad E(e_0e_1)=f_1CC_x^2,\end{aligned}\right\} \tag{2}$$

where $f_1=(1/n-1/N)$, $C=\rho C_y/C_x$ and

$C_y^2=S_y^2/\bar{Y}^2$, $C_x^2=S_x^2/\bar{X}^2$, $\rho=S_{xy}/S_xS_y$, $S_x^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i-\bar{X}\right)^2$, $S_y^2=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)^2$ and

$S_{xy}=\frac{1}{N-1}\sum_{i=1}^{N}\left(y_i-\bar{Y}\right)\left(x_i-\bar{X}\right)$.

Notes

Expressing $\bar{y}_{PR}^{*}$ in terms of e's, we obtain

$$\bar{y}_{PR}^{*}=\bar{Y}\left(1+e_0\right)\left\{\alpha\left(1-ge_1\right)^{-1}+\left(1-\alpha\right)\left(1-ge_1\right)\right\}.$$

We now assume that $|ge_1|<1$, so that we may expand $\left(1-ge_1\right)^{-1}$ as a series in powers of ge_1. Expanding, multiplying out and retaining terms of e's to the second degree, we obtain

Notes

$$\bar{y}_{PR}^{*}-\bar{Y}\cong\bar{Y}\left\{e_0+g\left(2\alpha-1\right)\left(e_1+e_0e_1\right)+\alpha g^2e_1^2\right\} \tag{3}$$

Taking expectation on both the sides of equation (3) and using the results of equation (2) we get the bias of $\bar{y}_{PR}^{*}$ as

$$B\left(\bar{y}_{PR}^{*}\right)=f_1\bar{Y}C_x^2g\left\{\left(2\alpha-1\right)C+\alpha g\right\} \tag{4}$$

The Bias, $B\left(\bar{y}_{PR}^{*}\right)$ in (4) is 'zero' if $\alpha=C/\left(2C+g\right)$.

Thus, the estimator $\bar{y}_{PR}^{*}$ with $\alpha=C/\left(2C+g\right)$ is almost unbiased.

Squaring both the sides of equation (3), taking expectation of the second- degree terms of order n^{-1} and using the results of (2), we obtain the MSE of $\bar{y}_{PR}^{*}$ as

$$M\left(\bar{y}_{PR}^{*}\right)=f_1\bar{Y}^2\left[C_y^2+g\left(2\alpha-1\right)\left\{g\left(2\alpha-1\right)+2C\right\}C_x^2\right] \tag{5}$$

which is minimized when

$$\alpha=\left(1-C/g\right)/2=\alpha_{opt.}\ \text{(say)} \tag{6}$$

Substituting the value of α from equation (6) in equation (1) yields the 'asymptotically optimum estimator' (AOE) as

$$\bar{y}_{PR}^{*opt.}=\bar{y}\frac{1}{2}\left[\left(1-\frac{1}{g}C\right)\frac{\bar{X}}{\bar{x}^*}+\left(1+\frac{1}{g}C\right)\frac{\bar{x}^*}{\bar{X}}\right]$$

Thus, the resulting bias and MSE of $\bar{y}_{PR}^{opt.}$ respectively are given as

$$B\left(\bar{y}_{PR}^{*opt.}\right)=f_1\bar{Y}C_x^2\left(g^2-2C^2-gC\right)/2 \tag{7}$$

and

$$M\left(\bar{y}_{PR}^{*opt.}\right)=f_1\bar{Y}^2C_y^2\left(1-\rho^2\right). \tag{8}$$

Equation (8) shows that mean squared error of $\bar{y}_{PR}^{*opt.}$ is same as the MSE of the linear regression estimator $\bar{y}_{reg.}=\bar{y}+b_{yx}\left(\bar{X}-\bar{x}\right)$, where b_{yx} is the sample regression coefficient of y on x.

From equation (7), we note that the bias, $B\left(\bar{y}_{PR}^{*opt.}\right)$ is 'zero' if either $g=2C$, or $g=-C$.

Remark 2.1

For $\alpha=0$, the estimator $\bar{y}_{PR}^{*}$ in (1) boils down to the dual to ratio estimator $\bar{y}_R^*$, proposed by Srivenkataramana (1980). The bias and MSE of $\bar{y}_R^*$ can be obtained by putting $\alpha=0$ in (4) and (5) respectively as

$$B\left(\bar{y}_R^*\right)=-\bar{Y}f_1gCC_x^2$$

and

$$M\left(\bar{y}_R^*\right)=f_1\bar{Y}^2\left\{C_y^2+gC_x^2\left(g-2C\right)\right\}. \tag{9}$$

Remark 2.2

For $\alpha=1$, the estimator $\bar{y}_{PR}^*$ in (1) boils down to the dual to product estimator $\bar{y}_p^*$, proposed by Bandyopadhyay (1980). The bias and MSE of $\bar{y}_p^*$ can be obtained by putting $\alpha=1$ in (4) and (5) respectively as

$$B\left(\bar{y}_p^*\right)=f_1\bar{Y}C_x^2g\left(g+C\right)$$

and

$$M\left(\bar{y}_p^*\right)=f_1\bar{Y}^2\left\{C_y^2+gC_x^2\left(g+2C\right)\right\} \tag{10}$$

III. Efficiency Comparisons

a) Comparison of $\bar{y}_{PR}^*$

In this section, firstly, we compare MSE of traditional estimators $\bar{y}$, $\bar{y}_R$ and $\bar{y}_P$ with MSE of proposed estimator $\bar{y}_{PR}^*$.

The MSE of sample mean $\bar{y}$ under SRSWOR sampling scheme is given by

$$M\left(\bar{y}\right)=f_1\bar{Y}^2C_y^2. \tag{11}$$

From equations (5) and (11), it is found that the proposed estimator $\bar{y}_{PR}^*$ is better than $\bar{y}$ if $-\left(2\alpha-1\right)\left\{2C+g\left(2\alpha-1\right)\right\}>0$

This condition holds if

either $1/2>\alpha$ and $1/2-C/g<\alpha$, or $1/2<\alpha$ and $1/2-C/g>\alpha$.

Therefore, the range of α under which the proposed estimator $\bar{y}_{PR}^*$ is more efficient than $\bar{y}$ is $\left[\min\left\{1/2,\ \left(1/2-C/g\right)\right\},\ \max\left\{1/2,\ \left(1/2-C/g\right)\right\}\right]$.

To compare the usual ratio estimator $\bar{y}_R$ and product estimator $\bar{y}_P$, we write the MSEs of $\bar{y}_R$ and $\bar{y}_P$ up to the first degree of approximation respectively as

$$M\left(\bar{y}_R\right)=f_1\bar{Y}^2\left\{C_y^2+C_x^2\left(1-2C\right)\right\} \tag{12}$$

and

$$M\left(\bar{y}_P\right)=f_1\bar{Y}^2\left\{C_y^2+C_x^2\left(1+2C\right)\right\} \tag{13}$$

From equations (5) and (12), we note that the proposed estimator $\bar{y}_{PR}^*$ has smaller MSE than that of the usual ratio estimator $\bar{y}_R$ if $\left\{1+g\left(2\alpha-1\right)\right\}\left\{1-g\left(2\alpha-1\right)-2C\right\}>0$.

This condition holds if

either $\left(1-1/g\right)/2>\alpha$ and $\left(1+1/g\right)/2-C/g<\alpha$, or $\left(1-1/g\right)/2<\alpha$ and $\left(1+1/g\right)/2-C/g>\alpha$.

Therefore, the range of α under which the proposed estimator $\bar{y}_{PR}^*$ is better than $\bar{y}_R$ is $\left[\min\left\{\left(1-1/g\right)/2,\ \left(1+1/g\right)/2-C/g\right\},\max\left\{\left(1-1/g\right)/2,\ \left(1+1/g\right)/2-C/g\right\}\right]$.

Notes

Notes

Further, we note from equations (5) and (13) that the estimator $\bar{y}^*_{PR}$ will dominate over usual product estimator $\bar{y}_P$ if $\{1-g(2\alpha-1)\}\{1+g(2\alpha-1)+2C\}>0$.

This condition holds if either

$(1+1/g)/2>\alpha$ and $(1-1/g)/2-C/g<\alpha$, or $(1+1/g)/2<\alpha$ and $(1-1/g)/2-C/g>\alpha$.

Therefore, the range of α under which the proposed estimator $\bar{y}^*_{PR}$ is better than $\bar{y}_P$ is $\left[\min\{(1+1/g)/2,\ (1-1/g)/2-C/g\},\max\{(1+1/g)/2,\ (1-1/g)/2-C/g\}\right]$.

Secondly, we compare the MSE of the proposed estimator with the MSE of dual to ratio estimator.

From equations (5) and (9), it is found that the proposed estimator $\bar{y}^*_{PR}$ will dominate over Srivenkataramana (1980) estimator $\bar{y}^*_R$ if $\alpha\{g(1-\alpha)-C\}>0$.

This condition holds if

either $0>\alpha$ and $(1-C/g)<\alpha$, or $0<\alpha$ and $(1-C/g)>\alpha$.

Therefore, the range of α under which the proposed estimator $\bar{y}^*_{PR}$ is more efficient than dual to ratio estimator $\bar{y}^*_R$ is $\{\min(0,\ 1-C/g),\ \max(0,\ 1-C/g)\}$.

Lastly, we compare MSE of the proposed estimator $\bar{y}^*_{PR}$ with those of dual to product estimator $\bar{y}^*_p$ of Bandyopadhyay (1980).

We note from equations (5) and (10) that $M(\bar{y}^*_p)>M(\bar{y}^*_{PR})$ if $(1-\alpha)(C+g\alpha)>0$.

This condition holds if

either $1>\alpha$ and $-C/g<\alpha$, or $1<\alpha$ and $-C/g>\alpha$.

Therefore, the range of α under which the proposed estimator $\bar{y}^*_{PR}$ is more efficient than dual to product estimator $\bar{y}^*_p$ is $\{\min(-C/g,\ 1),\ \max(-C/g,\ 1)\}$.

Thus, it seems from the above results that the proposed estimator $\bar{y}^*_{PR}$ may be made better than other estimators by making a suitable choice of the values of α.

b) Comparison of 'AOE' of $\bar{y}^{*opt.}_{PR}$

From equations (8)-(13), it is found that the 'AOE'

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Table 1: Description of the populations

Population	Source	Study variate y	Auxiliary variate x	N	n	ρ	C_y	C_x	$\bar{Y}$
1	Kadilar and Cingi (2006) pp. 1054	Level of apple production	Number of apple trees	106	20	0.82	4.18	2.02	15.37
2	Steel and Torrie (1960)	Log of leaf burn in secs	Chlorine percentage	30	6	-0.500	0.7001	0.7493	0.6860
3	Maddala (1977)	Consump-tion per capita.	Deflated prices of veal	30	6	-0.682	0.2278	0.0986	7.6375
4	Murthy (1967)	Output	Fixed capital	80	20	0.941	0.3542	0.7507	51.8264
5	Murthy (1967)	Output	Number of workers	80	20	0.915	0.3542	0.9484	51.8264
6	Kadilar and Cingi (2006) pp. 78	---	---	106	20	0.86	5.22	2.1	2212.59

Table 2 : Effective ranges of α and its optimum values of $\bar{y}^*_{PR}$.

Population	Ranges of α under which the proposed estimator $\bar{y}^*_{PR}$ is better than					Optimum value
	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}^*_R$	$\bar{y}^*_P$	α_{opt}
1	(-6.80,0.50)	(-4.65, -1.65)	(-8.95, 2.65)	(-6.30, 0.0)	(-7.30, 1.00)	-3.1482
2	(0.50, 2.37)	(-1.50, 4.37)	(0.37, 2.50)	(0.00, 2.87)	(1.00, 1.87)	1.4336
3	(0.50, 6.81)	(-1.50, 8.81)	(2.50, 4.81)	(0.00, 7.31)	(1.00, 6.31)	3.6527
4	(-0.83,0.50)	(-1.00, 0.67)	(-2.33, 2.00)	(-0.33, 0.0)	(-1.33, 1.00)	-0.1662
5	(-0.53,0.50)	(-1.00, 0.97)	(-2.03, 2.00)	(-0.03, 0.0)	(-1.03, 1.00)	-0.0126
6	(-8.69,0.50)	(-6.54, -1.65)	(-10.8, 2.65)	(-8.19, 0.0)	(-9.19, 1.00)	-4.0961

Notes

Table 3 : Percentage relative efficiency of different estimators with respect to $\bar{y}$.

Population	$\bar{y}$	$\bar{y}_R$	$\bar{y}_P$	$\bar{y}_R^*$	$\bar{y}_P^*$	$\bar{y}_{PR}^*$ or $\bar{y}_{PR}^{*opt.}$
1	100.00	226.76	†	120.73	†	305.25
2	100.00	†	†	†	124.34	133.26
3	100.00	†	167.59	†	115.73	187.10
4	100.00	†	†	591.38	†	877.54
5	100.00	†	†	612.44	†	614.34
6	100.00	212.82	†	117.95	†	384.02

†Relative efficiency less than 100%.

Notes

$\bar{y}_{PR}^{*opt.}$ is more efficient than the other existing estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$. Since

$$M\left(\bar{y}\right)-M\left(\bar{y}_{PR}^{*opt.}\right)=\frac{1-f}{n}\bar{Y}^2\rho^2C_y^2>0 ,$$

$$M\left(\bar{y}_R\right)-M\left(\bar{y}_{PR}^{*opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(1-C\right)^2>0 ,$$

$$M\left(\bar{y}_P\right)-M\left(\bar{y}_{PR}^{*opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(1+C\right)^2>0 ,$$

$$M\left(\bar{y}_R^*\right)-M\left(\bar{y}_{PR}^{*opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(C-g\right)^2>0 ,$$

$$M\left(\bar{y}_p^*\right)-M\left(\bar{y}_{PR}^{*opt.}\right)=\frac{1-f}{n}\bar{Y}^2C_x^2\left(C+g\right)^2>0 .$$

Hence, we conclude that the proposed class of estimator '$\bar{y}_{PR}^*$' is more efficient than other estimators in case of its optimality

Now we state the following theorem

Theorem 1

To the first degree of approximation, the proposed strategy '$\bar{y}_{PR}^*$' under optimality condition (6) is always more efficient than $M\left(\bar{y}\right)$, $M\left(\bar{y}_R\right)$, $M\left(\bar{y}_P\right)$, $M\left(\bar{y}_R^*\right)$, $M\left(\bar{y}_P^*\right)$ and equally efficient to $M\left(\bar{y}_{reg.}\right)$.

IV. Numerical Illustrations

To examine the merits of the constructed estimator over its competitors numerically, we consider six sets of population data. The sources of the population, the nature of the variates y and x and the values of the various parameters are listed in **Table 1**.

To reflect the gain in the efficiency of the proposed estimator $\bar{y}_{PR}^*$ over the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$, the effective ranges of α along with its optimum values are presented in **Table 2** with respect to the population data sets.

To observe the relative performance of different estimators, we have computed the percentage relative efficiency of different estimators of $\bar{Y}$ with respect to usual unbiased estimator $\bar{y}$ and this is presented in **Table 3**.

V. Conclusion

We have proposed an estimator of the combination of dual to product and dual to ratio estimators as in equation (1) and obtained 'AOE' for the proposed estimator. Theoretically, we have demonstrated that proposed estimator is always more efficient than other estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$ under the effective ranges of α and its optimum values.

In addition, we support these theoretical results numerically using the data sets as shown in **Table 1**.

Table 2 provides the wide ranges of α along with its optimum values for which the proposed estimator $\bar{y}_{PR}^*$ or $\bar{y}_{PR}^{*opt.}$ is more efficient than all other estimators considered in this paper as far as the mean squared error criterion is considered. It is also observed from **Table 2** that there is a scope for choosing α to obtain better estimators than $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$.

Table 3 exhibits that there is a considerable gain in efficiency by using proposed estimator $\bar{y}_{PR}^*$ or $\bar{y}_{PR}^{*opt.}$ over the estimators $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$. This shows that even if the scalar α deviates from its optimum value $(\alpha_{opt.})$, the suggested estimator $\bar{y}_{PR}^*$ will yield better estimates than $\bar{y}$, $\bar{y}_R$, $\bar{y}_P$, $\bar{y}_R^*$ and $\bar{y}_P^*$. Thus, it is preferred to use the proposed class of estimators $\bar{y}_{PR}^*$ or $\bar{y}_{PR}^{*opt.}$ in practice.

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Heat Conductance, a Boundary Value Problem Involving Certain Product of Special Functions

By Poonia M.S.
C.R.M. Jat P.G. College, Hisar

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Ref.

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Heat Conductance, a Boundary Value Problem Involving Certain Product of Special Functions

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Keywords : The product of Fox's H-function, M-series, a general class of polynomials and the multivariable H-function.

I. Introduction

Boundary value problem with Fox's H-function, M-series & multivariable H-function were studied by many authors, Churchill, R.V.[1], Mohammed, T.[3], Shrivastava, H.M. [6], Sharma, M.[4] etc.

Further, an integral involving Fox's H-function & heat conduction and on simultaneous operational calculus involving a product of Fox's H-function and the multivariable were studied by Bajpai [7], Chourasia [9] respectively.

This paper deals the problem of determining a function θ(x,t), representing the temperature in a non-homogeneous bar with ends at x = ± in which the thermal conductivity is proportional to $(1 - x^2)$ and if the lateral surface of the bar is insulated, it satisfies the partial differential equation of heat conduction Churchill [1] ,

$$\frac{\partial\theta}{\partial t} = b\frac{\partial}{\partial x}\left[(1-x^2)\frac{\partial\theta}{\partial x}\right], \tag{1}$$

where b is a constant, provided thermal coefficient is constant. The boundary conditions of the problem are that both ends of a bar at

$$x = \pm 1 \tag{2}$$

are also insulated because the conductivity vanishes there and the initial conditions

$$\theta\ (x,0) = f(x);\ -1\ ¡\ x\ ¡\ 1, \tag{3}$$

II. Result Required

(i) *The finite integral*

$$\int_{-1}^{1} (1-x^2)^{\alpha-1} P_\nu^\mu(x)\ {}_PF_Q\left[\begin{matrix}A_P\\B_Q\end{matrix};\beta(1-x^2)^d\right]$$

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$$H_{p,q}^{m,n}\left[M(1-x^2)^k\left|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right.\right]{}_{P_1}M_{Q_1}^{\alpha'}[M_1(1-x^2)^{k_1}]$$

$$.S_{v'}^{u'}[M_2(1-x^2)]H\left[\prod_{i=1}^{r} z_i(1-x^2)^{\sigma_i}\right]dx$$

$$=\sum_{G=1}\sum_{s,s',s'',t=0}^{\infty}\frac{(A_1)_t\ldots(A_p)_t\,\beta^t(-1)^s M^{g_s}\phi(g_s)}{(B_1)_t\ldots(B_Q)_t\ t!\ s!\ F_G\ s'!}$$

$$.\frac{\pi 2^{\mu}\ (-v')_{u's'}\ A_{v's'}\ M_2^{s'}(a_1)_{s''}\ldots(a_{P_1})_{s''}\ M_1^{s''}}{\Gamma\left(1-\frac{\mu}{2}+\frac{\nu}{2}\right)\Gamma\left(\frac{1}{2}-\frac{\nu}{2}-\frac{\mu}{2}\right)(b_1)_{s''}\ldots(b_{Q_1})_{s''}\ \Gamma(\alpha's''+1)}$$

$$.H_{A+2,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\lambda+2\quad:(u',v');\ldots;(u^{(r)},v^{(r)})}\left[\begin{matrix}[---------],[1-\alpha-td-kg_s-k_1s''-s'\pm\frac{\mu}{2}:\sigma_1,\ldots,\sigma_r],\\ [(c):\psi',\ldots,\psi^{(r)}]:[-\alpha-td-kgs\ -k_1s''-s'-\frac{\nu}{2}:\sigma_1,\ldots,\sigma_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}):\ [(b');\phi'];\ldots;[(b^{(r)}):\phi^{(r)}];\\ [1-\alpha-td-kg_s-k_1s''-s'+\frac{\nu}{2}:\sigma_1,\ldots,\sigma_r],[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];\end{matrix}\ z_1,\ldots,z_r\right], \tag{4}$$

where $\mathrm{Re}\left(\alpha+k\frac{f_j'}{F_j'}+\sum_{i=1}^{r}\sigma_i\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>\frac{1}{2}|\mathrm{Re}(\mu)|,\ j'=1,\ldots,m,\ j=1,\ldots,u^{(i)},\sigma_i>0,k>0,$

$k_1>0,|\arg(z_i)|<\frac{1}{2}T_i\pi,|\arg M|<\frac{1}{2}T'\pi,T'>0,u'$ is an arbitrary positive integer, the coefficients $A_{v',s'}(v',s'>0)$ are arbitrary constants, real or complex.

(ii) *Orthogonality property of the associated Legendre polynomials*

$$\int_{-1}^{1} P_n^m(t)\,P_k^m(t)\,dt=\frac{2(m+1)!}{(2n+1)(n-m)!}\delta_{nk} \tag{5}$$

where δ_{nk} is the Kroneckar delta defined by

$$\delta_{nk}\begin{cases}0,\text{if } n\neq k\\ 1\ \text{if } n=k\end{cases} \tag{6}$$

Solution of (1):-

Assuming the following

$$f(x)=(1-x^2)^{\alpha-1}\ {}_PF_Q[A_P;B_Q;\beta(1-x^2)^d]$$

$$H_{p,q}^{m,n}\left[M(1-x^2)^k\left|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right.\right]{}_{P_1}M_{Q_1}^{\alpha'}[M_2(1-x^2)^{k_1}]$$

$$S_{v'}^{u'}(M_1(1-x^2))H\left(\prod_{i=1}^{r} z_i(1-x^2)^{\sigma_i}\right), \tag{7}$$

Notes

Notes

The solution of the problem (4) can be written as

$$\theta(x,t)=\sum_{N=0}^{\infty} A_N P_N^{\mu}(x)\, e^{-bN(N+1)t'}, \tag{8}$$

If $t'=0$ in (8), then by virtue of (7)

$$f(x)=(1-x^2)^{\alpha-1}\,{}_PF_Q[A_P;B_Q;\beta(1-x^2)^d]$$

$$.\,H_{p,q}^{m,n}\left[M(1-x^2)^k \left|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right.\right]{}_{P_1}M_{Q_1}^{\alpha'}[M_2(1-x^2)^{k''}]$$

$$.\,S_{v'}^{u'}(M_1(1-x^2))H\left(\prod_{i=1}^{r} z_i(1-x^2)^{\sigma_i}\right)$$

$$=\sum_{N=0}^{\infty} A_N^{\mu} P_N^{\mu}(x), \tag{9}$$

Equation (7) is valid since f(x) is continuous in the closed interval $-1\le x\le 1$ and has a piecewise continuous derivative there, the Legendre series (9) associated with f(x) converges uniformly to f(x) in $-1+\in\le x\le 1-\in$, $0\le\in\le 1$.

Now multiplying both sides of (9) by $P_v^{\mu}(x)$ and integrating from − 1 to +1 with respect to x, we find

$$\int_{-1}^{1}(1-x^2)^{\alpha-1}\,{}_PF_Q[A_P;B_Q;\beta(1-x^2)^d]$$

$$.\,H_{p,q}^{m,n}\left[M(1-x^2)^k \left|\begin{matrix}(e_p,E_p)\\(f_q,F_q)\end{matrix}\right.\right]{}_{P_1}M_{Q_1}^{\alpha'}[M_2(1-x^2)^{k''}]$$

$$.\,S_{v'}^{u'}(M_1(1-x^2))H\left[\prod_{i=1}^{r} z_i(1-x^2)^{\sigma_i}\right]P_v^{\mu}(x)\,dx$$

$$=\sum_{N=0}^{\infty} A_N \int_{-1}^{1} P_N^{\mu}(x)\, P_{\nu}^{\mu}(x)\,dx, \tag{10}$$

Now using (4) and the orthogonal property of Legendre polynomials, (5) and (6), we get

$$A_N=\frac{2^{\mu-1}\pi(2\nu+1)(\nu-\mu)!}{(\nu+\mu)!\,\Gamma\left(1-\frac{\mu}{2}\pm\frac{\nu}{2}\right)}$$

$$.\sum_{G=1}^{m}\sum_{s,s',s'',t=0}^{\infty}\frac{(A_1)_t\ldots(A_p)_t\,\beta^t(-1)^s M^{g_s}\phi(g_s)}{(B_1)_t\ldots(B_Q)_t\; t!\; s!\; F_G\; s!}$$

$$.\frac{(a_1)_{s''}\ldots(a_{P_1})_{s''}\,M_2^{s''}(-v')_{u's'}\,A_{v',s'}\,M_1^{s'}}{(b_1)_{s''}\ldots(b_{Q_1})_{s''}\,\Gamma(\alpha's''+1)}$$

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$$.H^{0,\lambda+2\;:(u',v');...;(u^{(r)},v^{(r)})}_{A+2,C+2:(B',D');...;(B^{(r)},D^{(r)})}\left[\begin{matrix}[---------],[1-\alpha-td-kg_s\pm\frac{\mu}{2}-k''s''-s':\sigma_1,...,\sigma_r],\\ [(c):\psi',...,\psi^{(r)}]:[-\alpha-td-kg_s-\frac{\nu}{2}-k''s''-s':\sigma_1,...,\sigma_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',...,\theta^{(r)}):[(b');\phi'];...;[(b^{(r)}):\phi^{(r)}];\\ [1-\alpha-td-kg_s+\frac{\nu}{2}-k''s''-s':\sigma_1,...,\sigma_r],[(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1,...,z_r\right], \tag{11}$$

Notes

With the help of (8) and (9) the solution of the problem (1) is obtained in the form

$$\theta(x,t)=\pi 2^{\mu-1}\frac{(2\nu+1)(\nu-\mu)!}{(\nu+\mu)!\Gamma\left(1-\frac{\mu}{2}\pm\frac{\nu}{2}\right)}$$

$$.\sum_{G=1}^{m}\sum_{s,s',s'',t=0}^{\infty}e^{-bN(N+1)t'}\frac{(A_1)_t...(A_p)_t\,\beta^t}{(B_1)_t...(B_Q)_t\;\;t!\,s!}\frac{P_N^{\mu}(x)(-1)^s M^{g_s}\phi(g_s)}{f_G\;\;s!\,s'!}$$

$$.\frac{(a_1)_{s''}...(a_{P_1})_{s''}\;M_2^{s''}(-v')_{u's'}\,A_{v',s'}\,M_1^{s'}}{(b_1)_{s''}...(b_{Q_1})_{s''}\;\Gamma(\alpha's''+1)}$$

$$.H^{0,\lambda+2\;:(u',v');...;(u^{(r)},v^{(r)})}_{A+2,C+2:(B',D');...;(B^{(r)},D^{(r)})}\left[\begin{matrix}[---------],[1-\alpha-td-kg_s-k''s'-s'\pm\frac{\mu}{2}:\sigma_1,...,\sigma_r],\\ [(c):\psi',...,\psi^{(r)}]:[-\alpha-td-kg_s-k''s''-s'-\frac{\nu}{2}:\sigma_1,...,\sigma_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',...,\theta^{(r)}):[(b');\phi'];...;[(b^{(r)}):\phi^{(r)}];\\ [1-\alpha-td-kg_s+-k''s''-s'+\frac{\nu}{2}:\sigma_1,...,\sigma_r],[(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}];\end{matrix}\; z_1,...,z_r\right] \tag{12}$$

where $\mathrm{Re}\left(\alpha+k\frac{f_{j'}}{F_{j'}}+\sum_{i=1}^{r}\sigma_i\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>\frac{1}{2}|\mathrm{Re}\,(w)|,\,j=1,...,m;\sigma_i,k,k'',T_i>0,$

$$|\arg z_i|<\frac{1}{2}T_i\pi,i=1,...,r,\,\alpha>0,P\le Q,|M_2|<1,P_1\le Q,|\beta|<1,\arg M|<\frac{1}{2}T'\pi,T'>0.$$

Special Cases :-

(1) Putting $\lambda=A,u^{(i)}=1,v^{(i)}=B^{(i)},D^{(i)}=D^{(i)}+1,\forall i=1,...,r$ in (12), we obtain

$$\theta(x,t)=\frac{\pi 2^{\mu-1}(2\nu+1)(\nu-\mu)}{(\nu+\mu)!\Gamma\left(1-\frac{\mu}{2}\pm\frac{\nu}{2}\right)}$$

$$=\sum_{G=1}^{m}\sum_{s,s',s'',N,t=0}^{\infty}\frac{e^{-b\,N(N+1)t'}(A_1)_t...(A_p)_t\,\beta t}{(B_1)_t...(B_q)_t}$$

$$.\frac{P_N^{\mu}(x)(-1)^s M^{g_s}\phi(g_s)\,(a_1)_{s''}...(a_{P_1})_{s''}\,M_2^{s''}(-v')_{u's'}\,A_{v',s'}\,M_1^{s'}}{t!\,s!\,s'!\,(b_1)_{s''}...(b_{Q_1})_{s''}\;\;s'!}$$

$$.F^{A+2:B';...;B^{(r)}}_{C+2:D';...;D^{(r)}}\left[\begin{array}{l}[--------],[1-\alpha-td-kg_s-M_2s''-s'\pm\frac{\mu}{2}:\sigma_1,...,\sigma_r],\\ [1-(c):\psi',...,\psi^{(r)}]:[-\alpha-td-kg_s-M_2s''-s'-\frac{\nu}{2}:\sigma_1,...,\sigma_r],\end{array}\right.$$

$$\left.\begin{array}{l}[(a):\theta',...,\theta^{(r)}):\ [1-(b');\phi'];...;[(1-b^{(r)}):\phi^{(r)}];\\ [1-\alpha-td-kg_s-M_2s''-s'+\frac{\nu}{2}:\sigma_1,...,\sigma_r],\ [(1-d'):\delta'];...;[(d^{(r)}):\delta^{(r)}];\end{array}\ -z_1,...,-z_r\right], \qquad (13)$$

valid under the same conditions as derivable from (12).

(2) Letting r = 2 in (13), we have

$$\theta(x,t)=\pi 2^{\mu-1}\frac{(2\nu+1)(\nu-\mu)!}{(\nu+\mu)!\Gamma\left(1-\frac{\mu}{2}\pm\frac{\nu}{2}\right)}$$

$$.\sum_{G=1}^{m}\sum_{s,s',s'',t=0}^{\infty}e^{-bN(N+1)t'}\frac{(A_1)_t...(A_p)_t\,\beta^t}{(B_1)_t...(B_Q)_t\ t!}\frac{P_N^{\mu}(x)(-1)^s M^{g_s}\phi(g_s)(-v')_{u's'}A_{v',s'}M_1^{s'}}{f_G\ s!\ s'!}$$

$$.\frac{(a_1)_{s''}...(a_{P_1})_{s''}\ M_2^{s''}}{(b_1)_{s''}...(b_{Q_1})_{s''}\ \Gamma(\alpha's''+1)}$$

$$.S^{A+2:B';...;B''}_{C+2:D';...;D''}\left[\begin{array}{l}[--------],[1-\alpha-td-kg_s-M_2s''-s'\pm\frac{\mu}{2}:\sigma_1,\sigma_2],\\ [1-(c):\psi',...,\psi^{(r)}]:[-\alpha-td-kg_s-M_2s''-s'-\frac{\nu}{2}:\sigma_1,\sigma_2],\end{array}\begin{array}{l}[(a):\theta',\theta''):\ [1-(b');\phi'];[(b''):\phi''];\\ [1-\alpha-td-kg_s-s''-s'+\frac{\nu}{2}:\sigma_1.\sigma_2],\ [1-(d'):\delta'].;[(d''):\delta''];\end{array}\ -z_1,-z_2\right],(14)$$

valid under the same conditions as derivable from (15).

(3) Taking λ = A = C = 0 the results in (12) reduces to the following result

$$\theta(x,t)=\pi 2^{\mu-1}\frac{(2\nu+1)(\nu-\mu)!}{(\nu+\mu)!\Gamma\left(1-\frac{\mu}{2}\pm\frac{\nu}{2}\right)}$$

$$.\sum_{G=1}^{m}\sum_{s,s',s'',N,t'=0}^{\infty}e^{-bN(N+1)t'}\frac{(A_1)_t...(A_p)_t\,\beta^t}{(B_1)_t...(B_Q)_t\ t!\ s!}\ .\ \frac{P_N^{\mu}(x)(-1)^s M^{g_s}\phi(g_s)}{f_G}$$

$$.\frac{(a_1)_{s''}...(a_{P_1})_{s''}\ M_2^{s''}(-v')_{u's'}\ A_{v',s'}\ M_1^{s'}}{(b_1)_{s''}...(b_{Q_1})_{s''}\ \Gamma(\alpha's''+1)s'!}$$

$$.H^{0,2:(u',v');...;(u^{(r)},v^{(r)})}_{2,2:(B',D');...;(B^{(r)},D^{(r)})}\left[\begin{array}{l}[--------],[1-\alpha-td-kg_s-M_2s''-s'\pm\frac{\mu}{2}:\sigma_1,...,\sigma_r],\\ [(c):\psi',...,\psi^{(r)}]:[-\alpha-td-kg_s-M_2s''-s'-\frac{\nu}{2}:\sigma_1,...,\sigma_r],\end{array}\right.$$

$$\left.\begin{array}{l}[(a):\theta',...,\theta^{(r)}):\ [(b');\phi'];...;[(b^{(r)}):\phi^{(r)}];\\ [1-\alpha-td-kg_s+-M_2s''-s'+\frac{\nu}{2}:\sigma_1,...,\sigma_r],\ [(d'):\delta'];...;[(d^{(r)}):\delta^{(r)}];\end{array}\ z_1,...,z_r\right], \qquad (15)$$

valid under the same conditions as derivable from (12).

(4) Letting $k, \alpha', v' \to 0$ in (4), we have a known result given in ([8], eq.(1.3), p.227).

(5) Also taking $k, \alpha', v' \to 0$ in (12), we get a result given in ([8], eq. (2.1), p.228).

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Accelerating and Decelerating Hypersurface-Homogeneous Cosmological Models in Barber's Second Self-Creation Theory

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Abstract - We study the hypersurface-homogeneous cosmological model in presence of perfect fluid within the framework of Barber's [1982, GRG, 14, 117] second self-creation theory of gravitation. We have shown that the field equations are solvable for any arbitrary cosmic scale function and then obtained exact solutions for two values of a specific parameter. While doing so, we have used the general equation of state $p = m\rho$ where m $(0 \leq m \leq 1)$ is a constant. We also discussed the physical aspects of the models of the universe.

Keywords : *Hypersurface-homogeneous. Perfect fluid. Barber's self-creation theory.*

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S. D. Katore[α], R. S. Rane[σ], K. S. Wankhade[σ] & S.A.Bhaskar[α]

Abstract - We study the hypersurface-homogeneous cosmological model in presence of perfect fluid within the framework of Barber's [1982, GRG, 14, 117] second self-creation theory of gravitation. We have shown that the field equations are solvable for any arbitrary cosmic scale function and then obtained exact solutions for two values of a specific parameter. While doing so, we have used the general equation of state $p = m\rho$ where m $(0 \le m \le 1)$ is a constant. We also discussed the physical aspects of the models of the universe.

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I. Introduction

In recent years, there has been a considerable interest in alternative theories of gravitation. Brans-Dicke (1961) theory develops Mach's principle in a relativistic framework by assuming interaction of inertial masses of fundamental particles with some cosmic scalar field coupled with the large scale distribution of matter in motion. Brans-Dicke theory is a scalar-tensor theory of gravitation in which the tensor field is identified with the space-time of Riemannian geometry and scalar field is alien to geometry. This theory does not allow the scalar field to interact with fundamental principles and photons. However, Barber (1982) has modified scalar-tensor Brans-Dicke theory to develop a continuous creation of matter in the large scale structure of the universe. As a result, two self-creation theories are proposed by Barber (1982) in which the mass of the universe is seem to be created out of self-contained gravitational, scalar and matter field. Brans (1987) has pointed out that Barber's first theory is in disagreement with experiment as well as inconsistent, in general. Hence Barber's first theory is not accepted because this theory violets the equivalence principle.

The second theory retains the attractive features of the first theory and overcomes previous objections. These modified theories create the universe out of self-contained gravitational and matter fields. In Barber's second self-creation theory, the gravitational coupling of the Einstein field equations is allowed to be a variable scalar on the space-time manifold. Barber's second theory is a modification of general relativity to include continuous creation and is within observational limits, thus it modifies general relativity to become a variable G-theory. In this theory the scalar field does not directly gravitate, but simply divides the matter tensor, with the scalar acting as a reciprocal gravitational constant. The scalar field is postulated to couple to the trace of the energy-momentum

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tensor. Moreover, the most significant feature of self-creation is that it is as consistent with cosmological constraints in the distant supernovae data, the Cosmic microwave Background anisotropies and primordial nucleo-synthesis, as the standard paradigm. Unlike that model, however, it does not require the addition of the undiscovered physics of Inflation, dark non-baryonic matter or dark energy. Nevertheless, it does demand an exotic equation of state, which requires the presence of false vacuum energy at a moderate density determined by the Einstein's field equations. The consistency of Barber's second theory motivates us to study cosmological model in this theory.

Notes

Astronomical observations of the large-scale distribution of galaxies in the universe show that the distribution of matter can be satisfactorily described by a perfect fluid. Many authors have studied the Barber's self-creation theory of gravitation to produce mass creation in presence of perfect fluid satisfying the equation of state in the context of different space times. Pimentel (1985) and Soleng (1987) have discussed FRW models by using a power law relation between the expansion factor of the universe and the scalar field. Singh (1984), Reddy (1987) and Reddy *et al.* (1987) have studied Bianchi type space-times solutions in Barber's second theory of gravitation while Reddy and Venkateswarlu (1989) presented Bianchi type $-\mathrm{VI}_0$ cosmological model in Barber's second self-creation theory of gravitation. Shanti and Rao (1991) studied Bianchi type II and III space-times in this theory, both in vacuum as well as in presence of stiff fluid. Ram and Singh (1998) have discussed the spatially homogeneous and isotropic Robertson-Walker and Bianchi type-II models of the universe in Barber's self-creation theory in presence of perfect fluid by using gamma law equation of state. Pradhan and Pandey (2002), Pradhan and Vishwakarma (2002), Panigrahi and Sahu (2004), Vishwakarma and Narlikar (2005), Sahu and Mohanty (2006), Singh and Kumar (2007), Singh *et al.* (2008) , Venkateswarlu *et al.* (2008), Reddy and Naidu (2008) and Katore *et al.* (2010), are some of the authors who have studied various aspects of different cosmological models in Barber's second self-creation theory. In recent years, Verma and Shri Ram (2010) studied Hypersurface-homogeneous bulk viscous fluid space-times with time-dependent cosmological term and Shri Ram and Verma (2010) have discussed bulk viscous fluid hypersurface-homogeneous cosmological models with time varying G and Λ.

Motivated by these works, in this paper, we have investigated hypersurface-homogeneous cosmological model in presence of perfect fluid within the framework of Barber's second self-creation theory of gravitation. We first show that the field equations are solvable for any arbitrary cosmic scale function and then we obtain exact solutions for two values of a specific parameter. While doing so, we have used the general equation of state $p = m\rho$ where m $(0 \le m \le 1)$ is a constant. We also discuss the physical aspects of the models of the universe. This paper is organized as follows: The metric and field equations are considered in Sect. 2. In Section 3, solutions of Barber's field equations are obtained while the models are considered in Sect. 3.1 and 3.2. Some concluding remarks are given in Sect. 4.

II. The Metric and Field Equations

Stewart and Ellis (1968) have obtained general solutions of Einstein's field equations for a perfect fluid distribution satisfying a barotropic equation of state for the Hypersurface-homogeneous space time given by the metric

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[dy^2 + f_K^2(y)dz^2\right], \tag{1}$$

Notes

where $f_K(y) = \sin y,\ y,\ \sinh y$ respectively when $K = 1,\ 0,\ \text{-}1$.

Hajj-Boutros (1985) developed a method to build exact solutions of field equations in case of the metric (1) in presence of perfect fluid and obtained exact solutions of the field equations which add to the rare solutions not satisfying the barotropic equation of state. Recently Verma and Shri Ram (2010b) obtained some hypersurface-homogeneous bulk viscous fluid cosmological models with time-dependent cosmological term. Very recently Shri Ram and Verma (2010) presented bulk viscous fluid hypersurface-homogeneous cosmological models with time varying G and Λ term. The energy-momentum tensor in presence of perfect fluid has the form

$$T_{ij} = (\rho + p)u_i u_j - p\, g_{ij} \tag{2}$$

together with the relation

$$g_{ij} u^i u^j = 1 \tag{3}$$

and perfect fluid obeys the equation of state

$$p = m\rho \tag{4}$$

where m ($0 \le m \le 1$) is a constant. Here p is the pressure in the fluid and ρ is the energy density of the fluid and u^i is the four velocity vector defined by $u^i = \delta^i_4$, where i = 1, 2, 3, 4. We use the co-ordinate to be co-moving so that u^i = (0,0,0,1). For a universe field with perfect fluid, from (2) one finds

$$T_1^1 = T_2^2 = T_3^3 = -p, \quad T_4^4 = \rho \quad \text{and} \quad T = \rho - 3p\,. \tag{5}$$

The field equation in Barber's second self-creation theory of gravitation are

$$R_{ij} - \frac{1}{2} R\, g_{ij} = -8\pi\varphi^{-1} T_{ij} \tag{6}$$

and the scalar field equation is defined by

$$\varphi = \frac{8\pi}{3}\lambda T\,, \tag{7}$$

where R_{ij} is the Ricci tensor and R is the scalar curvature. $\varphi = \varphi_{;k}^{k}$ is the invariant d'Alembertian and T is the trace of the energy-momentum tensor that describes all non-gravitational and non-scalar field theory. Barber scalar field φ is a function of t due to the nature of space-time which plays the role analogous to the reciprocal of Newtonian gravitational constant i. e. $\varphi = \frac{1}{G}$. λ is a coupling constant to be determined from the experiment $|\lambda|$ ¡ 10^{-1} . In the limit as $\lambda \to 0$, this theory approaches the standard general relativity theory in every respect.

In a commoving coordinate system the Barber's field equations (6) and (7) for the metric (1) with the help of (5) take the form

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = -8\pi\varphi^{-1} p\,, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -8\pi\varphi^{-1} p \tag{9}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB} = -8\pi\varphi^{-1}p \tag{9}$$

$$2\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}^2}{B^2}+\frac{K}{B^2} = 8\pi\varphi^{-1}\rho \tag{10}$$

$$\ddot{\varphi}+\dot{\varphi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) = \frac{8\pi\lambda}{3}(\rho-3p) \tag{11}$$

where λ is a coupling constant to be determined from experiments ($|\lambda|\le 10^{-1}$, $\lambda=0$) and $G_N = \varphi^{-1}$. Equation (11) is the scalar field cosmological equation. Here overhead dots (.) indicate the differentiation with respect to t The energy conservation equation of general relativity $T^{ij}_{;j}=0$ takes the form

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)=0 \tag{12}$$

For the line element (1), we define the following physical and geometrical parameters, to be used in solving the Barber's field equations. The average scale factor (S), Volume scale factor (V), expansion scalar (θ) and shear scalar (σ) are

$$S=(AB^2)^{\frac{1}{3}} \tag{13}$$

$$V=S^3=AB^2 \tag{14}$$

$$\theta=v^i_{;i}=\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right) \tag{15}$$

and
$$\sigma^2=\frac{1}{3}\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)^2 \tag{16}$$

The generalized mean Hubble parameter H is given by

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{17}$$

where $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}=H_3$.

An important observational quantity is the deceleration parameter q which is defined as

$$q=\frac{-V\ddot{V}}{\dot{V}^2} \tag{18}$$

Notes

The sign of q indicates whether the model inflates or not. The positive sign corresponds to the standard decelerating model whereas the negative sign indicates inflation.

III. Solution of the Field Equations

Notes

Recently, Shri Ram and Verma (2010) have investigated the hypersurface-homogeneous cosmological models with time varying G and Λ term in the presence of bulk viscous fluid. They showed that the field equations are solvable for any arbitrary cosmic scale function. We follow the same approach to find exact solutions of the field equations.

From (8) and (9), we obtain

$$\frac{\ddot{B}}{B}-\frac{\ddot{A}}{A}+\frac{\dot{B}^2}{B^2}-\frac{\dot{A}\dot{B}}{AB}+\frac{K}{B^2}=0 \tag{19}$$

which on integration, yields

$$-B^2\dot{A}+AB\dot{B}=-K\int A dt+c_1 \tag{20}$$

where c_1 is an arbitrary constant.

We can write (20) in the form

$$\frac{d}{dt}(B^2)-2\frac{\dot{A}}{A}B^2=F(t), \tag{21}$$

where $F(t)=-2\frac{K}{A}\int A dt+c_1$ (22)

The linear differential equation (22) has the general solution given by

$$B^2=A^2\left[\int\frac{F(t)}{A^2}dt+c_2\right], \tag{23}$$

where c_2 is an integration constant. It is clear that the solution of Barber's field equations reduces to integration of (23) if $A(t)$ is known as a explicit function of time. We now obtain a particular solution of the field equations for a simple choice of the function $A(t)$. we choose

$$A=t^n \tag{24}$$

where n is a real number.

Integrating (23), we obtain

$$B^2=\frac{Kt^2}{n^2-1}+c_1t^{1+2n}+c_2t^{2n} \tag{25}$$

Without loss of generality, we take $c_1=c_2=0$. The solution (25) becomes

$$B^2=\frac{Kt^2}{n^2-1} \quad , \quad n\neq\pm1 \tag{26}$$

Hence the geometry of the universe in Barber's second self-creation theory for the hypersurface-homogeneous space-time corresponding to the solution (24) and (26) takes the form

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{Kt^2}{n^2-1}\left[dy^2 + f_K^{\,2}(y)dz^2\right] \tag{27}$$

We also consider the usual barotropic equation of state relating the perfect fluid pressure p to energy density ρ i.e. Equation (4).

Using (24), (26) and (4) in (12), we obtain the explicit form of the physical quantities p and ρ as

$$p = m\,c_3 t^{-(m+1)(n+2)} \tag{29}$$

and

$$\rho = c_3 t^{-(m+1)(n+2)} \tag{28}$$

From (11) and (4), we obtain the solution for scalar field $\varphi(t)$ in Barber's second self-creation theory is given by

$$\varphi(t) = c_4 t^{2-(m+2)(n+2)} \tag{30}$$

3.1 Model I:

For $K = 1$, the hypersurface-homogeneous cosmological model in (27) reduces to

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{n^2-1}\left[dy^2 + \sin^2 y\, dz^2\right] \tag{31}$$

This model is well defined for $n^2 - 1 > 0$.

For the model (31), the physical and geometrical parameters are given by

$$\theta = \frac{(n+2)}{t} \tag{32}$$

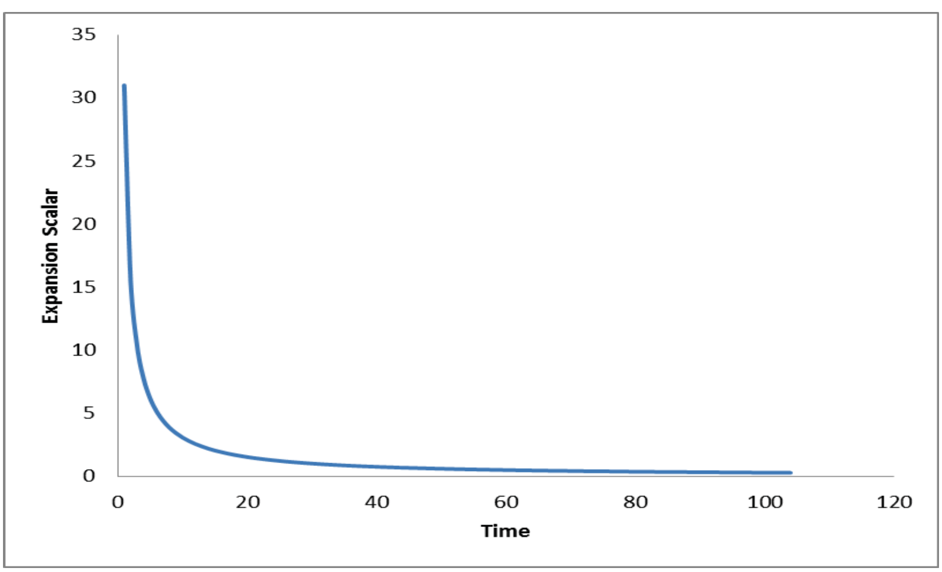

Figure 1 : Expansion Scalar Vs Time

Notes

$$\sigma^2 = \frac{(n-1)^2}{3t^2} \tag{33}$$

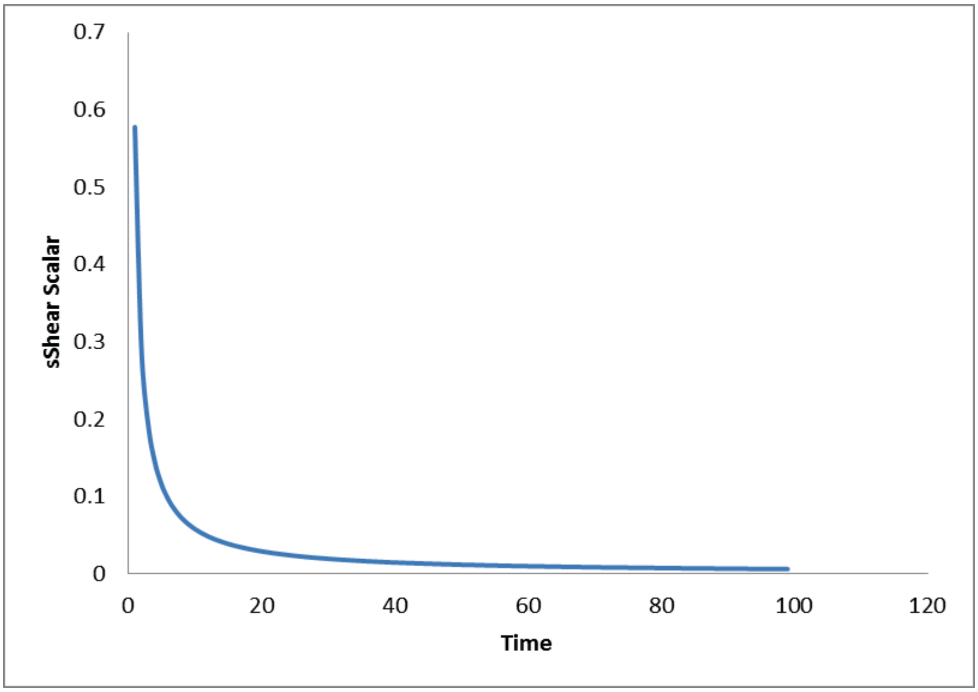

Figure 2 : Shear Scalar Vs Time .

$$S = \left(\frac{t^{n+2}}{n^2-1}\right)^{1/3} \tag{34}$$

$$V^3 = \frac{t^{n+2}}{n^2-1} \tag{35}$$

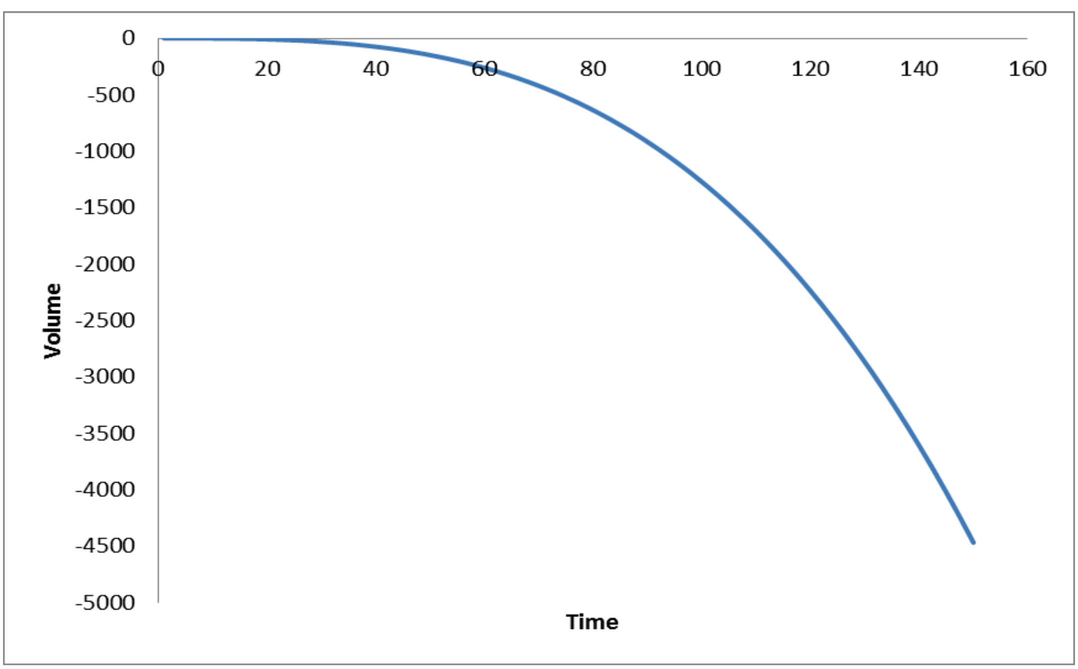

Figure 3 : Volume Vs Time.

Notes

$$H = \frac{n+2}{3t} \quad (36)$$

The deceleration parameter q is given by

$$q = \frac{1-n}{2+n} \quad (37)$$

For model (31), we observed that the spatial volume increases with time when $(n+2) > 0$ and it becomes infinite for large value of t. At $t = 0$, all the physical parameters ρ, σ, θ are infinite and the spatial volume is zero. Therefore, the cosmological model starts evolving with a big-bang at $t = 0$. Also the physical parameters decreases as time increases and tend to zero for large time. Since $\frac{\sigma}{\theta} = \frac{1}{\sqrt{3}}\left(\frac{n-1}{n+2}\right)$ the anisotropy in the universe is maintained throughout. The deceleration parameter q is positive for $-2 < n < -1$. In this case, the model (31) represents a decelerating universe. When n ¿1 , the value of deceleration parameter q is negative and thus (31) corresponds to an inflationary model of the universe.

3.2 Model II

For K = -1, the metric (27) of our solution can be written in the form

$$ds^2 = dt^2 - t^{2n}dx^2 - \frac{t^2}{1-n^2}\left[dy^2 + \sinh^2 y\, dz^2\right] \quad (38)$$

For the model (38), the expansion scalar θ, shear scalar σ and the generalized mean Hubble's parameter have the expressions given by (32), (33) and (36) respectively. The spatial volume V and the deceleration parameter q are given by the following expressions

$$V^3 = \frac{t^{n+2}}{1-n^2} \quad (39)$$

$$q = \frac{1-n}{2+n}, \quad (40)$$

where n is not less than -2. The deceleration parameter q is positive since $-1 < n < 1$. The model decelerates because of the fact that the deceleration parameter is positive constant. The cosmological model (38) starts with a big-bang at $t = 0$. The physical behaviors of this model are same as of the model (31).

IV. Conclusion

In this paper, we have obtained hypersurface-homogeneous cosmological model in presence of perfect fluid within the frame work of Barber's second self-creation theory. The cosmic fluid satisfies the barotropic equation of state. It is shown that Barber's field equations for hypersurface-homogeneous cosmological model are solvable for any arbitrary cosmic scale function. Two classes of exact solutions of Barber s field equations are presented for $K = 1$ and $K = -1$ which represent expanding, shearing, non-rotating, decelerating / accelerating models of the universe. In present models of the universe the anisotropy is maintained throughout. It is also observed that the Barber scalar field φ increases when t increases.

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An Oscillatory Free Convective Flow Through Porous Medium in a Rotating Vertical Porous Channel

By K.D.Singh & Alphonsa Mathew
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Abstract - A theoretical analysis of the effects of permeability and the injection/suction on an oscillatory free convective flow of a viscous incompressible fluid through a highly porous medium bounded between two infinite vertical porous plates is presented. The entire system rotates about the axis normal to the planes of the plates with uniform angular velocity Ω. For small and large rotations the dependence of the steady and unsteady resultant velocities and their phase differences on various parameters are discussed in detail.

Keywords : *Oscillatory, rotating, porous channel, Porous medium, Free convection.*

GJSFR-F Classication : *American Mathematical Society (2000) subject classification: 76 W*

AN OSCILLATORYFREECONVECTIVE FLOW THROUGHPOROUSMEDIUMINAROTATING VERTICALPOROUS CHANNEL

An Oscillatory Free Convective Flow Through Porous Medium in a Rotating Vertical Porous Channel

K.D.Singh[α] & Alphonsa Mathew[σ]

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I. Introduction

Free convection flows in a rotating porous medium are of great interest in a number of industrial applications such as fiber and granular insulation, geothermal systems etc. Buoyancy is also of importance in an environment where difference of temperatures can give rise to complicated flow patterns. In recent years, the problems of free convection have attracted the attention of a large number of scholars due to its diverse applications.

The flow of fluids through highly porous medium bounded by vertical porous plates find numerous engineering and geophysical applications, viz. in the fields of agricultural engineering to study the underground water resources, in petroleum technology to study the movement of natural gas, oil and water through the oil channels/reservoirs[1,2,10]. A series of investigations have been made by different scholars where the porous medium is either bounded by horizontal, vertical surfaces or parallel porous plates. Raptis [8] analyzed the unsteady flow through a porous medium bounded by an infinite porous plate subjected to a constant suction and variable temperature. Raptis and Perdikis [9] further studied the problem of free convective flow through a porous medium bounded by a vertical porous plate with constant suction when the free stream velocity oscillates in time about a constant mean value.

Apart from the above two dimensional studies a number of three dimensional flows through porous medium have also been studied. Singh *et al.* [16] analyzed the effects of periodic permeability on the three dimensional flow through highly porous medium bounded by an infinite porous surface. Singh *et al.* [15] also investigated the effect of permeability variation on the heat transfer and three dimensional flow through a highly porous medium bounded by an infinite porous plate with constant suction. Singh and Verma [13] studied further the flow of a viscous incompressible fluid through porous medium when the free stream velocity oscillates in time about a non-zero constant mean.

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In the recent years a number of studies have appeared in the literature involving rotation to a greater or lesser extent viz. Vidyanidhu and Nigam [19], Gupta [4], Jana and Datta [5], Singh [11,17]. Injection/suction effects have also been studied extensively for horizontal porous plate in rotating frame of references by Ganapathy [3], Mazumder [7], Mazumder *et al.* [6], Soundalgekar and Pop [18], Singh [12] for different physical situation. The flows of fluids through porous medium bounded by rotating porous channels find many industrial applications particularly in the fields of centrifugation, filtration and purification processes. In view of these applications Singh and Sharma [14] studied the effect of the permeability of the porous medium on the three dimensional Couette flow and heat transfer. In the present paper an attempt has been made to study the effects of the permeability of the porous medium and injection/suction through the porous parallel vertical plates on the free convective flow through a highly porous medium. The entire system rotates about an axis perpendicular to the planes of the plates.

II. Mathematical Analysis

Consider an oscillatory free convective flow of a viscous incompressible fluid through a highly porous medium bounded between two infinite vertical porous plates distance d apart. A constant injection velocity, w_0, is applied at the stationary plate $z^* = 0$ and the same constant suction velocity, w_0, is applied at the plate $z^* = d$, which is oscillating in its own plane with a velocity $U^*(t^*)$ about a non-zero constant mean velocity U_0. The origin is assumed to be at the plate $z^* = 0$ and the channel is oriented vertically upward along the x^*-axis. The channel rotates as a rigid body with uniform angular velocity Ω^* about the z^*-axis. Since the plates are infinite in extent, all the physical quantities except the pressure, depend only on z^* and t^*. Denoting the velocity components u^*, v^*, w^* in the x^*, y^*, z^* directions, respectively and temperature by T^*, the flow in the rotating system is governed by the following equations:

$$w_z^* = 0, \tag{1}$$

$$u_t^* + w_0^* u_z^* = -p_x^*/\rho + \upsilon u_{zz}^* + 2\Omega^* v^* + g\beta(T^* - T_d) - \upsilon u^*/\mathrm{K}^*, \tag{2}$$

$$v_t^* + w_0^* v_z^* = -p_y^*/\rho + \upsilon v_{zz}^* - 2\Omega^* u^* - \upsilon v^*/\mathrm{K}^* \tag{3}$$

$$T_t^* + w_0 T_z^* = \frac{k}{\rho C_p} T_{zz}^*, \tag{4}$$

where υ is the kinematic viscosity, t is the time, ρ is the density and p^* is the modified pressure, T^* is the temperature, C_p is the specific heat at constant pressure, k is the thermal conductivity, g is the acceleration due to gravity, β the coefficient of volume expansion and K^* is the permeability of the medium.

The boundary conditions for the problem are

$$\left.\begin{aligned} u^* = v^* = 0, \quad T^* = T_0 + \varepsilon(T_0 - T_d)\cos\omega^* t^* \qquad &\text{at} \quad z^* = 0, \\ u^* = U^*(t^*) = U_0(1 + \varepsilon\cos\omega^* t^*), \quad v^* = 0, \quad T^* = T_d, \qquad &\text{at} \quad z^* = d \end{aligned}\right\} \tag{5}$$

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5. Jana, R.N. and Datta, N. (1977), Couette flow and heat transfer in a rotating system, Acta Mech., **26**, 301-306.
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where ω^* is the frequency of oscillations and ε is a very small positive constant. By introducing the following non-dimensional quantities

$\eta = z^*/d$, $t = \omega^* t^*$, $u = u^*/U_0$, $v = v^*/U_0$, $\Omega = \Omega^* d^2/\upsilon$ the rotation parameter, $\omega = \omega^* d^2/\upsilon$ the frequency parameter, $\lambda = w_0 d/\upsilon$ the injection/suction parameter, $\mathrm{K} = \mathrm{K}^*/d^2$ the permeability parameter, $\theta = \dfrac{T^* - T_d}{T_0 - T_d}$, $Gr = \dfrac{\upsilon g\beta(T_0 - T_d)}{U_0 w_0^2}$ the Grashof number, $\Pr = \dfrac{\mu C_p}{k}$ the Prandtl number and suppressing the stars '*' the equations (2) to (4) become

$$\omega q_t + \lambda q_\eta = q_{\eta\eta} + \omega U_t + Gr\lambda^2\theta - 2i\Omega(q-U) - (q-U)/\mathrm{K}, \tag{6}$$

$$\omega\theta_t + \lambda\theta_\eta = \frac{1}{\Pr}\theta_{\eta\eta}, \tag{7}$$

where $q = u + iv$.

The boundary conditions (5) can also be written in complex notations as

$$\left.\begin{aligned} &q = 0, \quad \theta = 1 + \frac{\varepsilon}{2}(e^{it} + e^{-it}) \quad && at \quad \eta = 0, \\ &q = U(t) = 1 + \frac{\varepsilon}{2}\left(e^{it} + e^{-it}\right), \quad \theta = 0 \quad && at \quad \eta = 1. \end{aligned}\right\} \tag{8}$$

In order to solve the system of equations (6) and (7) subject to the boundary conditions (8), we assume,

$$q(\eta,t) = q_o(\eta) + \frac{\varepsilon}{2}\left\{q_1(\eta)e^{it} + q_2(\eta)e^{-it}\right\}, \tag{9}$$

$$\theta(\eta,t) = \theta_o(\eta) + \frac{\varepsilon}{2}\left\{\theta_1(\eta)e^{it} + \theta_2(\eta)e^{-it}\right\}. \tag{10}$$

Substituting (9) and (10) into (6) and (7) and comparing the harmonic and non-harmonic terms, we get

$$q_0'' - \lambda q_0' - (l^2 + \frac{1}{\mathrm{K}})q_0 = -(l^2 + \frac{1}{\mathrm{K}}) - Gr\lambda^2\theta_0, \tag{11}$$

$$q_1'' - \lambda q_1' - (m^2 + \frac{1}{\mathrm{K}})q_1 = -(m^2 + \frac{1}{\mathrm{K}}) - Gr\lambda^2\theta_1, \tag{12}$$

$$q_2'' - \lambda q_2' - (n^2 + \frac{1}{\mathrm{K}})q_2 = -(n^2 + \frac{1}{\mathrm{K}}) - Gr\lambda^2\theta_2, \tag{13}$$

$$\theta_0'' - \Pr\lambda\theta_0' = 0, \tag{14}$$

$$\theta_1'' - \Pr\lambda\theta_1' - \Pr\omega i\theta_1 = 0, \tag{15}$$

$$\theta_2'' - \Pr\lambda\theta_2' + \Pr\omega i\theta_2 = 0, \tag{16}$$

where $l^2 = i2\Omega$, $m^2 = i(2\Omega + \omega)$ and $n^2 = i(2\Omega - \omega)$.

The corresponding transformed boundary conditions reduce to

$$\left.\begin{array}{l} q_0 = q_1 = q_2 = 0, \quad \theta_0 = \theta_1 = \theta_2 = 1 \quad at \quad \eta = 0, \\ q_0 = q_1 = q_2 = 1, \quad \theta_0 = \theta_1 = \theta_2 = 0 \quad at \quad \eta = 1. \end{array}\right\} \tag{17}$$

The solutions of equations (11) to (16) under the boundary conditions (17) are

$$q_0(\eta) = 1 + B_1 e^{n_1\eta} + B_2 e^{n_2\eta} + A_1 e^{\lambda \Pr \eta} \ , \tag{18}$$

$$q_1(\eta) = 1 + B_3 e^{n_3\eta} + B_4 e^{n_4\eta} + A_2 e^{m_2\eta} + A_3 e^{m_1\eta} \ , \tag{19}$$

$$q_2(\eta) = 1 + B_5 e^{n_5\eta} + B_6 e^{n_6\eta} + A_4 e^{m_4\eta} + A_5 e^{m_3\eta} \ , \tag{20}$$

$$\theta_0(\eta) = \frac{e^{\lambda \Pr \eta} - e^{\lambda \Pr}}{1 - e^{\lambda \Pr}} \ , \tag{21}$$

$$\theta_1(\eta) = \frac{e^{m_1 + m_2\eta} - e^{m_2 + m_1\eta}}{e^{m_1} - e^{m_2}} \ , \tag{22}$$

$$\theta_2(\eta) = \frac{e^{m_3 + m_4\eta} - e^{m_4 + m_3\eta}}{e^{m_3} - e^{m_4}} , \tag{23}$$

where

$$m_1 = \frac{\Pr\lambda + \sqrt{\Pr^2\lambda^2 + 4i\omega\Pr}}{2}, \qquad m_2 = \frac{\Pr\lambda - \sqrt{\Pr^2\lambda^2 + 4i\omega\Pr}}{2},$$

$$m_3 = \frac{\Pr\lambda + \sqrt{\Pr^2\lambda^2 - 4i\omega\Pr}}{2}, \qquad m_4 = \frac{\Pr\lambda - \sqrt{\Pr^2\lambda^2 - 4i\omega\Pr}}{2},$$

$$n_1 = \frac{\lambda + \sqrt{\lambda^2 + 4(l^2 + \frac{1}{K})}}{2}, \qquad n_2 = \frac{\lambda - \sqrt{\lambda^2 + 4(l^2 + \frac{1}{K})}}{2},$$

$$n_3 = \frac{\lambda + \sqrt{\lambda^2 + 4(m^2 + \frac{1}{K})}}{2}, \qquad n_4 = \frac{\lambda - \sqrt{\lambda^2 + 4(m^2 + \frac{1}{K})}}{2},$$

$$n_5 = \frac{\lambda + \sqrt{\lambda^2 + 4(n^2 + \frac{1}{K})}}{2}, \qquad n_6 = \frac{\lambda - \sqrt{\lambda^2 + 4(n^2 + \frac{1}{K})}}{2},$$

$$A_1 = \frac{-Gr\lambda^2}{(1 - e^{\lambda\Pr})[\lambda^2\Pr(\Pr - 1) - (l^2 + \frac{1}{K})]}, \qquad A_2 = \frac{-Gr\lambda^2 e^{m_1}}{(e^{m_1} - e^{m_2})[m_2(m_2 - \lambda) - (m^2 + \frac{1}{K})]}$$

$$A_3 = \frac{Gr\lambda^2 e^{m_2}}{(e^{m_1} - e^{m_2})[m_1(m_1 - \lambda) - (m^2 + \frac{1}{K})]}, \qquad A_4 = \frac{-Gr\lambda^2 e^{m_3}}{(e^{m_3} - e^{m_4})[m_4(m_4 - \lambda) - (n^2 + \frac{1}{K})]},$$

Notes

$$A_5 = \frac{Gr\lambda^2 e^{m_4}}{(e^{m_3} - e^{m_4})[m_3(m_3 - \lambda) - (n^2 + \frac{1}{\mathrm{K}})]}, \quad B_1 = -\left[\frac{e^{n_2} + A_1(e^{n_2} - e^{\lambda \Pr})}{e^{n_2} - e^{n_1}}\right],$$

$$B_2 = \left[\frac{e^{n_1} + A_1(e^{n_1} - e^{\lambda \Pr})}{e^{n_2} - e^{n_1}}\right], \quad B_3 = -\left[\frac{e^{n_4} + A_2(e^{n_4} - e^{m_2}) + A_3(e^{n_4} - e^{m_1})}{e^{n_4} - e^{n_3}}\right],$$

$$B_4 = \left[\frac{e^{n_3} - A_2(e^{m_2} - e^{n_3}) - A_3(e^{m_1} - e^{n_3})}{e^{n_4} - e^{n_3}}\right], \quad B_5 = -\left[\frac{e^{n_6} + (e^{n_6} - e^{m_4}) + A_5(e^{n_6} - e^{m_3})}{e^{n_6} - e^{n_5}}\right],$$

$$B_6 = \left[\frac{e^{n_5} - A_4(e^{m_4} - e^{n_5}) - A_5(e^{m_3} - e^{n_5})}{e^{n_6} - e^{n_5}}\right].$$

Notes

III. Results and Discussion

Now for the resultant velocities and the shear stresses of the steady and unsteady flow, we write

$$u_0(\eta) + iv_0(\eta) = q_0(\eta) \quad \text{and} \tag{24}$$

$$u_1(\eta) + iv_1(\eta) = q_1(\eta)e^{it} + q_2(\eta)e^{-it}. \tag{25}$$

The solution (18) corresponds to the steady part which gives u_0 as the primary and v_0 as the secondary velocity components. The amplitude and the phase difference due to these primary and secondary velocities for the steady flow are given by

$$R_0 = \sqrt{u_0^2 + v_0^2}, \qquad \phi_0 = \tan^{-1}(v_0 / u_0) \tag{26}$$

The resultant velocity R_0 for the steady part is presented in Fig.1.a, b for small and large values of rotations respectively of the vertical porous channel. The two values of the Prandtl number $\Pr$ as 0.7 and 7.0 are chosen to represent air and water respectively. In Fig.1.a, b the curve I corresponds to the flow through an ordinary medium. It is very clear from Fig.1.a that R_0 increases with the Grashof number Gr, the rotation of the channel Ω, suction velocity λ, and the permeability parameter K. In the case of Prandtl number $\Pr$, R_0 is increasing near the oscillating plate.

Similarly for large rotations Ω shown in Fig 1.b., the amplitude R_0 increases with Gr, the free convection currents, and the permeability parameter K and R_0 also oscillates with the increase of the rotation Ω of the channel. It is interesting to note that increase of Prandtl number $\Pr$ leads to an increase of R_0 near the oscillating plate, but to a decrease near the stationary plate. However, the effects of λ, the suction/injection at the plates are reversed i.e. the amplitude R_0 increases near the stationary plate and decreases thereafter.

The phase difference ϕ_0 for the steady flow is shown graphically in Fig 2.a, b for small and large rotations respectively. Fig.2.a shows that the phase angle ϕ_0 is decreasing near the oscillating plate with the increase of Gr or $\Pr$ or λ and Ω, but increases with the permeability parameter K. Similarly for large rotations Ω shown in Fig 2.b., the phase difference decreases with rotation Ω and Prandtl number $\Pr$. But the increase of permeability parameter K, Grashof number Gr and the suction/injection at the plates λ leads to an increase of ϕ_0. The amplitude and the phase difference of shear stresses at the stationary plate $(\eta = 0)$ for the steady flow can be obtained as,

$$\tau_{0r} = \sqrt{\tau_{0x}^2 + \tau_{0y}^2}\text{, and} \qquad \phi_{or} = \tan^{-1}\left(\tau_{oy}/\tau_{ox}\right), \tag{27}$$

$$\text{where,} \quad \tau_{ox} + i\,\tau_{oy} = \left(\partial q/\partial \eta\right)_{\eta=0} = n_1 B_1 + n_2 B_2 + \lambda \Pr A_1. \tag{28}$$

Here τ_{ox} and τ_{oy} are, respectively, the shear stresses at the stationary plate due to the primary and secondary velocity components. The numerical values of the amplitude τ_{0r} of the steady shear stress and the phase difference of the shear stresses at the stationary plate ($\eta = 0$) for the

$\Pr$	Gr	Ω	λ	K	τ_{0r}	ϕ_{0r}
0.7	5	5	2	∞	3.717	1.351
0.7	5	5	2	1	3.44	1.765
7.0	5	5	2	1	2.498	0.967
0.7	10	5	2	1	5.461	-1.042
0.7	5	10	2	1	4.372	1.314
0.7	5	5	3	1	3.304	-1.269
0.7	5	5	2	2	3.575	-1.363
0.7	5	25	2	∞	6.607	1.029
0.7	5	25	2	1	6.581	1.02
7.0	5	25	2	1	6.389	0.873
0.7	10	25	2	1	6.902	1.154
0.7	5	50	2	1	9.392	0.922
0.7	5	25	3	1	6.313	1.07
7.0	5	25	2	2	6.594	1.024

Table 1: Values of τ_{0r} and ϕ_{0r} for various $\Pr$, Gr, Ω, λ, and K.

steady flow are presented in Table -1. The permeability parameter K, the Grashof number Gr, and the rotation parameter Ω lead to an increase of τ_{0r} for both the cases

Notes

of small or large rotations. It is also observed that τ_{0r} decreases with Pr and λ for small and large rotations. Similarly the values for ϕ_{0r}, the steady phase difference, increases with the suction parameter λ and the permeability parameter K for both the cases of small or large rotations. But the effect is reverse in the case of Prandtl number Pr. The increase of Ω leads to an increase in ϕ_{0r} for small rotations. But the effect will be reverse in the case of large rotations.

Notes

The solutions (19) and (20) together give the unsteady part of the flow. The unsteady primary and secondary velocity components $u_1(\eta)$ and $v_1(\eta)$, respectively, for the fluctuating flow can be obtained as

$$u_1(\eta,t)=\{\mathrm{Re}\,al\,q_1(\eta)+\mathrm{Re}\,al\,q_2(\eta)\}\cos t-\{\mathrm{Im}\,q_1(\eta)-\mathrm{Im}\,q_2(\eta)\}\sin t \quad , \tag{29}$$

$$v_1(\eta,t)=\{\mathrm{Re}\,al\,q_1(\eta)-\mathrm{Re}\,al\,q_2(\eta)\}\sin t+\{\mathrm{Im}\,q_1(\eta)+\mathrm{Im}\,q_2(\eta)\}\cos t \quad , \tag{30}$$

The resultant velocity or amplitude and the phase difference of the unsteady flow are given by

$$R_1=\sqrt{u_1^2+v_1^2} \quad , \quad \phi_1=\tan^{-1}(v_1/u_1) \tag{31}$$

For the unsteady part, the resultant velocity or the amplitude R_1 are presented in Fig.3.a, b. for the two cases of rotation Ω small and large. In Fig.3.a, b the curve I corresponds to the flow through an ordinary medium. It is observed from figure 3.a, for small rotations Ω that R_1 increases with Prandtl number Pr, free convection current Gr, the suction/injection parameter λ and permeability parameter K, but decreases with the rotation parameter Ω and the frequency of oscillations ω. Fig. 3.b, for large rotations Ω clearly shows that the amplitude R_1 increases with all the parameter Gr, Pr, λ, K, ω except that with the rotation parameter Ω, R_1 decreases near the oscillating plates.

The phase difference ϕ_1 for the unsteady part is shown in Figure 4. a, b. In Fig.4.a, b the curve I corresponds to the flow through an ordinary medium. Figure 4.*a* for small rotations Ω shows that the phase difference ϕ_1 increases with the Prandtl number Pr and the frequency of oscillations ω, but decreases with the Grashof number Gr, the suction parameter λ, the permeability parameter K. And, with the faster rotation of the channel Ω, ϕ_1 increases near the stationary plate. It is also evident from Figure 4.*b* that increase of Pr, or Gr, or λ or K leads to a decrease in ϕ_1 but the increase of the rotation parameter Ω, frequency of oscillations ω both lead to an increase in ϕ_1.

For the unsteady part of the flow, the amplitude and the phase difference of shear stresses at the stationary plate ($\eta=0$) can be obtained as

$$\tau_{1x}+i\tau_{1y}=(\partial u_1/\partial\eta)_{\eta=0}+i(\partial v_1/\partial\eta)_{\eta=0} \tag{32}$$

which gives

$$\tau_{1r}=\sqrt{\tau_{1x}^2+\tau_{1y}^2} \quad , \quad \phi_{1r}=\tan^{-1}(\tau_{1y}/\tau_{1x}) \tag{29}$$

The amplitude τ_{1r} of the unsteady shear stress are shown graphically in Figure 5.a, b respectively for small and large rotations. Fig.5.a, b the curve I corresponds to the flow through an ordinary medium. It is interesting to note that the shear stress increases sharply for small oscillations of the frequency and thereafter decreases abruptly for larger frequency of oscillations. This figure shows clearly that the shear stress τ_{1r} increases with increasing Gr, or λ, or Ω. However, the effects of Prandtl number Pr and the permeability parameter K are reversed. For larger rotation Ω the variations of shear stress τ_{1r} are presented in Figure 5. b. This figure shows that the amplitude τ_{1r} increases with the free convection current Gr, the Prandtl number Pr the suction parameter λ, the rotation parameter Ω and permeability parameter K.

The phase difference ϕ_{1r} of the unsteady shear stress is shown graphically in Figure 6.a, b respectively for small and large rotations. It is interesting to note from these figures that ϕ_{1r} goes on increasing with increasing frequency of oscillations for both small and large rotations. The phase difference ϕ_{1r} decreases for both small and large rotations with the increase of Grashof number Gr and suction parameter λ. However for small rotations Ω, ϕ_{1r} increases for all values of frequency of oscillations and for large rotations ϕ_{1r} decreases very near the oscillating plate. The effects of Prandtl number Pr and the permeability parameter K, lead to an increase in ϕ_{1r} every where for large or small rotations.

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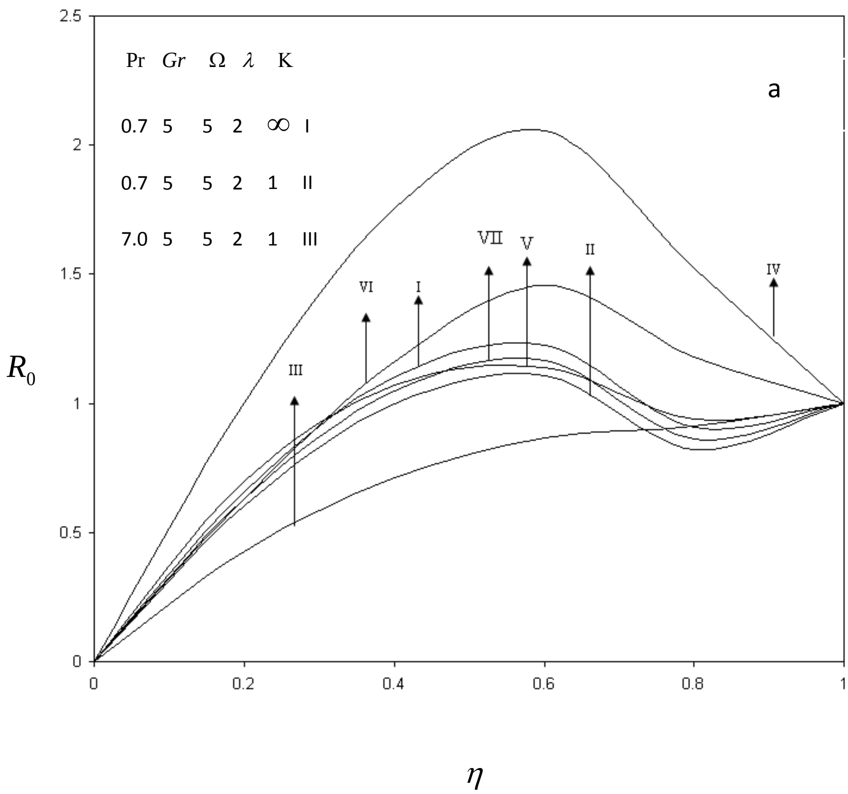

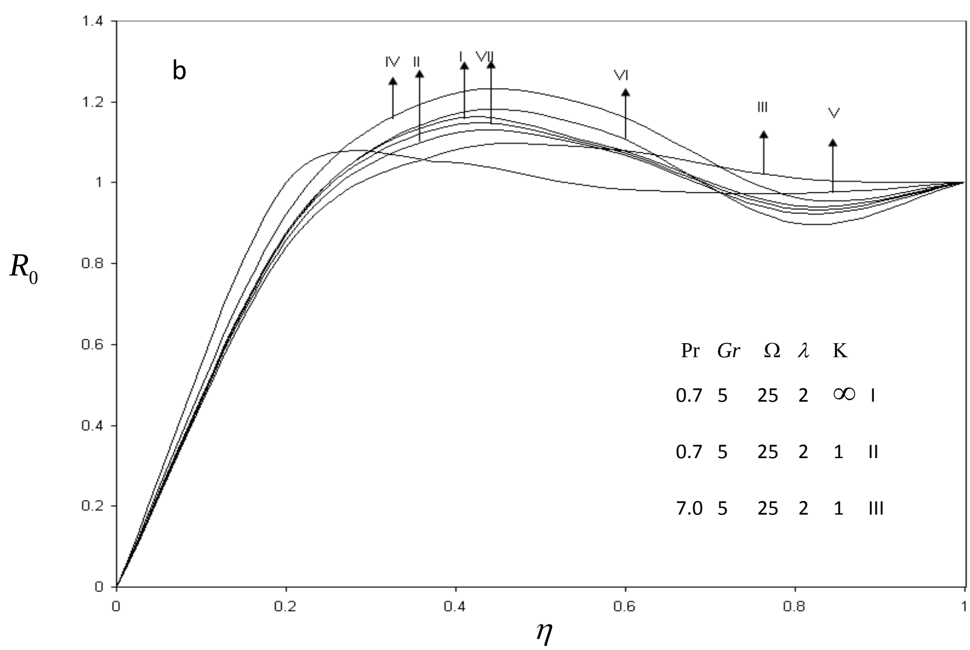

Fig.1 a, b : Resultant velocity R_0 for small and large rotations due to u_0 and v_0

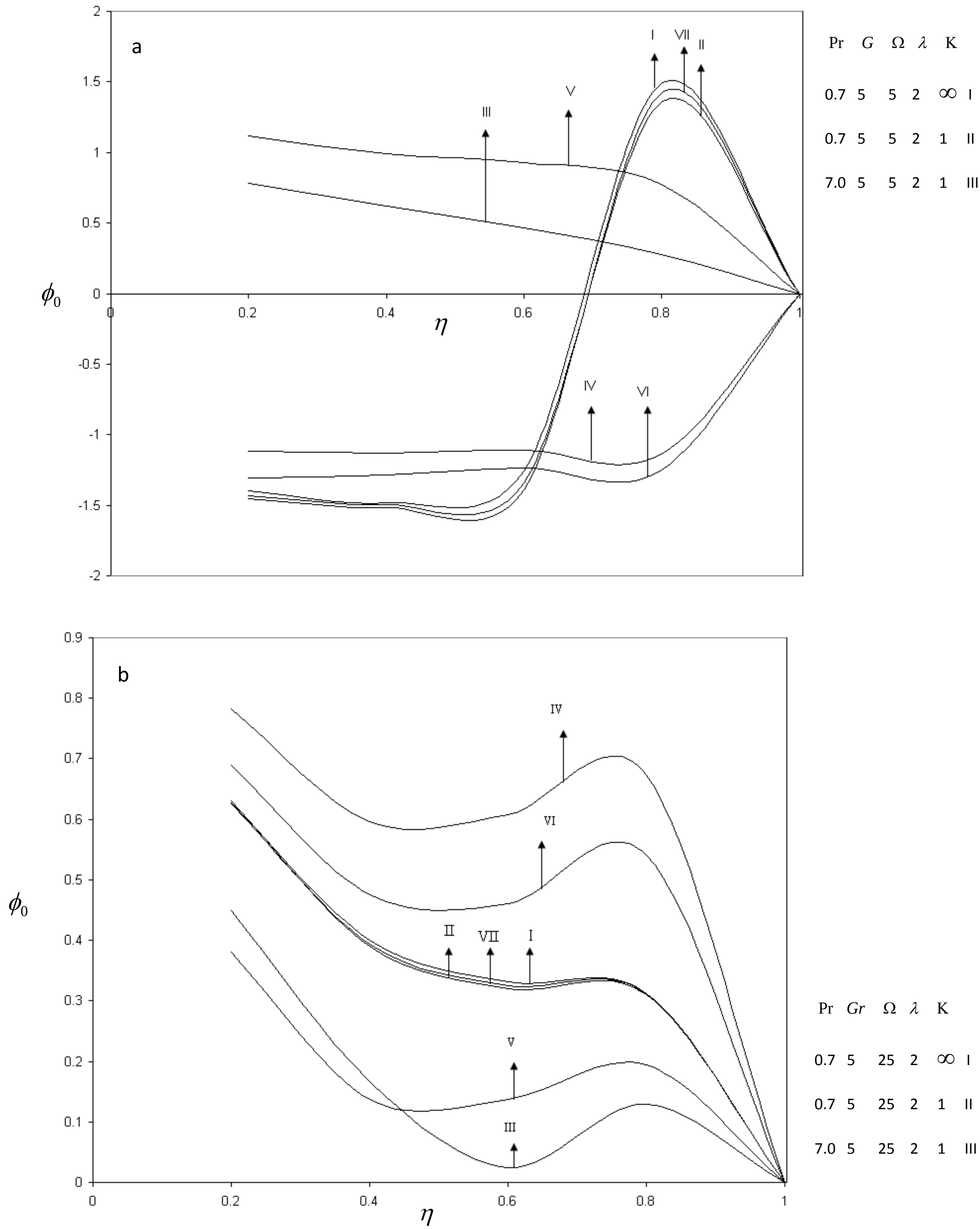

Fig.2 a, b : Phase angle ϕ_0 for small and large rotations due to u_0 and v_0

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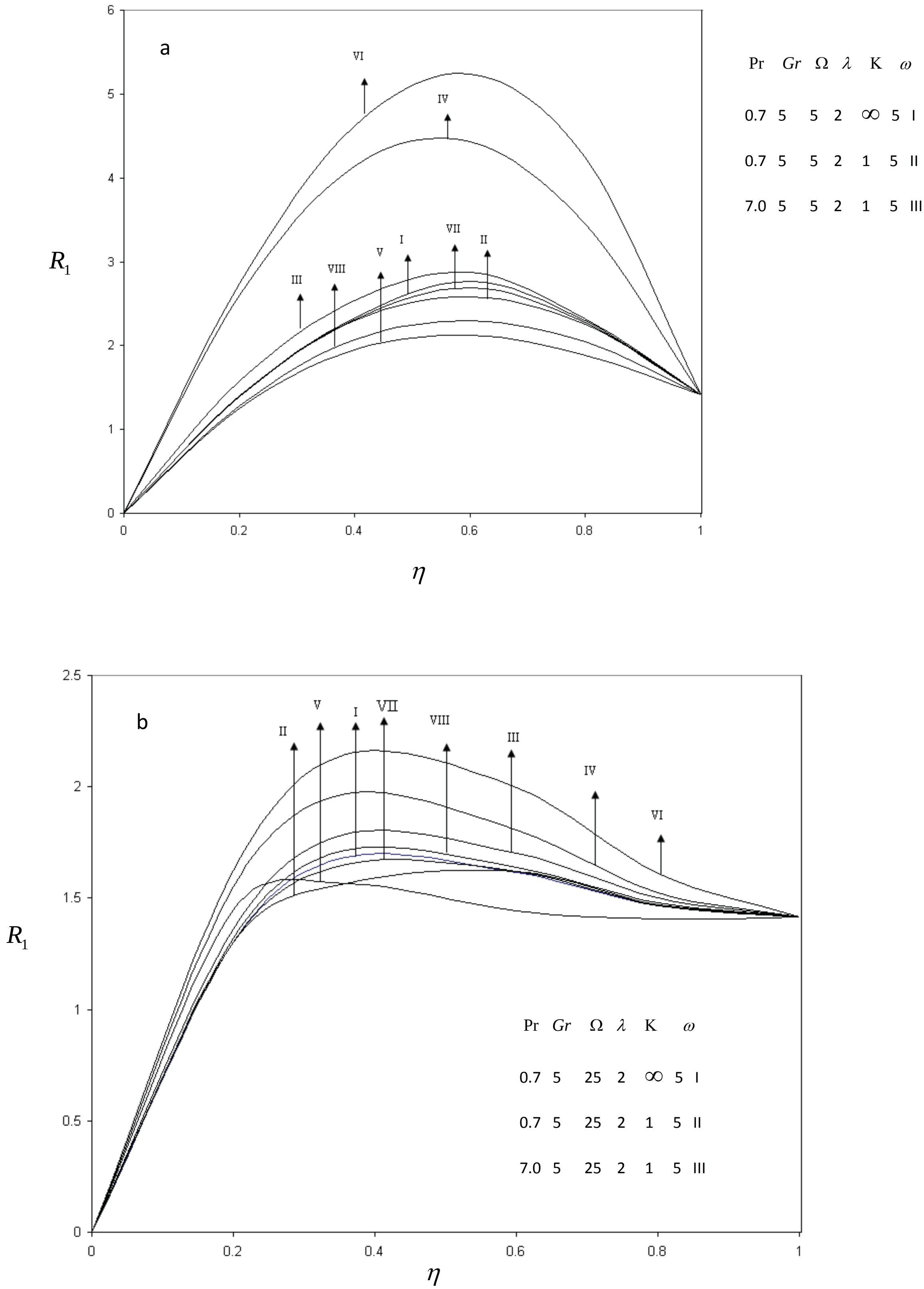

Fig.3 a, b : Resultant velocity R_1 for small and large rotations due to u_1 and v_1

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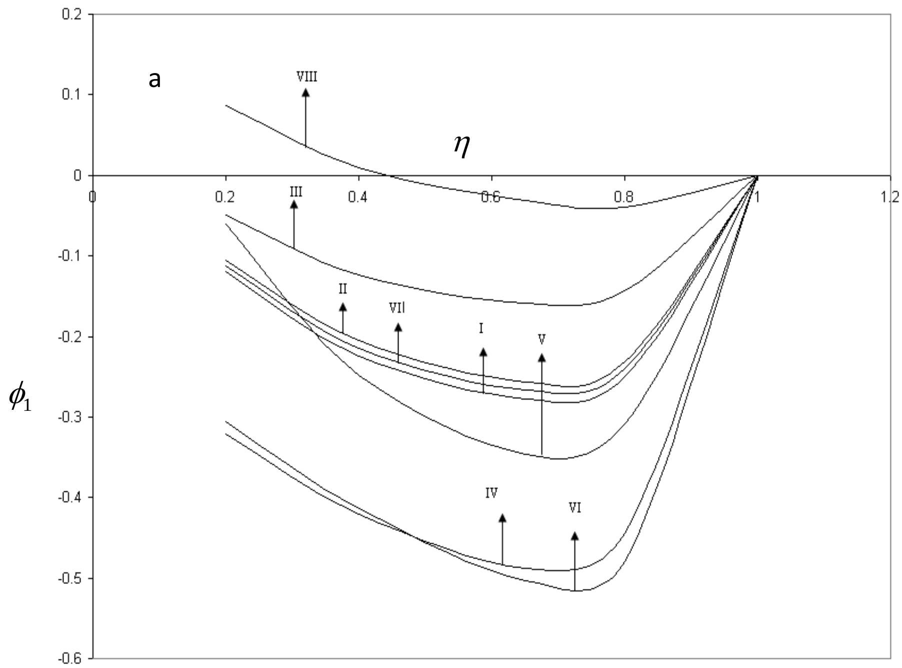
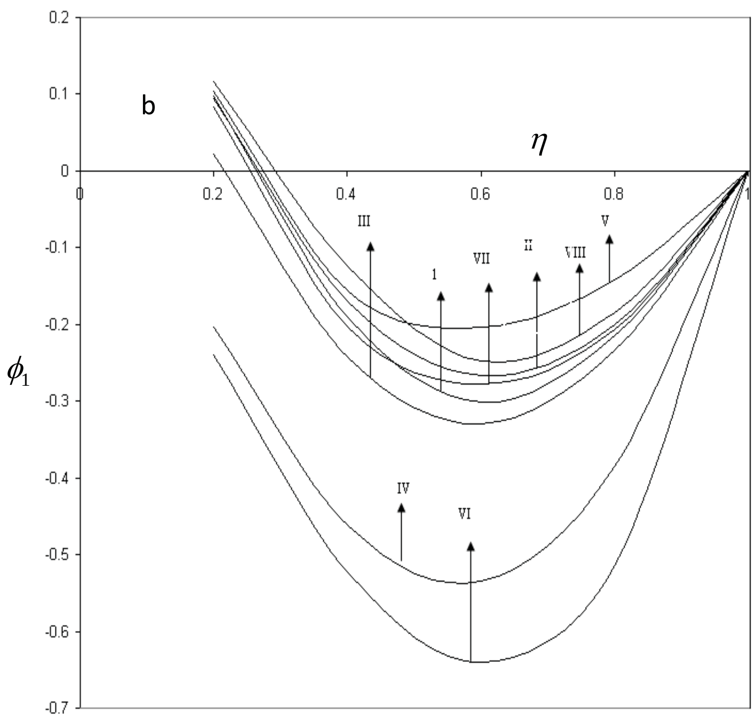
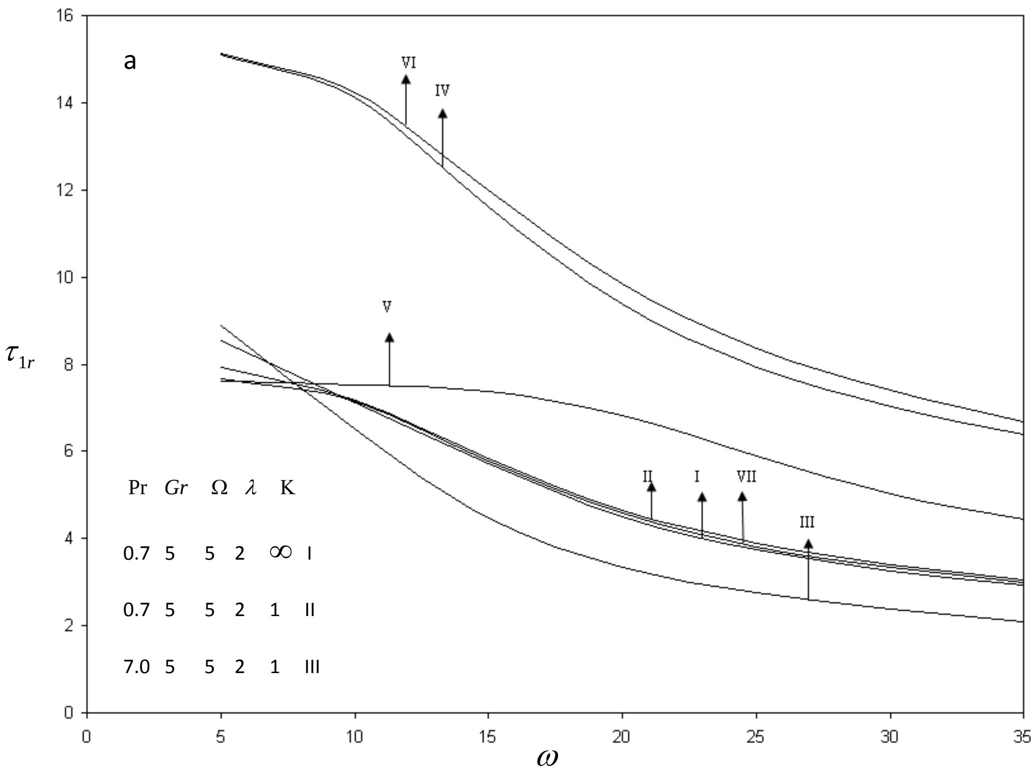


Pr	Gr	Ω	λ	K	ω	
0.7	5	5	2	∞	5	I
0.7	5	5	2	1	5	II
7.0	5	5	2	1	5	III



Pr	Gr	Ω	λ	K	ω	
0.7	5	25	2	∞	5	I
0.7	5	25	2	1	5	II
7.0	5	25	2	1	5	III
0.7	10	25	2	1	5	IV
0.7	5	50	2	1	5	V

Fig.4 a, b : Phase angle ϕ_1 for small and large rotations due to u_1 and v_1



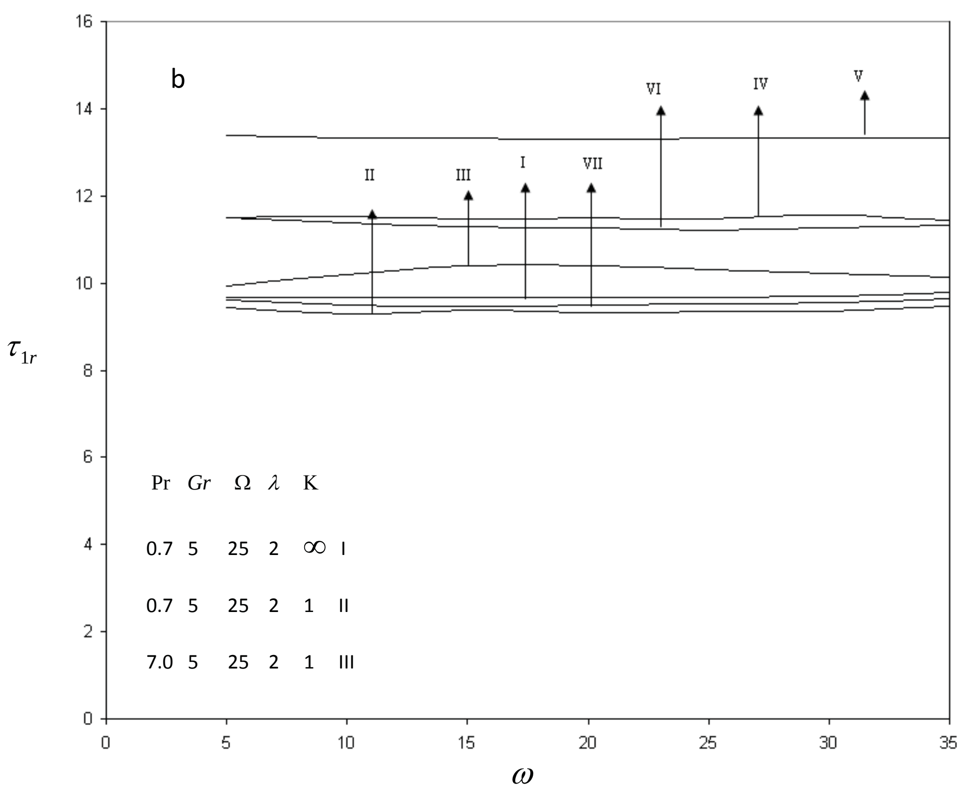


Fig. 5 a,b : The amplitude τ_{1r} of unsteady shear stresses for small and large rotations at $t = \frac{\pi}{4}$.

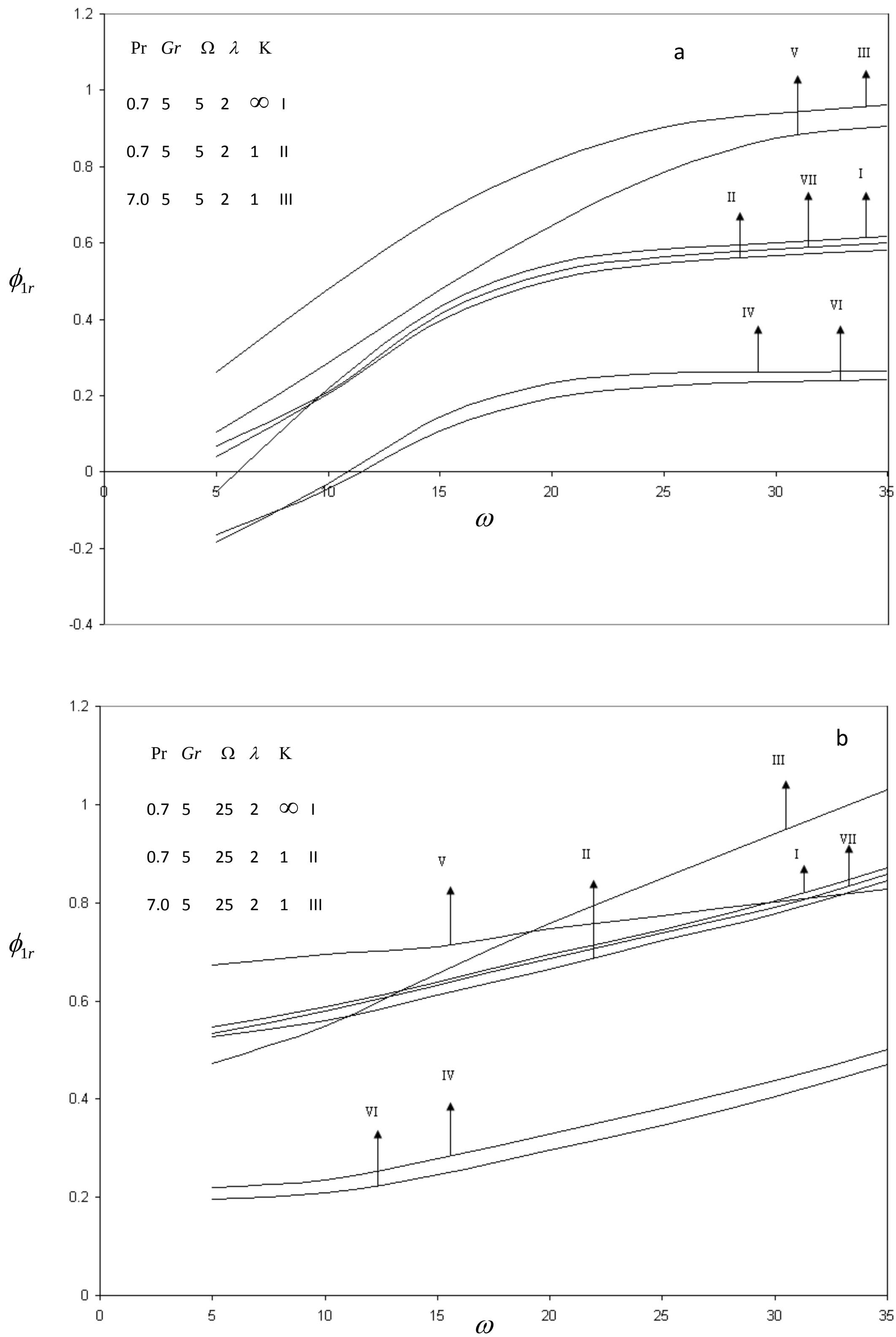


Fig. 6 a, b : The phase difference ϕ_{1r} of unsteady shear stresses for small and large rotations at $t = \frac{\pi}{4}$.

Notes

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New Finite Integrals of Generalized Meliin-Barnes Type of Contour Integrals

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Abstract - In the present paper, we obtain three new finite integral formulas. These formulas involve the product of a general class of polynomials and the generalized Meliin-Barnes type of contour integrals. Mainly we are using series representation of the $\bar{H}$-function given by Agarwal [14], Agarwal and Jain [13]. These integral formulas are unified in nature and act as the key formulas from which we can obtain as their special cases. By giving suitable values to the parameters, our main integral formulas are reduces to the Fox H-function, the G-function and generalized wright hypergeometric function.

Keywords : *$\bar{H}$-function, general class of polynomial, generalized wright hypergeometric function.*

GJSFR-F Classication : (MSC 2000) 33C45, 33C60

NEW FINITE INTEGRALS OF GENERALIZED MELIIN-BARNES TYPE OF CONTOUR INTEGRALS

Strictly as per the compliance and regulations of :

Research | Diversity | Ethics

Ref.
1. A.A. Inayat-Hussain, New properties of hypergeometric series derivable from Feynman integrals: I. Transformation and reeducation formulae, J.Phys.A:Math.Gen.20 (1987), 4109-4117.
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New Finite Integrals of Generalized Meliin-Barnes Type of Contour Integrals

Praveen Agarwal [α] & Mehar Chand [σ]

Abstract - In the present paper, we obtain three new finite integral formulas. These formulas involve the product of a general class of polynomials and the generalized Meliin- Barnes type of contour integrals. Mainly we are using series representation of the $\bar{H}$-function given by Agarwal [14], Agarwal and Jain [13]. These integral formulas are unified in nature and act as the key formulas from which we can obtain as their special cases. By giving suitable values to the parameters, our main integral formulas are reduces to the Fox H-function, the G-function and generalized wright hypergeometric function.

Keywords : $\bar{H}$ -function, general class of polynomial, generalized wright hypergeometric function.

I. Introduction

In 1987, Inayat-Hussain [1,2] was introduced generalization form of Fox's H-function, which is popularly known as $\bar{H}$-function. Now $\bar{H}$-function stands on fairly firm footing through the research contributions of various authors [1-3, 9- 10, 13-15]. $\bar{H}$-function is defined and represented in the following manner [10].

$$\bar{H}^{m,n}_{p,q}[z] = \bar{H}^{m,n}_{p,q}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right. \right] = \frac{1}{2\pi i}\int_L z^{\xi}\bar{\phi}(\xi)\,d\xi \qquad (z \neq 0) \tag{1.1}$$

where

$$\bar{\phi}(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

It may be noted that the $\bar{\phi}(\xi)$ contains fractional powers of some of the gamma function and m,n,p,q are integers such that $1 \le m \le q, 1 \le n \le p$ $(\alpha_j)_{1,p},(\beta_j)_{1,q}$ are positive real numbers and $(A_j)_{1,n},(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(\alpha_j)_{1,p}$ and $(\beta_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\bar{H}$-function can be seen in the papers [9, 10]. The behavior of the $\bar{H}$-function for small values of $|z|$ follows easily from a result given by Rathie [3]:

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$\overline{\mathrm{H}}_{\mathrm{p,q}}^{\mathrm{m,n}}[z]=o\left(|z|^{\alpha}\right);$ Where

$$\alpha=\min_{1\le j\le m}\operatorname{Re}\left(\frac{b_j}{\alpha_j}\right),|z|\to 0 \tag{1.3}$$

$$\mu_1=\sum_{j=1}^{m}|B_j|+\sum_{j=m+1}^{q}|b_jB_j|-\sum_{j=1}^{n}|a_jA_j|-\sum_{j=n+1}^{q}|A_j|>0\,,\,0<|z|<\infty \tag{1.4}$$

The following function which follows as special cases of the $\overline{\mathrm{H}}$-function will be required in the sequel [10]

$${}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};z\right]=\overline{H}_{p,q+1}^{1,p}\left[-z\left|\begin{matrix}(1-a_j,\alpha_j,A_j)_{1,p}\\(0,1),(1-b_j,\beta_j,B_j)_{1,q}\end{matrix}\right.\right] \tag{1.5}$$

The general class of polynomials $S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]$ will be defined and represented as follows [6, p.185, eqn. (7)]:

$$S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]=\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}x^{l_i} \tag{1.6}$$

where $n_1,\dots,n_r=0,1,2,\dots;m_1,\dots m_r$ are arbitrary positive integers, the coefficients $A_{n_i,l_i}(n_i,l_i\ge 0)$ are arbitrary constants, real or complex. $S_{n_1,\dots,n_r}^{m_1,\dots,m_r}[x]$ yields a number of known polynomials as its special cases. These includes, among other, the Jacobi polynomials, the Bessel Polynomials, the Lagurre Polynomials, the Brafman Polynomials and several others [8, p. 158-161].

The following formulas [11, p.77, Eqs. (3.1), (3.2) & (3.3)] will be required in our investigation.

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad\left(a>0;b\ge 0;c+4ab>0;\operatorname{Re}(p)+1/2>0\right) \tag{1.7}$$

$$\int_0^\infty\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad\left(a\ge 0;b>0;c+4ab>0;\operatorname{Re}(p)+1/2>0\right) \tag{1.8}$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1}dx=\frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\quad\left(a>0;b>0;c+4ab>0;\operatorname{Re}(p)+1/2>0\right) \tag{1.9}$$

II. Main Integrals

First Integral

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i\,!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}$$

Ref.

6. H.M.Srivastava, A multilinear generating function for the Konhauser sets of biorthogonal polynomials suggested by the Laguerre polynomials, Paci c J.Math.117, (1985), 183-191.
10. K.C. Gupta, R. Jain and R. Agarwal, On existence conditions for a generalized Mellin-Barnes type integral Natl Acad Sci Lett. 30(5-6) (2007), 169-172.

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{w}}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_i l_i,w;1\right),\left(a_j,\alpha_j,A_j\right)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},\left(-u-\sum_{i=1}^{r}v_i l_i,w;1\right)\end{matrix}\right.\right] \tag{2.1}$$

The above result will be converge under the following conditions

i. $a>0;b\geq 0;c+4ab>0$ and v_i,w are positive integers.

ii. $\operatorname{Re}\left[u+w\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z|<\frac{1}{2}\mu_1\pi$, where μ_1 is given by equation (1.4)

Second Integral

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_i l_i}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{w}}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_i l_i,w;1\right),\left(a_j,\alpha_j,A_j\right)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},\left(-u-\sum_{i=1}^{r}v_i l_i,w;1\right)\end{matrix}\right.\right] \tag{2.2}$$

The above result will be converge under the following conditions

i. $a\geq 0;b>0;c+4ab>0$ and v_i,w are positive integers.

ii. $\operatorname{Re}\left[u+w\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

iii. $|\arg z|<\frac{1}{2}\mu_1\pi$, where μ_1 is given by equation (1.4)

Third Integral

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_i l_i}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{w}}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_i l_i,w;1\right),\left(a_j,\alpha_j,A_j\right)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \left(b_j,\beta_j\right)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},\left(-u-\sum_{i=1}^{r}v_i l_i,w;1\right)\end{matrix}\right.\right] \tag{2.3}$$

The above result will be converge under the following conditions

i. $a>0;b>0;c+4ab>0$ and v_i,w are positive integers.

ii. $\operatorname{Re}\left[u+w\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

Notes

iii. $|\arg z| < \frac{1}{2}\mu_1\pi$, where μ_1 is given by equation (1.4)

Proof : To prove the first integral, we express $\overline{\mathrm{H}}$-function occurring on the L.H.S. of equation (2.1) in terms of Mellin-Barnes type of contour integral given by equation (1.1) and the general class of polynomials $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ in series form with the help of equation (1.6) and then interchanging the order of integration and summation, we get:

$$\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{2\pi i}\int_L\bar{\phi}(\xi)\left\{\int_0^\infty\left[\left(ax+\frac{b}{x^2}\right)^2+c\right]^{-u-\sum_{i=1}^{r}v_il_i-w\xi-1}dx\right\}z^\xi d\xi \tag{2.4}$$

Further using the result (1.7) the above integral becomes

$$\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}\frac{1}{2\pi i}\int_L\bar{\phi}(\xi)\frac{\left\{\Gamma\left(1/2+u+\sum_{i=1}^{r}v_il_i+w\xi\right)\right\}^1}{\left\{\Gamma\left(1+u+\sum_{i=1}^{r}v_il_i+w\xi\right)\right\}^1}\left[\frac{z}{(4ab+c)^w}\right]^\xi d\xi \tag{2.5}$$

Then interpreting with the help of (1.1) and (2.5) provides first integral.

The proof of second and third integral can be developing on the lines similar to those given with first integral with the help of the result (1.8) and (1.9) respectively.

(3.1) If we put $A_j = B_j = 1$, $\overline{\mathrm{H}}$ - function reduces to Fox's H-function [7, p. 10, Eqn. (2.1.1)], then the equation (2.1), (2.2) and (2.3) takes the following form.

$$\int_0^\infty\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]H_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \tag{3.1.1}$$

$$\int_0^\infty\frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]H_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \tag{3.1.2}$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r}y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]H_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right] \tag{3.1.3}$$

Ref.

7. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publishers, New Dehli, Madras (1982).

The Conditions of validity of (3.1.1), (3.1.2) and (3.1.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.2) By applying the results given in (2.1), (2.2) and (2.3) to the case of Hermite polynomials [4, 5] by setting $S_n^2(x) \to x^{n/2} H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $m_1,...,m_r = 2; n_1,...,n_r = n; r = 1; v_i = v, y_i = y, A_{n_i,l_i} = (-1)^l$, we have the following interesting results.

Notes

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{1}{y}\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{v}}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{vl}}\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \quad (3.2.1)$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{1}{y}\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{v}}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{vl}}\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \quad (3.2.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{1}{y}\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{v}}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{vl}}\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \quad (3.2.3)$$

The Conditions of validity of (3.2.1), (3.2.2) and (3.2.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.3) By applying the our results given in (2.1), (2.2) and (2.3) to the case of Lagurre polynomials [4, 5] by setting $S_n^2(x) \to L_n^{(\alpha)}[x]$ in which $m_1,...,m_r = 1; n_1,...,n_r = n; r = 1; v_i = v, y_i = y, A_{n_i,l_i} = \binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}$, we have the following interesting results.

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} L_n^{(\alpha)}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}\frac{(y)^l}{(4ab+c)^{vl}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j,A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j,B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \quad (3.3.1)$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} L_n^{(\alpha)}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}\frac{(y)^l}{(4ab+c)^{vl}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \qquad (3.3.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} L_n^{(\alpha)}\left[y\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v}\right]\overline{H}_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_l}\frac{(y)^l}{(4ab+c)^{vl}}$$

$$\overline{H}_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}(1/2-u-vl,w;1),(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q},(-u-vl,w;1)\end{matrix}\right.\right] \qquad (3.3.3)$$

The Conditions of validity of (3.3.1), (3.3.2) and (3.3.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.4) If we put $A_j = B_j = 1; \alpha_j = \beta_j = 1$, in (1.1) then the $\overline{\mathrm{H}}$-function reduces to the general type of G-function [12] i.e. $\overline{H}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,1,1)_{1,n},(a_j,1)_{n+1,p}\\(b_j,1,1)_{1,m},(b_j,1)_{m+1,q}\end{matrix}\right.\right]=G\left[z\left|\begin{matrix}(a_j,1)_{1,p}\\(b_j,1)_{1,q}\end{matrix}\right.\right]$, So using same assumptions in the equations (2.1), (2.2) and (2.3) then they takes the following form.

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]G_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right]$$

(3.4.1)

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]G_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_r=0}^{[n_r/m_r]}\prod_{i=1}^{r}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}\frac{(y_i)^{l_i}}{(4ab+c)^{v_il_i}}G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r}v_il_i,w;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-u-\sum_{i=1}^{r}v_il_i,w;1\right)\end{matrix}\right.\right]$$

(3.4.2)

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right]G_{p,q}^{m,n}\left[z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

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$$= \frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} \frac{(y_i)^{l_i}}{(4ab+c)^{v_i l_i}} G^{m,n+1}_{p+1,q+1}\left[\frac{z}{(4ab+c)^w}\left|\begin{matrix}\left(1/2-u-\sum_{i=1}^{r} v_i l_i, w;1\right),(a_j,1)_{1,p}\\ (b_j,1)_{1,q},\left(-u-\sum_{i=1}^{r} v_i l_i, w;1\right)\end{matrix}\right.\right] \quad (3.4.3)$$

The Conditions of validity of (3.4.1), (3.4.2) and (3.4.3) easily follow from those given in (2.1), (2.2) and (2.3).

(3.5) If we put $n=p, m=1, q=q+1, b_1=0, \beta_1=1, a_j=1-a_j, b_j=1-b_j$, in (1.1) then the $\overline{\mathrm{H}}$-function reduces to generalized wright hypergeometric function [16] i.e. $\overline{H}^{1,p}_{p,q+1}\left[z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right] = {}_p\overline{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\right]$, using same assumptions in the equations (2.1), (2.2) and (2.3) then they takes the following form.

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right] {}_p\overline{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{u+1/2}} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} \frac{(y_i)^{l_i}}{(4ab+c)^{v_i l_i}} {}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-u-\sum_{i=1}^{r} v_i l_i, w;1\right),(a_j,\alpha_j;A_j)_{1,p}\\ (b_j,\beta_j;B_j)_{1,q},\left(-u-\sum_{i=1}^{r} v_i l_i, w;1\right)\end{matrix};-\frac{z}{(4ab+c)^w}\right] \quad (3.5.1)$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right] {}_p\overline{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{u+1/2}} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} \frac{(y_i)^{l_i}}{(4ab+c)^{v_i l_i}} {}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-u-\sum_{i=1}^{r} v_i l_i, w;1\right),(a_j,\alpha_j;A_j)_{1,p}\\ (b_j,\beta_j;B_j)_{1,q},\left(-u-\sum_{i=1}^{r} v_i l_i, w;1\right)\end{matrix};-\frac{z}{(4ab+c)^w}\right] \quad (3.5.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-u-1} S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[\prod_{i=1}^{r} y_i\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-v_i}\right] {}_p\overline{\psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\left\{\left(ax+\frac{b}{x}\right)^2+c\right\}^{-w}\right]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{u+1/2}} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} \frac{(y_i)^{l_i}}{(4ab+c)^{v_i l_i}} {}_{p+1}\overline{\psi}_{q+1}\left[\begin{matrix}\left(1/2-u-\sum_{i=1}^{r} v_i l_i, w;1\right),(a_j,\alpha_j;A_j)_{1,p}\\ (b_j,\beta_j;B_j)_{1,q},\left(-u-\sum_{i=1}^{r} v_i l_i, w;1\right)\end{matrix};-\frac{z}{(4ab+c)^w}\right] \quad (3.5.3)$$

The Conditions of validity of (3.5.1), (3.5.2) and (3.5.3) easily follow from those given in (2.1), (2.2) and (2.3).

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Notes

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An Integral Transformation Involving a Certain Product of Special Functions

By Poonia,M.S.
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Abstract - The main object of this paper is to obtain integral transformation using certain product of multivariable H-function with a general class of polynomials. The result established in this paper are of general nature and hence encompass several cases of interest.

Keywords : *H-function, Lauricella function and M-series.*

GJSFR-F Classication : *MSC 2010: 33C65*

AN INTEGRALTRANSFORMATION INVOLVING A CERTAIN PRODUCT OF SPECIAL FUNCTIONS

Strictly as per the compliance and regulations of :

An Integral Transformation Involving a Certain Product of Special Functions

Poonia,M.S.

Abstract - The main object of this paper is to obtain integral transformation using certain product of multivariable H-function with a general class of polynomials. The result established in this paper are of general nature and hence encompass several cases of interest.

Keywords : H-function, Lauricella function and M-series.

I. Introduction

Integrals with Fox's H-function, M-series and multi variable H-function were studied by many authors. We have the following series representation of the H-function by Skibińki [4]

$$H_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_p,e_p)\\(b_q,f_q)\end{matrix}\right.\right]=\sum_{N=1}^{m}\sum_{s=0}^{\infty}\frac{(-1)^s z^{\eta_s}}{f_N\, N!}\phi(\eta_s), \tag{1}$$

where

$$\phi(\eta_s)=\prod_{i=1}^{m}\Gamma(b_i-f_i\eta_s)\prod_{i=1}^{n}\Gamma(1-a_i+e_i\eta_s)$$

$$\left\{\prod_{i=m+1}^{q}\Gamma(1-b_i+f_i\eta_s)\prod_{i=n+1}^{p}\Gamma(a_i-e_i\eta_s)\right\}^{-1}$$

and

$$\eta_s=\frac{b_N+s}{f_N}.$$

The following results of Srivastava and Daoust [1, eq.(1.2), p.15], Slater [2, p.79, eq. (2.5.27) and Chaurasia [3, p.194, eq. (2.3)] respectively also required in our investigations:

(a)

$$F_{\sigma\,:N';\ldots;N^{(s)};1,1}^{v:M';\ldots;M^{(s)};0,0}\left(\begin{matrix}[(\alpha_v):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:(m'):\rho];\ldots;\\ [(\beta_\sigma):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

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$$\left.\begin{matrix}[(m^{(s)}:\rho^{(s)}]; --- \quad ;---- \quad ; \\ [(\ell^{(s)}:\tau^{(s)}];[\alpha+1,1];[\beta+1,1)];\end{matrix} z_1^{'},\ldots,z_s^{'},-xt,(1-x)t\right)$$

$$=\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{v}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\sigma}(\beta_j)_{n\mu_j}}P_n^{(\alpha,\beta)}(1-2x)$$

$$F_{\sigma:N',\ldots,N^{(s)}}^{\nu:M';\ldots;M^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_u):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots; \\ [(\beta_\sigma+n\mu_\sigma):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;\end{matrix}\begin{matrix}[(m^{(s)}):\rho^{(s)}]; \\ [(\ell^{(s)}):\tau^{(s)}];\end{matrix} z_1^{'},\ldots,z_s^{'}\right)t^n \tag{2}$$

(b)

$${}_4F_3\left[\begin{matrix}a,b,\frac{m+d}{2},\frac{m+b-1}{2} \\ a+b,m,d\end{matrix};4y(1-y)\right]$$

$$=\sum_{k=0}^{\infty}\frac{(m+d-1)_k}{(a+b)_k}m_k\,y^k, \tag{3}$$

where m_k is given by

$${}_2F_1(a,b;m;y)\,{}_2F_1(a,b;d;y)=\sum_{k=0}^{\infty}m_k\,y^k, \tag{4}$$

(c)

$$\int_0^1 y^k H\left(y^{h_1}z_1,\ldots,y^{h_r}z_r\right)H_{P_1,Q_1}^{M_1,N_1}\left(xy^{L_1}\left|\begin{matrix}(a_p,e_p) \\ (b_q,f_q)\end{matrix}\right.\right){}_{P_2}M_{Q_2}^{\alpha'}(\tau_1 y^{L_2})\;S_V^U[\tau_2 y^{L_3}]\,dy$$

$$=\sum_{\sigma=0}^{M_1}\sum_{k_1,k_2=0}^{\infty}\sum_{k_3=0}^{[V/U]}\frac{(-1)^{k_1}x^{\eta k_1}}{f_\sigma^{'}}\phi(\eta k_1)\frac{(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_1)^{k_2}(-V)_{Uk_3}A_{V,k_3}}{(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\Gamma(\alpha' k_2+1)\;k_2!\,k_3!}\tau_2^{k_3}$$

$$.\;H_{A+1,C+1:(B',D');\ldots;(B^{(r)},D^{(r)})}^{0,\lambda+1\;:(u',v')\;;\ldots;(u^{(r)},v^{(r)})}\left[\begin{matrix}[-k-L_1\eta_{k_1}-L_2k_2-k_3L_3:h_1,\ldots,h_r], \\ [-k_1-1-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}] \\ [(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]\end{matrix},z_1,\ldots,z_r\right], \tag{5}$$

where $h_i>0,\mathrm{Re}\left(1+L_1\frac{b_{j'}}{f_{j'}}+\sum_{i=1}^{r}h_i\,d_j^{(i)}/\delta_j^{(i)}\right)>0,|\arg(z_i)|<\frac{T_i\pi}{2},T_i>0,$ $i=1,\ldots,r;$ $j=1,\ldots,u^{(i)},j'=1,\ldots,P_1,P_2\le Q_2,|\tau_2|<1,$ U is an arbitrary positive integer, the

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coefficients A_{V,k_3} (V, k_3 ¿ 0) are arbitrary constants, real or complex. L_1, L_2, L_3 ¿ 0 ,

$$|\arg \tau_1| < \frac{1}{2}\pi T', T' = \sum_{i=1}^{N_1} e_i - \sum_{i=N_1+1}^{P_1} e_i + \sum_{i=1}^{M_1} f_i - \sum_{i=M_1+1}^{Q_1} f_i \tag{6}$$

The result (5) is a generalization of a result of Chaurasia [2, p.194, eq. (2.3)]. Proof process used is the same.

II. Main Results

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta} F^{v:M';\ldots;M^{(s)};0,0}_{\in:N';\ldots;N^{(s)};1,1}\left(\begin{matrix}[(\alpha_V):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:(m'):\rho];\ldots; \\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}:\rho^{(s)}];\ ---\ \ ;----\ \ ; \\ [(\ell^{(s)}:\tau^{(s)}];[\alpha+1,1];[\beta+1,1)];\end{matrix} z_1',\ldots,z_s',-xt,(1-x)t\right) H\left(z_1 x^{h_1},\ldots,z_r x^{h_r}\right)$$

$$\cdot H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1}) \\ (b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2 x^{L_2}]S^U_V[\tau_3 x^{L_3}]dx$$

$$= \sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{k_1,k_2,n=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})}{\tau_4 f_{\tau_4}k_1!k_3!k_2!}$$

$$\cdot\frac{(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}}{(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\Gamma(\alpha' k_2+1)\,k_3!}$$

$$\cdot F^{\nu:M';\ldots;M^{(s)}}_{\sigma:N',\ldots,N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_V):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}]; \\ [(\beta_\sigma+n\mu_\sigma):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau']\ \ ;\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix} z_1',\ldots,z_s'\right)$$

$$\frac{(-t)^{\eta}\Gamma(\beta+n+1)}{n!}H^{0,\lambda+2\ \ :(u',v')\ ;\ldots;(u^{(r)},v^{(r)})}_{A+2,C+2:(B',D');\ldots;(B^{(r)},D^{(r)})}\left(\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-k_3L_3:h_1,\ldots,h_r], \\ [-\sigma-1-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}] \\ [(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]\end{matrix},z_1,\ldots,z_r\right) \tag{7}$$

,

where $\mathrm{Re}(\beta)>-1, \mathrm{Re}\left(\sigma+L_1\frac{b_{j'}}{f_{j'}}+\sum_{i=1}^{r}h_i d_j^{(i)}/\delta_j^{(i)}\right)>0, |\arg(z_i)|<\frac{T_i\pi}{2}, T_i>0, i=1,\ldots,r;$ $j=1,\ldots,u^{(i)}, j'=1,\ldots,P_1, P_2\le Q_2, |\tau_2|<1,$ U is an arbitrary positive integer, the coefficients A_{V,k_3} (V, k_3 ¿ 0) are arbitrary constants, real or complex. L_1, L_2, L_3 ¿ 0 , $|\arg\tau_1|<\frac{1}{2}\pi T'$ $\left[T'=\sum_{i=1}^{N_1}e_i-\sum_{i=N_1+1}^{P_1}e_i+\sum_{i=1}^{M_1}f_i-\sum_{i=M_1+1}^{Q_1}f_i\right]$, and the series on the right is convergent.

Ref.

2. Slater, L.J. Generalized hypergeometric functions, Cambridge University Press (1966).

Proof of main result

Multiplying both sides of (2) by $x^{\sigma-1}(1-x)^{\beta}\, H(z_1 x^{h_1},\ldots,z_r x^{h_r})$. $H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]\,{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2 x^{L_2}]S^U_V[\tau_3 x^{L_3}]$ and integrating it with respect to x from 0 to 1. Evaluating the right hand side thus obtained by interchanging the order of integration and summations (which is justified due to the absolute convergence of the integral involved in the process) and then integrating the inner integral with the help of the following result [5, eq. 2.2, p.131]

$$\int_0^1 x^{\in}(1-x)^{\beta}P_n^{(\alpha,\beta)}(1-2x)\,H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2 x^{L_2}]S^U_V[\tau_3 x^{L_3}]$$

$$\cdot H(z_1 x^{\sigma_1},\ldots,z_r x^{\sigma_r})\,dx$$

$$=\sum_{\tau_4}^{M_1}\sum_{k_1,k_2=0}^{\infty}\sum_{k_3=0}^{[V/U]}\frac{(-1)^n\,\Gamma(\beta+n+1)(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{n!\,\tau_4!\,f_{\tau_4}\;k_1!k_2!k_3!(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\,\Gamma(\alpha' k_2+1)}$$

$$\cdot(-V)_{Uk_3}\,A_{V,k_3}(\tau_3)^{k_3}\,H^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A,C:(B',D');\ldots;(B^{(r)},D^{(r)})}\left(\begin{matrix}[-\in-L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\ldots,\sigma_r],\\ [-\beta-\in-n-1-L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\ldots,\sigma_r],\end{matrix}\right.$$

$$\left.\begin{matrix}[\alpha-\in-L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\ldots,\sigma_r],[(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi']\;;\ldots;[(b^{(r)}):\phi^{(r)}]:\\ [\alpha+n-\in L_1\eta_{k_1}-L_2k_2-L_3k_3:\sigma_1,\ldots,\sigma_r],[(c):\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}]:\end{matrix}\;z_1,\ldots,z_r\right),\tag{8}$$

where $\mathrm{Re}(\beta)>-1,\ \mathrm{Re}\left(\in+L_1\frac{b'_j}{f'_j}+\sum_{i=1}^{r}\sigma_i\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)>-1,\ \sigma_i>0, L_1,L_2,L_3>0, |\arg(z_i)|$ $<\frac{1}{2}T_i\pi, \tau_1,\tau_2>0, |\arg\tau_1|<\frac{1}{2}T'\pi, i=1,\ldots,r;\ j=1,\ldots,u^{(i)},\ j'=1,\ldots,Q_2$, we arrive the required result (7).

1.8 SPECIAL CASES OF (7)

(i) Letting $\lambda=A, u^{(i)}=1, V^{(i)}=B^{(i)}, D^{(i)}=D^{(i)}+1\,\forall\, i=1,\ldots,r$ in (7), we find

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta}\,F^{\nu:M',\ldots,M^{(s)};0;0}_{\in:N',\ldots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\ldots;\\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]:\quad -\quad;-\quad;\\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix}\;z'_1,\ldots,z'_s,-xt,(1-x)t\right)$$

$$.F^{A:B',\ldots,B^{(r)}}_{C:D',\ldots,D^{(r)}}\left(\begin{matrix}[1-(a):\theta',\ldots,\theta^{(r)}]\;:[1-(b'):\phi'];\ldots;[1\text{-}(b^{(r)}):\phi^{(r)}]\\ [1-(c):\psi',\ldots,\psi^{(r)}]:[1-(d'):=\delta'];\ldots;[1\text{-}(d^{(r)}):\delta^{(r)}]\end{matrix}\;-z_1x^{h_1},\ldots,z_rx^{h_r}\right)$$

Ref.

5. **Srivastava, H.M. and Panda, R.** Expansion theorem for the H-function of several complex variables, J. Reine Angew. Math. **288**, (1976), 129-145.

Notes

$$\cdot H_{P_1,Q_1}^{M_1,N_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right] {}_{P_2}\overset{\alpha'}{M}_{Q_2}[\tau_2 x^{L_2}]S_V^U[\tau_3 x^{L_3}]\,dx$$

$$=\sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}$$

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}k_1!k_2!k_3!(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\Gamma(\alpha'k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F_{\in:N';\ldots;N^{(s)}}^{\nu:M';\ldots;M^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_t+\eta\mu_t):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right)$$

$$\frac{(-t)^n\,\Gamma(\beta+n+1)}{n!}F_{C+2:D';\ldots;D^{(r)}}^{A+2:B';\ldots;B^{(r)}}\left(\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],\\ [1-(c):\psi',\ldots,\psi^{(r)}],\end{matrix}\right.$$

$$[1+\alpha+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,\ldots,h_r],[1-(a):\theta',\ldots,\theta^{(r)}];$$

$$[1-\sigma\text{-}L\,\eta_{k_1}-L_2k_2-L_3k_3+\alpha+n:h_1,\ldots,h_r],[-\beta-h-\sigma-L\,\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],$$

$$\left.\begin{matrix}[1-(b'):\phi'];\ldots;[1-(b^{(r)}):\phi^{(r)}];\\ [(1-(d'):\delta'];\ldots;[1\text{-}d^{(r)}):\delta^{(r)}];\end{matrix}\; -z_1,\ldots,-z_r\right) \tag{9}$$

provided that $\operatorname{Re}(\sigma)>0, \operatorname{Re}(\beta)<-1, h_i>0, L_1, L_2, L_3, \tau_2, \tau_3>0, i=1,\ldots,r, |t|<1$ and the series on the right is convergent.

(ii) Taking r = 2, the result in (9) reduces to the following integral

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta}F_{\in:N',\ldots,N^{(s)};1;1}^{\nu:M',\ldots,M^{(s)};0;0}\left(\begin{matrix}[(\alpha_\nu):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\ldots;\\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]:\;[\;-\;;\;-\;];\\ [(\ell^{(s)}):\tau^{(s)}];[\alpha+1,1];[\beta+1,1];\end{matrix}\; z_1',\ldots,z_s',-xt,(1-x)t\right)$$

$$.F_{C:D',D''}^{A:B',B''}\left(\begin{matrix}[1-(a):\theta',\theta'']\;:[1-(b'):\phi'];[1\text{-}(b''):\phi'']\\ [1-(c):\psi',\psi'']:[1-(d'):=d'];[1\text{-}(d''):\delta'']\end{matrix}\; -z_1x^{h_1},-z_2x^{h_2}\right)$$

$$.\;H_{P_1,Q_1}^{M_1,N_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right] {}_{P_2}\overset{\alpha'}{M}_{Q_2}[\tau_2 x^{L_2}]S_V^U[\tau_3 x^{L_3}]\,dx$$

$$=\sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}$$

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$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}\,k_1!\,k_2!\,k_3!(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\,\Gamma(\alpha'k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$.F^{\nu:M';\ldots;M^{(s)}}_{\in:N';\ldots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_\in+\eta\mu_\in):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\; z_1',\ldots,z_s'\right).\frac{(-t)^n\,\Gamma(\beta+n+1)}{n!}$$

$$.F^{A+2:B',B''}_{C+2:D',D''}\left[\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2],\\ [1-(c):\psi',\psi''],\end{matrix}\right.$$

$$\begin{matrix}[1+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,h_2],[1-(a):\theta',\theta''];\\ [1-\sigma-L\,\eta_{k_1}-L_2k_2-L_3k_3+\alpha+n:h_1,h_2],[-\beta-h-\sigma-L\,\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2],\end{matrix}$$

$$\left.\begin{matrix}[1-(b'):\phi'];\ldots;[1-(b''):\phi''];\\ [(1-(d'):\delta'];\ldots;[1-d''):\delta''];\end{matrix}\;-z_1,-z_2\right),\qquad(10)$$

where $\operatorname{Re}(\sigma)>0,\operatorname{Re}(\beta)<-1,h_i>0,L_1,L_2,L_3,\tau_2,\tau_3>0,i=1,\ldots,r,|t|<1$ and the multiple series on the right of (10) converges absolutely.

(iii) Putting A = C = λ = 0 in (9), we have

$$\int_0^1 x^{\sigma-1}(1-x)^\beta\,F^{\nu:M',\ldots,M^{(s)};0;0}_{\in:N',\ldots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\ldots;\\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]:\;-\;;-\;;\\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix}\;z_1',\ldots,z_s',-xt,(1-x)t\right)$$

$$\cdot H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}\overset{\alpha'}{M}{}_{Q_2}[\tau_2x^{L_2}]S^U_V[\tau_3x^{L_3}]$$

$$\cdot\prod_{i=1}^r\left\{H^{(u^{(i)},v^{(i)})}_{B^{(i)},D^{(i)}}\left[z_1x^{h_i}\left|\begin{matrix}[(b^{(i)}):\phi^{(i)}]\\ [(d^{(i)}):\delta^{(i)}]\end{matrix}\right.\right]\right\}dx$$

$$=\sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}\frac{(-t)^n\Gamma(\beta+n+1)}{n!}$$

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}\,k_1!\,k_2!\,k_3!(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\,\Gamma(\alpha'k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F^{\nu:M';\ldots;M^{(s)}}_{\in:N';\ldots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_t+\eta\mu_t):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix}\;z_1',\ldots,z_s'\right)$$

$$H^{0,2:(u',v');\ldots;(u^{(r)},v^{(r)})}_{2,2:(B',D');\ldots;(B^{(r)},D^{(r)})}\left[\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,\ldots,h_r],\\ [(c):\psi',\ldots,\psi^{(r)}],\end{matrix}\right.$$

Notes

$$[1+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,...,h_r],[1-(a):\theta',...,\theta^{(r)}];$$
$$[1+\sigma+\alpha+n:h_1,...,h_r],[-\beta-n-L\eta_{k_1}-L_2k_2-L_3k_3:h_1,...,h_r],$$
$$\left.\begin{matrix}[(b'):\phi'];...;[(b^{r)}'):\phi^{(r)}']; \\ [((d'):\delta'];...;[d^{(r)}):\delta^{(r)}'];\end{matrix} -z_1,...,z_r\right], \tag{11}$$

Notes

valid under the conditions obtainable from (9).

(iv) Taking r = 2 in (11), we have

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta}\, F^{\nu:M',...,M^{(s)};0;0}_{\in:N',...,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',...,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];...; \\ [(\beta_\in):\zeta',...,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];...;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]: \quad - \quad ;- \quad ; \\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix} z_1^{'},...,z_s^{'},-xt,(1-x)t\right)$$

$$\cdot H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1 x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1}) \\ (b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}M^{\alpha'}_{Q_2}[\tau_2 x^{L_2}]S^U_V[\tau_3 x^{L_3}]$$

$$\cdot H^{0,\lambda:(u',v');(u'',v'')}_{A,C:(B',D');(B'',D'')}\left[\begin{matrix}[(a):\theta',\theta'']:[(b'):\phi'];[(b''):\phi''] ; \\ [(c):=\psi',\psi'']:[(d'):\delta'];[(d''):\delta''];\end{matrix} z_1x^{h_1},z_2x^{h_2}\right]dx$$

$$=\sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{n\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}\frac{(-t)^n\Gamma(\beta+n+1)}{n!}$$

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}...(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}\,k_1!\,k_2!\,k_3!(b_1)_{k_2}...(b_{Q_2})_{k_2}\,\Gamma(\alpha'k_2+1)}(-V)_{Uk_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F^{\nu:M';...;M^{(s)}}_{\in:N';...;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',...,\eta^{(s)}]:[(m'):\rho'];...;[(m^{(s)}):\rho^{(s)}]; \\ [(\beta_t+n\mu_t):\zeta',...,\zeta^{(s)}]:[(\ell'):\tau'];...;[(\ell^{(s)}):\tau^{(s)}];\end{matrix} z_1^{'},...,z_s^{'}\right)$$

$$H^{0,\lambda+2:(u',v');(u'',v'')}_{A+2,C+2:(B',D');(B'',D'')}\left[\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2], \\ [(c):\psi',\psi''],\end{matrix}\right.$$

$$[1+\sigma+L_1\eta_{k_1}+L_2k_2+L_3k_3:h_1,...,h_r],[1-(a):\theta',\theta''];$$

$$[1+\sigma+\alpha+n:h_1,h_2],[-\beta-n-L\eta_{k_1}-L_2k_2-L_3k_3:h_1,h_2],$$

$$\left.\begin{matrix}[(b'):\phi'];[(b''):\phi'']; \\ [((d'):\delta'];[d''):\delta''];\end{matrix} z_1,z_2\right], \tag{12}$$

where $\quad \mathrm{Re}(\beta)>-1, \mathrm{Re}\left(\alpha+h_1\frac{d'_j}{\delta'_j}+h_2\frac{d''_j}{\delta''_j}+L\frac{b_{j'''}}{f_{j'''}}\right)>0, j'=1,\ldots,u^{(i)}, j''=1,\ldots,u'',$

$j'''=1,\ldots,Q_2, T_1>0, T_2>0, |\arg(z_1)|<\frac{1}{2}T_1\pi, |\arg(z_2)|<\frac{1}{2}T_2\pi, |t|<1$ and the series on the right of (12) absolutely convergent.

Notes

(v) putting r=1 in (11),we find

$$\int_0^1 x^{\sigma-1}(1-x)^{\beta} F^{\nu:M',\ldots,M^{(s)};0;0}_{\in:N',\ldots,N^{(s)};1;1}\left(\begin{matrix}[(\alpha_\nu):\eta',\ldots,\eta^{(s)},\gamma,\gamma]:[(m'):\rho'];\ldots;\\ [(\beta_\in):\zeta',\ldots,\zeta^{(s)},\mu,\mu]:[(\ell'):\tau'];\ldots;\end{matrix}\right.$$

$$\left.\begin{matrix}[(m^{(s)}):\rho^{(s)}]: \quad - \quad ;- \quad ;\\ [(\ell^{(s)}):\tau^{(s)}][[\alpha+1,1];[\beta+1,1];\end{matrix} z'_1,\ldots,z'_s,-xt,(1-x)t\right)$$

$$.H^{u',v'}_{B',D'}\left[z_1x^{h_1}\left|\begin{matrix}(b'):\phi'\\(d'):\delta'\end{matrix}\right.\right] H^{M_1,N_1}_{P_1,Q_1}\left[\tau_1x^{L_1}\left|\begin{matrix}(a_{P_1},e_{P_1})\\(b_{Q_1},f_{Q_1})\end{matrix}\right.\right]{}_{P_2}\overset{\alpha'}{M}_{Q_2}[\tau_2x^{L_2}]S^U_V[\tau_3x^{L_3}]dx$$

$$=\sum_{\tau_4=1}^{M_1}\sum_{k_3=0}^{[V/U]}\sum_{n,k_1,k_2=0}^{\infty}\frac{\prod_{j=1}^{\nu}(\alpha_j)_{m\gamma_j}}{(\alpha+1)_n(\beta+1)_n\prod_{j=1}^{\in}(\beta_j)_{n\mu_j}}\frac{(-t)^n\Gamma(\beta+n+1)}{n!(\alpha+1)_n(\beta+1)_n}$$

$$\cdot\frac{(-1)^{k_1}(\tau_1)^{\eta_{k_1}}\phi(\eta_{k_1})(a_1)_{k_2}\ldots(a_{P_2})_{k_2}(\tau_2)^{k_2}}{\tau_4!f_{\tau_4}k_1!k_2!k_3!(b_1)_{k_2}\ldots(b_{Q_2})_{k_2}\Gamma(\alpha'k_2+1)}(-V)_{U,k_3}A_{V,k_3}(\tau_3)^{k_3}$$

$$\cdot F^{\nu:M';\ldots;M^{(s)}}_{\in:N';\ldots;N^{(s)}}\left(\begin{matrix}[(\alpha_\nu+n\gamma_\nu):\eta',\ldots,\eta^{(s)}]:[(m'):\rho'];\ldots;[(m^{(s)}):\rho^{(s)}];\\ [(\beta_t+n\mu_t):\zeta',\ldots,\zeta^{(s)}]:[(\ell'):\tau'];\ldots;[(\ell^{(s)}):\tau^{(s)}];\end{matrix} z'_1,\ldots,z'_s\right)$$

$$H^{u',v'+2}_{B'+2,D'+2}\left[z_1\left|\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3:h_1],\\ [(d'):\delta'],\end{matrix}\right.\right.$$

$$\left.\begin{matrix}[1-\sigma-L_1\eta_{k_1}-L_2k_2-L_3k_3+\alpha:h_1],[(b'):\phi']\\ [1-\sigma+\alpha+n+L\eta_{k_1}+L_2k_2+Lk_3:h_1],[-\sigma-\beta-n-L_1\eta_{k_1}-L_2k_2-Lk_3:h_1]\end{matrix}\right] \quad (13)$$

where $\quad \mathrm{Re}(\beta)>-1, h_1>0, \tau_2,\tau_3>0, L_1,L_2,L_3>0, \mathrm{Re}\left(\sigma+h_1d'_j/\delta'_j+L_1\frac{b'_j}{f'_j}\right)¿ \quad 0,$

$j=1,\ldots,u', j'=1,\ldots,Q_2, T_1>0, T_2>0, |t|<1, |\arg(z_1)|<\frac{1}{2}T_1\pi, |\arg\tau_1|<\frac{1}{2}T_2\pi$ and the series on the right of (13) is absolutely convergent.

Notes

III. Acknowledgement

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Mathematical Modeling of Thin-Layer Drying of Shrimp

By Hosain Darvishi, Asie Farhang, & Eisa Hazbavi
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Abstract - In this study, microwave drying behaviour of shrimp was investigated. The drying study showed that the times taken for drying of shrimp from the initial moisture contents of 3.103% (d.b.) to final moisture content of around 0.01% (d.b.) were 11.75, 7, 4.75 and 4 min in 200, 300, 400 and 500W, respectively. The drying data were fitted to 7 thin-layer drying models. The performances of these models were compared using the determination of coefficient (R^2), reduced chi-square (χ^2) and root mean square error (RMSE) between the observed and predicted moisture ratios. The results showed that Midilli model was found to satisfactorily describe the microwave drying curves of shrimp. The activation energy for moisture diffusion was found to be 12.834W/g.

Keywords : *microwave drying; shrimp; modeling.*

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MATHEMATICAL MODELING OF THIN-LAYER DRYING OF SHRIMP

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Hosain Darvishi[α], Asie Farhang[σ] & Eisa Hazbavi[ρ]

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Keywords : microwave drying; shrimp; modeling.

I. Introduction

Drying is probably the oldest and the most important method of food preservation practiced by humans. This process improves the food stability, since it reduces considerably the water and microbiological activity of the material and minimizes physical and chemical changes during its storage.

Dried shrimp is one of the most important exported marine products in many countries such as Thailand, China, Malaysia and United States. Most of the studies on drying kinetics of shrimp have focused on convective, superheated steam and heat-pump drying methods [1-8]. There is no available report regarding the effectiveness of intermittent microwave drying of shrimp compared to conventional drying techniques.

One of the most important aspects of drying technology is the modeling of the drying process. Drying is a complex thermal process in which unsteady heat and moisture transfer occur simultaneously. From an engineering point of view, it is important to develop a better understanding of the controlling parameters of this complex process. Mathematical models of the drying processes are used for designing new or improving existing drying systems or even for the control of the drying process. Therefore, the objective of this work was to evaluate a suitable drying model for describing the microwave drying process of shrimp.

II. Materials and Method

The shrimp samples used in this study were obtained from a local fish market, Tehran, Iran during the summer season of 2010. The selected samples were cleaned with tap water to make samples free from dust and foreign materials. In order to preserve its original quality, they were stored in a refrigerator at 4 ﬂC until drying experiments. The

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1. Namsanguan Y., Tia W., Devahastin S., and Soponronnarit, S; Drying kinetics and quality of shrimp undergoing different two-stage drying processes. Dry Tech, 22(4) (2004), 759-778.

average initial moisture contents of the shrimp samples were found to be 3.103% (d.b.), as determined by using convective oven at at 103ffi1 ˚C for 4h [3].

The drying was done in a microwave dryer developed for this purpose. The schematic of the experimental microwave drying set-up is given in Fig. 1. The dryer consists of a microwave oven M945, Samsung Electronics Ins, a variable speed fan and a digital balance. The dimensions of the microwave cavity were 327×370×207 mm.

The microwave oven was operated by a control terminal which could control both microwave power level and emission time. The microwave oven was operated by a control terminal which could control both microwave power level and emission time. In order to weigh the samples without taking them out of the oven, a weighing system was integrated to the oven. A digital balance (GF-600, A & D, Japan) which has a sensitivity of 0.01 g and a plastic disc was mounted to the bottom of the microwave oven. The disc was rotated at 5 rpm on a ball bearing shaft driven by an electrical motor. The presence of the rotating disc was necessary to obtain homogeneous drying and to decrease the level of the reflected microwaves on to the magnetrons. The oven has ventilation holes on the top as well as on the bottom. Air velocity was kept at a constant value of 1 m/s with an accuracy of ffi0.1 m/s measured with a Vane Probe anemometer AM- 4202 Lutron flowed perpendicular to the bed. Drying experiments were carried out with 200, 300, 400 and 500 W microwave power levels to investigate the effects of microwave power on drying of shrimp. Samples (46 ffi 0.5 g) were placed in a single layer on a rotating glass plate in the oven. Moisture loss of the samples was recorded by means of the balance at 15 s intervals until no discernible weight change was observed. Rotating was stopped by pulling back the driving disc when recording the weight data.

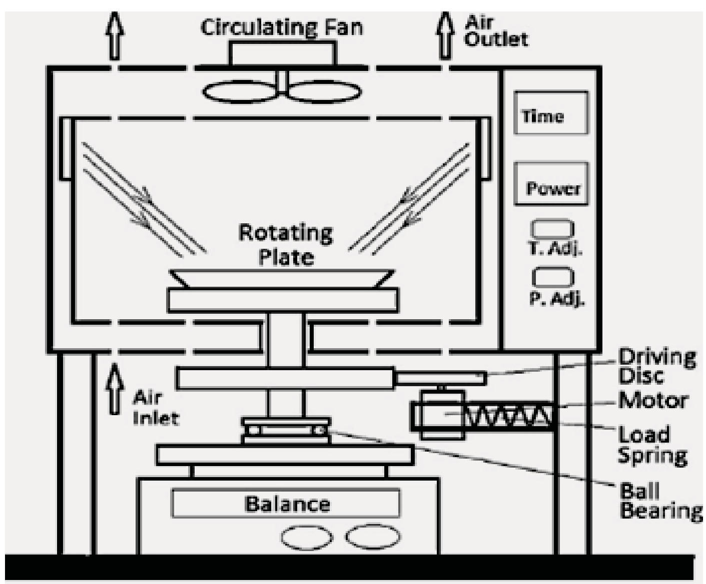

Fig. 1 : Schematic illustration of the microwave drying set-up

The moisture content of drying sample at time t can be transformed to be moisture ratio (MR):

$$MR = \frac{M_t - M_e}{M_0 - M_e} \tag{1}$$

where M_t, M_0 and M_e are moisture content at any time of drying (kg water/kg dry matter), initial moisture content (kg water/kg dry matter) and equilibrium moisture content (kg water/kg dry matter), respectively.

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The moisture ratio was simplified to Mt/M0 instead of Eq. (1) by some investigators [9-11] due to the continuous fluctuation of the relative humidity of the drying air during microwave drying process.

Table 1: Mathematical models given by various authors for drying curves

Model name	Model	Refe
Lewis	MR=exp(-kt)	[16]
Henderson and Pabis	MR=a exp(-kt)	[17]
Page	MR=exp(-kt^n)	[18]
Wang and Singh	MR =1 + bt + at^2	[19]
Parabolic	MR =c + bt + at^2	[20]
Logarithmic	MR=a exp(-kt) + b	[11]
Midilli	MR=a exp(-kt^n)+ bt	[21]

where, k is the drying constant and a, b, n are equation constants.

The drying data obtained were fitted to seven thin-layer drying models detailed in Table 1 using the nonlinear least squares regression analysis. Statistical analyses of the experimental data were performed by using the software SPSS 17.0. The coefficient of determination (R^2) is one of the primary criteria for selecting the best model to define the drying curves. In addition to R^2, reduced chi-square (χ^2) and root mean square error (RMSE) are used to determine the quality of the fit. These parameters can be calculated as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(MR_{pre,i} - MR_{exp,i}\right)^2}{\sum_{i=1}^{N}\left(\overline{MR_{pre,i}} - MR_{exp,i}\right)^2} \tag{2}$$

$$\chi^2 = \frac{\sum_{i=1}^{N}\left(MR_{pre,i} - MR_{exp,i}\right)^2}{N - z} \tag{3}$$

$$RMSE = \left(\frac{\sum_{i=1}^{N}\left(MR_{pre,i} - MR_{exp,i}\right)^2}{N}\right)^{\frac{1}{2}} \tag{4}$$

where $MR_{exp,i}$ is experimental moisture ratio; $MR_{pre,i}$ is predicted moisture ratio; N is number of observations; z is number of constants. The best model describing the drying characteristics of samples was chosen as the one with the highest coefficient of determination, the least reduced chi square, root mean square error and mean relative percent error [12-15].

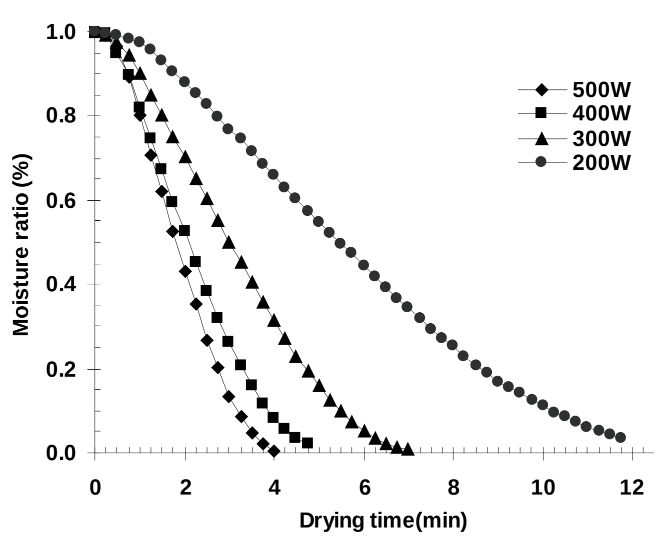

Fig 2 : Moisture ratio versus drying time of shrimp at different microwave powers

III. Results and Discussion

The changes in moisture ratio with drying time of shrimp samples in microwave drying are presented in Fig. 2. According to the results in Fig. 2, the drying microwave power a significant effect on the moisture content of the shrimp samples as expected. The results showed that drying time decreased greatly when drying temperature increased. The drying time required to reach the final moisture content of samples were 11.75, 7, 4.75 and 4 min at the microwave powers of 200, 300, 400 and 500W, respectively. The results indicates that mass transfer within the sample was more rapid during higher microwave power heating because more heat was generated within the sample creating a

Model no	P (W)	Model constants	R2	χ 2	RMSE
Lewis	500	k=0.7917	0.7454	0.0471	0.2105
	400	k=0.5803	0.8447	0.0265	0.1586
	300	k=0.4531	0.7816	0.0351	0.1840
	200	k=0.2019	0.8508	0.0203	0.1409
Henderson and Pabis	500	a= 2.254, k=1.0873	0.8297	0.1599	0.3756
	400	a= 1.7673, k=0.7556	0.9118	0.0549	0.2224
	300	a= 2.1411, k=0.6133	0.8614	0.1053	0.3131
	200	a= 1.6307, k=0.2636	0.9188	0.0295	0.1682
Page	500	k=0.203, n=2.077	0.999	0.0002	0.0123
	400	k=0.182,n=1.847	0.999	0.0001	0.0109
	300	k=0.092, n=1.865	0.998	0.0003	0.0158
	200	k=0.033, n=1.804	0.999	0.0001	0.0116
Wang and Singh	500	a=0.0002, b= - 0.2704	0.9759	0.0034	0.0552
	400	a=0.0068, b= - 0.2521	0.9834	0.0021	0.0437
	300	a= 0.0035, b= - 0.1772	0.9847	0.0019	0.0422
	200	a= 0.0003, b=- 0.0913	0.9869	0.0014	0.0369
Parabolic	500	a= 0.019, b= - 0.3636, c= 1.0962	0.9861	0.0021	0.0419
	400	a= 0.0185, b= - 0.321, c= 1.084	0.9917	0.0011	0.0310
	300	a= 0.0091, b= - 0.2252, c= 1.086	0.9927	0.0009	0.0291
	200	a= 0.0022, b= - 0.1185, c= 1.081	0.9945	0.0006	0.0239
Logarithmic	500	k= 0.115, a=3.125, b= - 2.033	0.985	0.0022	0.0429
	400	k= 0.141, a=2.284, b= - 1.204	0.991	0.0013	0.0329
	300	k= 0.101, a= 2.239, b= - 1.156	0.992	0.0011	0.0314
	200	k= 0.044, a=2.711, b=- 1.632	0.997	0.0007	0.0250
Midilli	500	k= 0.207, a= 1.008, b= -0.012, n=1.944	0.999	0.00006	0.0067
	400	k= 0.187, a= 1.007, b= -0.011, n=1.718	0.999	0.00003	0.0053
	300	k= 0.095, a= 1.002, b= -0.01, n=1.718	0.999	0.00011	0.0096
	200	k= 0.038, a= 1.008, b= -0.006, n=1.657	0.999	0.00005	0.0066

large vapor pressure difference between the centre and the surface of the product due to characteristic microwave volumetric heating.

The statistical results from models are summarised in Tables 2. The best model describing the thin-layer drying characteristics of shrimp was chosen as the one with the highest R^2 values and the lowest χ^2 and RMSE values. The R^2 for Henderson and Pabis, Logarithmic, Wang and Singh, Page, and Midilli models were all above 0.99, and that for Lewis model was lower, but still above 0.745. The statistical parameter estimations showed that R^2, χ^2 and RMSE values were ranged from 0.7454 to 0.9999, 0.00003 to 0.1599, and 0.0053 to 0.3756, respectively. Of all the models tested, the Midilli model gives the highest value of R2 and the lowest values of χ^2 and RMSE. Fig. 3 compares

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experimental data with those predicted with the Midilli model for shrimp samples at 200, 300, 400 and 500W. The prediction using the model showed MR values banded along the straight line, which showed the suitability of these models in describing drying characteristics of shrimp.

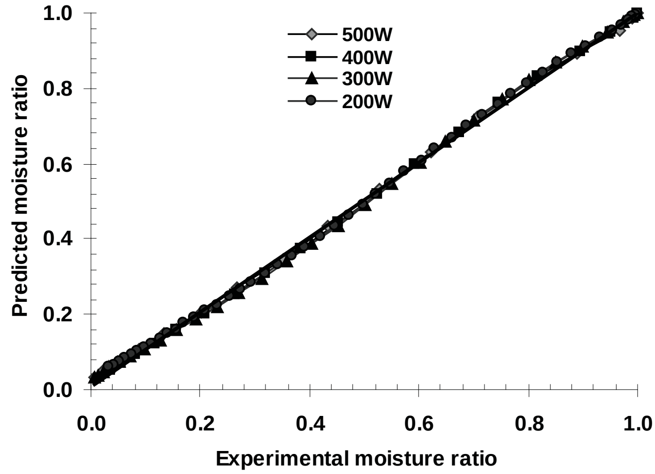

Fig 3 : Experimental versus predicted moisture ratio (MR) values for shrimp drying

It was determined that the value of k increased with the increase in the microwave power. This data indicates that with increase in microwave power drying curve becomes steeper indicating faster drying of the product. A similar trend was observed by Ozkan et al. [22] for spinach; Sharma and Prasad [20] for garlic cloves.

In this study, as the temperature is not measurable variable in the standard microwave oven used for drying process, the Arrhenius equation was used in a modified form to illustrate the relationship between the kinetic rate constant and the ratio of the microwave output power to sample amount instead of the temperature for calculation of the activation energy. After evaluation of the data, the dependence of the kinetic rate constant on the ratio of microwave output power to sample amount was represented with an exponential equation (6) derived by Ozbek and Dadali [23]:

$$k = k_0 \exp\left(\frac{-E_a.m}{P}\right) \tag{6}$$

where k is the drying rate constant obtained by using Midilli model (1/min), k_0 is the pre-exponential constant (1/min), E_a is the activation energy (W/g), P is the microwave output power (W) and m is the mass of raw sample (g). The values of k versus m/P shown in Fig. 4 accurately fit to Eq. (6) with coefficient of determination (R^2) of 0.9869. Then, k_0 and E_a values were estimated as 0.722 (1/min) and 12.834W/g.

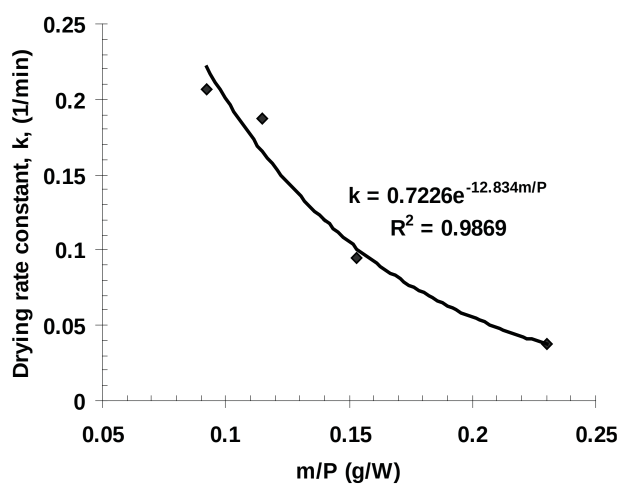

Fig 4 : The relationship between the values of drying rate constant versus sample amount/power

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23. Ozbek B. and Dadali G; Thin-layer drying characteristics and modelling of mint leaves undergoing microwave treatment. J Food Eng, 83(2007), 541-549

IV. Conclusion

The drying kinetics of shrimp was investigated in a microwave dryer as a single layer at the drying microwave powers of 200, 300, 400 and 500 W. Based on non-linear regression analysis, the Midilli model was considered adequate to describe the thin-layer drying behavior of shrimp. The drying rate constant increased with increasing microwave power and it followed an Arrhenius relationship. The activation energy for moisture diffusion was found to be 12.834W/g.

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Solving Third Order Three-Point Boundary Value Problem on Time Scales by Solution Matching Using Differential Inequalities

By K. R. Prasad, N. V. V. S. Suryanarayana, & P. Murali
Andhra University

Abstract - We consider the third order boundary value problem associated with the di®erential equation on time scales

$$y^{\Delta^3} = f(t, y, y^{\Delta}, y^{\Delta^2}),\ t \in [t_1, \sigma^3(t_3)]$$

on time scales satisfying the conditions

$$y(t_1) = y_1,\ y(t_2) = y_2,\ y(\sigma^3(t_3)) = t_3$$

We establish the solution of the three point boundary value problem on time scales on $[t_1, \sigma^3(t_3)]$ by matching solutions on $[t_1, t_2]$ with solutions on $[t_2, \sigma^3(t_3)]$.

Keywords and phrases : Time scales, boundary value problem, dynamical equation, matching methods.

GJSFR-F Classication : MSC 2010: 34A40

SOLVING THIRD ORDER THREE-POINT BOUNDARY VALUE PROBLEM ON TIME SCALES BY SOLUTION MATCHING USING DIFFERENTIAL INEQUALITIES

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Solving Third Order Three-Point Boundary Value Problem on Time Scales by Solution Matching Using Differential Inequalities

K. R. Prasad[α], N. V. V. S. Suryanarayana[σ], & P. Murali[ρ]

Abstract - We consider the third order boundary value problem associated with the differential equation on time scales

$$y^{\Delta^3} = f(t, y, y^{\Delta}, y^{\Delta^2}),\ t \in [t_1, \sigma^3(t_3)]$$

on time scales satisfying the conditions

$$y(t_1) = y_1,\ y(t_2) = y_2,\ y(\sigma^3(t_3)) = t_3.$$

We establish the solution of the three point boundary value problem on time scales on $[t_1, \sigma^3(t_3)]$ by matching solutions on $[t_1, t_2]$ with solutions on $[t_2, \mathbb{R}^3(t_3)]$.

Keywords and phrases : Time scales, boundary value problem, dynamical equation, matching methods.

I. Introduction

In this paper we consider, the existence and uniqueness of solutions of the three point boundary value problems associated with the differential equation on time scales

$$y^{\Delta^3}(t) = f(t, y(t), y^{\Delta}(t), y^{\Delta^2}(t)) \tag{1.1}$$

With

$$y(t_1) = y_1,\ y(t_2) = y_2,\ y(\sigma^3(t_3)) = y_3 \tag{1.2}$$

where $f \in C_{rd}[[t_1, \sigma^3(t_3)] \times \mathbb{R}^3, \mathbb{R}]$ and we assume through out that solutions of initial value problems associated with (1.1) exist, are unique and extend through out a fixed interval of $\mathbb{R}$. A monotonicity restriction on f assumes that the two point boundary value problem for (1.1) satisfying any one of

$$y(t_1) = y_1,\ y(t_2) = y_2,\ y^{\Delta}(t_2) = m \tag{1.3}$$

$$y(t_1) = y_1,\ y(t_2) = y_2,\ y^{\Delta^2}(t_2) = m \tag{1.4}$$

$$y(t_2) = y_2,\ y^{\Delta}(t_2) = m,\ y(\sigma^3(t_3)) = y_3 \tag{1.5}$$

or

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$$y(t_2) = y_2,\ y^{\Delta^2}(t_2) = m,\ y(\sigma^3(t_3)) = y_3 \qquad (1.6)$$

have at most one solution and with added hypothesis, a unique solution of the three point boundary value problem (1.1), (1.2) is constructed by using differential inequalities. This is acheived by matching solutions of the boundary value problem (1.1), (1.3) with solutions of (1.1), (1.5) or solutions of the boundary value problem (1.1), (1.4) with solutions of (1.1), (1.6).

The technique of matching solutions was discussed by Bailey, Shamphine and Waltman [2] to obtain solutions of two-point boundary value problems for the second order equation by matching solutions of initial value problems. Later, many authors like Barr and Sherman [4], Barr and Miletta [3], Das and Lalli [8], Henderson [10, 11], Henderson and Taunton [13], Lakshmikantham and Murty [16], Moorti and Garner [17], Rao, Murty and Rao [18] have used this technique and obtained solutions three point bound-ary value problems by matching solutions of two two-point boundary value problems for ordinary differential equations. Henderson and Prasad [12] and Eggensperger, Kaufmann and Kasmatov [9] obtained solutions of three point boundary value problems using matching methods for boundary value problems on time scales.

In this paper, we are concerned with the existence and uniqueness of solutions of three point boundary value problems for a differential equation on time scales using differential inequalities. We state some basic definitions of time scales for ready reference.

Definition 1.1. A nonempty closed subset of $\mathbb{R}$ is called a time scale. It is denoted by $\mathbb{T}$. By an interval we mean the intersection of the given interval with a time scale. For $t < \sup\mathbb{T}$ and $r > \inf\mathbb{T}$, define the forward jump operator, σ and backward jump operator, ρ, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\} \in \mathbb{T},$$

$$\rho(r) = \sup\{s \in \mathbb{T} : s < r\} \in \mathbb{T},$$

for all $t, r \in \mathbb{T}$. If $\sigma(t) = t$, t is said to be right dense,(otherwise t is said to be right scattered) and if $\rho(r) = r$, r is said to be left dense, (otherwise r is said to be left scattered).

Definition 1.2. For $x : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}$ (if $t = \sup\mathbb{T}$, assume t is not left scattered), define the delta derivative of $x(t)$, denoted by $x^{\Delta}(t)$, to be the number(when it exists), with the property that, for any $\epsilon > 0$, there is a neighborhood U of t such that

$$|\, [x(\sigma(t)) - x(s)] - x^{\Delta}(t)[\sigma(t) - s]\, | \le \epsilon\, |\, \sigma(t) - s\, |,$$

for all $s \in U$.

If x is delta differentiable for every $t \in \mathbb{T}$; we say that $x : \mathbb{T} \to \mathbb{R}$ is delta differentiable on $\mathbb{T}$.

Definition 1.3. If the time scale $\mathbb{T}$ has a maximal element which is also left scattered, that point is called a degenerate point. Any subset of non- degenerate points of $\mathbb{T}$ is denoted by $\mathbb{T}^k$.

Definition 1.4. A function $x : \mathbb{T} \to \mathbb{R}$ is right dense continuous (rd- continuous) if it is continuous at every right dense point $t \in \mathbb{T}$ and its left hand limit exists at each left dense point $t \in \mathbb{T}$.

Ref.

3. D. Barr and P. Miletta, An existence and uniqueness criterion for solutions of boundary value problems *J. Diff. Eqns.*, 16(1974), 460-471.
4. D. Barr and T. Sherman, Existence and uniqueness of solutions of three point boundary value problems, *J. Diff. Eqns.*, 13(1973), 197-212.

The forward jump operator $\sigma : \mathbb{T} \to \mathbb{R}$ is right dense continuous and more generally if $x : \mathbb{T} \to \mathbb{R}$ is continuous, then $x(\sigma) : \mathbb{T} \to \mathbb{R}$ is right dense continuous. moreover, we say that f is delta differentiable on $\mathbb{T}^k$ provided $f^{\Delta}(t)$ exists for all $t \in \mathbb{T}^k$: The function $f^{\Delta} : \mathbb{T}^k \to \mathbb{R}$ is then called the delta derivative of f on $\mathbb{T}^k$.

Notes

Definition 1.5. A function $\mathbb{F} : \mathbb{T}^k \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T}^k \to \mathbb{R}$ provided $\mathbb{F}^{\Delta}(t) = f(t)$ holds for all $t \in \mathbb{T}^k$: We then define the integral of f by

$$\int_a^t f(s)\Delta s = F(t) - F(a).$$

Definition 1.6. The point t_0 is a generalized zero of the function $y(t)$ if either $y(t_0) = 0$ or $y(t_0)y(\sigma(t_0)) < 0$.

Theorem 1.1. Mean value theorem: if $y : \mathbb{T} \to \mathbb{R}$ is continuous and $y(t)$ has generalized zeros at a and b, then there exists $p \in [a, b]$ such that y^{Δ} has a generalized zero at p.

Proof. We refer to Bohner and Eloe [5].

II. Differential Inequalities

In this section, we develop the theory of differential inequalities on time scales associated with the second order differential equation

$$y^{\Delta^2}(t) = f(t, y(t), y^{\Delta}(t)) \qquad (2.1)$$

For this, we need the following set.

Definition 2.1. Let $y \in C^2_{rd}[[t_1, \sigma^2(t_2)], \mathbb{R}]$. We say that a point $t_0 \in (t_1, \sigma^2(t_2))$ is in the set Ω if $y(t_0) \le 0$ and y^{Δ} has a generalized zero at t_0.

Lemma 2.1. Assume that $y \in C^2_{rd}[[t_1, \sigma 2(t_2)], \mathbb{R}]$ and y has a generalized zero at t_1 and suppose that $y^{\Delta 2}(t_0) < 0$ whenever $t_0 \in \Omega$. If $y(t_0) \neq 0$ on $[t_1, \sigma^2(t_2))$, then y^{Δ} has a generalized zero at t_2 if and only if y has a generalized zero at t_2.

Proof. Suppose that y has a generalized zero at t_2 and $y(t) \neq 0$ on $[t_1, \sigma 2(t_2))$. For the sake of contradiction, we assume that y^{Δ} has no generalized zero at t_2. Since y has generalized zeros at t_1 and t_2, y^{Δ} will have a generalized zero at some $r \in (t_1; \sigma 2(t_2))$ such that y^{Δ} has no generalized zero in $(r, \sigma^2(t_2))\cap\mathbb{T}$. From the definition of generalized zero, we have either $y^{\Delta}(r) = 0$ or $y^{\Delta}(r)y^{\Delta}(\sigma(r)) < 0$. If r is right dense, then $y^{\Delta}(r) = 0$ and if r is right scattered, then $y^{\Delta}(r)y^{\Delta}(\sigma(r)) < 0$. Let $y^{\Delta}(t) = 0$ on $(r, \sigma^2(t_2)] \cap \mathbb{T}$. (otherwise use $-y^{\Delta}(t)$.) Then $0 < \int_r^{t2} y^{\Delta}(t)\Delta(t) = y(t_2) - y(r) \le -y(r)$ which implies $y(r) < 0$. Since $y(r) < 0$, y^{Δ} has a generalized zero at r and hence $r \in \Omega$ which implies by hypothesis that$y^{\Delta 2}(r) < 0$. However, if r is right dense (i.e. $\sigma(r) = r$),then

$$y^{\Delta^2}(r) = \lim_{t\to\sigma(r)} \frac{y^{\Delta}(t)}{t-r} > 0$$

and if r is right scattered (i.e. $\sigma(r) > r$) then

$$y^{\Delta^2}(r) = \frac{y^{\Delta}(\sigma(r)) - y^{\Delta}(r)}{\sigma(r) - r} > 0$$

Thus, in either case,we have obtained $y^{\Delta 2}(r) > 0$, which is a contradiction. Thus, y^{Δ} has a generalized zero at t_2. A similar argument holds if y^{Δ} has a generalized zero at t_2.

Lemma 2.2. Assume that $y \in C^2_{rd}[[t_1, \sigma^2(t_2)],R]$ and y has a generalized zero at t_2 and further suppose that $y^{\Delta 2}(r) < 0$ whenever $r \in \Omega$. If $y(t) \neq 0$ on $[t_1, \sigma^2(t_2))$ then y^{Δ} has a generalized zero at t_1 if and only if y has a generalized zero t_1.

Proof is analogous to the proof of the Lemma 2.1.

Notes

Lemma 2.3. If $y(t)$ is any solution of (2.1) such that y has generalized zeros at t_1 and t_2 and if $y^{\Delta 2}(t_0) < 0$ whenever $t_0 \in \Omega$, then $y(t) = 0$ on $[t_1, \sigma^2(t_2)]$.

Proof. For the sake of contradiction, we assume that $y(t) \neq 0$ on $[t_1, \sigma^2(t_2)]$. Since $y(t) \neq 0$ at any point in $(t_1, \sigma^2(t_2))$, y must have a non zero extremum in $(t_1, \sigma^2(t_2))$ =) y^{Δ} has a generalized zero at some $t_0 \in (t_1, \sigma(t_2))$. i.e. either $y^{\Delta}(t_0) = 0$ or $y^{\Delta}(t_0)y^{\Delta}(\sigma(t_0)) < 0$. If t_0 is right dense, then $y^{\Delta}(t\ ) = 0$

and

if t_0 is right scattered, then $y^{\Delta}(t_0)y^{\Delta}(\sigma(t_0)) < 0$. Assume with out loss generality that $y^{\Delta}(t_0) > 0$ on $(t_0, œ(t_2)]$. Then $0 < \int_{t0}^{t2} y^{\Delta}(t)\Delta(t) = y(t_2) - y(t_0) \leq -y(t_0)$ which implies $y(t_0) < 0$. Now $y(t_0) < 0$ and $y^{\Delta}(t_0) \geq 0$ which implies by hypothesis that $y^{\Delta 2}(t_0) < 0$. How ever, if t_0 is right dense ,then

$$y^{\Delta^2}(t_0) = \lim_{t \to \sigma(t_0)} \frac{y^{\Delta}(t)}{t - t_0} > 0$$

and if t_0 is right scattered,then

$$y^{\Delta^2}(t_0) = \frac{y^{\Delta}(\sigma(t_0)) - y^{\Delta}(t_0)}{\sigma(t_0) - t_0} > 0.$$

Hence, a contradiction. Thus, $y(t) = 0$ on $[t_1, \sigma^2(t_2)]$.

Consider the boundary value problem

$$\begin{aligned} y^{\Delta^2}(t) &= f(t, y(t), y^{\Delta}(t)) \\ y(t_1) &= y_1,\ y(\sigma^2(t_2)) = y_2 \end{aligned} \tag{2.3}$$

Suppose $\Phi(t)$ and $\Psi(t)$ are two solutions of the above boundary value problem. Write $\chi(t) = \Phi(t) - \Psi(t)$. Then

$$\begin{aligned} \chi^{\Delta^2}(t) &= \Phi^{\Delta^2}(t) - \Psi^{\Delta^2}(t) \\ &= f(t, \Phi(t), \Phi^{\Delta}(t)) - f(t, \Psi(t), \Psi^{\Delta}(t)) \\ &= f(t, \chi(t) + \Psi(t), \chi^{\Delta}(t) + \psi^{\Delta}(t)) - f(t, \Psi(t), \Psi^{\Delta}(t)) \\ &= F(t, \chi(t), \chi^{\Delta}(t)) \end{aligned}$$

Clearly $F(t, 0, 0) = 0$, $\chi(t_1) = 0$, $\chi(t_2) = 0$. Thus, we have the following theorem.

Theorem 2.1. The boundary value problem

$$y^{\Delta^2}(t) = F(t, y(t), y^{\Delta}(t))$$
$$y(t_1) = 0,\ \ y(\sigma^2(t_2)) = 0 \tag{2.2}$$

where F(t, 0, 0) = 0 has only the trivial solution if and only if the following boundary value problem

Notes

$$y^{\Delta^2}(t) = f(t, y(t), y^{\Delta}(t))$$
$$y(t_1) = y_1,\ \ y(\sigma^2(t_2)) = y_2 \tag{2.3}$$

has a unique solution.

Proof. Suppose (2.2) has only the trivial solution $\chi(t)$. Then $\chi(t) = 0\ \forall\ t \in [t_1, \sigma^2(t_2)]$ and hence $\Phi(t) = \Psi(t)$. Conversely, suppose that (2.3) has a unique solution. Then, $\chi(t) = \Phi(t) - \Psi(t)$. Obviously $\chi(t_1) = \chi(t_2) = 0$ and $\chi^{\Delta2}(t) = F(t, \chi(t), \chi^{\Delta}(t))$ and $F(t, 0, 0) = 0$. Hence $\chi(t)$ is the only solution of (2.2). Thus, the proof of the theorem is complete.

We now develop the theory of differential inequalities associated with the third order differential equation. For this, we need the following sets and classes of functions.

Definition 2.2. Let $y \in C^3_{rd}[[t_1, \sigma^3(t_3)],\mathbb{R}]$. We say that a point $t_0 \in \Omega_1$ if $y(t_0) \le 0$, $y^{\Delta}(t_0) > 0$ and $y^{\Delta2}$ has a generalized zero at t_0 for some $t_0 \in [t_1, t_2]$ and $t_0 \in \Omega_2$ if $y(t_0) \ge 0$, $y^{\Delta}(t_0) > 0$ and $y^{\Delta2}$ has a generalized zero at t_0 for some $t_0 \in [t_2, \sigma^3(t_3)]$.

Definition 2.3. We say that a function

$f(t, y(t), y^{\Delta}(t), y^{\Delta2}(t)) \in \mathbb{C}rd[[t_1, \sigma^3(t_3)] \times \mathbb{R}^3,\mathbb{R}]$ is in the set G_1 if $f(t, y(t), y^{\Delta}(t), y^{\Delta2}(t)) \ge 0\ \forall\ t \in [t_1, t_2]$, $f(t, y(t), y^{\Delta}(t), y^{\Delta2}(t))$ is non decreasing in y and strictly increasing in y^{Δ} and belongs to the set G_2 if $f(t, y(t), y^{\Delta}(t), y^{\Delta2}(t)) \ge 0\ \forall\ t \in [t_2, \sigma^3(t_3)]$, $f(t, y(t), y^{\Delta}(t), y^{\Delta2}(t))$ is non decreasing in y and strictly increasing in y^{Δ}.

Lemma 2.4. Let $y(t)$ be a solution of (1.1) such that y has generalized zeros at t_1 and t_2 with $f(t, 0, 0, 0) = 0$. Further suppose that $y^{\Delta3}(t_0) > 0$ whenever $t_0 \in \Omega_1$. If either y^{Δ} or $y^{\Delta2}$ has a generalized zero at t_2, then $y(t) = 0$ for all $t \in [t_1, t_2]$.

Proof. We first suppose that y^{Δ} has a generalized zero at t_2. Then we claim that $y^{\Delta2}$ also has a generalized zero at t_2. To the contrary, assume that $y^{\Delta2}$ has no generalized zero at t_2. With out loss of generality we can assume that $y^{\Delta2}(t2) < 0$. So, $\exists$ a $q \in [t_1, t_2)$ such that $y^{\Delta2}$ has a generalized zero at q and $y^{\Delta2}(t) < 0\ \forall\ t \in (q, t_2]$. Then

$$0 > \int_t^{t_2} y^{\Delta^2}(t)\Delta t = y^{\Delta}(t_2) - y^{\Delta}(t) \ge -y^{\Delta}(t)$$

which implies $y^{\Delta}(t) > 0\ \forall\ t \in (q, t_2]$. Since $y^{\Delta2}(t) < 0\ \forall\ t \in (q, t_2]$, it follows that $y^{\Delta}(t)$ is decreasing for $t > q$ and $y^{\Delta}(q)$ is positive. Again $0 < \int_q^{t2} y^{\Delta}(t)\Delta t = y(t2) - y(q) \le -y(q)$ which implies $y(q) < 0$. Thus, $y(q) < 0$, $y^{\Delta}(q) > 0$ and $y^{\Delta2}$ has a generalized zero at q, which implies $q \in \Omega_1$ which implies by hypothesis that $y^{\Delta3}(q) > 0$.

However, if q is right dense,

$$y^{\Delta^3}(q) = \lim_{t\to\sigma(q)} \frac{y^{\Delta^2}(t) - y^{\Delta^2}(q)}{t-q} = \lim_{t\to\sigma(q)} \frac{y^{\Delta^2}(t)}{t-q} \le 0$$

and if q is right scattered,

$$y^{\Delta^3}(q) = \frac{y^{\Delta^2}(\sigma(q)) - y^{\Delta^2}(q)}{\sigma(q)-q} \le 0.$$

Hence, a contradiction.

Thus, $y^{\Delta 2}$ has a generalized zero at t_2. Since y has a generalized zero at t_2, y^{Δ} has a generalized zero at t_2, $y^{\Delta 2}$ has a generalized zero at t_2 and $f(t, 0, 0, 0) = 0$, it follows that $y(t) = 0\ \forall\ t \in [t_1, t_2]$.

Next, we suppose that $y^{\Delta 2}$ has a generalized zero at t_2. Then it is claimed that y^{Δ} has a generalized zero at t_2. To the contrary, suppose that y^{Δ} has no generalized zero at t_2. With out loss of generality we can assume that $y^{\Delta}(t_2) > 0$. Since y has generalized zeros at t_1 and t_2, it follows from mean value theorem that there exists an $r \in (t_1, t_2)$ such that y^{Δ} has a generalized zero at r and y^{Δ} has no generalized zero in (r, t_2). Assume without loss of generality that $y^{\Delta}(t) > 0$ forall $t \in (r, t_2)$. We claim that there exists a $p \in [r, t_2)$ such that $y^{\Delta 2}(p) > 0$. To the contrary, suppose that $y^{\Delta 2}(p) \le 0$. Then $0 \ge \int_r^t y^{\Delta 2}(t)\Delta t = y^{\Delta}(t) - y^{\Delta}(r) \ge -y^{\Delta}(r)$ which implies $y^{\Delta}(t) \le 0\ \forall\ t \in (r, t_2)$, which is a contradiction. Hence, the claim. Now, there exists a $q \in (p, t_2)$ such that $y^{\Delta 2}$ has a generalized zero at q and $y^{\Delta 2}(t) > 0$ forall $t \in (p, q)$. Again $0 < \int_q^{t2} y^{\Delta}(t)\Delta t = y(t_2) - y(q) \ge -y(q)$.

Thus $y(q) < 0$, $y^{\Delta}(q) > 0$ and $y^{\Delta 2}$ has a generalized zero at q and hence $q \in \Omega_1$ which implies by hypothesis, $y^{\Delta 3}(q) > 0$. However, if q is right dense,

$$y^{\Delta^3}(q) = \lim_{t\to\rho(q)} \frac{y^{\Delta^2}(t) - y^{\Delta^2}(q)}{t-q} = \lim_{t\to\rho(q)} \frac{y^{\Delta^2}(t)}{t-q} \le 0$$

and if q is right scattered,

$$y^{\Delta^3}(q) = \frac{y^{\Delta^2}(\rho(q)) - y^{\Delta^2}(q)}{\rho(q)-q} \le 0.$$

Hence, a contradiction. Thus y^{Δ}has a generalized zero at t_2. Since y, y^{Δ}, $y^{\Delta 2}$ has generalized zeros at t_2 and $f(t, 0, 0, 0) = 0$, it follows that $y(t) = 0\ \forall\ t \in [t_1, t_2]$.

Lemma 2.5. Let $y(t)$ be a solution of (1.1) such that y has a generalized zero at t_1 and t_2 with $f(t, 0, 0, 0) = 0$. Further suppose that $y^{\Delta 3}(t_0) > 0$ whenever $t_0 \in \Omega_2$. If either y^{Δ} or $y^{\Delta 2}$ has a generalized zero at t_1, then $y(t) = 0$ for all $t \in [t_1, t_2]$.

Proof. We first suppose that y^{Δ} has a generalized zero at t_1. Then we claim that $y^{\Delta 2}$ also has a generalized zero at t_1. To the contrary, assume that $y^{\Delta 2}$ has no generalized zero at t_1. With out loss of generality we can assume that $y^{\Delta 2}(t_1) > 0$. So, $\exists$ a $q \in [t_1, t_2)$ such that $y^{\Delta 2}$ has a generalized zero at q and $y^{\Delta 2}(t) > 0\ \forall\ t \in [t_1, q)$. Then

$$0 < \int_{t_1}^{t} y^{\Delta^2}(t)\Delta t = y^{\Delta}(t) - y^{\Delta}(t_1) \le y^{\Delta}(t)$$

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which implies $y^{\Delta}(t) > 0\ \forall\ t \in [t_1, q)$. Since $y^{\Delta\Delta}(t) > 0\ \forall\ t \in [t_1, q)$, it follows that $y^{\Delta}(t)$ is decreasing for $t < q$ and $y^{\Delta}(t)$ is positive. Again $0 < \int_{t1}^{q} y^{\Delta}(t)\Delta t = y(q) - y(t_1) \leq y(q)$ which implies $y(q) > 0$. Thus, $y(q) > 0$, $y^{\Delta}(q) > 0$ and $y^{\Delta 2}$ has a generalized zero at q, which implies $q \in \Omega_2$ which implies by hypothesis that $y^{\Delta 3}(q) > 0$.

However, if q is right dense,

Notes

$$y^{\Delta^3}(q) = \lim_{t\to\rho(q)} \frac{y^{\Delta^2}(t) - y^{\Delta^2}(q)}{t-q} = \lim_{t\to\rho(q)} \frac{y^{\Delta^2}(t)}{t-q} \leq 0$$

and if q is right scattered,

$$y^{\Delta^3}(q) = \frac{y^{\Delta^2}(\rho(q)) - y^{\Delta^2}(q)}{\rho(q)-q} \leq 0.$$

Hence, a contradiction. Thus, $y^{\Delta 2}$ has a generalized zero at t_1. Since y has a generalized zero at t_1, y^{Δ} has a generalized zero at t_1, $y^{\Delta 2}$ has a generalized zero at t_1 and $f(t, 0, 0, 0) = 0$, it follows that $y(t) = 0\ \forall\ t \in [t_1, t_2]$. Next, we suppose that $y^{\Delta 2}$ has a generalized zero at t_1. Then it is claimed that y^{Δ} has a generalized zero at t_1. To the contrary, suppose that y^{Δ} has a generalized zero at t_1. With out loss of generality we can assume that $y^{\Delta}(t_1) > 0$. Since y has generalized zeros at t_1 and t_2, it follows from mean value theorem that there exists an $r \in (t_1, t_2)$ such that y^{Δ} has a generalized zero at r and y^{Δ} has no generalized zero in $[t_1, r)$. Assume with out loss of generality that $y^{\Delta}(t) > 0$ forall t 2 $[t_1, r)$. We claim that $\exists$ a $p \in [t_1, r)$ such that $y^{\Delta 2}(p) < 0$. To the contrary, suppose that $y^{\Delta 2}(p) \geq 0$. Then

$$0 \leq \int_t^r y^{\Delta^2}(t)\Delta t = y^{\Delta}(r) - y^{\Delta}(t) \leq -y^{\Delta}(t)$$

which implies $y^{\Delta}(t) \leq 0\ \forall\ t \in [t_1, r)$, which is a contradiction. Hence, the claim. Now, $\exists$ a $q \in [t_1, p)$ such that $y^{\Delta 2}$ has a generalized zero at q and $y^{\Delta 2}(t) < 0\ \forall\ t \in (q, p)$. Again

$$0 < \int_{t_1}^{q} y^{\Delta}(t)\Delta t = y(q) - y(t_1) \leq y(q).$$

Thus $y(q) > 0$, $y^{\Delta}(q) > 0$ and $y^{\Delta 2}$ has a generalized zero at q and hence $q \in \Omega_2$, which implies by hypothesis that $y^{\Delta 3}(q) > 0$. However, if q is right dense,

$$y^{\Delta^3}(q) = \lim_{t\to\sigma(q)} \frac{y^{\Delta^2}(t) - y^{\Delta^2}(q)}{t-q} = \lim_{t\to\sigma(q)} \frac{y^{\Delta^2}(t)}{t-q} \leq 0$$

and if q is right scattered,

$$y^{\Delta^3}(q) = \frac{y^{\Delta^2}(\sigma(q)) - y^{\Delta^2}(q)}{\sigma(q)-q} \leq 0.$$

Hence, a contradiction. Thus y^{Δ} has a generalized zero at t_1. Since y, y^{Δ}, $y^{\Delta 2}$ has generalized zeros at t_1 and $f(t, 0, 0, 0) = 0$, it follows that $y(t) = 0$ forall $t \in [t_1, t_2]$.

Lemma 2.6. Let $y(t)$ be a solution of (1.1) such that y has a generalized zero at t_2 and t_3 with $f(t, 0, 0, 0) = 0$. Further suppose that $y^{\Delta 3}(t_0) > 0$ whenever $t_0 \in \Omega_2$. If either y^{Δ} or $y^{\Delta 2}$ has a generalized zero at t_2, then $y(t) = 0$ for all $t \in [t_2, \sigma^3(t_3)]$.

Lemma 2.7. Let $y(t) \in C^3_{rd}[[t_1, t_2],\mathbb{R}] \ni y$ has generalized zeros at t_1 and t_2, $y^{\Delta 3}(t_0) > 0$ for some $t_0 \in \Omega_1$ and either $y^{\Delta}(t_2) < 0$ or $y^{\Delta 2}(t_2) < 0$. Then, $\exists$ a $p \in [t_1, t_2)$ such that y^{Δ} has a generalized zero at p and $y^{\Delta}(t) < 0$ on $(p, t_2]$ and $y(t) > 0$ on $[p, t_2)$.

Proof. We first suppose that $y^{\Delta 2}(t_2) < 0$. Then, there exists a $q \in [t_1, t_2)$ such that $y^{\Delta 2}$ has a generalized zero at q and $y^{\Delta 2}(t) < 0 \ \forall\ t \in (q, t_2]$.We first claim that $y^{\Delta}(t) < 0 \ \forall\ t \in [q, t_2]$. To the contrary, suppose that $y^{\Delta}(t) > 0 \ \forall\ t \in [q, t_2]$. Then $0 < \int_q^{t2} y^{\Delta}(t)\Delta t = y(t_2) - y(q) \leq - y(q)$ which implies $y(q) < 0$. Thus $y(q) < 0$, $y^{\Delta}(q) > 0$ and $y^{\Delta 2}$ has a generalized zero at q and hence $q \in \Omega_1$, implies by hypothesis that $y^{\Delta 3}(q) > 0$.

However, if q is right dense, then

$$y^{\Delta^3}(q) = \lim_{t\to\sigma(q)} \frac{y^{\Delta^2}(t) - y^{\Delta^2}(q)}{t-q} \leq 0$$

and if q is right scattered, then

$$y^{\Delta^3}(q) = \frac{y^{\Delta^2}(\sigma(q)) - y^{\Delta^2}(q)}{\sigma(q)-q} \leq 0,$$

which is a contradiction.Hence, $y^{\Delta}(t) \leq 0$ forall $t \in [q, t_2)$. therefore there exists *a* $p \in [q, t_2)$ such that y^{Δ} has a generalized zero at p and $y^{\Delta}(t) < 0$ forall $t \in (p, t_2]$ which implies $0 > \int_t^{t2} y^{\Delta}(t)\Delta t = y(t_2) - y(t) \geq -y(t)$ which implies $y(t) < 0$ on $[p, t_2)$. A similar argument holds if $y^{\Delta}(t_2) < 0$.

Lemma 2.8. Let $y(t) \in C^{3}_{rd}[[t_2, \sigma^3(t_3)],\mathbb{R}] \ni y$ has generalized zeros at t_2 and t_3, $y^{\Delta 3}(t_0) > 0$ for some $t_0 \in \Omega_2$ and either $y^{\Delta}(t_2) > 0$ or $y^{\Delta 2}(t_2) > 0$.Then, $\exists$ *a* $p \in (t_2, \sigma^3(t_3)]$ such that y^{Δ} has a generalized zero at p and $y^{\Delta}(t) < 0$ on $[t_2, p)$ and $y(t) < 0$ on $(t_2, p]$.

Proof. We first suppose that $y^{\Delta 2}(t_2) > 0$. Then, there exists a $q \in (t_2, \sigma^3(t_3)]$ such that $y^{\Delta 2}$ has a generalized zero at q and $y^{\Delta 2}(t) > 0$ forall $t \in [t_2, q)$. Then it is claimed that $y^{\Delta}(t) \leq 0$ forall $t \in (t_2, q]$. For the sake of contradiction, we assume that $y^{\Delta}(t) > 0 \ \forall\ t \in (t_2, q]$. Then $0 < \int_{t2}^{q} y^{\Delta}(t)\Delta t = y(q) - y(t_2) \leq y(q)$ which implies $y(q) > 0$. Thus $y(q) > 0$, $y^{\Delta}(q) > 0$ and $y^{\Delta 2}$ has a generalized zero at q and hence $q \in \Omega_2$, implies by hypothesis that $y^{\Delta 3}(q) > 0$.

However, if q is right dense, then

$$y^{\Delta^3}(q) = \lim_{t\to\rho(q)} \frac{y^{\Delta^2}(t) - y^{\Delta^2}(q)}{t-q} \leq 0$$

and if q is right scattered, then

$$y^{\Delta^3}(q) = \frac{y^{\Delta^2}(\rho(q)) - y^{\Delta^2}(q)}{\rho(q)-q} \leq 0,$$

which is a contradiction. Hence, $y^{\Delta}(t) \leq 0$ forall $t \in (t_2, q]$. Therefore there exists a $p \in (t_2, q]$ such that y^{Δ} has a generalized zero at p and $y^{\Delta}(t) < 0$ forall $t \in [t_2, p)$ which implies $0 > \int_{t2}^{t} y^{\Delta}(t)\Delta t = y(t) - y(t_2) \geq y(t)$ which implies $y(t) < 0$ on $(t_2, p]$. A similar argument holds if $y^{\Delta}(t_2) > 0$.

Notes

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III. Main Result

In this section we establish existence and uniqueness of solutions (1.1),(1.2). We first show that there exists at most one solution to (1.1) satisfying one of (1.3) ,(1.4) ,(1.5) or (1.6).

Lemma 3.1. Assume that $f \in C_{rd}[[t_1, \sigma^3(t_3)] \times \mathbb{R}_3, \mathbb{R}]$ and let $f \in G_1$, f $_2$ G_2.

Assume that when $u_1 \le u_2$, $v_1 > v_2$ and $w_1 = w_2$, then $f(t, u_1, v_1, w_1) - f(t, u_2, v_2, w_2) \ge 0$ $\forall$ $t \in [t_1, t_2)$.

Notes

Also assume that when $u_1 \ge u_2$, $v_1 > v_2$ and $w_1 = w_2$, then $f(t, u_1, v_1, w_1) - f(t, u_2, v_2, w_2) \ge 0$ $\forall$ $t \in (t_2, \sigma^3(t_3)]$.

Then for each $m \in \mathbb{R}$, $\exists$ at most one solution to (1.1) satisfying one of (1.3) ,(1.4) ,(1.5) or (1.6).

Proof. The proof of (1.1),(1.4) will be given. Similar argument holds for other boundary problems. Suppose that $\Phi(t)$ and $\Psi(t)$ are each solutions of the boundary value problem (1.1),(1.4).

Write $\chi(t) = \Phi(t) - \Psi(t)$.

Clearly $\chi(t_1) = 0$, $\chi(t_2) = 0$ and $\chi^{\Delta 2}(t_2) = 0$.

Hence

$$\chi^{\Delta^3}(t) = \Phi^{\Delta^3}(t) - \Psi^{\Delta^3}(t)$$
$$= f(t, \Phi(t), \Phi^{\Delta}(t), \Phi^{\Delta^2}(t)) - f(t, \Psi(t), \Psi^{\Delta}(t), \Psi^{\Delta^2}(t)) > 0.$$

Now $\chi(t)$ satisfies the hypothesis of Lemma 2.4.

So, $\chi(t) = 0$ or $\Phi(t) = \Psi(t)$.

Theorem 3.1. Assume that

(i) for each $m \in \mathbb{R}$, $\exists$ solutions for each of the boundary value problem (1.1) satisfying one of (1.3) ,(1.4), (1.5) or (1.6).

(ii) $f \in G_1$ and if $u_1 \le u_2$, $v_1 > v_2$ and $w_1 = w_2$, then $f(t, u_1, v_1, w_1) - f(t, u_2, v_2, w_2) \ge 0$ $\forall$ $t \in [t_1, t_2)$

(iii) $f \in G_2$ and if $u_1 \ge u_2$, $v_1 > v_2$ and $w_1 = w_2$, then $f(t, u_1, v_1, w_1) - f(t, u_2, v_2, w_2) \ge 0$ $\forall$ $t \in (t_2, \sigma 3(t_3)]$.

Then the boundary value problem (1.1),(1.2) has a unique solution.

Proof. By Lemma 3.1, the solutions of (1.1) satisfying one of (1.3) ,(1.4) ,(1.5) or (1.6), whenever they exists, are unique. Let $\Phi(t, m)$ denotes the solution of the boundary value problem (1.1), (1.4).

Set $\chi(t) = \Phi(t, m_1) - \Phi(t, m_2)$.

Clearly if $m_2 > m_1$, $\chi(t_1) = 0$, $\chi(t_2) = 0$, and $\chi^{\Delta 2}(t_2) = 0$. If $t \in \Omega_1$, then $\chi(t) \le 0$, $\chi^{\Delta}(t) > 0$ and $\chi^{\Delta 2}(t)$ has a generalized zero at t and hence using (ii),

$$\chi^{\Delta^3}(t) = f(t, \Phi(t, m_1), \Phi^{\Delta}(t, m_1), \Phi^{\Delta^2}(t, m_1))$$
$$- f(t, \Phi(t, m_2), \Phi^{\Delta}(t, m_2), \Phi^{\Delta^2}(t, m_2)) \ge 0$$

Thus Lemma 2.7 yields $\chi^{\Delta}(t) < 0$, $t \in (p, t_2]$. In particular,

$$\chi^{\Delta}(t_2) = \Phi^{\Delta}(t_2, m_1) - \Phi^{\Delta}(t_2, m_2) < 0.$$

Hence, it follows that $\Phi^{\Delta}(t_2, m)$ is a strictly increasing function of m. A similar reasoning given above demonstrates that $\Psi^{\Delta}(t_2, m)$ is a strictly decreasing function of m, where $\Psi(t, m)$ is the solution of the boundary value problem (1.1),(1.6).

It now follows from the fact that solutions of (1.1),(1.4) and (1.1),(1.6) are unique and the ranges of $\Phi^{\Delta}(t_2, m)$ and $\Psi^{\Delta}(t_2, m)$ are the set of all reals, that there exists a unique $m_0 \in \mathbb{R}$ such that $\Phi^{\Delta}(t_2, m_0) = \Psi^{\Delta}(t_2, m_0)$. Thus $y(t)$ defined by

$$y(t) = \begin{cases} \Phi(t, m), & t_1 \le t \le t_2, \\ \Psi(t, m), & t_2 \le t \le \sigma^3(t_3), \end{cases}$$

is a solution of (1.1),(1.2).

Notes

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the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

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10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

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15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

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18. **Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page

- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

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This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
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- In spite of position, each table must be titled, numbered one after the other and complete with heading
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Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
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- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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