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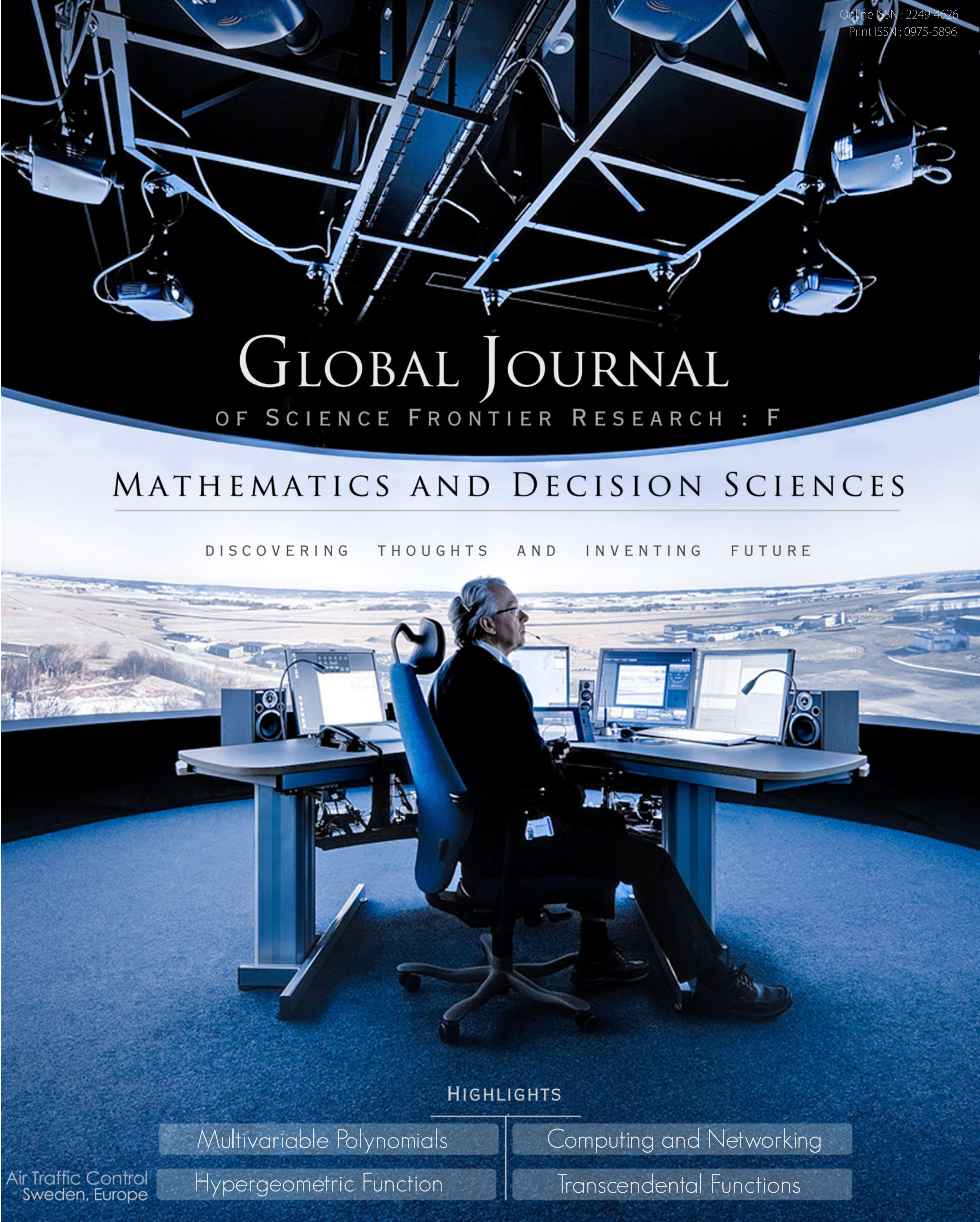

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The Integration of Certain Products of the $\overline{H}$ - function with Extended Jaboci Polynomials

By V.B.L. Chaurasia & Gulshan Chand
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Abstract – The object of this paper is to derive a finite integral pertaining to two $\overline{H}$-functions with extended Jacobi-polynomial. In the particular cases we have discussed the integration of product of a certain class of Feynman integral with our main integral. Application of the main result have also been discussed with the Riemann-Liouville type fractional integral operator. The results derived here are basic in nature and they are likely to be useful applications into several fields notably electromagnetic theory, statistical mechanics and probability theory.

Keywords : *Fractional integral, Feynman integrals, $\overline{H}$ -function, Extended Jaboci polynomials.*

Mathematics Subject Classification 2000 : *26A33, 44A10, 33C60*

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The Integration of Certain Products of the $\overline{\mathrm{H}}$-function with Extended Jaboci Polynomials

V.B.L. Chaurasia [α] & Gulshan Chand [σ]

Abstract - The object of this paper is to derive a finite integral pertaining to two $\overline{\mathrm{H}}$-functions with extended Jacobi-polynomial. In the particular cases we have discussed the integration of product of a certain class of Feynman integral with our main integral. Application of the main result have also been discussed with the Riemann-Liouville type fractional integral operator. The results derived here are basic in nature and they are likely to be useful applications into several fields notably electromagnetic theory, statistical mechanics and probability theory.

Keywords : Fractional integral, Feynman integrals, $\overline{H}$-function, Extended Jaboci polynomials.

I. Introduction

The $\overline{H}$-function introduced by Inayat-Hussain ([9], see also [1]) in terms of Mellin-Barnes type contour integral is defined as follows

$$\overline{\mathrm{H}}_{P,Q}^{M,N}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{N+1,Q}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\Phi(\xi)z^{\xi}\,d\xi, \tag{1.1}$$

where

$$\Phi(\xi)=\frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=M+1}^{Q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=N+1}^{P}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

which contains fractional powers of some of the Γ−functions. Here and throughout the paper a_j (j=1,…,P) and b_j(j = 1,…,Q) are complex parameters, $\alpha_j \geq 0$ (j=1,..., P, $\beta_j \geq 0$ (j=1,..., Q), (not all zero simultaneously) and throughout A_j (j = 1,…,N) and B_j (j=M+1,…,Q) can take on non-integer values.

The contour in (1.2) is imaginary and Re(ξ) = 0. It is suitably idented in Γ−function and to keep these singularities on appropriate side. Again, for A_j (j=1,…,N)

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Ref.

9. Hai, N.T. and Yakubovich, S.B. – The Double Mellin-Barnes Type Integrals and Their Application to Convolution Theory, World Scientific Publishing Co. Pvt. Ltd., Singapore, New Jersey, London, Hongkong, 1992.

not an integer, the poles of the Γ−functions of the numerator in (1.2) are converted to branch points. However, as long as there is no coincidence of poles from any $\Gamma(b_j-\beta_j\xi)$ (j = 1,…,M) and $\Gamma(1-a_j+\alpha_j\xi)$ (j = 1,…,N) pair, the branch cuts can be chosen so that the path of integration can be distorted in the usual manner.

For the sake of brevity

$$T=\sum_{j=1}^{M}|\beta_j|+\sum_{j=1}^{N}A_j\alpha_j-\sum_{j=M+1}^{Q}|B_j\beta_j|-\sum_{j=N+1}^{P}\alpha_j>0.$$

Notes

II. Main Integral

$$\int_a^b (x-a)^{\beta}(b-x)^{\sigma}F_n(\beta,\alpha;x)$$

$$.\ \overline{H}_{P,Q}^{M,N}\left[z(b-x)^k\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$.\ \overline{H}_{P',Q'}^{M',N'}\left[z'(b-x)^{k'}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N'},(a'_j,\alpha'_j)_{N'+1,P'}\\(b'_j,\beta'_j)_{1,M'},(b'_j,\beta'_j;B'_j)_{M'+1,Q'}\end{matrix}\right.\right]dx$$

$$=\sum_{h=1}^{M'}\sum_{r=0}^{\infty}\frac{\prod_{\substack{j=1\\j\neq h}}^{M'}\Gamma(b'_j-\beta'_j\xi_{h,r})\prod_{j=1}^{N'}\{\Gamma(1-a'_j+\alpha'_j\xi_{h,r})\}^{A'_j}}{\prod_{j=1+M'}^{Q'}\{\Gamma(1-b'_j+\beta'_j\xi_{h,r})\}^{B'_j}\prod_{j=1+N'}^{P'}\Gamma(a'_j-\alpha'_j\xi_{h,r})}$$

$$.\ \frac{(z')^{k'\xi_{h,r}}(-1)^{r+n}\lambda^n(b-a)^{\beta+\sigma+1}\Gamma(1+\beta+n)}{r!\ \ \beta_h\ \ n!}$$

$$.\ \overline{H}_{P+2,Q+2}^{M,N+2}\left[z(b-a)^k\left|\begin{matrix}(\alpha-\sigma-k'\xi_{h,r},k;1),(-\sigma-k'\xi_{h,r},k;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(\alpha+n-\sigma-k'\xi_{h,r},k;1),(-1-\sigma-\beta-n-k'\xi_{h,r},k;1)\end{matrix}\right.\right],\qquad(2.1)$$

where

(i) $k\geq 0, k'\geq 0;$

(ii) $\mathrm{Re}\left(\sigma+k\frac{b_j}{\beta_j}+k'\frac{b'_j}{\beta'_j}\right)>0; j=1,\ldots, M$

(iii) $|\arg(z)| < \frac{1}{2} T\pi, T > 0, |\arg(z')| < \frac{1}{2} T'\pi$

(iv) $F_n(\beta, \alpha; x)$ is Fujiwara polynomials [8].

(v) $\lambda = u(b - a)$.

Proof

To establish (2.1), we express the $\bar{H}$-functions in series from and contour form as in (1.2) respectively, and then interchanging the order of summations and integrations which is permissible under the conditions stated, solving the remaining integral with the help of a known result Chiney and Bhonsle ([4], p.9, eqn. (3.1)), and thus, interpreting the result in the desired form.

Special Cases

(i) Putting M = 1, N = 3 = P = Q, and replacing z by – z in (1.2), and using

$$g(\gamma,\eta,\tau,p;y) = \frac{E_{d-1}\Gamma(p+1)\Gamma(\frac{1}{2}+\frac{\tau}{2})}{(-1)^p\, 2^{2+p}\pi^{1/2}\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}$$

$$. \, H^{1,3}_{3,3}\left[-y\left|\begin{matrix}(1-\gamma,1;1),(1-\gamma+\frac{\tau}{2},1;1),(1-\eta,1);1+p)\\(0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p)\end{matrix}\right.\right], \tag{3.1}$$

where $E_d = \frac{2^{1-d}\pi^{-d/2}}{\Gamma(d/2)}$ ([11], p.4121,eqn. (5))

The above function is connected with a certain class of Feynman integrals. We get

$$\int_a^b (x-a)^{\beta}(b-x)^{\sigma} F_n(\beta,\alpha;x)\, g(\gamma,\eta,\tau,p;g(b-x)^k)$$

$$. \, \bar{H}^{M',N'}_{P',Q'}\left[z'(b-x)^{k'}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N'},(a'_j,\alpha'_j)_{N'+1,P'}\\(b'_j,\beta'_j)_{1,M'},(b'_j,\beta'_j;B'_j)_{M'+1,Q'}\end{matrix}\right.\right]dx$$

$$= \sum_{h=1}^{M'}\sum_{r=0}^{\infty}\frac{\prod_{\substack{j=1\\j\neq h}}^{M'}\Gamma(b'_j-\beta'_j\xi_{h,r})\prod_{j=1}^{N'}\{\Gamma(1-a'_j+\alpha'_j\xi_{h,r})\}^{A'_j}}{\prod_{j=1+M'}^{Q'}\{\Gamma(1-b'_j+\beta'_j\xi_{h,r})\}^{B'_j}\prod_{j=1+N'}^{P'}\Gamma(a'_j-\alpha'_j\xi_{h,r})}$$

$$. \, \frac{(z')^{k'\xi_{h,r}}(-1)^{r+n}\lambda^n(b-a)^{\beta+\sigma+1}\Gamma(1+\beta+n)E_{d-1}\Gamma(p+1)\Gamma(\frac{1}{2}+\frac{\tau}{2})}{r!\;\; \beta_h\;\; n!(-1)^p 2^{2+p}\pi^{1/2}\Gamma(\gamma)\Gamma(\gamma-\frac{\tau}{2})}$$

Ref.

4. Chiney, S.P.and Bhonsle, B.R. – Some results involving extended Jacobi polynomials Rev. Univ. Nac. Tucumán, A, mat. fiu.teor. Tucumán, ISSN0080-2360,V-25, No 1-(1975), 7-11.

$$.\,\overline{H}^{1,5}_{5,5}\left[-z(b-a)^k \left| \begin{matrix} (\alpha-\sigma-k'\xi_{h,r},k;1),(-\sigma-k'\xi_{h,r},k;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P},(1-\gamma,1;1) \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(\alpha+n-\sigma-k'\xi_{h,r},k;1),(-1-\sigma-\beta-n-k'\xi_{h,r},k;1) \end{matrix}\right.\right.$$

$$\left.\begin{matrix} (1-\gamma+\frac{\tau}{2},1;1),(1-\eta,1;1+p) \\ (0,1),(-\frac{\tau}{2},1;1),(-\eta,1;1+p) \end{matrix}\right], \qquad (3.2)$$

valid under the condition as surrounding (2.1).

III. Application

We shall define the Riemann-Liouville fractional derivative of function f(x) of order σ (or, alternatively, −σth order fractional integral) ([5], p.181, 11, p.49) by

$$_{\alpha}D_x^{\sigma}\{f(x)\} = \begin{cases} \dfrac{1}{\Gamma(\sigma)}\displaystyle\int_0^x (x-t)^{-\sigma-1} f(t)\,dt, & Re(\sigma) \geq 0 \\ \dfrac{d^q}{dx^q}\, {}_{\alpha}D_x^{\sigma-q}\{f(x)\}, & (q-1) \leq Re(\sigma) < q, \end{cases} \qquad (4.1)$$

where q is a positive integer and the integral exists.

For $\alpha = 0$, we have $D_x^{\sigma} \equiv {}_0D_x^{\sigma}$.

Now, replacing b by x and a = 0 in the main result, it can be rewritten as the following fractional integral formula

$$.\; D_x^{-\sigma}\left\{ x^{\beta}\, \overline{H}^{M,N}_{P,Q}\left[z(x-t)^k \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q} \end{matrix} \right.\right]\right.$$

$$.\; \left.\overline{H}^{M',N'}_{P',Q'}\left[z'(x-t)^{k'} \left| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N'},(a'_j,\alpha'_j)_{N'+1,P'} \\ (b'_j,\beta'_j)_{1,M'},(b'_j,\beta'_j;B'_j)_{M'+1,Q'} \end{matrix} \right.\right] F_n(\beta,\alpha;t) \right\}$$

$$= \sum_{h=1}^{M'} \sum_{r=0}^{\infty} \frac{\prod\limits_{\substack{j=1 \\ j\neq h}}^{M'} \Gamma(b'_j - \beta'_j \xi_{h,r}) \prod\limits_{j=1}^{N'} \{\Gamma(1-a'_j+\alpha'_j\xi_{h,r})\}^{A'_j}}{\prod\limits_{j=1+M'}^{Q'} \{\Gamma(1-b'_j+\beta'_j\xi_{h,r})\}^{B'_j} \prod\limits_{j=1+N'}^{P'} \Gamma(a'_j - \alpha'_j\xi_{h,r})}$$

$$.\; \frac{(-1)^{r+n}\,\lambda^n\,(z')^{k'\xi_{h,r}}\, x^{\beta+\sigma+k'\xi_{h,r}}\,\Gamma(1+\beta+n)}{r!\; n!\,\beta_h\;\Gamma(\sigma)}$$

Ref.

5. Erdélyi, A. et al. – Tables of Integral Transforms, Vol.II, McGraw-Hill, New York, 1954.

$$. \overline{H}^{M,N+2}_{P+2,Q+2}\left[z\,x^k \left| \begin{matrix} (\alpha-\sigma-k'\xi_{h,r},k;1),(-\sigma-k'\xi_{h,r},k;1),(a_j,\alpha_j;A_j)_{1,N},(a_j,\alpha_j)_{N+1,P} \\ (b_j,\beta_j)_{1,M},(b_j,\beta_j;B_j)_{M+1,Q},(\alpha+n-\sigma-k'\xi_{h,r},k;1),(-1-\sigma-\beta-n-k'\xi_{h,r},k;1) \end{matrix} \right. \right], \quad (4.2)$$

where $Re(\sigma) > 0$ and all other conditions of validity mentioned with (2.1) are satisfied.

Notes

The results recently derived by Gupta and Soni in [6], Chaurasia and Srivastava in [2] and Chaurasia and Pandey in [3] can be obtained on giving suitable values to the parameters and arguments. The result given in (4.2) is also quite general in nature and can easily yield Riemann-Liouville fractional integrals of large number of simpler functions and polynomials merely by specializing the parameters of H and F_n appearing in it which may find applications in electromagnetic theory, statistical mechanics and probability theory.

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Notes

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Hypergraph-Based Edge Detection in Gray Images by Suppression of Interior Pixels

By R.Dharmarajan & K.Kannan
SASTRA University

Abstract – This paper presents a new two-stage hypergraph-based algorithm for edge detection in noise-free gray images. The first stage consists of mapping the input image onto a hypergraph called the Intensity Interval Hypergraph (IIHG) associated with the image. In the second stage, each hyperedge is partitioned into two disjoint subsets, namely, the interior pixels and the edge pixels. The interior pixels are then suppressed, so that the edge pixels trace out the edges in the image. These edges are then sharpened using an edge sharpener function to eliminate all the duplicated edges. The algorithm is validated on a number of images of largely varying details, and shows promising results. Other hypergraph-based algorithms are of computational complexity $O(n^2)$ or $O(n^3)$ whereas the IIHG model works at a reduced computational complexity of $O(n)$.

Keywords : *Hypergraph, hyperedge, chessboard metric, interior point, edge.*

AMS subject classification : *05C65, 68U10*

HYPERGRAPH-BASED EDGEDETECTION IN GRAY IMAGES BY SUPPRESSION OF INTERIOR PIXELS

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Ref.

2. D.J. Morris, M. de J. Lee and A.G. Constantinides, A unified method for segmentation and edge detection using graph theory, ICASSP '86, TOKYO, (1986) 2051-2054.

Hypergraph-Based Edge Detection in Gray Images by Suppression of Interior Pixels

R.Dharmarajan [α] & K.Kannan [α]

Abstract - This paper presents a new two-stage hypergraph-based algorithm for edge detection in noise-free gray images. The first stage consists of mapping the input image onto a hypergraph called the Intensity Interval Hypergraph (IIHG) associated with the image. In the second stage, each hyperedge is partitioned into two disjoint subsets, namely, the interior pixels and the edge pixels. The interior pixels are then suppressed, so that the edge pixels trace out the edges in the image. These edges are then sharpened using an edge sharpener function to eliminate all the duplicated edges. The algorithm is validated on a number of images of largely varying details, and shows promising results. Other hypergraph-based algorithms are of computational complexity $O(n^2)$ or $O(n^3)$ whereas the IIHG model works at a reduced computational complexity of $O(n)$.

Keywords / phrases : Hypergraph, hyperedge, chessboard metric, interior point, edge.

I. Introduction

In edge detection, one approach is to track pixels column wise (or, row wise) before using statistical measures for the processing [1]. Graph-based approach [2] identifies binary-related pixels before processing them. Graphs are mathematical modeling tools for low-level image processing applications because graphs are essentially about relationships between objects (these are pixels in images). But graphs do not go beyond binary relations, and pixel relations in images are, in most applications, complex and not necessarily binary. Hence a model that can accommodate higher order relations would be desirable and valuable.

Hypergraphs do precisely that – they accommodate higher order object relations. Hypergraph theory is an original work of Claude Berge [3]. As mathematical entities, hypergraphs are rich and extensive in theory. They also have applications, and published research works [4-7] have shown hypergraphs to be excellent tools in image processing.

The concept of edge is a very familiar one, yet there is no precise rigorous definition of an edge in an arbitrary image. Indeed, the concept as we use it is an abstract one, and so it can give different meaning in different contexts [8]. Several widely accepted ideas of edges and edge detection methods are reported in literature [9, 10]. Essentially, edges in an image correspond to intensity discontinuities or visible intensity changes. The average human eye sees edges in the form of boundaries of objects in the target image. Edge detection, therefore, can be thought of as the process of bringing into view these boundaries while *suppressing* the rest of the image. Broadly, edge detection can be considered a two-stage process: first, the characterization of intensity changes; and second, the use of some structural knowledge to find the edges [11]. Some widely known edge detectors are the Sobel, the Laplacian-of-Gaussian (or LoG) and the Canny edge detectors. However, they do have drawbacks: appearance of undesirable double edges, large and complicated set of rules, and generation of speckles, to mention a few.

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A premise in this paper is that edges are consequences of pixel features and pixel relations. As regards gray images, we have only two aspects at our disposal: the intensity of each pixel and the spatial relationship between pixels. Many algorithms in the traditional class tend to ignore the important spatial relationship aspect [2]. This problem is addressed in the hypergraph framework in this paper.

Wide and thick edges (roughly speaking, these are edges upon edges without any separating features, with one edge following exactly the course of the other) hamper edge detection processes even in clean images by producing undesirable duplication effects in the output image (or, the edge image). So there is a need to characterize not just edges but also duplication of edges to identify the undesirable thick edges before eliminating such. This hypergraph-based work brings some properties of sets and functions into a hypergraph model towards such characterization in a clean image.

The contribution of this article is a novel hypergraph model (called the IIHG, detailed in section 3) for edge detection (in noise-free gray images) with reduced complexity O (n). To the best of our knowledge, the proposed algorithm is the first hypergraph-based one for edge detection with complexity O (n).

The remainder of this paper is organized in sections 2 through 7. Section 2 mentions some published and widely-cited graph- and hypergraph-based edge detection works. Section 3 introduces the hypergraph model that is the base for our algorithm. Section 4 presents the flow of the algorithm in a compact form. Results of experiments on standard test images and real world images are reported and discussed in section 5. This section also features comparative studies to establish the excellent performance and potential of the proposed algorithm. Features of the algorithm are presented in section 6. Concluding remarks form section 7.

II. Related Information on Graph- and Hypergraph-Based Edge Detection

A unified graph-based method for segmentation and edge detection is given in [2], which is in a way a pioneering shift from the traditional approach (of row or column tracking). In [2], mapping of the image onto a graph and computation of shortest spanning trees are important preludes to the process of segmentation and edge detection. However, [2] does not go into the computational complexity of the algorithm . Also, this approach conveys an impression that pixel relations in any image could be simplistic enough to be binary, which impression finds no support in published research.

Bretto and others [4-7] based their research on hypergraph models where patches of pixels (rather than pairs of pixels) are processed by algorithms that are guided principally by the pixel intensity values. This approach is reflective of the fact that hypergraphs are generalizations of graphs. Besides mapping the image onto a hypergraph structure, Bretto et al use the idea of stars and star aggregates. These illustrate possibilities of application of higher order pixel relations in hypergraphs to image processing. But these algorithms are computationally expensive (O (n^3)) and tend to leave unprocessed pixels behind.

III. Hypergraph Representation of a Gray Image

A *hypergraph* is a couple H = (V, E), where V is a nonempty finite set and E is a family of nonempty subsets of V that fills out V. Since E is finite, we index it by a set J = {1, …, k}, k ∈ N, and so we have $E = \{X_1, \ldots, X_k\}$ and $X_1 \cup \ldots \cup X_k = V$. The set V is called the *vertex set* of H. The family E is called a *hyperedge family* on the vertex set V, and each member of E is called a *hyperedge* (in H).

Ref.

4. Lei Ding and Alper Yilmaz, Interactive image segmentation using probabilistic hypergraphs, Pattern Recognition 43 (2010) 1863-1873.

If the members of E are distinct (meaning: $i \neq j \Longrightarrow Xi \neq X_j$), then H is *simple*. In this case, E is a *set* of nonempty subsets of V. H = (V, E) is called a *partitioned hypergraph* if its hyperedges form a partition of V – i.e., $V = X_1 \cup \ldots \cup X_k$ and $X_i \cap X_j = \varphi$ for $i \neq j$ (where φ denotes the empty set).

To begin with, the input image is represented as a partitioned hypergraph. The hyperedges for this representation are constructed as follows:

A digital gray image labeled I (and assumed noise-free) is mathematically represented by the function $I: V \rightarrow W$ (where $V \subseteq N \times N$ and W is the set of non-negative integers), where for $a = (x, y) \in V$, I (a) is the gray scale intensity value of the pixel a located at $(x, y) \in N \times N$, so that it is natural to think of the image I as a nonempty finite subset V of N x N. Let V be endowed with the chessboard metric ρ.

Let L be a positive integer, $L \leq 254$ and $q = [255 - 255(\text{mod } L)] / L$. We set

(a) $E_1 = \{a \in V \mid 0 \leq I(a) \leq L\}$,

(b) $E_k = \{a \in V \mid (k-1) L + 1 \leq I(a) \leq k L\}$ for k = 2, …, q,

(c) $E_{q+1} = \{a \in V \mid q L + 1 \leq I(a) \leq 255\}$. Obviously the E_t (t = 1, . . ., q + 1) are subsets of V, some possibly empty (φ).

Let $E = \{E_t \mid t = 1$ through $q + 1$; and $E_t \neq \varphi''$. Then E is a set of nonempty subsets of V, and E fills out V. We take H = (V, E). Then H is a hypergraph on the set V, and thereby is a hypergraph representation of the image I. We call this the *Intensity Interval Hypergraph* (IIHG) associated with the image I. This hypergraph is a partitioned one.

The essential mathematics for the algorithm is given in the appendix (after the references), where all the theory (A1 through A5) is within the framework of the IIHG on V detailed above.

IV. Proposed Algorithm

Figure 1 below gives the flow of the proposed IIHG algorithm. The input image data (box numbered 1 in Fig. 1) are as follows:

1(a) V = set of pixels of the image I (as a finite nonempty subset of $N \times N$)

1(b) Gray scale intensity matrix of V.

1(c) Domain distance metric ρ (Chessboard metric) on V

1(d) Parameter L (called 'intensity interval')

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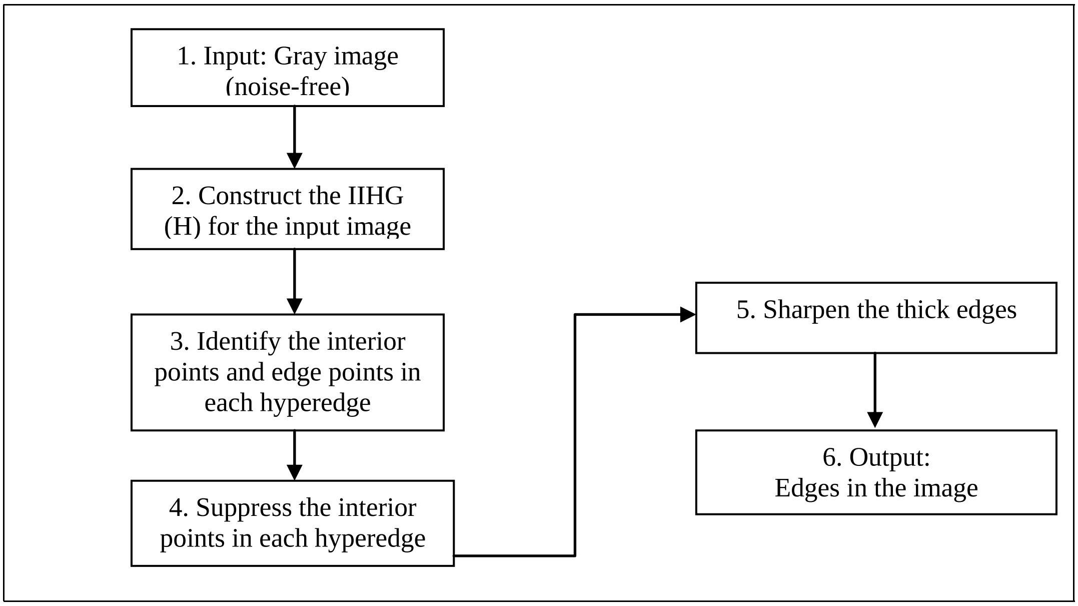


Figure 1 : The IIHG algorithm flow diagram

V. Experiments and Discussion

The computing environment for coding the proposed IIHG algorithm has the following principal components:

(i) Computer category: Micro

(ii) Processor: Intel i13, 3.2 GHz

(iii) Software: MATLAB® 7.0.1

In the proposed algorithm, the output showing the edges depends on the number of hyperedges. The more the number of hyperedges, the denser the edges in the output image.

a) Test reports

Figure 2 below is a simple illustration of how the edge detection algorithm works on a 10 x 10 image patch for L = 90. This patch is a part of a test image from [12].

Ref.

12. http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/BSDS300/html/dataset/images.html

207 P_1	158 P_{11}	121 P_{21}	138 P_{31}	131 P_{41}	128 P_{51}	132 P_{61}	133 P_{71}	129 P_{81}	128 P_{91}
127 P_2	142 P_{12}	172 P_{22}	193 P_{32}	161 P_{42}	139 P_{52}	130 P_{62}	124 P_{72}	127 P_{82}	132 P_{92}
157 P_3	171 P_{13}	169 P_{23}	201 P_{33}	155 P_{43}	128 P_{53}	131 P_{63}	133 P_{73}	129 P_{83}	127 P_{93}
183 P_4	157 P_{14}	109 P_{24}	101 P_{34}	75 P_{44}	81 P_{54}	114 P_{64}	128 P_{74}	123 P_{84}	123 P_{94}
112 P_5	95 P_{15}	92 P_{25}	72 P_{35}	56 P_{45}	69 P_{55}	102 P_{65}	121 P_{75}	126 P_{85}	125 P_{95}
62 P_6	65 P_{16}	87 P_{26}	104 P_{36}	85 P_{46}	75 P_{56}	80 P_{66}	98 P_{76}	125 P_{86}	134 P_{96}
73 P_7	67 P_{17}	54 P_{27}	74 P_{37}	86 P_{47}	92 P_{57}	76 P_{67}	66 P_{77}	91 P_{87}	123 P_{97}
60 P_8	61 P_{18}	58 P_{28}	59 P_{38}	85 P_{48}	106 P_{58}	90 P_{68}	59 P_{78}	66 P_{88}	101 P_{98}
66 P_9	61 P_{19}	65 P_{29}	60 P_{39}	55 P_{49}	60 P_{59}	63 P_{69}	61 P_{79}	78 P_{89}	107 P_{99}
68 P_{10}	61 P_{20}	56 P_{30}	51 P_{40}	61 P_{50}	61 P_{60}	57 P_{70}	59 P_{80}	68 P_{90}	92 P_{100}

Figure 2 : The input image is the patch of size 10 x 10. The hundred pixels in this patch are labeled P_1 through P_{100}. The intensity value of each pixel is just above its label. The pixels filled with black are the edge pixels as identified by the algorithm. The pixels without any filling color are the suppressed pixels – these are either the interior pixels suppressed in the partitioning of the hyperedges or the edge pixels suppressed in the sharpening of thick edges.

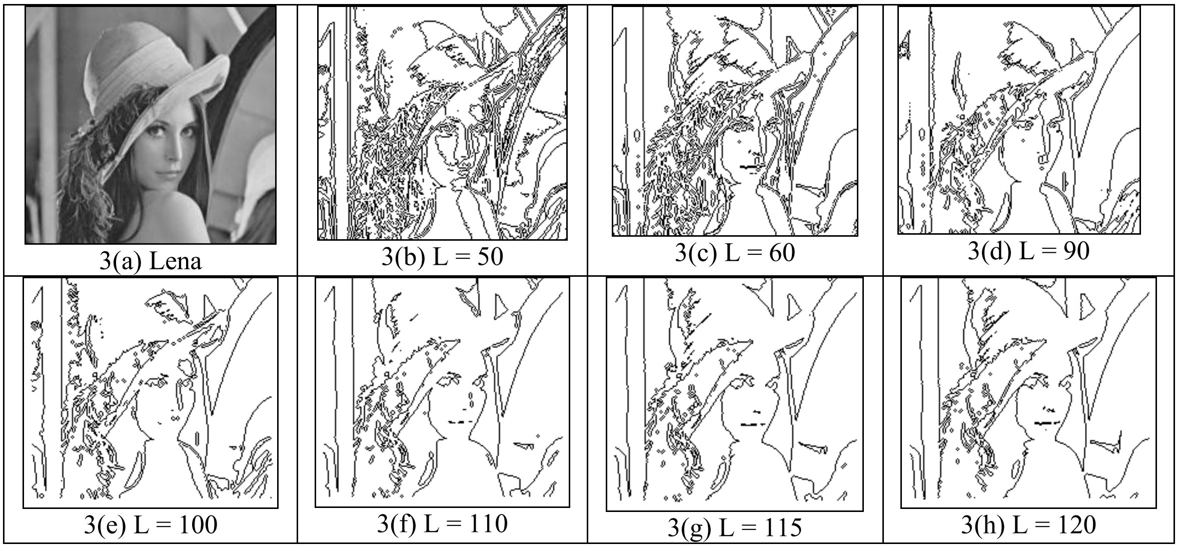
Over six hundred images were taken from [12, 13] and Google Earth which contain ranges of gray images with widely varying features and details. Several of these images appear in published works, and are standard test images – for instance, Lena, Photographer and Peppers – in image processing research. Tests on five widely used images are reported and discussed in this section. The values of the parameter L specified in the reports have been selected after exhaustive testing covering the entire range of L ($1 \leq L \leq 254$). For each image reported here, the selected values of L produce visually more credible results (to the subjective human eye) than its other values. As is always the case in any low level image processing, the judgment of the visual results shown in the examples is subjective.

In fig. 3(b) and 3(c), the outputs show 'cluttering' of edges for $L = 50$ and $L = 60$, respectively. For $L > 80$, the edges become more distinguishable. However, as L is increased, some of the edges may actually disappear- for instance, in the edge image for $L = 100$, the outline of the lips has all but vanished. It is inferred that large values of L could result in loss of edges. And this is in direct contrast to 'too many edges' (or, cluttering) resulting from a low value of L. As regards the Lena image, our inference is that $80 \leq L \leq 95$ is a good range for the edge image to be a reliable representative of the true edges in the original (to the human eye). A similar inference can be made for each image tested.

Ref.

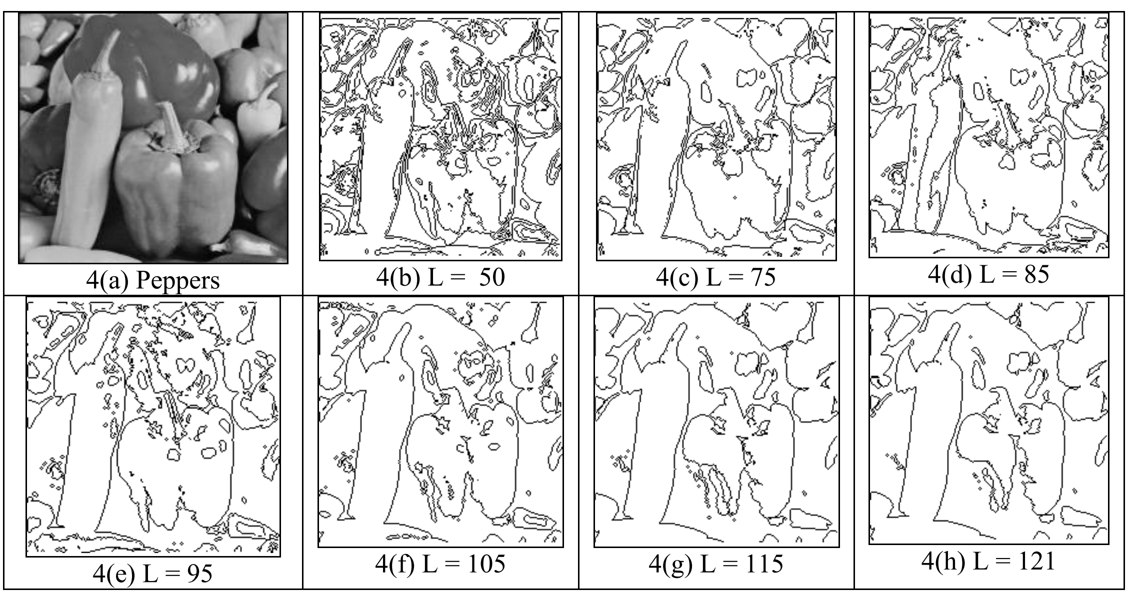
13. http://www.vision.caltech.edu/htmlfiles/archive.html

In each of the following test reports, the first image (a) is the input (original). The others are the output edge images for the specified values of L.

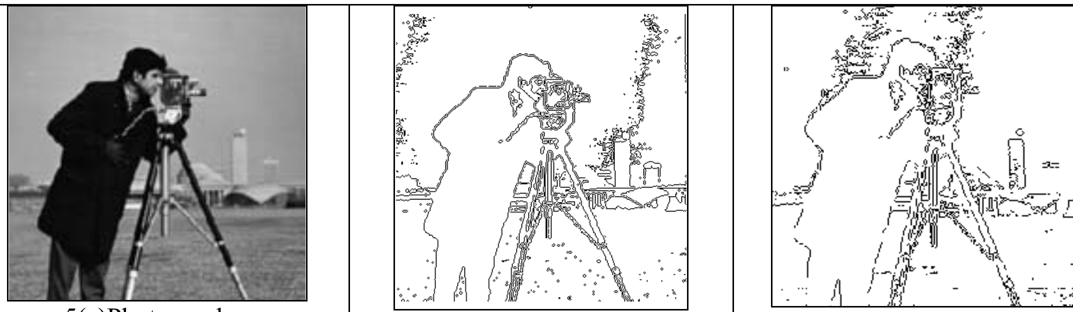


3(a) Lena 3(b) L = 50 3(c) L = 60 3(d) L = 90
3(e) L = 100 3(f) L = 110 3(g) L = 115 3(h) L = 120

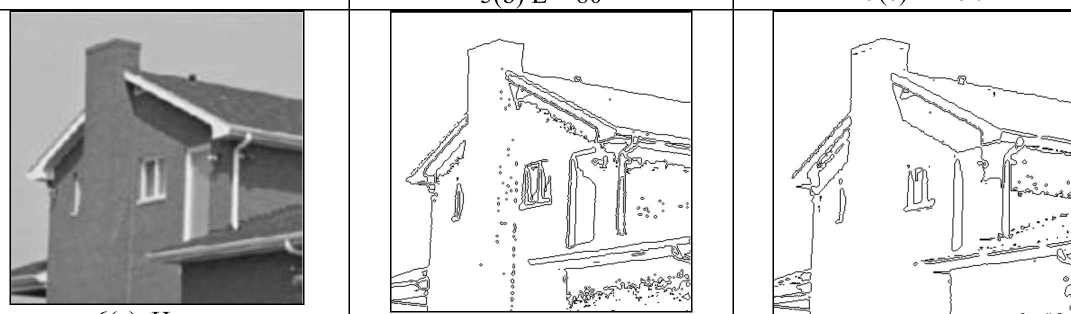
Another widely used test image – Peppers – features in figure 4. The range 90 ≤ L ≤ 120 gives better edge representations. The Lena and the Peppers images also feature in the comparison of our proposed algorithm with three other edge detection algorithms, shown in section 5.2



4(a) Peppers 4(b) L = 50 4(c) L = 75 4(d) L = 85
4(e) L = 95 4(f) L = 105 4(g) L = 115 4(h) L = 121

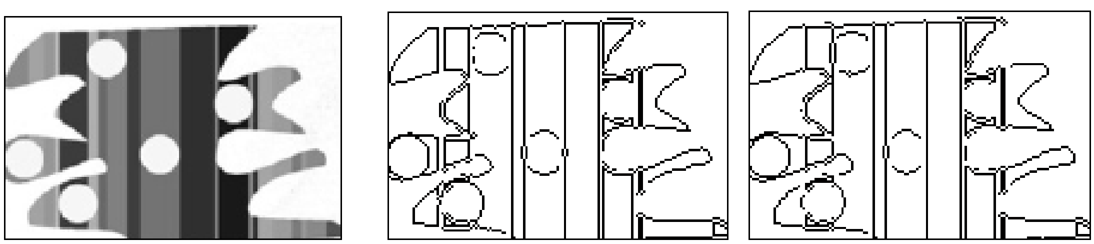


5(a)Photographer | 5(b) L = 80 | 5(c) L = 90

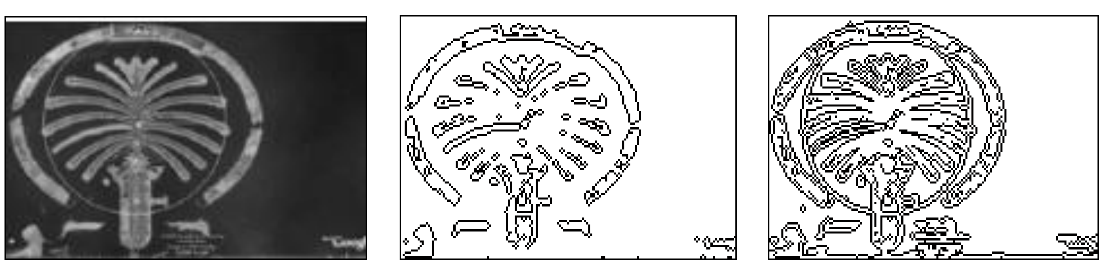


6(a) House | 6(b) L = 70 | 6(c) L = 90

Synthetic and real world images were tested with a view to stress-testing the code. Results on two such images are seen in figures 7 and 8 below.



7(a) Synthetic image (Syn 1) | 7(b) Edges in Syn 1 (L = 90) | 7(c) Edges in Syn 1 (L = 100)



8(a) Real world image (RW1) (Acquired from Google Earth) | 8(b) Edges in RW 1 (L = 128) | 8(c) Edges in RW 1 (L = 65)

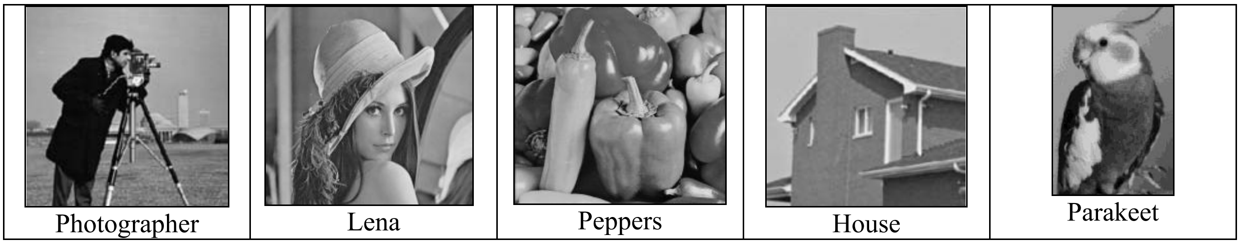
The images reported here are of different sizes (80 x 80 to 512 x 512) and detail contents. CPU run time is more for some of these images because of their larger size.

b) Comparisons and performance reports

The proposed IIHG algorithm was compared for edge detection results with three other published algorithms – namely, the Sobel [14], the Canny [15], and the MG-IT2FIS [16]. The original images (inputs) are in panel 1. The results of each of the four algorithms on these five images are in panel 2. Figures in these panels have not been numbered. As can be seen from panel 2, the comparison works out, to a significant extent, in favor of the proposed IIHG-based algorithm.

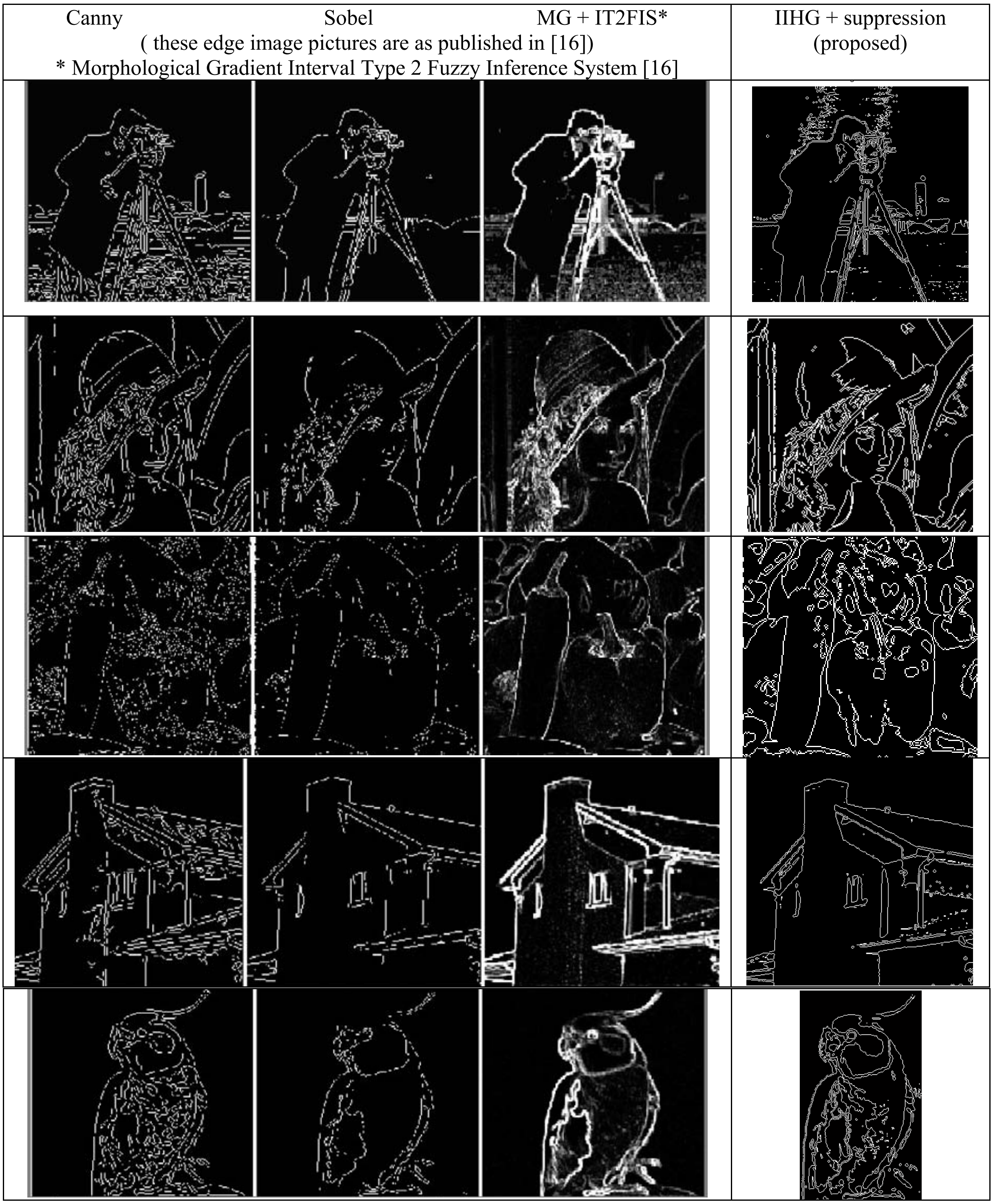
Ref.

15. John Canny, A computational approach to edge detection, IEEE transactions on Pattern Analysis and Machine Intelligence, 6 (1986) 679-698.



Panel 1 : The five original images that feature in the comparison (shown in panel 2)

Notes



Panel 2 : Comparison of the proposed IIHG algorithm with Sobel, Canny and MG+IT2

Notes

In the above comparison experiment, we used L = 90 (for the Photographer image), 80 (Lena), 95 (Peppers), 90 (House) and 60 (Parakeet).

Table 1 : Performance of the proposed algorithm (Standard test images) #H: Number of hyperedges; *Run time in seconds, rounded to one place after the decimal

S.no.	Image	L	#H	Run time*	S.no.	Image	L	#H	Run time*
1	Lena (200 x 200; Bitmap)	50	6	13.8	4	Peppers (200 x 200; JPEG)	50	6	12.8
		60	5	12.7			75	4	10.5
		90	3	10.3			85	3	10.9
		100	3	11.2			95	3	9.5
		110	3	11			105	3	8.8
		115	3	10			115	3	8.4
2	Photographer (333 x 336;Bitmap)	80	4	15.8	5	House (333 x 333; Bitmap)	70	4	10
		90	3	17.4			90	3	8.1
		105	3	17.9			100	3	7
3	Parakeet (328 x 198; Bitmap)	60	5	15	6	Other images** TIF / JPEG / Bitmap	40 to 150	2 to 7	3 to 40
		70	4	13.2					
		85	3	12.4					

** More than 350 images from [12] and [13] (size: 80 x 80 to 512 x 512)

Table 2 : Performance of the proposed algorithm (Synthetic and real world images)

S.no.	Image & size	L	#H	Run time*
1	Syn 1 (94 x 150; Bitmap)	90	3	8.2
		100	3	8.6
		55	5	7.1
2	RW1 (108 x 168; Bitmap)	128	2	10.5
		93	3	14
		65	4	16.6
3	Other images*** TIF / JPEG / Bitmap	40 to 150	2 to 7	6 to 40

*** More than 300 images from [12] and [13] and Google Earth (size: 90 x 90 to 512 x 512)

c) *Computational complexity of the proposed algorithm*

The number of hyperedges in the first stage does not exceed q + 1, where q = [255 – 255 (mod L)] / L. So for any positive integer value of L, we have $q \leq 255$. Since the hyperedges are non-intersecting, each pixel is visited exactly once in the first stage.

In the second stage, in each hyperedge, each pixel is visited at most four times for segregating the edge points from the interior points. Then, to suppress each interior pixel, we need exactly one assignment of the value 255 to the pixel. Subsequently, to identify the thick edges, each edge pixel is visited at most four times. And the sharpening process that follows takes one assignment operation (the one mentioned above) for every pair of thick edge pixels that correspond in the required bijective way (see A3 of appendix). Hence the number of computations in the algorithm is λn, with $1 \leq \lambda \leq 10$, where n = number of pixels in the input image. Let f(n) = n and g(n) = λ n. As n tends to ∞, the limit of f(n) / g(n) is $1/\lambda$, and that of g(n)/ f(n) is λ. Since both λ and $1/\lambda$ are finite and nonzero, we aver that the complexity of the proposed algorithm is O(n).

VI. Algorithm Features

(i) The sequential combination of two functions – thick edge identifier and thick edge sharpener, in that order – ensures that no redundancies appear in any edge. This combination is effective principally because of the IIHG model.

(ii) The first stage (construction of the IIHG) ends when the empty set (φ) takes the place of V, and this happens in at most q + 1 steps. The second stage (edge detection) ends when each hyperedge has been cleared of its thick edges, which happens in at most S steps (but in most images well below S because of interior points being excluded from this process), where $S = \Sigma\,\Sigma\,(|E_j| \times |E_k|)$, the sums running over the indices j (second) and k (first) with $j, k \in \{1, . . , |E|\}$ and $j < k$; and whatever the value of L, $|E|$ does not exceed 255. Thus the algorithm is convergent.

(iii) The algorithm handles large sized images of varying dimensions and for all values of L in its stipulated range ($1 \leq L \leq 254$), and so is robust.

(iv) The algorithm is fast for test images that are widely used as standards by researchers in image processing (for instance, Lena and Peppers).

(v) Since the output is always viewed rather subjectively, edges that are considered 'not desirable' can be removed by tuning L. While this is a facility that is in-built in the algorithm, tuning L to eliminate such 'undesirable' edges could accidentally rub out true edges also. This is one limitation of the algorithm.

VII. Concluding Remarks

(i) We have presented a hypergraph-based one-parameter-driven partitioning algorithm for edge detection in clean gray images. The algorithm processes patches of pixels of arbitrary (finite) size and distribution efficiently. The computational complexity is O(n), which is an outstanding feature here.

(ii) From the tests reported in section 5.1, we have arrived at an apparently good range for L for a large number of images, standard or real-time, and this is $60 \leq L \leq 120$. However, L is image-dependent. Going by our tests (on hundreds of standard, real-time and synthetic images), we report that $L < 60$ tends to clutter the output figure with too many edges because false edges are shown among the true ones. On the other hand, for $L > 120$ could accidentally suppress considerable number of edge pixels, resulting in loss of true edges. Since performance of parameter-driven algorithms are application-dependent, we have not gone into the question of optimizing L.

(iii) In image engineering applications, the input image may have to be first subjected to a noise removal scheme before the IIHG algorithm is applied. As for noise removal, adequate schemes are available [5, 6, 17-20].

VIII. Acknowledgements

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Ref.

16. Patricia Melin, Olivia Mendoza, Oscar Castillo, An improved method for edge detection based on interval type-2 fuzzy logic, Expert Systems with Applications, 37 (2010) 8527–8535.

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Notes

APPENDIX

Bonded sets

Let N × N denote the Cartesian square of the set N of positive integers. For $(x_1, y_1), (x_2, y_2) \in N \times N$, we define $\rho((x_1, y_1), (x_2, y_2)) = \max\{|x_1 - x_2|, |y_1 - y_2|\}$. The function ρ is a metric on N x N – and hence on any nonempty subset of N x N – and ρ is called the *chessboard metric*.

For a given nonempty set V, by 2^V we mean the power set of V; and by 2^{V*} we mean the set of all nonempty subsets of V. Let V be a finite nonempty subset of N x N endowed with the chessboard metric.

If $X \in 2^{V*}$ and $a \in V$, we define $\rho(a, X) = \min\{\rho(a, b): b \in X''$. If $X, Y \in 2^{V*}$ then we define $\rho(X, Y) = \min\{\rho(a, b): a \in X, b \in Y''$.

Let A be a nonempty subset of V. A finite sequence $x_1, \ldots, x_k$ of elements of A is called a *1-step sequence* (1-ss) in A if $\rho(x_i, x_{i+1}) = 1$ for each $i = 1, \ldots, k-1$. If a, b ∈ A, then we say a is *bonded* to b in A if $\rho(a, b) \leq 1$ or if there exist points $z_1, \ldots, z_k$ in A such that the sequence $a, z_1, \ldots, z_k, b$ is a 1-ss in A. In this case we write –a: b″A.

Clearly: (i) {a: a″A, (ii) {a: b″A ⟹ {b: a″A and (iii) {a: b″a and {b: c″A ⟹{a: c}A, for all a, b, c ∈ A. Further, {a: b″A ⟹ {a: b″B whenever A ⊆ B. A is called a *bonded set* if {a: b″A for every a, b ∈ A. A singleton set is obviously bonded.

Interior points and edge points in an image

Given a = (x, y) ∈ V, we define the neighborhood $B_4(a)$ as:

$B_4(a) = \{b = (p, q) \in V \mid \rho(a, b) \leq 1$ and (x = p or y = q)″. Clearly $a \in B_4(a)$ for each a ∈ V.

Let $A \in 2^{V*}$and a = (x, y) ∈ A. We say a is an *interior point* of A if and only if B_4 (a) ⊆A. We let IntA denote the set of all the interior points of a given set A. If a is not an interior point of A then we call it an *edge point* of A.

Edges in an image

By $|A|$ we mean the cardinality (or, size) of the set A. An *edge* in V is a nonempty subset e (V) of V with the following properties:

(ed-1) e (V) ⊆ X for some (hence unique) hyperedge X in H;

(ed-2) $|e(V)| > 1$;

(ed-3) no point of e (V) is an interior point of X;

(ed-4) e (V) is bonded, and

(ed-5) if Y satisfies (i) e (V) ⊂ Y ⊆ X, (ii) e (V) ≠ Y and (iii) Y ∩ IntX = φ, then Y is not bonded.

A *thick edge* (or, a duplicated edge) in V is a nonempty subset r(V) of V that can be partitioned as $r(V) = r_1(V) \cup r_2(V)$ (i.e., $r_1(V)$ and $r_2(V)$ are nonempty subsets of r(V) such that $r_1(V) \cap r_2(V) = \varphi$) with the following properties:

(t-ed-1) $|r_1(V)| = |r_2(V)|$,

Notes

(t-ed-2) $r_1(V) \subseteq X_1$ and $r_2(V) \subseteq X_2$ for some distinct (hence disjoint) hyperedges X_1 and X_2 in H (we call X_1 and X_2 the *source hyperedges* of $r_1(V)$ and $r_2(V)$, respectively), and

(t-ed-3) there exists a bijective map f: $r_1(V) \to r_2(V)$ such that for each $a \in r_1(V)$ we have $a \in B_4(f(a))$ as well as $f(a) \in B_4(a)$.

Notes

Suppression of interior points

Let $A \in 2^{V^*}$ and $b^*(A) = A - \text{Int}A$, where IntA denotes the set of all the interior points of A. Let. Evidently $b^*(A)$ is nonempty unless A = V. We call the computation of $b^*(A)$ the *suppression* of the interior points of A. Notice that if $A \in E$, then $b^*(A)$ is either an edge in V or a union of edges in V.

Sharpening of thick edges

Given two distinct hyperedges X_1 and X_2 in H, we write $X_1 < X_2$ if $I(a) < I(b)$ for every $a \in X_1$ and $b \in X_2$. Let $r(V) = r_1(V) \cup r_2(V)$ be a thick edge in V with source hyperedges X_1 and X_2, respectively, such that $X_1 < X_2$. Let $\psi: r_2(V) \to W$ be the constant function $\psi(b) = 255$. The function ψ is called the edge sharpener function. It suppresses one half of the targeted thick edge out of the picture, so that only the other half is seen in the edge image.

Notes

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Integral Formulae's Involving Two $\bar{H}$-function and Multivariable Polynomials

By Praveen Agarwal
Anand International College of Engineering

Abstract – The aim of the present paper is to derive a new Integral formulae's for the $\bar{H}$ - function due to Inayat-Hussain whose based upon some integral formulae due to Qureshi et.al. The results are obtained in a compact form containing the multivariable Polynomials.

Keywords : $\bar{H}$-function, general class of polynomials, generalized wright hypergeometric function.

GJSFR-F Classification : (MSC 2000) 33C45, 33C60

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Ref.

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Praveen Agarwal

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I. Introduction

In 1987, Inayat-Hussain [1, 2] introduced generalization form of Fox's H-function, which is popularly known as $\bar{\text{H}}$-function. Now $\bar{\text{H}}$-function stands on fairly firm footing through the research contributions of various authors [1-3, 9, 10, 13-15].

$\bar{\text{H}}$-function is defined and represented in the following manner [10].

$$\bar{H}^{m,n}_{p,q}[z] = \bar{H}^{m,n}_{p,q}\left[z \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right. \right] = \frac{1}{2\pi i}\int_L z^{\xi}\bar{\phi}(\xi)d\xi \ (z \neq 0) \tag{1.1}$$

where

$$\bar{\phi}(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \tag{1.2}$$

It may be noted that the $\bar{\phi}(\xi)$ contains fractional powers of some of the gamma function and m,n,p,q are integers such that $1 \leq m \leq q, 1 \leq n \leq p$ $(\alpha_j)_{1,p},(\beta_j)_{1,q}$ are positive real numbers and $(A_j)_{1,n},(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(\alpha_j)_{1,p}$ and $(\beta_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\bar{\text{H}}$-function can be seen in the papers [9, 10].

The behavior of the $\bar{\text{H}}$-function for small values of $|z|$ follows easily from a result given by Rathie [3]:

$$\bar{H}^{m,n}_{p,q}[z] = o(|z|^{\alpha}); \text{ Where}$$

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$$\alpha = \min_{1\le j\le m} \operatorname{Re}\left(\frac{b_j}{\alpha_j}\right), |z| \to 0 \quad (1.3)$$

$$\Omega = \sum_{j=1}^{m} |B_j| + \sum_{j=m+1}^{q} |b_j B_j| - \sum_{j=1}^{n} |a_j A_j| - \sum_{j=n+1}^{q} |A_j| > 0\,,\, 0 < |z| < \infty \quad (1.4)$$

The following function which follows as special cases of the $\bar{\mathrm{H}}$-function will be required in the sequel [9]

$${}_p\bar{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};z\right] = \bar{H}^{1,p}_{p,q+1}\left[-z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p}\\(0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right] \quad (1.5)$$

The general class of multivariable polynomials is defined by Srivastava and Garg [7]:

$$S_L^{h_1,\dots,h_r}[x_1,\dots,x_r] = \sum_{k_1,\dots,k_r=0}^{h_1k_1+\dots+h_rk_r\le L} (-L)_{h_1k_1+\dots+h_rk_r} A(L;k_1,\dots k_r)\frac{x_1^{k_1}}{k_1!}\cdots\frac{x_r^{k_r}}{k_r!} \quad (1.6)$$

Where $h_1,\dots,h_r$ are arbitrary positive integers and the coefficients $A(L;k_1,\dots,k_r)$, $(L;h_i\in N; i=1,\dots,r)$ are arbitrary constant, real or complex.

Evidently the case $r=1$ of the polynomials (1.6)

Would correspond to the polynomials given by Srivastava [5]

$$S_L^h[x] = \sum_{k=0}^{[L/h]} \frac{(-L)_{hk}}{k!} A_{L,k} x^k \{L\in N=(0,1,2,\dots)\} \quad (1.7)$$

Where h is arbitrary positive integers and the coefficient $A_{L,k}(L,k\ge 0)$ are arbitrary constant, real or complex.

The following formulas [11 , p.77, Ens. (3.1), (3.2) & (3.3)] will be required in our investigation.

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\ (a>0; b\ge 0; c+4ab>0; \operatorname{Re}(p)+1/2>0) \quad (1.8)$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\ (a\ge 0; b>0; c+4ab>0; \operatorname{Re}(p)+1/2>0) \quad (1.9)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2 + c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}}\frac{\Gamma(p+1/2)}{\Gamma(p+1)},\ (a>0; b>0; c+4ab>0; \operatorname{Re}(p)+1/2>0) \quad (1.10)$$

II. Main Integral Formulae's

Let X stands for $\left(ax+\frac{b}{x}\right)^2 + c$

Ref.

7. H.M. Srivastava and M. Garg, Some integrals involving a general class of polynomials and multivariable H-function, RevRoumaine Phys 32(1987), 685-692.

Notes

First Integral Formulae:

$$\int_0^\infty X^{-\eta-1} S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}]\overline{H}_{P,Q}^{M,N}\left[X^\sigma\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}_{p,q}^{m,n}\left[zX^\rho\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+1/2}}\sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\le L}(-L)_{h_1k_1+\ldots+h_rk_r}A(L;k_1,\ldots k_r)\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}k_1!}\cdots\frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r}k_r!}$$

$$\frac{1}{2\pi!}\int_L\bar{\phi}(\xi)\overline{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^\rho\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},\left(-\eta+\sigma\xi+\sum_{i=1}^r k_i\delta_i,\rho;1\right)\\\left(1/2-\eta+\sigma\xi+\sum_{i=1}^r k_i\delta_i,\rho;1\right),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}d\xi \quad (2.1)$$

The above result will be converging under the following conditions:

(I) $a>0; b\ge 0; c+4ab>0$ and $\eta>0,\delta_i\ge 0,\sigma>0,\rho\ge 0$

(II) $-\eta+\sigma\min_{1\le j\le M}\mathrm{Re}\left(\frac{b'_j}{\beta'_j}\right)+\rho\min_{1\le j\le m}\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)<\frac{1}{2}$

(III) $|\arg z|<\frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

Second Integral Formulae:

$$\int_0^\infty \frac{1}{x^2}X^{-\eta-1}S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}]\overline{H}_{P,Q}^{M,N}\left[X^\sigma\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}_{p,q}^{m,n}\left[zX^\rho\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\eta+1/2}}\sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\le L}(-L)_{h_1k_1+\ldots+h_rk_r}A(L;k_1,\ldots k_r)\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}k_1!}\cdots\frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r}k_r!}$$

$$\frac{1}{2\pi!}\int_L\bar{\phi}(\xi)\overline{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^\rho\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},\left(-\eta+\sigma\xi+\sum_{i=1}^r k_i\delta_i,\rho;1\right)\\\left(1/2-\eta+\sigma\xi+\sum_{i=1}^r k_i\delta_i,\rho;1\right),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}d\xi \quad (2.2)$$

The above result will be converging under the following conditions:

(I) $a\ge 0; b>0; c+4ab>0$ and $\eta>0,\delta_i\ge 0,\sigma>0,\rho\ge 0$

(II) $-\eta+\sigma\min_{1\le j\le M}\mathrm{Re}\left(\frac{b'_j}{\beta'_j}\right)+\rho\min_{1\le j\le m}\mathrm{Re}\left(\frac{b_j}{\beta_j}\right)<\frac{1}{2}$

(III) $|\arg z|<\frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

Third Integral Formulae:

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)X^{-\eta-1}S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}]\overline{H}_{P,Q}^{M,N}\left[X^\sigma\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}_{p,q}^{m,n}\left[zX^\rho\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\eta+1/2}} \sum_{k_1,\dots,k_r=0}^{h_1k_1+\dots+h_rk_r\le L} (-L)_{h_1k_1+\dots+h_rk_r} A(L;k_1,\dots k_r) \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1} k_1!} \cdots \frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r} k_r!}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi) \bar{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^{\rho} \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},\left(-\eta+\sigma\xi+\sum_{i=1}^{r}k_i\delta_i,\rho;1\right) \\ \left(1/2-\eta+\sigma\xi+\sum_{i=1}^{r}k_i\delta_i,\rho;1\right),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right. \right] (4ab+c)^{\xi\sigma} d\xi \tag{2.3}$$

The above result will be converging under the following conditions:

(I) $a>0; b>0; c+4ab>0$ and $\eta>0, \delta_i\ge 0, \sigma>0, \rho\ge 0$

(II) $-\eta+\sigma \min_{1\le j\le M} \mathrm{Re}\left(\frac{b'_j}{\beta'_j}\right)+\rho \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{\beta_j}\right)<\frac{1}{2}$

(III) $|\arg z|<\frac{1}{2}\Omega\pi$, where Ω is given by equation (1.4)

Proof: To prove the first integral, we first express $\bar{\mathrm{H}}$-function occurring on the L.H.S. of equation (2.1) in terms of Mellin-Barnes type of contour integral given by equation (1.1) and general class of multivariable polynomials $S_L^{h_1,\dots,h_r}[x_1,\dots,x_r]$ in series form with the help of (1.6) and then interchanging the order of integration and summation.

We get:

$$\sum_{k_1,\dots,k_r=0}^{h_1k_1+\dots+h_rk_r\le L} (-L)_{h_1k_1+\dots+h_rk_r} A(L;k_1,\dots k_r) \frac{c_1^{k_1}}{k_1!}\cdots\frac{c_r^{k_r}}{k_r!} \frac{1}{2\pi i}\int_L \bar{\phi}(\xi) \frac{1}{2\pi i}\int_L \bar{\varphi}(\zeta) z^{\zeta} \left[\int_0^{\infty} \left[\left(ax+\frac{b}{x}\right)^2 + c \right]^{-\eta+\sigma\xi+\rho\zeta+\sum_{i=1}^{r}k_i\delta_i-1} dx \right] d\xi d\zeta \tag{2.4}$$

Further using the result (1.8) the above integral becomes

$$\sum_{k_1,\dots,k_r=0}^{h_1k_1+\dots+h_rk_r\le L} (-L)_{h_1k_1+\dots+h_rk_r} A(L;k_1,\dots k_r) \frac{c_1^{k_1}}{k_1!}\cdots\frac{c_r^{k_r}}{k_r!} \frac{1}{2\pi i}\int_L \bar{\phi}(\xi)$$

$$\left\{ \frac{1}{2\pi i}\int_L \bar{\varphi}(\zeta) z^{\zeta} \left[\frac{\sqrt{\pi}}{2a(4ab+c)^{\eta-\sigma\xi-\rho\zeta-\sum_{i=1}^{r}k_i\delta_i+1/2}} \frac{\Gamma\left(\eta-\sigma\xi-\rho\zeta-\sum_{i=1}^{r}k_i\delta_i+1/2\right)}{\Gamma\left(\eta-\sigma\xi-\rho\zeta-\sum_{i=1}^{r}k_i\delta_i+1\right)} \right] d\zeta \right\} d\xi \tag{2.5}$$

Then interpreting with the help of (1.1) and (2.5) provides first integral.

Proceeding on the same parallel lines, integral second and third given by equation (2.2) and (2.3) can be easily obtained by using the results (1.9) and (1.10) respectively.

III. Special Cases

(3.1) If we put $A'_j=B'_j=A_j=B_j=1$, $\bar{\mathrm{H}}$-function reduces to Fox's H-function [6, p. 10, Eqn. (2.1.1)], then the equations (2.1), (2.2) and (2.3) take the following form.

$$\int_0^{\infty} X^{-\eta-1} S_L^{h_1,\dots,h_r}[c_1X^{\delta_1},\dots,c_rX^{\delta_r}] H_{P,Q}^{M,N}\left[X^{\sigma} \left| \begin{matrix} (a'_j,\alpha'_j)_{1,P} \\ (b'_j,\beta'_j)_{1,Q} \end{matrix} \right. \right] H_{p,q}^{m,n}\left[zX^{\rho} \left| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] dx$$

Ref.

6. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publishers, New Delhi, Madras (1982).

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+1/2}} \sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\le L} (-L)_{h_1k_1+\ldots+h_rk_r} A(L;k_1,\ldots k_r) \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1} k_1!} \cdots \frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r} k_r!}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi) H_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^{\rho} \left| \begin{matrix} (a_j,\alpha_j)_{1,p}, \left(-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho\right) \\ \left(1/2-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho\right),(b_j,\beta_j)_{1,q} \end{matrix} \right. \right] (4ab+c)^{\xi\sigma} d\xi \qquad (3.1.1)$$

$$\int_0^{\infty} \frac{1}{x^2} X^{-\eta-1} S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}] H_{P,Q}^{M,N}\left[X^{\sigma} \left| \begin{matrix} (a'_j,\alpha'_j)_{1,P} \\ (b'_j,\beta'_j)_{1,Q} \end{matrix} \right. \right] H_{p,q}^{m,n}\left[zX^{\rho} \left| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\eta+1/2}} \sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\le L} (-L)_{h_1k_1+\ldots+h_rk_r} A(L;k_1,\ldots k_r) \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1} k_1!} \cdots \frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r} k_r!}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi) H_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^{\rho} \left| \begin{matrix} (a_j,\alpha_j)_{1,p}, \left(-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho\right) \\ \left(1/2-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho\right),(b_j,\beta_j)_{1,q} \end{matrix} \right. \right] (4ab+c)^{\xi\sigma} d\xi \qquad (3.1.2)$$

$$\int_0^{\infty} \left(a+\frac{b}{x^2}\right) X^{-\eta-1} S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}] H_{P,Q}^{M,N}\left[X^{\sigma} \left| \begin{matrix} (a'_j,\alpha'_j)_{1,P} \\ (b'_j,\beta'_j)_{1,Q} \end{matrix} \right. \right] H_{p,q}^{m,n}\left[zX^{\rho} \left| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right. \right] dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\eta+1/2}} \sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r\le L} (-L)_{h_1k_1+\ldots+h_rk_r} A(L;k_1,\ldots k_r) \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1} k_1!} \cdots \frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r} k_r!}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi) H_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^{\rho} \left| \begin{matrix} (a_j,\alpha_j)_{1,p}, \left(-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho\right) \\ \left(1/2-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho\right),(b_j,\beta_j)_{1,q} \end{matrix} \right. \right] (4ab+c)^{\xi\sigma} d\xi \qquad (3.1.3)$$

The conditions of convergence of (3.1.1), (3.1.2) and (3.1.3) can be easily obtained from those of (2.1), (2.2) and (2.3) respectively.

Further If we put $A'_j = B'_j = A_j = B_j = 1; \alpha'_j = \beta'_j = \alpha_j = \beta_j = 1$, then the $\overline{\text{H}}$-function reduces to general type of G-function [12], which is also the new special case.

(3.2) If we put $n = p,\ m = 1,\ q = q+1,\ b_1 = 0, \beta_1 = 1,\ a_j = 1-a_j,\ b_j = 1-b_j$, then the $\overline{\text{H}}$-function reduces to generalized wright hypergeometric function [9] i.e. $\overline{H}_{p,q+1}^{1,p}\left[z \left| \begin{matrix} (1-a_j,\alpha_j;A_j)_{1,p} \\ (0,1),(1-b_j,\beta_j;B_j)_{1,q} \end{matrix} \right. \right] = {}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ;-z \right]$, the equations (2.1), (2.2) and (2.3) take the following form.

$$\int_0^{\infty} X^{-\eta-1} S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}] \overline{H}_{P,Q}^{M,N}\left[X^{\sigma} \left| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P} \\ (b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q} \end{matrix} \right. \right] {}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ;-zX^{\rho} \right] dx$$

Ref.

9. K.C. Gupta and R.C. Soni, On a basic integral formula involving the product of the H-function and Fox H-function, J.Raj.Acad.Phy. Sci., 4 (3) (2006), 157-164.

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+1/2}} \sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r \le L} (-L)_{h_1k_1+\ldots+h_rk_r} A(L;k_1,\ldots k_r) \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}\, k_1!} \cdots \frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r}\, k_r!}$$

$$\frac{1}{2\pi !}\int_L \bar{\phi}(\xi)\, {}_{p+1}\bar{\Psi}_{q+1}\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p}, \left(-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho;1\right) \\ \left(1/2-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho;1\right), (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -z(4ab+c)^{\rho} \right] (4ab+c)^{\xi\sigma}\, d\xi \qquad (3.2.1)$$

$$\int_0^\infty \frac{1}{x^2} X^{-\eta-1} S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}]\, \bar{H}_{P,Q}^{M,N}\left[X^{\sigma} \middle| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N}, (a'_j,\alpha'_j)_{N+1,P} \\ (b'_j,\beta'_j;B'_j)_{1,M}, (b'_j,\beta'_j)_{M+1,Q} \end{matrix} \right] {}_p\bar{\Psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -zX^{\rho} \right] dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\eta+1/2}} \sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r \le L} (-L)_{h_1k_1+\ldots+h_rk_r} A(L;k_1,\ldots k_r) \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}\, k_1!} \cdots \frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r}\, k_r!}$$

$$\frac{1}{2\pi !}\int_L \bar{\phi}(\xi)\, {}_{p+1}\bar{\Psi}_{q+1}\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p}, \left(-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho;1\right) \\ \left(1/2-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho;1\right), (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -z(4ab+c)^{\rho} \right] (4ab+c)^{\xi\sigma}\, d\xi \qquad (3.2.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\eta-1} S_L^{h_1,\ldots,h_r}[c_1X^{\delta_1},\ldots,c_rX^{\delta_r}]\, \bar{H}_{P,Q}^{M,N}\left[X^{\sigma} \middle| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N}, (a'_j,\alpha'_j)_{N+1,P} \\ (b'_j,\beta'_j;B'_j)_{1,M}, (b'_j,\beta'_j)_{M+1,Q} \end{matrix} \right] {}_p\bar{\Psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -zX^{\rho} \right] dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\eta+1/2}} \sum_{k_1,\ldots,k_r=0}^{h_1k_1+\ldots+h_rk_r \le L} (-L)_{h_1k_1+\ldots+h_rk_r} A(L;k_1,\ldots k_r) \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}\, k_1!} \cdots \frac{c_r^{k_r}}{(4ab+c)^{k_r\delta_r}\, k_r!}$$

$$\frac{1}{2\pi !}\int_L \bar{\phi}(\xi)\, {}_{p+1}\bar{\Psi}_{q+1}\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p}, \left(-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho;1\right) \\ \left(1/2-\eta+\sigma\xi+\sum_{i=1}^{r} k_i\delta_i,\rho;1\right), (b_j,\beta_j;B_j)_{1,q} \end{matrix} ; -z(4ab+c)^{\rho} \right] (4ab+c)^{\xi\sigma}\, d\xi \qquad (3.2.3)$$

The conditions of convergence of (3.2.1), (3.2.2) and (3.2.3) can be easily obtained from those of (2.1), (2.2) and (2.3) respectively.

(3.3) If we put r=1 the general class of multivariable polynomials given by Srivastava and Garg [7] reduces to the polynomials given by Srivastava [5], the equations (2.1), (2.2) and (2.3) take the following form:

$$\int_0^\infty X^{-\eta-1} S_L^{h_1}[c_1X^{\delta_1}]\, \bar{H}_{P,Q}^{M,N}\left[X^{\sigma} \middle| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N}, (a'_j,\alpha'_j)_{N+1,P} \\ (b'_j,\beta'_j;B'_j)_{1,M}, (b'_j,\beta'_j)_{M+1,Q} \end{matrix} \right] \bar{H}_{p,q}^{m,n}\left[zX^{\rho} \middle| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n}, (a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m}, (b_j,\beta_j)_{m+1,q} \end{matrix} \right] dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+1/2}} \sum_{k_1=0}^{[L/h_1]} \frac{(-L)_{h_1k_1} A_{L,k_1}}{k_1!} \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

Ref.

5. H.M. Sarivastava, A contour integral involving Fox's H-function, Indian J. Math. 14(1972), 1-6.

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi)\overline{H}^{m+1,n}_{p+1,q+1}\left[z(4ab+c)^{\rho}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1)\\(1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}\,d\xi$$

(3.3.1)

$$\int_0^\infty \frac{1}{x^2}X^{-\eta-1}S_L^{h_1}[c_1X^{\delta_1}]\overline{H}^{M,N}_{P,Q}\left[X^{\sigma}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}^{m,n}_{p,q}\left[zX^{\rho}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\eta+1/2}}\sum_{k_1=0}^{[L/h_1]}\frac{(-L)_{h_1k_1}A_{L,k_1}}{k_1!}\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi)\overline{H}^{m+1,n}_{p+1,q+1}\left[z(4ab+c)^{\rho}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1)\\(1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}\,d\xi$$

(3.3.2)

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)X^{-\eta-1}S_L^{h_1}[c_1X^{\delta_1}]\overline{H}^{M,N}_{P,Q}\left[X^{\sigma}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}^{m,n}_{p,q}\left[zX^{\rho}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\eta+1/2}}\sum_{k_1=0}^{[L/h_1]}\frac{(-L)_{h_1k_1}A_{L,k_1}}{k_1!}\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi)\overline{H}^{m+1,n}_{p+1,q+1}\left[z(4ab+c)^{\rho}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1)\\(1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}\,d\xi$$

(3.3.3)

The conditions of convergence of (3.3.1), (3.3.2) and (3.3.3) can be easily obtained from those of (2.1), (2.2) and (2.3) respectively.

(3.4) By applying the our results given in (3.3.1), (3.3.2) and (3.3.3) to the case of Hermite polynomials [8] by setting $S_n^2[x]=x^{n/2}H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $L=n, h_1=2, A_{L,k_1}=(-1)^{k_1}$, we have the following interesting results:

$$\int_0^\infty X^{-\eta-1}\left(c_1X^{\delta_1}\right)^{n/2}H_n\left[\frac{1}{2\sqrt{c_1X^{\delta_1}}}\right]\overline{H}^{M,N}_{P,Q}\left[X^{\sigma}\left|\begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}^{m,n}_{p,q}\left[zX^{\rho}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+1/2}}\sum_{k_1=0}^{[n/2]}\frac{(-n)_{2k_1}(-1)^{k_1}}{k_1!}\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi)\overline{H}^{m+1,n}_{p+1,q+1}\left[z(4ab+c)^{\rho}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1)\\(1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}\,d\xi \quad (3.4.1)$$

Notes

$$\int_0^\infty \frac{1}{x^2} X^{-\eta-1}\left(c_1 X^{\delta_1}\right)^{n/2} H_n\left[\frac{1}{2\sqrt{c_1 X^{\delta_1}}}\right]\overline{H}_{P,Q}^{M,N}\left[X^\sigma \left| \begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}_{p,q}^{m,n}\left[zX^\rho \left| \begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\eta+1/2}}\sum_{k_1=0}^{[n/2]}\frac{(-n)_{2k_1}(-1)^{k_1}}{k_1!}\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi)\overline{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^\rho \left| \begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1)\\(1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}d\xi \tag{3.4.2}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\eta-1}\left(c_1 X^{\delta_1}\right)^{n/2} H_n\left[\frac{1}{2\sqrt{c_1 X^{\delta_1}}}\right]\overline{H}_{P,Q}^{M,N}\left[X^\sigma \left| \begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$\overline{H}_{p,q}^{m,n}\left[zX^\rho \left| \begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\eta+1/2}}\sum_{k_1=0}^{[n/2]}\frac{(-n)_{2k_1}(-1)^{k_1}}{k_1!}\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi)\overline{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^\rho \left| \begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1)\\(1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}d\xi \tag{3.4.3}$$

The conditions of convergence of (3.4.1), (3.4.2) and (3.4.3) can be easily obtained from those of (2.1), (2.2) and (2.3) respectively.

(3.5) By applying the our results given in (3.3.1), (3.3.2) and (3.3.3) to the case of Lagurre polynomials [8] by setting $S_n^1[x]\to L_n^{(\alpha)}[x]$ in which $L=n, h_1=1, A_{L,k_1}=\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_{k_1}}$, we have the following interesting results:

$$\int_0^\infty X^{-\eta-1}L_n^{(\alpha)}\left[c_1X^{\delta_1}\right]\overline{H}_{P,Q}^{M,N}\left[X^\sigma \left| \begin{matrix}(a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P}\\(b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q}\end{matrix}\right.\right]\overline{H}_{p,q}^{m,n}\left[zX^\rho \left| \begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\eta+1/2}}\sum_{k_1=0}^{[n]}\frac{(-n)_{k_1}}{k_1!}\binom{n+\alpha}{n}\frac{1}{(\alpha+1)_{k_1}}\frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi)\overline{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^\rho \left| \begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1)\\(1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q}\end{matrix}\right.\right](4ab+c)^{\xi\sigma}d\xi \tag{3.5.1}$$

Notes

$$\int_0^\infty \frac{1}{x^2} X^{-\eta-1} L_n^{(\alpha)}\left[c_1 X^{\delta_1}\right] \overline{H}_{P,Q}^{M,N}\left[X^\sigma \left| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P} \\ (b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q} \end{matrix} \right.\right] \overline{H}_{p,q}^{m,n}\left[zX^\rho \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right.\right] dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\eta+1/2}} \sum_{k_1=0}^{[n]} \frac{(-n)_{k_1}}{k_1!} \binom{n+\alpha}{n} \frac{1}{(\alpha+1)_{k_1}} \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

Notes

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi) \overline{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^\rho \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1) \\ (1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right.\right] (4ab+c)^{\xi\sigma} d\xi \qquad (3.5.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\eta-1} L_n^{(\alpha)}\left[c_1 X^{\delta_1}\right] \overline{H}_{P,Q}^{M,N}\left[X^\sigma \left| \begin{matrix} (a'_j,\alpha'_j;A'_j)_{1,N},(a'_j,\alpha'_j)_{N+1,P} \\ (b'_j,\beta'_j;B'_j)_{1,M},(b'_j,\beta'_j)_{M+1,Q} \end{matrix} \right.\right] \overline{H}_{p,q}^{m,n}\left[zX^\rho \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p} \\ (b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right.\right] dx$$

$$= \frac{\sqrt{\pi}}{(4ab+c)^{\eta+1/2}} \sum_{k_1=0}^{[n]} \frac{(-n)_{k_1}}{k_1!} \binom{n+\alpha}{n} \frac{1}{(\alpha+1)_{k_1}} \frac{c_1^{k_1}}{(4ab+c)^{k_1\delta_1}}$$

$$\frac{1}{2\pi!}\int_L \bar{\phi}(\xi) \overline{H}_{p+1,q+1}^{m+1,n}\left[z(4ab+c)^\rho \left| \begin{matrix} (a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p},(-\eta+\sigma\xi+k_1\delta_1,\rho;1) \\ (1/2-\eta+\sigma\xi+k_1\delta_1,\rho;1),(b_j,\beta_j;B_j)_{1,m},(b_j,\beta_j)_{m+1,q} \end{matrix} \right.\right] (4ab+c)^{\xi\sigma} d\xi \qquad (3.5.3)$$

The conditions of convergence of (3.5.1), (3.5.2) and (3.5.3) can be easily obtained from those of (2.1), (2.2) and (2.3) respectively.

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Notes

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Applications of Continued Fraction Identities

By M.P. Chaudhary
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Abstract – In present paper we established four new expressions on q-product identities with the applications of continued fractions in recent results established by the author [7].

Keywords : *Triple product identities, q-product identities, continued fractions.*

AMS Subject Classifications : *05A17, 05A15, 11P83*

APPLICATIONS OF CONTINUED FRACTION IDENTITIES

Strictly as per the compliance and regulations of :

Applications of Continued Fraction Identities

M.P. Chaudhary

Abstract - In present paper we established four new expressions on q-product identities with the applications of continued fractions in recent results established by the author [7].

Keywords : Triple product identities, q-product identities, continued fractions.

I. Introduction

For $|q| < 1$,

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1 - aq^n) \tag{1.1}$$

$$(a;q)_\infty = \prod_{n=1}^{\infty}(1 - aq^{(n-1)}) \tag{1.2}$$

$$(a_1, a_2, a_3, ..., a_k; q)_\infty = (a_1;q)_\infty(a_2;q)_\infty(a_3;q)_\infty...(a_k;q)_\infty \tag{1.3}$$

Ramanujan has defined general theta function, as

$$f(a,b) = \sum_{-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} \;;\; |ab| < 1, \tag{1.4}$$

Jacobi's triple product identity [1, p.35] is given, as

$$f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty \tag{1.5}$$

Special cases of Jacobi's triple products identity are given, as

$$\Phi(q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)_\infty^2(q^2;q^2)_\infty \tag{1.6}$$

$$\Psi(q) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.7}$$

$$f(-q) = \sum_{n=-\infty}^{\infty} (-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.8}$$

Equation (1.8) is known as Euler's pentagonal number theorem. Euler's another well known identity is as

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Ref.
(1) B.C. Berndt : *Ramanujan's Notebooks, Part III*, Springer-Verlag, New York, 1991.

$$(q;q^2)_\infty^{-1} = (-q;q)_\infty \tag{1.9}$$

Throughout this paper we use the following representations

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty = (q^a,q^b,q^c\cdots q^t;q^n)_\infty \tag{1.13}$$

$$(q^a;q^n)_\infty(q^a;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty = (q^a,q^a,q^c\cdots q^t;q^n)_\infty \tag{1.14}$$

Notes

Computation of q-product identities:

we can have following q-products identities, as

$$\begin{aligned}(q^2;q^2)_\infty &= \prod_{n=0}^{\infty}(1-q^{2n+2})\\
&= \prod_{n=0}^{\infty}(1-q^{2(4n)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+1)+2})\times\\
&\times \prod_{n=0}^{\infty}(1-q^{2(4n+2)+2}) \times \prod_{n=0}^{\infty}(1-q^{2(4n+3)+2})\\
&= \prod_{n=0}^{\infty}(1-q^{8n+2}) \times \prod_{n=0}^{\infty}(1-q^{8n+4})\times\\
&\times \prod_{n=0}^{\infty}(1-q^{8n+6}) \times \prod_{n=0}^{\infty}(1-q^{8n+8})\\
&= (q^2;q^8)_\infty(q^4;q^8)_\infty(q^6;q^8)_\infty(q^8;q^8)_\infty\\
&= (q^2,q^4,q^6,q^8;q^8)_\infty \end{aligned} \tag{1.15}$$

$$\begin{aligned}(q^4;q^4)_\infty &= \prod_{n=0}^{\infty}(1-q^{4n+4})\\
&= \prod_{n=0}^{\infty}(1-q^{4(3n)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+1)+4}) \times \prod_{n=0}^{\infty}(1-q^{4(3n+2)+4})\\
&= \prod_{n=0}^{\infty}(1-q^{12n+4}) \times \prod_{n=0}^{\infty}(1-q^{12n+8}) \times \prod_{n=0}^{\infty}(1-q^{12n+12})\\
&= (q^4;q^{12})_\infty(q^8;q^{12})_\infty(q^{12};q^{12})_\infty\\
&= (q^4,q^8,q^{12};q^{12})_\infty \end{aligned} \tag{1.16}$$

$$\begin{aligned}(q^4;q^{12})_\infty &= \prod_{n=0}^{\infty}(1-q^{12n+4})\\
&= \prod_{n=0}^{\infty}(1-q^{12(5n)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+1)+4})\times \end{aligned}$$

Notes

$$\times \prod_{n=0}^{\infty}(1-q^{12(5n+2)+4}) \times \prod_{n=0}^{\infty}(1-q^{12(5n+3)+4})\times$$

$$\times \prod_{n=0}^{\infty}(1-q^{12(5n+4)+4})$$

$$= \prod_{n=0}^{\infty}(1-q^{60n+4}) \times \prod_{n=0}^{\infty}(1-q^{60n+16}) \times \prod_{n=0}^{\infty}(1-q^{60n+28})\times$$

$$\times \prod_{n=0}^{\infty}(1-q^{60n+40}) \times \prod_{n=0}^{\infty}(1-q^{60n+52})$$

$$= (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$
$$= (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \quad (1.17)$$

Similarly we can compute following, as

$$(q^4;q^{12})_\infty = (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$
$$= (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \quad (1.18)$$

$$(q^6;q^6)_\infty = (q^6;q^{24})_\infty(q^{12};q^{24})_\infty(q^{18};q^{24})_\infty(q^{24};q^{24})_\infty$$
$$= (q^6,q^{12},q^{18},q^{24};q^{24})_\infty \quad (1.19)$$

$$(q^6;q^{12})_\infty = (q^6;q^{60})_\infty(q^{18};q^{60})_\infty(q^{30};q^{60})_\infty(q^{42};q^{60})_\infty(q^{54};q^{60})_\infty$$
$$= (q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty \quad (1.20)$$

$$(q^8;q^8)_\infty = (q^8;q^{48})_\infty(q^{16};q^{48})_\infty(q^{24};q^{48})_\infty(q^{32};q^{48})_\infty(q^{40};q^{48})_\infty(q^{48};q^{48})_\infty$$
$$= (q^8,q^{16},q^{24},q^{32},q^{40},q^{48};q^{48})_\infty \quad (1.21)$$

$$(q^8;q^{12})_\infty = (q^8;q^{60})_\infty(q^{20};q^{60})_\infty(q^{32};q^{60})_\infty(q^{44};q^{60})_\infty(q^{56};q^{60})_\infty$$
$$= (q^8,q^{20},q^{32},q^{44},q^{56};q^{60})_\infty \quad (1.22)$$

$$(q^8;q^{16})_\infty = (q^8;q^{48})_\infty(q^{24};q^{48})_\infty(q^{40};q^{48})_\infty$$
$$= (q^8,q^{24},q^{40};q^{48})_\infty \quad (1.23)$$

$$(q^{10};q^{20})_\infty = (q^{10};q^{60})_\infty(q^{30};q^{60})_\infty(q^{50};q^{60})_\infty$$

$$= (q^{10},q^{30},q^{50};q^{60})_\infty \quad (1.24)$$

$$(q^{12};q^{12})_\infty = (q^{12};q^{60})_\infty(q^{24};q^{60})_\infty(q^{36};q^{60})_\infty(q^{48};q^{60})_\infty(q^{60};q^{60})_\infty$$

$$= (q^{12},q^{24},q^{36},q^{48},q^{60};q^{60})_\infty \quad (1.25)$$

$$(q^{16};q^{16})_\infty = (q^{16};q^{48})_\infty (q^{32};q^{48})_\infty (q^{48};q^{48})_\infty$$
$$= (q^{16},q^{32},q^{48};q^{48})_\infty \tag{1.26}$$

$$(q^{20};q^{20})_\infty = (q^{20};q^{60})_\infty (q^{40};q^{60})_\infty (q^{60};q^{60})_\infty$$
$$= (q^{20},q^{40},q^{60};q^{60})_\infty \tag{1.27}$$

The outline of this paper is as follows. In sections 2, we have recorded some well known results on continued fraction identities and recent results on q-products identities given by the author[7], those are useful to the rest of the paper. In section 3, we state and prove four new results related to q-product identities with theapplications of continued fraction identities.

II. Preliminaries

In 1983 Denis [5], has introduced following continued fraction identity

$$(q^2;q^2)_\infty(-q;q)_\infty = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} = \cfrac{1}{1-\cfrac{q}{1+\cfrac{q(1-q)}{1-\cfrac{q^3}{1+\cfrac{q^2(1-q^2)}{1-\cfrac{q^5}{1+\cfrac{q^3(1-q^3)}{1+\vdots}}}}}}} \tag{2.1}$$

The famous Rogers-Ramanujan continued fraction identity [3, (1.6)], is

$$\frac{(q;q^5)_\infty(q^4;q^5)_\infty}{(q^2;q^5)_\infty(q^3;q^5)_\infty} = \cfrac{1}{1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{1+\cfrac{q^4}{1+\vdots}}}}} \tag{2.2}$$

A well known continued fraction identity due to Ramanujan [4, (4.21)], is

$$\frac{(-q^3;q^4)_\infty}{(-q;q^4)_\infty} = \cfrac{1}{1+\cfrac{q}{1+\cfrac{q^3+q^2}{1+\cfrac{q^5}{1+\cfrac{q^7+q^4}{1+\cfrac{q^9}{1+\cfrac{q^{11}+q^6}{1+\vdots}}}}}}} \tag{2.3}$$

Ref.

(7) M.P. Chaudhary : *On q-product identities*, pre-print.

One of the most celebrated continued fractional identities associated with Ramanujan's academic career, given by Rogers-Ramanujan [6], is

$$C(q)=\frac{(q^2;q^5)_\infty(q^3;q^5)_\infty}{(q;q^5)_\infty(q^4;q^5)_\infty}=1+\cfrac{q}{1+\cfrac{q^2}{1+\cfrac{q^3}{1+\cfrac{q^4}{1+\cfrac{q^5}{1+\vdots}}}}} \tag{2.4}$$

Ref.

Recently Chaudhary [7], has introduce following q-product identities

$$(q^2,q^4,q^6;q^8)_\infty[(-q;q^2)^2_\infty+(q;q^2)^2_\infty]=2(-q^4;q^8)^2_\infty \tag{2.5}$$

$$(q^2,q^4,q^6,q^8;q^8)_\infty[(-q;q^2)^2_\infty-(q;q^2)^2_\infty]=4q\frac{(q^{16},q^{32},q^{48};q^{48})_\infty}{(q^8,q^{24},q^{40};q^{48})_\infty} \tag{2.6}$$

$$\frac{(-q;q^2)^2_\infty+(q;q^2)^2_\infty}{(-q;q^2)^2_\infty-(q;q^2)^2_\infty}=\frac{(-q^4;q^8)^2_\infty(q^8,q^8,q^{24},q^{24},q^{40},q^{40};q^{48})_\infty}{2q} \tag{2.7}$$

$$(-q;q^2)^2_\infty(q;q^2)^2_\infty(q^2;q^2)^2_\infty=(q^2,q^2,q^4;q^4)_\infty \tag{2.8}$$

$$\frac{(-q;q^2)_\infty(-q^3;q^6)_\infty-(q;q^2)_\infty(q^3;q^6)_\infty}{(-q;q^2)_\infty\times(-q^3;q^6)_\infty\times(q;q^2)_\infty\times(q^3;q^6)_\infty}$$

$$=\frac{2q(-q^2;q^4)^2_\infty(q^4,q^8,q^{16},q^{20},q^{24};q^{24})_\infty}{(q^2,q^4,q^6,q^8;q^8)_\infty(q^6,q^{12},q^{18};q^{24})_\infty} \tag{2.9}$$

$$\frac{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty-(q^3;q^6)_\infty(q^5;q^{10})_\infty}{(-q^3;q^6)_\infty\times(-q^5;q^{10})_\infty\times(q^3;q^6)_\infty\times(q^5;q^{10})_\infty}$$

$$=\frac{(q^4,q^8,q^{12};q^{12})_\infty}{(q^6,q^{12},q^{18},q^{24};q^{24})_\infty}\times$$

$$\times\frac{2q^3}{(q^2,q^6,q^{10};q^{12})_\infty(q^{10},q^{20},q^{30},q^{30},q^{40},q^{50};q^{60})_\infty} \tag{2.10}$$

5

And,

$$\frac{[(q;q^2)_\infty(q^{15};q^{30})_\infty]+[(-q;q^2)_\infty(-q^{15};q^{30})_\infty]}{[(q;q^2)_\infty(q^{15};q^{30})_\infty][(-q;q^2)_\infty(-q^{15};q^{30})_\infty]}$$

$$=\frac{(q^{12},q^{20},q^{24},q^{36},q^{40},q^{48},q^{60},q^{60};q^{60})_\infty}{(q^{10},q^{30},q^{30},q^{50},q^{60};q^{60})_\infty}\times$$

$$\times\frac{2}{(q^2,q^4,q^6,q^8,q^8;q^8)_\infty(q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty} \tag{2.11}$$

(7) M.P. Chaudhary : *On q-product identities*, pre-print.

III. Main Results

In this section, we established and proved following identities with the applications of continued fraction identities in the q-product identities, recently given by Chaudhary [7], as

$$(q^2,q^4,q^6,q^8;q^8)_\infty[(-q;q^2)_\infty^2-(q;q^2)_\infty^2]$$

$$=\frac{4q(q^{16},q^{32};q^{48})_\infty}{(q^8,q^{40};q^{48})_\infty}\times\cfrac{1}{1-\cfrac{q^{24}}{1+\cfrac{q^{24}(1-q^{24})}{1-\cfrac{q^{72}}{1+\cfrac{q^{48}(1-q^{48})}{1-\cfrac{q^{120}}{1+\cfrac{q^{72}(1-q^{72})}{1+\vdots}}}}}}} \tag{3.1}$$

$$\frac{(-q;q^2)_\infty(-q^3;q^6)_\infty-(q;q^2)_\infty(q^3;q^6)_\infty}{(-q;q^2)_\infty\times(-q^3;q^6)_\infty\times(q;q^2)_\infty\times(q^3;q^6)_\infty}$$

$$=\frac{2q(-q^2;q^4)_\infty^2(q^4,q^8,q^{16},q^{20};q^{24})_\infty}{(q^2,q^4,q^6,q^8;q^8)_\infty(q^6,q^{18};q^{24})_\infty}\times$$

$$\times\cfrac{1}{1-\cfrac{q^{12}}{1+\cfrac{q^{12}(1-q^{12})}{1-\cfrac{q^{36}}{1+\cfrac{q^{24}(1-q^{24})}{1-\cfrac{q^{60}}{1+\cfrac{q^{36}(1-q^{36})}{1+\vdots}}}}}}} \tag{3.2}$$

$$\frac{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty-(q^3;q^6)_\infty(q^5;q^{10})_\infty}{(-q^3;q^6)_\infty\times(-q^5;q^{10})_\infty\times(q^3;q^6)_\infty\times(q^5;q^{10})_\infty}$$

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$$=\frac{2q^3(q^8;q^{12})_\infty(q^4,q^{16},q^{28},q^{40},q^{54};q^{60})_\infty}{(q^6,q^{12},q^{18},q^{24};q^{24})_\infty(q^2,q^{10};q^{12})_\infty(q^{10},q^{20},q^{30},q^{30},q^{40},q^{50};q^{60})_\infty}\times$$

$$\times\cfrac{1}{1-\cfrac{q^{6}}{1+\cfrac{q^{6}(1-q^{6})}{1-\cfrac{q^{18}}{1+\cfrac{q^{12}(1-q^{12})}{1-\cfrac{q^{30}}{1+\cfrac{q^{18}(1-q^{18})}{1+\vdots}}}}}}} \tag{3.3}$$

Ref.

(7) M.P. Chaudhary : *On q-product identities*, pre-print.

And,

$$\frac{[(q;q^2)_\infty(q^{15};q^{30})_\infty]+[(-q;q^2)_\infty(-q^{15};q^{30})_\infty]}{[(q;q^2)_\infty(q^{15};q^{30})_\infty][(-q;q^2)_\infty(-q^{15};q^{30})_\infty]}$$

$$=\frac{2(q^{12},q^{20},q^{24},q^{36},q^{40},q^{48};q^{60})_\infty}{(q^2,q^4,q^6,q^8,q^8;q^8)_\infty(q^6,q^{10},q^{18},q^{30},q^{42},q^{50},q^{54},q^{60};q^{60})_\infty}\times$$

$$\times\left[\cfrac{1}{1-\cfrac{q^{30}}{1+\cfrac{q^{30}(1-q^{30})}{1-\cfrac{q^{90}}{1+\cfrac{q^{60}(1-q^{60})}{1-\cfrac{q^{150}}{1+\cfrac{q^{90}(1-q^{90})}{1+\vdots}}}}}}}\right]^2 \tag{3.4}$$

Notes

Proof of (3.1): Making suitable arrangements in the q-products identities given in the right hand side of (2.6), and further apply (2.1) for $q=q^{24}$, we get (3.1).

Proof of (3.2): Making suitable arrangements in the q-products identities given in the right hand side of (2.9), and further apply (2.1) for $q=q^{12}$, we get (3.2).

Proof of (3.3): Making suitable arrangements in the q-products identities given in the right hand side of (2.10), and further apply (1.17), and (2.1) for $q=q^{6}$, we get (3.3).

Proof of (3.4): Making suitable arrangements in the q-products identities given in the right hand side of (2.11), and further apply (2.1) for $q=q^{30}$, we get (3.4).

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On Geometric Models in Modern Computing and Networking

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Abstract – This paper discusses a few geometric models that have been of great utility in the modern technologically dominated society. Under the setting of topological manifolds the mathematical concepts underlying computer graphics have been explored. Other applications of the theory of manifolds in computer science and communication technology namely in codes, ciphers and networks are described. Using the concept of triangulations and homology we describe a discrete model concerning the networked environment.

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Notes

On Geometric Models in Modern Computing and Networking

J. V. Ramana Raju [α] & T. Venkatesh [σ]

Abstract - This paper discusses a few geometric models that have been of great utility in the modern technologically dominated society. Under the setting of topological manifolds the mathematical concepts underlying computer graphics have been explored. Other applications of the theory of manifolds in computer science and communication technology namely in codes, ciphers and networks are described. Using the concept of triangulations and homology we describe a discrete model concerning the networked environment.

Keywords : Riemannian, surveillance, Parameterization, Splines, Hamming distance.

I. Introduction

Geometric models have been playing a vital role in today's commercial enterprises albeit a fact quite unknown to the users of technology. These technologies have been essentially driven by the revolutions in e-commerce and internet usage. The Mathematical theory of communication which was initiated by C.E Shannon at Bell labs in the mid of 20th century can be regarded as a backbone of today's network revolution. To give a sample of the mathematical models driving our society, we have the theory of codes (both source codes as well as error correcting codes) which help achieve reliable and efficient transport of digital and analog data. Transactions have been taking place over the ATM Machines, Internet as well as swipe cards thanks to the mathematical description of money exchange. Security systems of e-commerce are basically operated by number theoretic and geometric models. Moreover the networked environment itself is topologically defined and several surveillance mechanisms are devised through combinatorial and geometric models. A global analysis of the same can be made through the theory of Riemann surfaces. Our basic structure in this paper is that of a manifold and we see several mathematical models that are currently used by industries world over to enable communication and trade. In section-I we look at the basic geometric spaces and their use in computer graphics and visualization. In Section -II we describe how transmission of huge digital data is possible in an error free manner by the use of geometric spaces. Also algebraic-geometry based ciphers have been discussed. In section-III we briefly describe the mathematical model for networking environment at a global scale. Finally in section-4 we look at quantum scale structures which are also motivated by the theory of manifolds and Hilbert spaces.

II. Geometric Spaces and Computer Graphics

Shapes that we need to convert into graphics can be perceived as a collection of curves. So naturally we are led to the definition of a manifold. For example if one

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revolves a unit line segment about a point then we see a unit disc formed. But the challenge to the computer graphics industry is to describe algorithms such that one constructs a required shape with a minimum amount of information to be fed to the computer. *Geometric Modeling* is the science of developing algorithms to construct geometric shapes and scenes as required by the graphics industry. One of the earliest methods developed in this direction is that of using polynomials to approximate surfaces. The basic idea here is to develop a mesh made of curves. This mesh is a discrete version of a manifold [2]. Using the so called algebraic splines a class of manifolds called cell polyhedral surfaces can be constructed. A suitable smoothing process then makes up the shape that we require. The crucial theoretical consideration here is that of parameterization of rational curves and surfaces. From a practical standpoint, parameterization requires many functions to be computed and hence it is a costly affair as far as computational complexity is concerned. Other alternatives are the implicit surface constructions and the conformal geometric algorithms [3].

a) Algorithms and Software

Warren and Moore's Algorithm based on triangulation of quadratic algebraic patches is one of the earliest algorithm being implemented on IBM 3D interactive accelerator. NURBS is an acronym for a class of algorithms that use the theory of algebraic curves and surfaces to generate computer graphics. It means non-uniform rational basis spline, which employs analytic techniques including Bezier curves. Pierre Bezier and Casteljau are the pioneers who developed mathematical models to build very flexible representations of curves and surfaces. If the surface or a scene to be created is already an algebraic manifold then it is relatively easy to build algorithms to represent the same on the computer screen However to get more realistic textures one uses softwares that generate fractal geometric sets. 'GANITH' is a software developed by Scientists at the Purdue University. The programmes written in C-language enable a wide range of computing with respect to graphics. The tasks that can be performed through GANITH include synthesis of graphics and rendering, spline generation, implicit surface generation and many kinds of animations.

b) Mathematical programming method

This method uses linear algebra and related algorithms to generate graphics. A scattered set of points, curves and derived jets are given as inputs. They are fed to the computer as a finite set of vectors. The output should ideally be a low degree algebraic surface fit through the scattered set of points, curves and derived jets with a prescribed higher order interpolation and least squares approximation. The mathematical problem to be solved can be expressed as follows:

Let X be a vector containing coefficients of an algebraic surface containing the points, curves and derived jets. Let M_I, M_A be the interpolation matrix and approximation matrix respectively. Then one needs to minimize $X^t(M^T{}_A, M_A X)$ subject to the constraints i) $M_I X=0$ and ii) $X^t X=0$

This linear programming problem which involves alignment of points curves and patches in a given pattern leads to a solution that is very helpful in the computer graphics industry.

III. Codes and Ciphers

In the modern era of ICT - Information and communication technology the right kind of processes are required for efficient transfer of data. The problems associated came to the fore when digital technology was in its infant stage. Mathematicians Richard

Ref.

2. J.Munkres, (1996) "Elements of Algebraic Topology", Addison Wessley.

Hamming and others began working towards a theory for reliable communication, which got fulfilled with the path breaking work of C.E.Shannon. He developed the so called "Mathematical Theory of communication" which became a foundation for a lot of future work to be done in this area.

Notes

a) Description of the Problem

When data has to travel through some medium which we shall call a channel, it is subjected to disturbances or 'Noise'. Errors may creep in at various positions of the digital data so much so that the receiver may not make any sense out of the scrambled data. Same is the case with the data/signals contained in a digital compact disc. So for reliable data communication one needs to encode the data in such a way that at the receiving end, a check can be performed to detect for errors and then correct all the errors that might have occurred. There is another type of encoding in literature namely source encoding. This is done whenever we need to achieve data compression. This is based on Shannon's theory of communication and the related Nyquist rate. Here we are only concerned with data encoding which is done for reliable transmission of data as described above.

b) The Solution

One of the popular methods to solve the above problem is to use vector spaces. Suppose one encodes all the message bits using an alphabet set say $\sum$ then this set need to have the structure of a finite field and the n-dimensional product space becomes a vector space of dimension 'n'. We fix the block size as 'n' so that the whole message is divided into blocks of size 'n'. In each block one deliberately keeps message bits of size n-k, thus making way for k positions for inserting check bits. The set of all meaningful words will be a subspace of the space Σ^n. Now after passing through the channel, the received word can be checked for errors by using the parity check matrix. The detection of error as well as the correction is done by making use of the concept of a distance on this space of (digital) words. The distance function called the Hamming distance is defined as follows:

$$d(\mathbf{x},\mathbf{y})=\{i:x_i\neq y_i,\ x_i,\ y_i \text{ are the } i\text{th position bits of x and y respectively}\}$$

c) Role of Geometry

Clearly the vector space structure and the parity check matrix play a central role in any such scheme of error correction. Now there are three parameters of any scheme of coding through vector spaces. The block size n, the number of check bits k and the minimum distance of the code. Here the minimum distance is defined as the smallest distance separating two meaningful words, among all the pairs of words, that is $D=Min\{d(x,y):x,y \in C,\ x\neq y\}$.Now for the code to be efficient in the sense of faster implementation, the value of 'd' should be large while keeping the value of n also large. This is accomplished by considering an algebraic curve. The set of all rational functions on this curve is a certain ring of polynomials. It is in fact an integral domain and hence one can define a mapping from this ring into the vector space Σ^n. This mapping makes use of evaluation of the polynomials at the points of the curve. This is the precise geometric argument and the injectivity of the above map means that the image is a subspace of Σ^n. Thus we arrive at an efficient code. In very recent developments one uses a very special kind of manifold namely a Grassman manifold to develop efficient codes.

d) Ciphers

The elliptic curve which is topologically a torus has a very interesting algebraic structure. The points on this surface can be realized as an abelian group. This geometric object and of late its generalization namely an abelian variety are used in the theory of cryptograms. Cryptography is a science that assumes a very central role in today's world of electronic transactions, digitally signed documents, virtual conferences etc. Primarily one needs to protect messages being sent on a public network from the so called ***eavesdroppers*** or illegal snoopers. Another problem to be tackled in the e-commerce environment is the 'authentication' of messages. Let us say a bank has to release money to a vendor on behalf of a customer. Now the bank should be sure that the customer has made the transaction and the customer should not be able to fool the bank saying that he has not entered into a contract with the vendor (This property is called non-repudiation).

So a cipher (or ciphertext) is a transformed text out of the original text so that only the intended recipient can recover the original message and the sender himself cannot alter its content once having made the communication.

e) Discrete Log problem

Let G be a cyclic group of a very large order generated by an element 'a'. Let y be any random element of G. Then the discrete log problem in this setting is to find 'n' satisfying the equation $y=a^n$ .One implements public key cryptosystem by using this generic mathematically hard problem. Let „a" be a random element such that $1\leq a\leq q-1$. Now let us compute the number $h=g^a(\mathrm{mod}p)$. The triple (p,g,h) is called a public key used by anybody who would like to encrypt and send messages digitally using this cipher scheme. Now the private key available only to the recipient is the number 'a'.

The Process: By using the public key (p,g,h) one encrypts a message m (plain text message is converted into a number say „m") by computing $r=g^k(\mathrm{mod}p)$, $s=h^k m$ (modp) . Note that here $0\leq m\leq p-1$

Now the required cipher text is c=(r,s). One can decrypt the message by using the secret key (i.e the private key „a") just by computing the value of $s.r^{-a}$

This simple analysis can be made extended to a more secure cryptosystem by making a judicial use of Geometric ***spaces.*** The torus alluded to earlier in this article contains a neat algebraic structure namely that of an abelian group. The discrete log problem described in the previous paragraph can be suitably modified to make analogous computations on the so called elliptic curve. By a suitable identification one can visualize the torus as the set $E=\{(x,y): x^3+ax+b-y^2=0, a,b \in \mathbf{F}\}$ where **F** is a very large finite field. This set of points is in the algebro-geometric language is called an Elliptic curve.

Elliptic curve cryptography has been made very popular by some firms involved in digital signatures and digital copyrights. Researchers in this area are trying to use higher dimensional geometric spaces in search of better security since they need to be always smarter than the hacking communities.

IV. Networking Environment Model

a) Sensor Networks

A network that we use in the communication system is basically made up of several nodes that are interconnected by physical or abstract linkages. Signals which may be of digital or analog form are transmitted across these nodes. If we view the entire domain that is inter-networked, one can imagine a manifold underlying the entire gamut of devices. Sensors are devices that measure features of a domain and return a signal from

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which information is extracted. More complex sensors involve video devices so as to extract visual, audio or textual data. While local topology is coarse in nature the continuum nature of a Riemann Surface is quite useful for a deeper understanding of the systems. The fundamental idea here is the **integration** of small networks to get a global surface. The local data is a triangulated domain. These discrete objects can be integrated to get a Riemannian surface. The emphasis on the Riemannian structure is to enable one to do a homological study and develop a suitable model. A network of sensors required for applications like global positioning systems, machine learning systems and other ad-hoc network devices consists of a simplicial complex made up of cloud points. These are essentially neighborhood systems made up of ε-balls. Let V be the set of points. In real world applications this is a finite set. However the mathematical abstraction has a provision of infinite points embedded in the Euclidean space R^n or on an oriented Riemann surface.With this convention we describe a discrete model.

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b) Discrete Model

Based on local communication a class of simple sensors helps in fast and pervasive computing. The discretization of the Riemannian surface is as follows: The whole global area is covered by triangles formed by nodes. Each node broadcasts a unique ID number and it can detect any other node due to connectivity. The nodes have radially symmetric covering domains. The nodes on the boundary have designated properties so that neighboring devices can interact. The theory of simplicial complexes leads to this mathematical model on the said Riemannian surface. A theorem of Rado asserts that every orientable Riemann surface can be triangulated. On the other hand given any surface with a Riemannian metric given, one can put isothermal coordinates on the surface thus getting a conformal structure which leads to a complex manifold of dimension-1. Thus we get a Riemann surface say X. Let this surface have a triangulation Π made up of points of the set V. Now consider a graph G(V,E) where E is the set of edges occurring in the triangulation. Now select a spanning tree i.e a subgraph Γ with the same vertex set V but edges are selected such that no non-trivial closed paths (circuits) exist. This spanning tree helps us to form a fundamental polygon for Π .The following theorem [9] then enables us to construct a homology basis.

Theorem 3.2 Let D be a canonical cell decomposition of a compact orientable surface M of genus g. ≥1 with n_2 cells and n edges. Then (there exists) a canonical homology basis for M such that any curve in the basis is homotopic to an edge path D having atmost n_1-n_2 edges.

Thus using the short geodesics guaranteed by the above theorem one constructs a homology basis.

V. Microstructures 'a Futuristic Proposition'

In this concluding section we delve into a futuristic proposition. While the networking environment is inundated by mathematical modeling proceedures, we seek to view what is in store in the pipeline of research. According to Moore's law, the size of the computing structures is decreasing at a rapid pace. So it is worthwhile to look at the kind of innovations taking place to reduce the size of the devices (nodes) themselves. Quantum computing is the buzzword in this direction, ever since Peter Shor demonstrated the power of this kind of computing. Here the fundamentals of quantum mechanics take over the semiconductor devices to perform computing. As all of us know a computing device is a finite state machine that takes as input a string of states and after processing in finite time a desired output is generated. While these states are processed as LOGIC gates operated by semiconductor chips, one is constantly looking for "lighter materials" in place of the bulkier ones. Thus nanotechnological advances are fast making inroads to develop miniature designs. If one uses principles of particle physics, then we lead to quantum particles and strings. Quantum computing is evolving by looking at energy states that are actually at the level of quantum packets. Non-zero equivalence classes of a Hilbert space represent all the energy states of a quantum particle. The theory of Riemann surfaces surprisingly is the right kind of mathematical abstraction to understand computing at this level. The Transition from Classical (Physics) to the Quantum setting as per Edward Witten , the Fields medalist , is very closely connected to the passage from Riemannian Geometry to Symplectic Geometry. Physicists discuss deformations mainly to look at one "The Quantum Theory" and the other to string Theory (Membranes). Symplectic Geometry was first explored because the classical equations of motions can be put in 'Hamiltonian form' and thereby symplectic properties can be utilized to solve these equations in certain important cases. Superdense coding is possible in this setting due to which in future the computing capabilities can become extremely fast almost comparable to the speed of light.

Notes

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A Unified Study of Fourier Series Involving Generalized Hypergeometric Function

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Abstract – In this paper, we make an application of an integral involving sine function, exponential function, the product of Kamp´e de F´eriet functions and the I-function to evaluate three fourier series. We also evaluate a multiple integral involving the Ifunction to make its application to derive a multiple exponential Fourier series. Some known and interesting particular cases are also given at the end.

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A Unified Study of Fourier Series Involving Generalized Hypergeometric Function

Yashwant Singh [α] & Naseem A.Khan [σ]

Abstract - In this paper, we make an application of an integral involving sine function, exponential function, the product of Kamp´e de F´eriet functions and the I-function to evaluate three fourier series. We also evaluate a multiple integral involving the Ifunction to make its application to derive a multiple exponential Fourier series. Some known and interesting particular cases are also given at the end.

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I. Introduction

$$I^{m,n}_{p_i,q_i:r}[z] = I^{m,n}_{p_i,q_i:r}\left[z \,\middle|\, \begin{matrix} (a_j,\alpha_j)_{1,n}, (a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m}, (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right]$$

$$= \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} \phi(\xi) z^{\xi}\, d\xi \tag{1.1}$$

where

$$\phi(\xi) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}}{\Sigma\{\prod_{j=m+1}^{q_i}\{\Gamma(1-b_{ji}+\beta_{ji}\xi)\}\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\}} \tag{1.2}$$

For the convergence and other details of the I-function, we refer the original paper of Saxena[5]. Saxena [5] has proved that the integral on the right hand side of (1.1) is absolutely convergent when $\Omega > 0$ and $|\arg z| < \frac{1}{2}\pi\Omega$, where
Kampé de Fériet hypergeometric function will be represented as follows.

$$F\left(\begin{matrix} p \\ \mu \\ q \\ \sigma \end{matrix} \,\middle|\, \begin{matrix} a_1,\cdots,a_p \\ b_1,b_1',\cdots,b_\mu,b_\mu' \\ c_1,\ldots,c_q \\ d_1,d_1',\cdots,d_\sigma,d_\sigma' \end{matrix} \,\middle|\, xy\right) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\prod_{j=1}^{p}(a_j)_{m+n}\prod_{j=1}^{\mu}\{(b_j)_m(b_j')_n\}}{\prod_{j=M+1}^{q}(c_j)_{m+n}\prod_{j=1}^{\sigma}\{(d_j)_m(d_j')_n\}}\frac{x^m y^n}{m!n!} \tag{1.3}$$

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$$(p+\nu<q+\sigma+1 \text{ or } p+\nu=q+\sigma+1 \text{ and } |\, x \,|+|\, y \,|< min(1,2^{q-p+1}));$$

$$= -\frac{1}{4\pi^2 K}\int_{-i\infty}^{+i\infty}\int_{-i\infty}^{+i\infty}\psi(s,t)\Gamma(-s)\Gamma(-t)(-x)^s(-y)^t ds\ dt$$

where

$$K=\frac{\prod_{j=1}^{p}\Gamma(a_j)\prod_{j=1}^{\mu}\{\Gamma(b_j)\Gamma(b'_j)\}}{\prod_{j=M+1}^{q}\Gamma(c_j)\prod_{j=1}^{\sigma}\{\Gamma(d_j)\Gamma(d'_j)\}} \tag{1.4}$$

and

$$\psi(s,t)=\frac{\prod_{j=1}^{p}(a_j+s+t)\prod_{j=1}^{\mu}\{\Gamma(b_j+s)\Gamma(b'_j+t)\}}{\prod_{j=M+1}^{q}\Gamma(c_j+s+t)\prod_{j=1}^{\sigma}\{\Gamma(d_j+s)\Gamma(d'_j+t)\}} \tag{1.5}$$

if we put $\nu=0=\sigma$,then it changes in the following form;

$$F\left(\begin{array}{c} p\\ \mu\\ q\\ \sigma\end{array}\left|\begin{array}{c} a_1,\cdots,a_p\\ ----\\ c_1,\ldots,c_q\\ ----\end{array}\right| xy\right)={}_pF_q\left(\begin{array}{c} a_1,\cdots,a_p\\ \\ c_1,\ldots,c_q\end{array};\ x+y\right) \tag{1.7}$$

For further detail one can refer the monography by Appell and Kampé de Fériet[1]. Mishra[4] has evaluated

$$\int_0^{\pi}(\sin x)^{w-1}e^{imx}{}_pF_q\left[\begin{array}{l}\alpha_p;\\ \beta_q;\end{array}C(sinx)^{2h}\right]dx\ =\ \frac{\pi e^{im\pi/2}}{2^{w-1}}\sum_{r=0}^{\infty}\frac{(\alpha_p)_r C^r\Gamma(w+2hr)}{(\beta_q)_r r!4^{hr}\Gamma(\frac{\omega+2hr\pm M+1}{2})} \tag{1.7}$$

Where $(\alpha)_p$ denotes $\alpha_1,\cdots,\alpha_p;\Gamma(a\pm b)$ represents$\Gamma(a+b),\Gamma(a-b)$; h is a positive integer;$p<q$ and $Re(w)>0$.Recall the following elementary integrals:

$$\int_0^{\pi}e^{i(m-n)x}\,dx=\begin{cases}\pi\ , & m=n\ ;\\ 0\ , & m\neq n\ ;\end{cases} \tag{1.8}$$

$$\int_0^{\pi}e^{imx}\cos nx\,dx=\begin{cases}\frac{\pi}{2}\ , & m=n\neq 0\ ;\\ \pi\ , & m=n=0\ ;\\ 0\ , & m=n\ ;\end{cases} \tag{1.9}$$

$$\int_0^{\pi}e^{imx}\sin nx\,dx=\begin{cases}i\frac{\pi}{2}\ , & m=n\ ;\\ 0\ , & m\neq n\ ;\end{cases} \tag{1.10}$$

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Provided either both m and n are odd or both m and n are even integers.

For brevity, we shall use the following notations.

$$\frac{\prod_{k=1}^{E}(e_k)_{r+t}\prod_{k=1}^{F}(f_k)_r\prod_{k=1}^{F'}(f'_k)_t}{\prod_{k=1}^{G}(g_k)_{r+t}\prod_{k=1}^{H}(h_k)_r\prod_{k=1}^{H'}(h'_k)_t}=\epsilon$$

$$\frac{\prod_{k_1=1}^{E_1}(e_{1k_1})_{r_1+t_1}\prod_{k_1=1}^{F_1}(f_{1k_1})_{r_1}\prod_{k_1=1}^{F'_1}(f'_{1j_1})_{t_1}}{\prod_{k_1=1}^{G_1}(g_{1k_1})_{r_1+t_1}\prod_{k_1=1}^{H_1}(h_{1k_1})_{r_1}\prod_{k_1=1}^{H'_1}(h_{1k_1})_{t_1}}=\epsilon_1$$

....

....

$$\frac{\prod_{k_n=1}^{E_n}(e_{nk_n})_{r_n+t_n}\prod_{k_n=1}^{F_n}(f_{nk_n})_{r_n}\prod_{k_n=1}^{F'_n}(f'_{nj_n})_{t_n}}{\prod_{k_n=1}^{G_n}(g_{nk_n})_{r_n+t_n}\prod_{k_n=1}^{H_n}(h_{nk_n})_{r_n}\prod_{k_n=1}^{H_n}(h'_{nk_n})_{t_n}}=\epsilon_n$$

II. Integral

The integrals to be evaluated are:

$$\int_0^{\pi}(\sin\ x)^{w-1}e^{imx}F^{E;F;F'}_{G;H;H'}\left[\begin{array}{ll}(e);(f);(f'); & \alpha(sin\ x)^{2\rho}\\ (g);(h);(h'); & \beta(sin\ x)^{2\gamma}\end{array}\right]$$

$$\times I^{p_i,q_i:r}_{m,n}\left[\ z(\sin\ x)^{2\sigma}\ \middle|\ \begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,P_i}\\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right]dx\ =\ \frac{\sqrt{(\pi)}e^{im\pi/2}}{2^{\omega-1}}\sum_{r,t=0}^{\infty}\epsilon\frac{(\alpha/4^{\rho})^r(\beta/4^{\gamma})^t}{r!\ t!}$$

$$\times I^{m,n+1}_{p_i+1,qi+2:r}\left[\ \frac{z}{4^{\sigma}}\ \middle|\ \begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,pi}\\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right]\tag{2.1}$$

provided that $|\arg z|<\frac{1}{2}\pi\Omega$,and $\mathbf{Re}(\mathbf{w})>\mathbf{0};\alpha,\beta,\rho,\gamma,\sigma,\mathbf{z}$ are positive integers, where

$$\Omega\ \equiv\ \sum_{j=1}^{m}\beta_j+\sum_{j=1}^{n}\alpha_j\ -\sum_{j=m+1}^{q_i}\beta_{ji}\ -\sum_{j=n+1}^{p}\alpha_{ji}\ >\ 0.$$

$$\int_0^{\pi}\cdots\int_0^{\pi}(\sin x)^{w_1-1}\cdots(sinx)^{w_n-1}e^{i(m_1x_1+\cdots+m_nx_n)}$$

$$\times F^{E_1;F_1;F_1'}_{G_1;H_1;H_1'}\left[\begin{matrix}(e_1);(f_1);(f_1); & \alpha_1(sinx_1)^{2\rho_1}\\ (g_1);(h_1);(h_1'); & \beta_1(sinx_1)^{2\gamma_1}\end{matrix}\right]\cdots F^{E_n;F_n;F_n'}_{G_n;H_n;H_n'}\left[\begin{matrix}(e_n);(f_n);(f_n'); & \alpha_n(sinx_n)^{2\rho_n}\\ (g_n);(h_n);(h_n'); & \beta_n(sinx_n)^{2\gamma_n}\end{matrix}\right]$$

$$\times I^{p_i,q_i:r}_{m,n}\left[\ z(\sin x_1)^{2\sigma_1}\cdots(\sin x_n)^{2\sigma_n}\ \right]dx_1\cdots dx_n$$

$$=\frac{(\pi)^n e^{i(m_1+\cdots+m_n)\pi/2}}{2^{(\omega_1+\cdots+\omega_n)-n}}\sum_{r_1,t_1=0}^{\infty}\cdots\sum_{r_n,t_n}^{\infty}(\epsilon_1\cdots\epsilon_n)\frac{(\alpha_1/4^{\rho_1})^{r_1}(\beta_1/4^{\gamma_1})^{t_1}}{r_1!\ t_1!}\cdots\frac{(\alpha_n/4^{\rho_n})^{r_n}(\beta_n/4^{\gamma_n})^{t_n}}{r_n!\ t_n!}$$

$$\times I^{m,n+n}_{p_i+n,q_i+2n:r}\left[\frac{z}{4^{(\sigma_1+\cdots+\sigma_n)}}\left|\begin{matrix}(1-\omega_1-2\rho_1r_1-2\gamma_1t_1,2\sigma_1;1)\cdots(1-\omega_n-2\rho_nr_n-2\gamma_nt_n,2\sigma_n;1),\\ (b_j,\beta_j)_{1,m},\quad(b_{ji},\beta_{ji})_{m+1,q_i}\quad\left(\frac{1-\omega_1-2\rho_1r_1-2\gamma_1t_1\pm m_1}{2},\sigma_1;1\right)\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \cdots\left(\frac{1-\omega_n-2\rho_nr_n-2\gamma_nt_n\pm m_n}{2},\sigma_n;1\right)\end{matrix}\right] \tag{2.2}$$

provided that all the conditions of (2.1) are satisfied and $\mathbf{Re(w_i)>0};\sigma_\mathbf{i},\alpha_\mathbf{i},\beta_\mathbf{i},\rho_\mathbf{i},\gamma_\mathbf{i},\mathbf{z_i}$ are positive integers $(i=1,\cdots,n)$

Proof: To prove (2.1), expand the I-Function into the mellin-Barnes type integral. Now, on changing the order of integration,which is permissible under the conditions stated with the integral, the integral readily follows from (1.7)

On applying the same procedure as above the integral (2.2) can be derived easily.

III. Exponential Fourier Series

Let

$$f(x)\ =\ (\sin x)^{w-1}F^{E;F;F'}_{G;H;H'}\left[\begin{matrix}(e);(f);(f'); & \alpha(sinx)^{2\rho}\\ (g);(h);(h'); & \beta(sinx)^{2\gamma}\end{matrix}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[\ z(\sin x)^{2\sigma}\ \left|\begin{matrix}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{matrix}\right.\right]dx\ =\ \sum_{p=-\infty}^{\infty}A_pe^{-ipx} \tag{3.1}$$

which is valid due to $f(x)$ is continuous and of bounded variation with interval $(0,\pi)$. Now,multiplying by e^{imx} both sides in (3.1) and integrating it with respect to x from 0 to π, and then making an appcal to (1.8)and (2.1),we get

$$A_p\ =\ \frac{e^{im\pi/2}}{2^{\omega-1}}\ \sum_{r,t=0}^{\infty}\ \epsilon\ \frac{(\alpha/4^{\rho})^r}{r!}\ \ \frac{(\beta/4^{\gamma})^t}{t!}$$

Notes

$$\times I^{m,n+1}_{p_i+1,q_i+1:r}\left[\frac{z}{4^{\sigma}}\left|\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right.\right] \tag{3.2}$$

An application to (3.1) and (3.2) gives the required exponential Fourier series

Notes

$$(2\sin x)^{w-1}F^{E;F;F'}_{G;H;H'}\left[\begin{array}{ll}(e);(f);(f'); & \alpha(sinx)^{2\rho}\\ (g);(h);(h'); & \beta(sinx)^{2\gamma}\end{array}\right]$$

$$\times \bar{H}^{P,Q}_{M,N}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right.\right]$$

$$=\sum_{p=-\infty}^{\infty}\sum_{r,t=0}^{\infty}e^{ip(\pi/2-x)}\ \epsilon\frac{(\alpha/4^{\rho})^r}{r!}\ \frac{(\beta/4^{\gamma})^t}{t!}$$

$$\times I^{m,n+1}_{p_i+1,q_i+1:r}\left[\frac{z}{4^{\sigma}}\left|\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right.\right]. \tag{3.3}$$

IV. Cosine Fourier Series

Let

$$f(x)=(\sin x)^{w-1}F^{E;F;F'}_{G;H;H'}\left[\begin{array}{ll}(e);(f);(f'); & \alpha(sinx)^{2\rho}\\ (g);(h);(h'); & \beta(sinx)^{2\gamma}\end{array}\right]$$

$$\times I^{m,n}_{pi,q_i:r}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right.\right]=\frac{B_0}{2}+\sum_{p=1}^{\infty}B_p\ cospx \tag{4.1}$$

Integrating both sides with respect to x from 0 to π, we get

$$\frac{B_0}{2}=\frac{1}{\sqrt{(\pi)}}\sum_{r,t=0}^{\infty}\ \epsilon\ \frac{(\alpha)^r}{r!}\ \frac{(\beta)^t}{t!}$$

$$\times I^{m,n+1}_{p_i+1,q_i+1:r}\left[z\left|\begin{array}{l}(\frac{2-\omega}{2}-\rho r-\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega}{2}-2\rho r-2\gamma t,\sigma;1\right)\end{array}\right.\right] \tag{4.2}$$

Now, multiplying by e^{imx} both sides in (4.1) and integrating it with respect to x frome 0 to π ,and finally ,making an application to (1.8),(1.9) and (2.1), we derive

$$B_p=\frac{e^{ip\pi/2}}{2^{\omega-1}}\sum_{r,t=0}^{\infty}\ \epsilon\ \frac{(\alpha/4^{\rho})^r}{r!}\ \frac{(\beta/4^{\gamma})^t}{t!}$$

$$\times I^{m,n+1}_{p_i+1,qi+2:r}\left[\frac{z}{4^{\sigma}}\left|\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right.\right] \quad (4.3)$$

using (4.2), (4.3), from(4.1) we get required cosine Fourier Series.

$$(\sin x)^{w-1}F^{E;F;F'}_{G;H;H'}\left[\begin{array}{ll}(e);(f);(f'); & \alpha(sinx)^{2\rho}\\ (g);(h);(h'); & \beta(sinx)^{2\gamma}\end{array}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \\ (b_j,\beta_j)_{1,m},(b_j,\beta_j)_{m+1,q_i}\end{array}\right.\right]=\frac{1}{\sqrt{(\pi)}}\sum_{r,t=0}^{\infty}\epsilon\ \frac{(\alpha)^r}{r!}\ \frac{(\beta)^t}{t!}$$

$$\times I^{m,n+1}_{p_i+1,q_i+1:r}\left[z\left|\begin{array}{l}(\frac{2-\omega}{2}-\rho r-\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega}{2}-2\rho r-2\gamma t),\sigma;1\right)\end{array}\right.\right]$$

$$+\sum_{p=-\infty}^{\infty}\sum_{r,t=0}^{\infty}\epsilon\ e^{ip\pi/2}\ cospx\ \frac{(\alpha/4^{\rho})^r}{r!}\ \frac{(\beta/4^{\gamma})^t}{t!}.\frac{1}{2^{\omega-2}}$$

$$\times I^{m,n+1}_{p_i+1,q_i+2:r}\left[\frac{z}{4^{\sigma}}\left|\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \\ (b_j,\beta_j)_{1,n},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right.\right]. \quad (4.4)$$

V. Sine Fourier Series

Let

$$f(x)=(\sin x)^{w-1}F^{E;F;F'}_{G;H;H'}\left[\begin{array}{ll}(e);(f);(f'); & \alpha(sinx)^{2\rho}\\ (g);(h);(h'); & \beta(sinx)^{2\gamma}\end{array}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right.\right]=\sum_{p=-\infty}^{\infty}C_p\ sin\ px. \quad (5.1)$$

Multiplying by e^{imx} both sides in (5.1) and the integrating it with respect to x frome 0 to π ,and making to (1.10) and (2.1), we obtain

$$C_p=\frac{e^{ip\pi/2}}{2^{\omega-1}}\sum_{r,t=0}^{\infty}\epsilon\ \frac{(\alpha/4^{\rho})^r}{r!}\ \frac{(\beta/4^{\gamma})^t}{t!}.$$

Notes

$$\times I^{m,n+1}_{p_i+1,q_i+2:r}\left[\frac{z}{4^\sigma}\;\middle|\;\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1.p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right]. \quad (5.2)$$

Now making an application of (5.1) and (5.2), we get required Sine Fourier Series.

Notes

$$(2\sin x)^{w-1}F^{E;F;F'}_{G;H;H'}\left[\begin{array}{ll}(e);(f);(f'); & \alpha(sinx)^{2\rho}\\ (g);(h);(h'); & \beta(sinx)^{2\gamma}\end{array}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z(\sin x)^{2\sigma}\;\middle|\;\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right]$$

$$=\sum_{p=-\infty}^{\infty}\sum_{r,t=0}^{\infty}\frac{2\;\epsilon\;e^{ip\pi/2}}{i}sin\;px\quad\epsilon\;\frac{(\alpha/4^\rho)^r}{r!}\quad\frac{(\beta/4^\gamma)^t}{t!}$$

$$\times I^{m,n+1}_{p_i+1,q_i+2:r}\left[\frac{z}{4^\sigma}\;\middle|\;\begin{array}{l}(1-\omega-2\rho r-2\gamma t,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega-2\rho r-2\gamma t\pm m}{2},\sigma;1\right)\end{array}\right]. \quad (5.3)$$

VI. Multiple Exponential Fourier Series

Let

$$f(x_1,\cdots,x_n)\;=\;(\sin x)^{w_1-1}\cdots(\sin x)^{w_n-1}F^{E_1;F_1;F'_1}_{G_1;H_1;H'_1}\left[\begin{array}{ll}(e_1);(f_1);(f'_1); & \alpha_1(sinx_1)^{2\rho_1}\\ (g_1);(h_1);(h'_1); & \beta_1(sinx_1)^{2\gamma_1}\end{array}\right]$$

$$\cdots F^{E_n;F_n;F'_n}_{G_n;H_n;H'_n}\left[\begin{array}{ll}(e_n);(f_n);(f'_n); & \alpha_n(sinx_n)^{2\rho_n}\\ (g_n);(h_n);(h'_n); & \beta_n(sinx_n)^{2\gamma_n}\end{array}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z(\sin x_1)^{2\sigma_1}\cdots(\sin x_n)^{2\sigma_n}\;\middle|\;\begin{array}{l}(a_j,\alpha_j)_{1,},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right]$$

$$=\sum_{p_1=-\infty}^{\infty}\cdots\sum_{p_n=-\infty}^{\infty}A_{p_1\cdots p_n}e^{-i(p_1x_1+\cdots+p_nx_n)}. \quad (6.1)$$

Equation (6.1) is valid, since $f(x_1,\cdots,x_n)$ is continuous and of bounded variation in the open interval $(0,\pi)$. In the series (6.1),to calculate $A_{p_1\cdots p_n}$we fix $x_1,\cdots,x_{n-1}$,so that

$$\sum_{p_1=-\infty}^{\infty}\cdots\sum_{p_{n-1}=-\infty}^{\infty}A_{p_1\cdots p_{n-1}}e^{-i(p_1x_1+\cdots+p_{n-1}x_{n-1})}$$

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depends only on p_n.

Furthermore ,it must be the coefficient of Fourier exponential series in x_n of $f(x_1, \cdots, x_n)$ over $0 < x_n < \pi$.

Now multiplying by $e^{im_n x_n}$ both sides in (6.1) and integrating with respect to x_n from 0 to π,we get

$$(\sin x_1)^{w_1-1} \cdots (\sin x_n)^{w_n-1} F^{E_1;F_1;F_1'}_{G_1;H_1;H_1'} \left[\begin{matrix} (e_1);(f_1);(f_1'); & \alpha_1(sinx_1)^{2\rho_1} \\ (g_1);(h_1);(h_1'); & \beta_1(sinx_1)^{2\gamma_1} \end{matrix} \right]$$

$$\cdots F^{E_{n-1};F_{n-1};F_{n-1}'}_{G_{n-1};H_{n-1};H_{n-1}'} \left[\begin{matrix} (e_{n-1});(f_{n-1});(f_{n-1}'); & \alpha_{n-1}(sinx_{n-1})^{2\rho_{n-1}} \\ (g_{n-1});(h_{n-1});(h_{n-1}'); & \beta_{n-1}(sinx_{n-1})^{2\gamma_{n-1}} \end{matrix} \right]$$

$$\times \int_0^{\pi} (sinx_n)^{w_n-1} e^{im_n x_n} F^{E_n;F_n;F_n'}_{G_n;H_n;H_n'} \left[\begin{matrix} (e_n);(f_n);(f_n'); & \alpha_n(sinx_n)^{2\rho_n} \\ (g_n);(h_n);(h_n'); & \beta_n(sinx_n)^{2\gamma_n} \end{matrix} \right]$$

$$\times I^{m,n}_{p_i,q_i:r} \left[z(\sin x_1)^{2\sigma_1} \cdots (\sin x_n)^{2\sigma_n} \left| \begin{matrix} (a_j,\alpha_j)_{1,n}, (a_{ji},\alpha_{ji})_{n+1,p_i} \\ \\ (b_j,\beta_j)_{1,m}, (b_{ji},\beta_{ji})_{m+1,q_i} \end{matrix} \right. \right] dx_n$$

$$= \sum_{p_1=-\infty}^{\infty} \cdots \sum_{p_{n-1}=-\infty}^{\infty} A_{p_1 \cdots p_{n-1}} e^{-i(p_1x_1+\cdots+p_nx_n)} + \sum_{p_n=-\infty}^{\infty} \int_0^{\pi} (e^{i(m_n-p_n)x_n} dx \qquad (6.2)$$

using(1.8) and (2.1),from (6.2), respectively,we get

$$A_{p_1 \cdots p_n} = \sum_{r_1,t_1=0}^{\infty} \cdots \sum_{r_n,t_n=0}^{\infty} \frac{e^{i(p_1+\cdots+p_n)\pi/2}}{2^{(\omega_1+\cdots+\omega_n)-n}} \quad (\epsilon_1 \cdots \epsilon_n)$$

$$\times \frac{(\alpha_1/4^{\rho_1})^{r_1}}{r_1!} \frac{(\beta_1/4^{\gamma_1})^{t_1}}{t_1!}, \cdots\cdots\cdots\cdots, \frac{(\alpha_n/4^{\rho_n})^{r_n}}{r_n!} \frac{(\beta_n/4^{\gamma_n})^{t_n}}{t_n!}$$

$$\times I^{m,n+1}_{p_i+1,q_i+1:r} \left[\frac{z}{4^{(\sigma_1+\cdots+\sigma_n)}} \left| \begin{matrix} (1-\omega_1-2\rho_1 r_1-2\gamma_1 t_1, 2\sigma_1;1) \cdots (1-\omega_n-2\rho_n r_n-2\gamma_n t_n, 2\sigma_n;1 \\ \\ (b_j,\beta_j)_{1,m}, \quad (b_{ji},\beta_{ji})_{m+1,q_i} \quad \left(\frac{1-\omega_1-2\rho_1 r_1-2\gamma_1 t_1 \pm m_1}{2}, \sigma_1;1\right) \end{matrix} \right. \right.$$

$$\left. \begin{matrix} (a_j,\alpha_j)_{1,n}, (a_{ji},\alpha_{ji})_{n+1,p_i} \\ \\ \cdots \left(\frac{1-\omega_n-2\rho_n r_n-2\gamma_n t_n \pm m_n}{2}, \sigma_n;1\right) \end{matrix} \right]. \qquad (6.3)$$

Using (6.3) in (6.1), we get required multiple exponential Fourier series.

Let

$$(\sin x_1)^{w_1-1} \cdots (\sin x_n)^{w_n-1} F^{E;F;F'}_{G;H;H'} \left[\begin{matrix} (e_1);(f_1);(f_1'); & \alpha_1(sinx_1)^{2\rho_1} \\ (g_1);(h_1);(h_1'); & \beta_1(sinx_1)^{2\gamma_1} \end{matrix} \right]$$

Notes

$$\times F^{E_n;F_n;F'_n}_{G_n;H_n;H'_n}\left[\begin{array}{ll}(e_n);(f_n);(f'_n); & \alpha_n(sinx_n)^{2\rho_n}\\(g_n);(h_n);(h'_n); & \beta_n(sinx_n)^{2\gamma_n}\end{array}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z(\sin x_1)^{2\sigma_1}\cdots(\sin x_n)^{2\sigma_n}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_j,\alpha_j)_{n+1,p_i}\\ \\(b_j,\beta_j)_{1},m,(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right.\right]$$

Notes

$$=\sum_{p_1\cdots p_n=-\infty}^{\infty}\cdots\sum_{r_1\cdots r_n,t_1\cdots t_n=0}^{\infty}\frac{(\epsilon_1\cdots\epsilon_n)}{2^{(\omega_1+\cdots+\omega_n)-n}}\times e^{-i(p_1n_1+\cdots+p_nn_n)}\ .\ e^{(p_1+\cdots+p_n)\ \ 2}$$

$$\frac{(\alpha_1/4^{\rho_1})^{r_1}}{r_1!}\ \frac{(\beta_1/4^{\gamma_1})^{t_1}}{t_1!},\cdots\cdots\cdots,\frac{(\alpha_n/4^{\rho_n})^{r_n}}{r_n!}\ \frac{(\beta_n/4^{\gamma_n})^{t_n}}{t_n!}$$

$$\times I^{m,n+n}_{p_i+n,q_i+n}\left[\frac{z}{4^{(\sigma_1+\cdots+\sigma_n)}}\left|\begin{array}{l}(1-\omega_1-2\rho_1r_1-2\gamma_1t_1,2\sigma_1;1)\cdots(1-\omega_n-2\rho_nr_n-2\gamma_nt_n,2\sigma_n;1),\\ \\(b_j,\beta_j)_{1,m},\quad(b_{ji},\beta_{ji})_{m+1,q_i}\quad\left(\frac{1-\omega_1-2\rho_1r_1-2\gamma_1t_1\pm m_1}{2},\sigma_1;1\right)\end{array}\right.\right.$$

$$\left.\begin{array}{l}(a_{ji},\alpha_{ji})_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \\ \cdots\left(\frac{1-\omega_n-2\rho_nr_n-2\gamma_nt_n\pm m_n}{2},\sigma_n;1\right)\end{array}\right]. \qquad (6.4)$$

VII. Particular Cases

Setting $\beta_1,\cdots,\beta_n=0$ in (2.2),we get

$$\int_0^{\pi}\cdots\int_0^{\pi}(\sin x_1)^{w_1-1}\cdots(sinx_n)^{w_n-1}e^{i(m_1x_1\cdots m_nx_n)}$$

$$\times{}_{E_1+F_1}F_{G_1+H_1}\left[\begin{array}{ll}(e_1);(f_1); & \\ & \alpha_1(sinx_1)^{2\rho_1}\\(g_1);(h_1); & \end{array}\right]\cdots{}_{E_n+F_n}F_{G_n+H_n}\left[\begin{array}{ll}(e_n);(f_n); & \\ & \alpha_n(sinx_n)^{2\rho_n}\\(g_n);(h_n); & \end{array}\right]$$

$$\times I^{m,n}_{p_i,q_i:r}\left[z(\sin x_1)^{2\sigma_1}\cdots(\sin x_n)^{2\sigma_n}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \\(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right.\right]dx_1\cdots dx_n$$

$$=\frac{(\pi)^ne^{i(m_1+\cdots+m_n)\pi/2}}{2^{(\omega_1+\cdots+\omega_n)-n}}\sum_{r_1\cdots r_n=0}^{\infty}\frac{\prod_{k_1=1}^{E_1}(e_{1k_1})_{r_1}\prod_{k_1=1}^{F_1}(f_{1k_1})_{r_1}}{\prod_{k_1=1}^{G_1}(g_{1k_1})_{r_1}\prod_{k_1=1}^{H_1}(h_{1k_1})_{r_1}}\cdots$$

$$\cdots \frac{\prod_{k_n=1}^{E_n}(e_{nk_n})_{r_n}\prod_{k_n=1}^{F_n}(f_{nk_n})_{r_n}}{\prod_{k_n=1}^{G_n}(g_{nk_n})_{r_n}\prod_{k_n=1}^{H_n}(h_{nk_n})_{r_n}}\frac{(\alpha_1/4^{\rho_1})^{r_1}}{r_1!}\cdots\frac{(\alpha_n/4^{\rho_n})^{r_n}}{r_n!}$$

$$\times I^{m,n+n}_{p_i+2,q_i+2:r}\left[\frac{z}{4^{(\sigma_1+\cdots+\sigma_n)}}\left|\begin{array}{l}(1-\omega_1-2\rho_1 r_1,2\sigma_1;1)\cdots(1-\omega_n-2\rho_n r_n,2\sigma_n;1),\\ (b_j,\beta_j)_{1,m},\quad (b_{ji},\beta_{ji})_{m+1,q_i}\quad \left(\frac{1-\omega_1-2\rho_1 r_1\pm m_1}{2},\sigma_1;1\right)\end{array}\right.\right.$$

$$\left.\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \cdots\left(\frac{1-\omega_n-2\rho_n r_n\pm m_n}{2},\sigma_n;1\right)\end{array}\right]. \tag{7.1}$$

Further setting $\alpha_1,\cdots,\alpha_n=0$ in (7.1),we obtain

$$\int_0^{\pi}\cdots\int_0^{\pi}(\sin x_1)^{w_1-1}\cdots(sinx_n)^{w_n-1}e^{i(m_1x_1+\cdots+m_nx_n)}$$

$$\times I^{n,N}_{p_i,q_i:r}\left[z(\sin x_1)^{2\sigma_1}\cdots(\sin x_n)^{2\sigma_n}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right.\right]dx_1\cdots dx_n$$

$$=\frac{(\pi)^n e^{i(m_1+\cdots+m_n)\pi/2}}{2^{(\omega_1+\cdots+\omega_2)-n}}$$

$$\times I^{m,n+1}_{p_i+n,q_i+n:r}\left[\frac{z}{4^{(\sigma_1+\cdots+\sigma_n)}}\left|\begin{array}{l}(1-\omega_1,2\sigma_1;1)\cdots(1-\omega_n,2\sigma_n;1),\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega_1\pm m_1}{2},\sigma_1;1\right)\end{array}\right.\right.$$

$$\left.\begin{array}{l}(a_j,\alpha_j)_{1,},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ \cdots\left(\frac{1-\omega_n\pm m_n}{2},\sigma_n;1\right)\end{array}\right]. \tag{7.2}$$

Now setting $\alpha=\beta=0$ in (3.3) we establish

$$(\sin x)^{w-1}I^{m,n}_{p_i,q_i}\left[z(\sin x)^{2\sigma}\left|\begin{array}{l}(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\end{array}\right.\right]$$

$$=\sum_{p=-\infty}^{\infty}\frac{e^{ip(\frac{\pi}{2}-x)}}{2^{w-1}}I^{m,n+1}_{P=p_i+1,q_i+2:r}\left[\frac{z}{4^{\sigma}}\left|\begin{array}{l}(1-\omega,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega_1\pm p}{2},\sigma;1\right)\end{array}\right.\right]. \tag{7.3}$$

Notes

Notes

Letting $p = 2l$ as l is a integer,from (7.3), we establish

$$L.H.S.\ of\ (7.3) = \frac{1}{\sqrt{\pi}} I^{m,n+1}_{p_i+1,q_i+1:r}\left[z \left| \begin{matrix} (\frac{2-\omega}{2},\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i} \\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega}{2},\sigma;1\right) \end{matrix} \right. \right]$$

$$+\frac{1}{2^{w-2}}\sum_{p_n=1}^{\infty} cosl\pi \quad cos2lx\ I^{m,n+1}_{p_i+1,q_i+2:r}\left[\frac{z}{4^{\sigma}} \left| \begin{matrix} (1-\omega,\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i} \\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega\pm 2l}{2},\sigma;1\right) \end{matrix} \right. \right] \tag{7.4}$$

Further letting $p = (2l+1)$ as l is an integer,from (7.3) we obtain

$$= \frac{1}{2^{w-2}}\sum_{p=1}^{\infty} sin(2l+1)\pi/2 \quad .sin(2l+1)x$$

$$\times I^{m,n+1}_{p_i+1,q_i+2:r}\left[\frac{z}{4^{\sigma}} \left| \begin{matrix} (1-\omega,2\sigma;1),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i} \\ \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i}\left(\frac{1-\omega\pm(2l+1)}{2},\sigma;1\right) \end{matrix} \right. \right] \tag{7.5}$$

Similarly, remaining particular cases can be evaluated by (4.4) and (5.3) applying the same techniques.

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Notes

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New Generating Fun ctions Pertaining To Generalized Mellin-Barnes Type of Contour Integrals

By Mehar Chand
Punjabi University

Abstract – The aim of the present paper is to evaluate new generating functions of $\overline{\mathrm{H}}$-function, using Truesdell's ascending and descending F-equation technique. These formulae are unified in nature and act as the key formulae from which we can obtain as their special cases. For the sake of illustration, we record here some special cases of our main formulae, which are believe to be new and important themselves.

Keywords : *$\overline{H}$-function, generating function, F-equations.*

Subject Classification : *(MSC 2010) 33C99, 33C60*

NEW GENERATING FUN CTIONS PERTAINING TO GENERALIZED MELLIN-BARNES TYPE OF CONTOUR INTEGRALS

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New Generating Functions Pertaining To Generalized Mellin-Barnes Type of Contour Integrals

Mehar Chand

Abstract - The aim of the present paper is to evaluate new generating functions of $\bar{\mathbf{H}}$-function, using Truesdell's ascending and descending F-equation technique. These formulae are unified in nature and act as the key formulae from which we can obtain as their special cases. For the sake of illustration, we record here some special cases of our main formulae, which are believe to be new and important themselves.

Keywords : $\bar{H}$*-function, generating function, F-equations.*

I. Introduction

In 1987, Inayat-Hussain [1, 2] introduced generalization form of Fox's H-function, which is popularly known as $\bar{H}$-function. Now $\bar{H}$-function stands on fairly firm footing through the research contributions of various authors [1-3, 6, 7, 9-12]. $\bar{H}$-function is defined and represented in the following manner [7]:

$$\bar{H}_{p,q}^{m,n}[z]=\bar{H}_{p,q}^{m,n}\left[z\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\(b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]=\frac{1}{2\pi i}\int_L z^{\xi}\bar{\phi}(\xi)d\xi\ (z\neq 0) \qquad (1.1)$$

where

$$\bar{\phi}(\xi)=\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}}{\prod_{j=m+1}^{q}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\xi)} \qquad (1.2)$$

It may be noted that the $\bar{\phi}(\xi)$ contains fractional powers of some of the gamma function and m,n,p,q are integers such that $1\le m\le q, 1\le n\le p$, $(\alpha_j)_{1,p},(\beta_j)_{1,q}$ are positive real numbers and $(A_j)_{1,n},(B_j)_{m+1,q}$ may take non-integer values, which we assume to be positive for standardization purpose. $(a_j)_{1,p}$ and $(b_j)_{1,q}$ are complex numbers.

The nature of contour L, sufficient conditions of convergence of defining integral (1.1) and other details about the $\bar{H}$-function can be seen in the papers [6, 7].

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Ref.

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The behavior of the $\bar{H}$-function for small values of $|z|$ follows easily from a result given by Rathie [3]:

$$\bar{H}_{p,q}^{m,n}[z] = o\left(|z|^{\alpha}\right);$$

Where

$$\alpha = \min_{1\le j\le m} \mathrm{Re}\left(\frac{b_j}{\alpha_j}\right), |z| \to 0 \tag{1.3}$$

$$\Omega = \sum_{j=1}^{m} |\beta_j| - \sum_{j=m+1}^{q} |\beta_j B_j| + \sum_{j=1}^{n} |\alpha_j A_j| - \sum_{j=n+1}^{p} |\alpha_j| > 0 \,,\, 0 < |z| < \infty \tag{1.4}$$

The following function which follows as special cases of the $\bar{H}$-function will be required in the sequel [7]:

$${}_p\bar{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};z\right] = \bar{H}_{p,q+1}^{1,p}\left[-z\left|\begin{matrix}(1-a_j,\alpha_j;A_j)_{1,p} \\ (0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right] \tag{1.5}$$

Truesdell's F-equations are defined and represented as:

a) The function $F(z,s)$ is said to satisfy the ascending F-equations if $D_z^r F(z,s) = F(z,s+r)$ where $D_z^r = \left(\frac{d}{dz}\right)^r$.

b) The function $Y(z,s)$ is said to satisfy the descending F-equations if $D_z^r Y(z,s) = Y(z,s-r)$, where r is a positive integer.

For $F(z,s)$ satisfying ascending F-equation, Truesdell [13] and Agarwal and Saxena [4] obtained the following generating functions using Tayler's series:

$$F(z+y,s) = \sum_{n=0}^{\infty} y^n \frac{F(z,s+n)}{n!} \tag{1.6}$$

$$Y(z+y,s) = \sum_{n=0}^{\infty} y^n \frac{Y(z,s-n)}{n!} \tag{1.7}$$

In order to obtain main results of this section, we will make use of the following well known results on multiplication formulae for the Gamma functions.

$$\prod_{k=0}^{m-1} \Gamma\left(\frac{s+r+k}{m}\right) = m^{-r}(s)_r \prod_{k=0}^{m-1} \Gamma\left(\frac{s+k}{m}\right) \tag{1.8}$$

$$\prod_{k=0}^{m-1} \Gamma\left(\frac{s-r-k}{m}\right) = \frac{(-m)^{-r}}{(m-s)_r} \prod_{k=0}^{m-1} \Gamma\left(\frac{s-k}{m}\right) \tag{1.9}$$

$$\prod_{k=0}^{m-1} \Gamma\left(\frac{s-r+k}{m}\right) = \frac{(-m)^{-r}}{(m-s)_r} \prod_{k=0}^{m-1} \Gamma\left(\frac{s+k}{m}\right) \tag{1.10}$$

Ref.

3. A.K. Rathie, A new generalization of generalized hypergeometric functions, Le Mathematic he Fasc. II 52 (1997), 297-310.

$$\prod_{k=0}^{m-1}\Gamma\left(\frac{s+r-k}{m}\right)=m^{-r}(s-m+1)_r\prod_{k=0}^{m-1}\Gamma\left(\frac{s-k}{m}\right) \tag{1.11}$$

$$\{\Delta(a,s),h\}=\left(\frac{s}{a},h\right),\left(\frac{s+1}{a},h\right),\left(\frac{s+2}{a},h\right),\ldots,\left(\frac{s+a-1}{a},h\right) \tag{1.12}$$

Notes

II. Different Forms of $\bar{H}$-Function

In this section, we have different forms of $\bar{H}$-Function which satisfy Truesdell's ascending and descending F-equation.

The following forms of $\bar{H}$-Function satisfy Truesdell's ascending F-equation:

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.1}$$

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s+1/2),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(2a,2s),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.2}$$

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s+2/3),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s+1/3),h\}\\ \{\Delta(3a,3s),h\},(b_j,\beta_j)_{3a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.3}$$

$$\left(\frac{z}{a}\right)^{-(s+1)}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(2a,2s+1/2),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p-2a},\{\Delta(2a,2s+1),h\}\\ \{\Delta(4a,4s+1),h\},(b_j,\beta_j)_{4a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1),h\}\end{matrix}\right.\right] \tag{2.4}$$

$$\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s),h\}\end{matrix}\right.\right] \tag{2.5}$$

$$\left(\frac{z}{a}\right)^{-s}2^{s}e^{i\pi s/2}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{2ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s/2),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,(s+1)/2),h\}\end{matrix}\right.\right] \tag{2.6}$$

$$\left(\frac{z}{a}\right)^{-s}e^{i\pi s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}\{\Delta(\rho,s),h\}\ (a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s),h\}(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.7}$$

$$\left(\frac{z}{a}\right)^{-s}e^{i\pi s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s),h\}\,(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.8}$$

Making use of the equation (2.1), we obtain:

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$$D_z^r[A(z,s)] = D_z^r\left(\frac{z}{a}\right)^{-s}\bar{H}_{p,q}^{m,n}\left[\left(\frac{zt}{a}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.9}$$

Replacing the $\bar{H}$-Function by its definition (1.1) and then interchanging the order of integration and differentiation and (2.9) transforms to

Notes

$$\frac{1}{2\pi i}\int_L \frac{\prod_{k=0}^{a-1}\Gamma\left(\frac{s+k}{a}-h\xi\right)\prod_{j=a+1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}\left(\frac{t^{ha\xi}}{a^{ha\xi-s}}\right)D_z^r(z)^{ha\xi-s}}{\prod_{j=m+1}^{q-\rho}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{k=0}^{\rho-1}\Gamma\left(1-\frac{s+k}{\rho}-h\xi\right)\prod_{j=n+1}^{p-\rho}\Gamma(a_j-\alpha_j\xi)\prod_{k=0}^{\rho-1}\Gamma\left(\frac{s+k}{\rho}-h\xi\right)}d\xi \tag{2.10}$$

Now using (1.8) and (1.9) lead to two identities:

$$\prod_{k=0}^{a-1}\Gamma\left(\frac{s+k}{a}-h\xi\right) = \frac{a^r}{(s-ha\xi)_r}\prod_{k=0}^{a-1}\Gamma\left(\frac{s+r+k}{a}-h\xi\right) \tag{2.11}$$

$$\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+k}{a}+h\xi\right) = \frac{(s-ha\xi)_r}{(-1)^r a^r}\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+r+k}{a}+h\xi\right) \tag{2.12}$$

Using these results, equation (2.10) takes the following form:

$$D_z^r[A(z,s)] = A[z,s+r]$$

Similarly, forms of $\bar{H}$-Function (2.2), (2.3), (2.4), (2.5), (2.6), (2.7) and (2.8) satisfy the Truesdell's ascending F-equation.

Also the following forms of $\bar{H}$-Function satisfy Truesdell's descending F-equation:

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.13}$$

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(2a,2s),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+1/2),h\}\,(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.14}$$

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(4a,4s+1),h\},(a_j,\alpha_j;A_j)_{4a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q-2a},\{\Delta(2a,2s),h\}\end{matrix}\right.\right] \tag{2.15}$$

$$\left(\frac{z}{a}\right)^{s-1}\bar{H}_{p,q}^{m,n}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(3a,3s),h\},(a_j,\alpha_j;A_j)_{3a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1/3),h\}\end{matrix}\right.\right] \tag{2.16}$$

$$\left(\frac{z}{a}\right)^{s-1} e^{i\pi s}\,\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.17}$$

$$\left(\frac{z}{a}\right)^{s-1} 2^{-s} e^{i\pi s/2}\,\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{2ha}\left|\begin{matrix}\{\Delta(a,s/2),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,(s+1)/2),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.18}$$

$$\left(\frac{z}{a}\right)^{s-1} \overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.19}$$

$$\left(\frac{z}{a}\right)^{s-1} e^{i\pi s}\,\overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(\rho,s),h\},(a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(\rho,s),h\}\ (b_j,\beta_j)_{\rho+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{2.20}$$

Making use of the equation (2.13), we obtain:

$$D^r_z[B(z,s)] = D^r_z\left(\frac{z}{a}\right)^{s-1} \overline{H}^{m,n}_{p,q}\left[\left(\frac{t}{za}\right)^{ha}\left|\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right] \tag{2.21}$$

Replacing the $\overline{H}$-Function by its definition (1.1) and then interchanging the order of integration and differentiation and (2.21) transforms to

$$\frac{1}{2\pi i}\int_L \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\xi)\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+k}{a}+h\xi\right)\prod_{j=1}^{n}\{\Gamma(1-a_j+\alpha_j\xi)\}^{A_j}\left(\frac{t^{ha\xi}}{a^{ha\xi+s-1}}\right)D^r_z(z)^{s-ha\xi-1}}{\prod_{j=m+1}^{q-\rho}\{\Gamma(1-b_j+\beta_j\xi)\}^{B_j}\prod_{k=0}^{\rho-1}\Gamma\left(1-\frac{s+k}{\rho}+h\xi\right)\prod_{j=n+1}^{p-\rho}\Gamma(a_j-\alpha_j\xi)\prod_{k=0}^{\rho-1}\Gamma\left(\frac{s+k}{\rho}-h\xi\right)}d\xi \tag{2.22}$$

Now using (1.10) and (1.11) lead to two identities:

$$\prod_{k=0}^{a-1}\Gamma\left(\frac{s+k}{a}-h\xi\right)=(-\lambda)^{-r}(ha\xi-s+1)_r\prod_{k=0}^{a-1}\Gamma\left(\frac{s-r+k}{a}-h\xi\right) \tag{2.23}$$

$$\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s+k}{a}+h\xi\right)=\frac{a^r}{(ah\xi-s+1)_r}\prod_{k=0}^{a-1}\Gamma\left(1-\frac{s-r+k}{a}+h\xi\right) \tag{2.24}$$

Using these results, (2.22) takes the following form:

$$D^r_z[B(z,s)] = B[z,s-r]$$

Similarly, forms of $\overline{H}$-function (2.14), (2.15), (2.16), (2.17), (2.18), (2.19) and (2.20) satisfy the Truesdell's descending F-equation.

Notes

III. Generating Functions

If $A=\left(1+\frac{h}{a}\right)$, then the generating functions obtained by employing forms (2.1) to (2.8):

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s+r),h\}\\ \{\Delta(a,s+r),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s+r),h\}\end{matrix}\right.\right] \tag{3.1}$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}\{\Delta(a,s+1/2),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(2a,2s),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}\{\Delta(a,s+r+1/2),h\}\ (a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(2a,2s+2r),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{3.2}$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}\{\Delta(a,s+2/3),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s+1/3),h\}\\ \{\Delta(3a,3s),h\},(b_j,\beta_j)_{3a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}\{\Delta(a,s+r+2/3),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s+r+1/3),h\}\\ \{\Delta(3a,3s+3r),h\},(b_j,\beta_j)_{3a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \tag{3.3}$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}\{\Delta(2a,2s+1/2),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p-2a},\{\Delta(2a,2s+1),h\}\\ \{\Delta(4a,4s+1),h\},(b_j,\beta_j)_{4a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}\{\Delta(2a,2s+2r+1/2),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p-2a},\{\Delta(2a,2s+2r+1),h\}\\ \{\Delta(4a,4s+4r+1),h\},(b_j,\beta_j)_{4a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+r+1),h\}\end{matrix}\right.\right] \tag{3.4}$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{ha}x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s),h\}\end{matrix}\right.\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}^{m,n}_{p,q}\left[x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+r),h\}\end{matrix}\right.\right] \tag{3.5}$$

$$A^{-s}\bar{H}^{m,n}_{p,q}\left[A^{2ha}x^2\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s/2),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,(s+1)/2),h\}\end{matrix}\right.\right]$$

Notes

$$=\sum_{r=0}^{\infty}\frac{(2h)^r(-1)^{r/2}}{r!}\bar{H}_{p,q}^{m,n}\left[x^2\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,(s+r)/2),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,(s+r+1)/2),h\}\end{matrix}\right.\right] \quad (3.6)$$

$$A^{-s}\bar{H}_{p,q}^{m,n}\left[A^{ha}x\left|\begin{matrix}\{\Delta(\rho,s),h\}\ (a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}\{\Delta(\rho,s+r),h\}\ (a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+r),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s+r),h\}\end{matrix}\right.\right] \quad (3.7)$$

$$A^{-s}\bar{H}_{p,q}^{m,n}\left[A^{ha}x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+r),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.8)$$

To prove the above generating function, we employ the forms (2.1) to (2.8) in equation (1.6) and then on replacing z by $\frac{ya}{h}$ and $\left(\frac{yt}{h}\right)^{ha}$ by x, we get the above result from (3.1) to (3.8).

Generating functions obtained by employing forms (2.13) to (2.20):

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}\{\Delta(a,s-r),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-\rho},\{\Delta(\rho,s-r),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q-\rho},\{\Delta(\rho,s-r),h\}\end{matrix}\right.\right] \quad (3.9)$$

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{matrix}\{\Delta(2a,2s),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+1/2),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}\{\Delta(2a,2s-2r),h\},(a_j,\alpha_j;A_j)_{2a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s-r+1/2),h\}\ (b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{matrix}\right.\right] \quad (3.10)$$

$$A^{s-1}\bar{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{matrix}\{\Delta(4a,4s+1),h\},(a_j,\alpha_j;A_j)_{4a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q-2a},\{\Delta(2a,2s),h\}\end{matrix}\right.\right]$$

$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\bar{H}_{p,q}^{m,n}\left[x\left|\begin{matrix}\{\Delta(4a,4s-4r+1),h\},(a_j,\alpha_j;A_j)_{4a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s-r),h\}\\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,\beta_j)_{2a+1,m},(b_j,\beta_j;B_j)_{m+1,q-2a},\{\Delta(2a,2s-2r),h\}\end{matrix}\right.\right] \quad (3.11)$$

$$
A^{s-1}\overline{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{array}{c}\{\Delta(3a,3s),h\},(a_j,\alpha_j;A_j)_{3a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s+1/3),h\}\end{array}\right.\right]
$$

$$
=\sum_{r=0}^{\infty}\frac{h^r}{r!}\overline{H}_{p,q}^{m,n}\left[x\left|\begin{array}{c}\{\Delta(3a,3s-3r),h\},(a_j,\alpha_j;A_j)_{3a+1,n},(a_j,\alpha_j)_{n+1,p}\\ \{\Delta(a,s-r+2/3),h\}\,(b_j,\beta_j)_{a+1,m},(b_j,\beta_j;B_j)_{m+1,q-a},\{\Delta(a,s-r+1/3),h\}\end{array}\right.\right] \tag{3.12}
$$

$$
A^{s-1}\overline{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{array}{c}\{\Delta(a,s),h\}\,(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right]
$$

$$
=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\overline{H}_{p,q}^{m,n}\left[x\left|\begin{array}{c}\{\Delta(a,s-r),h\}\,(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right] \tag{3.13}
$$

$$
A^{s-1}\overline{H}_{p,q}^{m,n}\left[A^{-2ha}x^2\left|\begin{array}{c}\{\Delta(a,s/2),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,(s+1)/2),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right]
$$

$$
=\sum_{r=0}^{\infty}\frac{(2h)^r(-1)^{r/2}}{r!}\overline{H}_{p,q}^{m,n}\left[x^2\left|\begin{array}{c}\{\Delta(a,(s-r)/2),h\},(a_j,\alpha_j;A_j)_{a+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,(s-r+1)/2),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right] \tag{3.14}
$$

$$
A^{s-1}\overline{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{array}{c}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right]
$$

$$
=\sum_{r=0}^{\infty}\frac{h^r}{r!}\overline{H}_{p,q}^{m,n}\left[x\left|\begin{array}{c}(a_j,\alpha_j;A_j)_{1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s-r),h\}\\ (b_j,\beta_j)_{1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right] \tag{3.15}
$$

$$
A^{s-1}\overline{H}_{p,q}^{m,n}\left[A^{-ha}x\left|\begin{array}{c}\{\Delta(\rho,s),h\},(a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(\rho,s),h\}\,(b_j,\beta_j)_{\rho+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right]
$$

$$
=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\overline{H}_{p,q}^{m,n}\left[x\left|\begin{array}{c}\{\Delta(\rho,s-r),h\},(a_j,\alpha_j;A_j)_{\rho+1,n},(a_j,\alpha_j)_{n+1,p-a},\{\Delta(a,s-r),h\}\\ \{\Delta(\rho,s-r),h\}\,(b_j,\beta_j)_{\rho+1,m},(b_j,\beta_j;B_j)_{m+1,q}\end{array}\right.\right] \tag{3.16}
$$

To prove the above generating function, we employ the forms (2.13) to (2.20) in equation (1.7) and then on replacing z by $\frac{ya}{h}$ and $\left(\frac{yt}{h}\right)^{ha}$ by x, we get the above result from (3.9) to (3.16).

Notes

IV. Special Cases

(4.1) When $A_j = B_j = 1$, the $\bar{\text{H}}$-Function reduces to the Fox's H-function [5, p. 10, Eqn. (2.1.1)], the above results (3.1) to (3.16) reduces to the following form:

$$A^{-s} H^{m,n}_{p,q}\left[A^{ha}x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p-\rho}, \{\Delta(\rho,s),h\} \\ \{\Delta(a,s),h\}, (b_j,\beta_j)_{1,q-\rho}, \{\Delta(\rho,s),h\} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H^{m,n}_{p,q}\left[x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p-\rho}, \{\Delta(\rho,s+r),h\} \\ \{\Delta(a,s+r),h\}, (b_j,\beta_j)_{a+1,q-\rho}, \{\Delta(\rho,s+r),h\} \end{matrix}\right] \quad (4.1.1)$$

$$A^{-s} H^{m,n}_{p,q}\left[A^{ha}x \middle| \begin{matrix} \{\Delta(a,s+1/2),h\}, (a_j,\alpha_j)_{a+1,p} \\ \{\Delta(2a,2s),h\}, (b_j,\beta_j)_{2a+1,q} \end{matrix}\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H^{m,n}_{p,q}\left[x \middle| \begin{matrix} \{\Delta(a,s+r+1/2),h\}, (a_j,\alpha_j)_{a+1,p} \\ \{\Delta(2a,2s+2r),h\}, (b_j,\beta_j)_{2a+1,q} \end{matrix}\right] \quad (4.1.2)$$

$$A^{-s} H^{m,n}_{p,q}\left[A^{ha}x \middle| \begin{matrix} \{\Delta(a,s+2/3),h\}, (a_j,\alpha_j)_{a+1,p-a}, \{\Delta(a,s+1/3),h\} \\ \{\Delta(3a,3s),h\}, (b_j,\beta_j)_{3a+1,q} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H^{m,n}_{p,q}\left[x \middle| \begin{matrix} \{\Delta(a,s+r+2/3),h\}, (a_j,\alpha_j)_{a+1,p-a}, \{\Delta(a,s+r+1/3),h\} \\ \{\Delta(3a,3s+3r),h\}, (b_j,\beta_j)_{3a+1,q} \end{matrix}\right] \quad (4.1.3)$$

$$A^{-s} H^{m,n}_{p,q}\left[A^{ha}x \middle| \begin{matrix} \{\Delta(2a,2s+1/2),h\}, (a_j,\alpha_j)_{2a+1,p-2a}, \{\Delta(2a,2s+1),h\} \\ \{\Delta(4a,4s+1),h\}, (b_j,\beta_j)_{4a+1,q-a}, \{\Delta(a,s+1),h\} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H^{m,n}_{p,q}\left[x \middle| \begin{matrix} \{\Delta(2a,2s+2r+1/2),h\}, (a_j,\alpha_j)_{2a+1,p-2a}, \{\Delta(2a,2s+2r+1),h\} \\ \{\Delta(4a,4s+4r+1),h\}, (b_j,\beta_j)_{4a+1,q-a}, \{\Delta(a,s+r+1),h\} \end{matrix}\right] \quad (4.1.4)$$

$$A^{-s} H^{m,n}_{p,q}\left[A^{ha}x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q-a}, \{\Delta(a,s),h\} \end{matrix}\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H^{m,n}_{p,q}\left[x \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q-a}, \{\Delta(a,s+r),h\} \end{matrix}\right] \quad (4.1.5)$$

$$A^{-s} H^{m,n}_{p,q}\left[A^{2ha}x^2 \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ \{\Delta(a,s/2),h\}, (b_j,\beta_j)_{a+1,q-a}, \{\Delta(a,(s+1)/2),h\} \end{matrix}\right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r (-1)^{r/2}}{r!} H^{m,n}_{p,q}\left[x^2 \middle| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ \{\Delta(a,(s+r)/2),h\}, (b_j,\beta_j)_{a+1,q-a}, \{\Delta(a,(s+r+1)/2),h\} \end{matrix}\right] \quad (4.1.6)$$

$$A^{-s} H^{m,n}_{p,q}\left[A^{ha}x \middle| \begin{matrix} \{\Delta(\rho,s),h\}, (a_j,\alpha_j)_{\rho+1,p} \\ \{\Delta(a,s),h\}, (b_j,\beta_j)_{a+1,q-\rho}, \{\Delta(\rho,s),h\} \end{matrix}\right]$$

Ref.

5. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-function of one and two variables with applications, South Asian Publishers, New Dehli, Madras (1982).

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(\rho,s+r),h\},(a_j,\alpha_j)_{\rho+1,p} \\ \{\Delta(a,s+r),h\},(b_j,\beta_j)_{a+1,q-\rho},\{\Delta(\rho,s+r),h\} \end{matrix} \right.\right] \quad (4.1.7)$$

$$A^{-s}H_{p,q}^{m,n}\left[A^{ha}x \left| \begin{matrix} (a_j,\alpha_j)_{1,p} \\ \{\Delta(a,s),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right.\right] = \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} (a_j,\alpha_j)_{1,q} \\ \{\Delta(a,s+r),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right.\right] \quad (4.1.8)$$

$$A^{s-1}H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,\alpha_j)_{a+1,p-\rho},\{\Delta(\rho,s),h\} \\ (b_j,\beta_j)_{1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,\alpha_j)_{a+1,p-\rho},\{\Delta(\rho,s-r),h\} \\ (b_j,\beta_j)_{1,q-\rho},\{\Delta(\rho,s-r),h\} \end{matrix} \right.\right] \quad (4.1.9)$$

$$A^{s-1}H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(2a,2s),h\},(a_j,\alpha_j)_{2a+1,p} \\ \{\Delta(a,s+1/2),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right.\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(2a,2s-2r),h\},(a_j,\alpha_j)_{2a+1,p} \\ \{\Delta(a,s-r+1/2),h\},(b_j,\beta_j)_{a+1,q} \end{matrix} \right.\right] \quad (4.1.10)$$

$$A^{s-1}H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(4a,4s+1),h\},(a_j,\alpha_j)_{4a+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j)_{2a+1,q-2a},\{\Delta(2a,2s),h\} \end{matrix} \right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(4a,4s-4r+1),h\},(a_j,\alpha_j)_{4a+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,\beta_j)_{2a+1,q-2a},\{\Delta(2a,2s-2r),h\} \end{matrix} \right.\right] \quad (4.1.11)$$

$$A^{s-1}H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(3a,3s),h\},(a_j,\alpha_j)_{3a+1,p} \\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j)_{a+1,q-a},\{\Delta(a,s+1/3),h\} \end{matrix} \right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(3a,3s-3r),h\},(a_j,\alpha_j)_{3a+1,p} \\ \{\Delta(a,s-r+2/3),h\},(b_j,\beta_j)_{a+1,q-a},\{\Delta(a,s-r+1/3),h\} \end{matrix} \right.\right] \quad (4.1.12)$$

$$A^{s-1}H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,\alpha_j)_{a+1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right.\right] = \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,\alpha_j)_{a+1,p} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right.\right] \quad (4.1.13)$$

$$A^{s-1}H_{p,q}^{m,n}\left[A^{-2ha}x^2 \left| \begin{matrix} \{\Delta(a,s/2),h\},(a_j,\alpha_j)_{a+1,p-a},\{\Delta(a,(s+1)/2),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right.\right]$$

Notes

$$= \sum_{r=0}^{\infty} \frac{(2h)^r (-1)^{r/2}}{r!} H_{p,q}^{m,n}\left[x^2 \left| \begin{matrix} \{\Delta(a,(s-r)/2),h\},(a_j,\alpha_j)_{a+1,p-a},\{\Delta(a,(s-r+1)/2),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right.\right] \qquad (4.1.14)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} (a_j,\alpha_j)_{1,p-a},\{\Delta(a,s),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right.\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} (a_j,\alpha_j)_{1,p-a},\{\Delta(a,s-r),h\} \\ (b_j,\beta_j)_{1,q} \end{matrix} \right.\right] \qquad (4.1.15)$$

$$A^{s-1} H_{p,q}^{m,n}\left[A^{-ha}x \left| \begin{matrix} \{\Delta(\rho,s),h\},(a_j,\alpha_j)_{\rho+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(\rho,s),h\},(b_j,\beta_j)_{\rho+1,q} \end{matrix} \right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r (-1)^r}{r!} H_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(\rho,s-r),h\},(a_j,\alpha_j)_{\rho+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(\rho,s-r),h\},(b_j,\beta_j)_{\rho+1,q} \end{matrix} \right.\right] \qquad (4.1.16)$$

(4.2) If we put $A_j = B_j = 1; \alpha_j = \beta_j = 1$, then the $\overline{H}$-function reduces to general type of G-function [8] i.e. $\overline{H}_{p,q}^{m,n}\left[z \left| \begin{matrix} (a_j,1,1)_{1,n},(a_j,1)_{n+1,p} \\ (b_j,1,1)_{1,m},(b_j,1)_{m+1,q} \end{matrix} \right.\right] = G\left[z \left| \begin{matrix} (a_j,1)_{1,p} \\ (b_j,1)_{1,q} \end{matrix} \right.\right]$, the above results (3.1) to (3.16) reduces to the following form:

$$A^{-s} G_{p,q}^{m,n}\left[A^{ha}x \left| \begin{matrix} (a_j,1)_{1,p-\rho},\{\Delta(\rho,s),h\} \\ \{\Delta(a,s),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} (a_j,1)_{1,p-\rho},\{\Delta(\rho,s+r),h\} \\ \{\Delta(a,s+r),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s+r),h\} \end{matrix} \right.\right] \qquad (4.2.1)$$

$$A^{-s} G_{p,q}^{m,n}\left[A^{ha}x \left| \begin{matrix} \{\Delta(a,s+1/2),h\},(a_j,1)_{a+1,n+1,p} \\ \{\Delta(2a,2s),h\},(b_j,1)_{2a+1,q} \end{matrix} \right.\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(a,s+r+1/2),h\},(a_j,1)_{a+1,p} \\ \{\Delta(2a,2s+2r),h\},(b_j,1)_{2a+1,q} \end{matrix} \right.\right] \qquad (4.2.2)$$

$$A^{-s} G_{p,q}^{m,n}\left[A^{ha}x \left| \begin{matrix} \{\Delta(a,s+2/3),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,s+1/3),h\} \\ \{\Delta(3a,3s),h\},(b_j,1)_{3a+1,q} \end{matrix} \right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G_{p,q}^{m,n}\left[x \left| \begin{matrix} \{\Delta(a,s+r+2/3),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,s+r+1/3),h\} \\ \{\Delta(3a,3s+3r),h\},(b_j,1)_{3a+1,q} \end{matrix} \right.\right] \qquad (4.2.3)$$

$$A^{-s} G_{p,q}^{m,n}\left[A^{ha}x \left| \begin{matrix} \{\Delta(2a,2s+1/2),h\},(a_j,1)_{2a+1,p-2a},\{\Delta(2a,2s+1),h\} \\ \{\Delta(4a,4s+1),h\},(b_j,1)_{4a+1,q-a},\{\Delta(a,s+1),h\} \end{matrix} \right.\right] \qquad (4.2.4)$$

Ref.

8. Meijer, C.S., On the G-function, Proc. Nat. Acad. Wetensch, 49, p. 227 (1946).

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(2a,2s+2r+1/2),h\},(a_j,1)_{2a+1,p-2a},\{\Delta(2a,2s+2r+1),h\} \\ \{\Delta(4a,4s+4r+1),h\},(b_j,1)_{4a+1,q-a},\{\Delta(a,s+r+1),h\} \end{matrix} \right. \right]$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{ha} x \left| \begin{matrix} (a_j,1)_{1,p} \\ (b_j,1)_{1,q-a},\{\Delta(a,s),h\} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} (a_j,1)_{1,p} \\ (b_j,1)_{1,q-a},\{\Delta(a,s+r),h\} \end{matrix} \right. \right] \quad (4.2.5)$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{2ha} x^2 \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,s/2),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,(s+1)/2),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r (-1)^{r/2}}{r!} G^{m,n}_{p,q}\left[x^2 \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,(s+r)/2),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,(s+r+1)/2),h\} \end{matrix} \right. \right] \quad (4.2.6)$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{ha} x \left| \begin{matrix} \{\Delta(\rho,s),h\},(a_j,1)_{\rho+1,p} \\ \{\Delta(a,s),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r (-1)^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(\rho,s+r),h\},(a_j,1)_{\rho+1,p} \\ \{\Delta(a,s+r),h\},(b_j,1)_{a+1,q-\rho},\{\Delta(\rho,s+r),h\} \end{matrix} \right. \right] \quad (4.2.7)$$

$$A^{-s} G^{m,n}_{p,q}\left[A^{ha} x \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,s),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r (-1)^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} (a_j,1)_{1,p} \\ \{\Delta(a,s+r),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] \quad (4.2.8)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha} x \left| \begin{matrix} \{\Delta(a,s),h\},(a_j,1)_{a+1,p-\rho},\{\Delta(\rho,s),h\} \\ (b_j,1)_{1,q-\rho},\{\Delta(\rho,s),h\} \end{matrix} \right. \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,1)_{a+1,p-\rho},\{\Delta(\rho,s-r),h\} \\ (b_j,1)_{1,q-\rho},\{\Delta(\rho,s-r),h\} \end{matrix} \right. \right] \quad (4.2.9)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha} x \left| \begin{matrix} \{\Delta(2a,2s),h\},(a_j,1)_{2a+1,p} \\ \{\Delta(a,s+1/2),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x \left| \begin{matrix} \{\Delta(2a,2s-2r),h\},(a_j,1)_{2a+1,p} \\ \{\Delta(a,s-r+1/2),h\},(b_j,1)_{a+1,q} \end{matrix} \right. \right] \quad (4.2.10)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha} x \left| \begin{matrix} \{\Delta(4a,4s+1),h\},(a_j,1)_{4a+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(2a,2s+1/2),h\},(b_j,1)_{2a+1,q-2a},\{\Delta(2a,2s),h\} \end{matrix} \right. \right]$$

Notes

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x\left| \begin{matrix} \{\Delta(4a,4s-4r+1),h\},(a_j,1)_{4a+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,1)_{2a+1,q-2a},\{\Delta(2a,2s-2r),h\}\end{matrix}\right.\right] \quad (4.2.11)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha}x\left| \begin{matrix} \{\Delta(3a,3s),h\},(a_j,1)_{3a+1,p} \\ \{\Delta(a,s+2/3),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,s+1/3),h\}\end{matrix}\right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x\left| \begin{matrix} \{\Delta(3a,3s-3r),h\},(a_j,1)_{3a+1,p} \\ \{\Delta(a,s-r+2/3),h\},(b_j,1)_{a+1,q-a},\{\Delta(a,s-r+1/3),h\}\end{matrix}\right.\right] \quad (4.2.12)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha}x\left| \begin{matrix} \{\Delta(a,s),h\},(a_j,1)_{a+1,p} \\ (b_j,1)_{1,q}\end{matrix}\right.\right] = \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} G^{m,n}_{p,q}\left[x\left| \begin{matrix} \{\Delta(a,s-r),h\},(a_j,1)_{a+1,p} \\ (b_j,1)_{1,q}\end{matrix}\right.\right] \quad (4.2.13)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-2ha}x^2\left| \begin{matrix} \{\Delta(a,s/2),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,(s+1)/2),h\} \\ (b_j,1)_{1,q}\end{matrix}\right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r(-1)^{r/2}}{r!} G^{m,n}_{p,q}\left[x^2\left| \begin{matrix} \{\Delta(a,(s-r)/2),h\},(a_j,1)_{a+1,p-a},\{\Delta(a,(s-r+1)/2),h\} \\ (b_j,1)_{1,q}\end{matrix}\right.\right] \quad (4.2.14)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha}x\left| \begin{matrix} (a_j,1)_{1,p-a},\{\Delta(a,s),h\} \\ (b_j,1)_{1,q}\end{matrix}\right.\right] = \sum_{r=0}^{\infty} \frac{h^r}{r!} G^{m,n}_{p,q}\left[x\left| \begin{matrix} (a_j,1)_{1,p-a},\{\Delta(a,s-r),h\} \\ (b_j,1)_{1,q}\end{matrix}\right.\right] \quad (4.2.15)$$

$$A^{s-1} G^{m,n}_{p,q}\left[A^{-ha}x\left| \begin{matrix} \{\Delta(\rho,s),h\},(a_j,1)_{\rho+1,p-a},\{\Delta(a,s),h\} \\ \{\Delta(\rho,s),h\},(b_j,1)_{\rho+1,q}\end{matrix}\right.\right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!} G^{m,n}_{p,q}\left[x\left| \begin{matrix} \{\Delta(\rho,s-r),h\},(a_j,1)_{\rho+1,p-a},\{\Delta(a,s-r),h\} \\ \{\Delta(\rho,s-r),h\},(b_j,1)_{\rho+1,q}\end{matrix}\right.\right] \quad (4.2.16)$$

(4.3) If we put $n=p, m=1, q=q+1, b_1=0, \beta_1=1, a_j=1-a_j, b_j=1-b_j$, then the $\overline{H}$-function reduces to generalized wright hypergeometric function [12] i.e. $\overline{H}^{1,p}_{p,q+1}\left[z\left| \begin{matrix} (1-a_j,\alpha_j;A_j)_{1,p} \\ (0,1),(1-b_j,\beta_j;B_j)_{1,q}\end{matrix}\right.\right] = {}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};-z\right]$, the above results (3.1) to (3.16) reduces to the following form:

$$A^{-s}\,{}_p\overline{\psi}_q\left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p-\rho}\,\{\Delta(\rho,s),h\} \\ \{\Delta(a,s),h\},(b_j,\beta_j;B_j)_{a+1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix};-A^{ha}x\right]$$

Ref.

12. R.G. Buschman and H.M. Srivastava, The H-function associated with a certain class of Feynman integrals, J.Phys.A:Math.Gen. 23(1990), 4707-4710.

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!}\, {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p-\rho}, \{\Delta(\rho,s+r),h\} \\ \{\Delta(a,s+r),h\}, (b_j,\beta_j;B_j)_{a+1,q-\rho}, \{\Delta(\rho,s+r),h\} \end{matrix} ;-x \right] \tag{4.3.1}$$

$$A^{-s}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+1/2),h\}, (a_j,\alpha_j;A_j)_{a+1,p} \\ \{\Delta(2a,2s),h\}, (b_j,\beta_j;B_j)_{2a+1,q} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+r+1/2),h\}, (a_j,\alpha_j;A_j)_{a+1,p} \\ \{\Delta(2a,2s+2r),h\}, (b_j,\beta_j;B_j)_{2a+1,q} \end{matrix} ;-x \right] \tag{4.3.2}$$

$$A^{-s}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+2/3),h\}, (a_j,\alpha_j;A_j)_{a+1,p-a}, \{\Delta(a,s+1/3),h\} \\ \{\Delta(3a,3s),h\}, (b_j,\beta_j;B_j)_{3a+1,q} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(a,s+r+2/3),h\}, (a_j,\alpha_j;A_j)_{a+1,p-a}, \{\Delta(a,s+r+1/3),h\} \\ \{\Delta(3a,3s+3r),h\}, (b_j,\beta_j;B_j)_{3a+1,q} \end{matrix} ;-x \right] \tag{4.3.3}$$

$$A^{-s}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(2a,2s+1/2),h\}, (a_j,\alpha_j;A_j)_{2a+1,p-2a}, \{\Delta(2a,2s+1),h\} \\ \{\Delta(4a,4s+1),h\}, (b_j,\beta_j;B_j)_{4a+1,q-a}, \{\Delta(a,s+1),h\} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r}{r!}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(2a,2s+2r+1/2),h\}, (a_j,\alpha_j;A_j)_{2a+1,p-2a}, \{\Delta(2a,2s+2r+1),h\} \\ \{\Delta(4a,4s+4r+1),h\}, (b_j,\beta_j;B_j)_{4a+1,q-a}, \{\Delta(a,s+r+1),h\} \end{matrix} ;-x \right] \tag{4.3.4}$$

$$A^{-s}\, {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q-a}, \{\Delta(a,s),h\} \end{matrix} ;-A^{ha}x \right] = \sum_{r=0}^{\infty} \frac{h^r}{r!}\, {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ (b_j,\beta_j;B_j)_{1,q-a}, \{\Delta(a,s+r),h\} \end{matrix} ;-x \right] \tag{4.3.5}$$

$$A^{-s}\, {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ \{\Delta(a,s/2),h\}, (b_j,\beta_j;B_j)_{a+1,q-a}, \{\Delta(a,(s+1)/2),h\} \end{matrix} ;-A^{2ha}x^2 \right]$$

$$= \sum_{r=0}^{\infty} \frac{(2h)^r(-1)^{r/2}}{r!}\, {}_p\overline{\Psi}_q \left[\begin{matrix} (a_j,\alpha_j;A_j)_{1,p} \\ \{\Delta(a,(s+r)/2),h\}, (b_j,\beta_j;B_j)_{a+1,q-a}, \{\Delta(a,(s+r+1)/2),h\} \end{matrix} ;-x^2 \right] \tag{4.3.6}$$

$$A^{-s}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(\rho,s),h\}, (a_j,\alpha_j;A_j)_{\rho+1,p} \\ \{\Delta(a,s),h\}, (b_j,\beta_j;B_j)_{a+1,q-\rho}, \{\Delta(\rho,s),h\} \end{matrix} ;-A^{ha}x \right]$$

$$= \sum_{r=0}^{\infty} \frac{h^r(-1)^r}{r!}\, {}_p\overline{\Psi}_q \left[\begin{matrix} \{\Delta(\rho,s+r),h\}, (a_j,\alpha_j;A_j)_{\rho+1,p} \\ \{\Delta(a,s+r),h\}, (b_j,\beta_j;B_j)_{a+1,q-\rho}, \{\Delta(\rho,s+r),h\} \end{matrix} ;-x \right] \tag{4.3.7}$$

Notes

Notes

$$A^{-s}\,{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p}\\ \{\Delta(a,s),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-A^{ha}x\right]=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,q}\\ \{\Delta(a,s+r),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-x\right] \quad (4.3.8)$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,p-\rho},\{\Delta(\rho,s),h\}\\ (b_j,\beta_j;B_j)_{1,q-\rho},\{\Delta(\rho,s),h\}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s-r),h\},(a_j,\alpha_j;A_j)_{a+1,p-\rho},\{\Delta(\rho,s-r),h\}\\ (b_j,\beta_j;B_j)_{1,q-\rho},\{\Delta(\rho,s-r),h\}\end{matrix};-x\right] \quad (4.3.9)$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(2a,2s),h\},(a_j,\alpha_j;A_j)_{2a+1,p}\\ \{\Delta(a,s+1/2),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(2a,2s-2r),h\},(a_j,\alpha_j;A_j)_{2a+1,p}\\ \{\Delta(a,s-r+1/2),h\},(b_j,\beta_j;B_j)_{a+1,q}\end{matrix};-x\right] \quad (4.3.10)$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(4a,4s+1),h\},(a_j,\alpha_j;A_j)_{4a+1,p-a},\{\Delta(a,s),h\}\\ \{\Delta(2a,2s+1/2),h\},(b_j,\beta_j;B_j)_{2a+1,q-2a},\{\Delta(2a,2s),h\}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(4a,4s-4r+1),h\},(a_j,\alpha_j;A_j)_{4a+1,p-a},\{\Delta(a,s-r),h\}\\ \{\Delta(2a,2s-2r+1/2),h\},(b_j,\beta_j;B_j)_{2a+1,q-2a},\{\Delta(2a,2s-2r),h\}\end{matrix};-x\right] \quad (4.3.11)$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(3a,3s),h\},(a_j,\alpha_j;A_j)_{3a+1,p}\\ \{\Delta(a,s+2/3),h\},(b_j,\beta_j;B_j)_{a+1,q-a},\{\Delta(a,s+1/3),h\}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(3a,3s-3r),h\},(a_j,\alpha_j;A_j)_{3a+1,p}\\ \{\Delta(a,s-r+2/3),h\},(b_j,\beta_j;B_j)_{a+1,q-a},\{\Delta(a,s-r+1/3),h\}\end{matrix};-x\right] \quad (4.3.12)$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s),h\},(a_j,\alpha_j;A_j)_{a+1,p}\\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};-A^{-ha}x\right]$$
$$=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s-r),h\},(a_j,\alpha_j;A_j)_{a+1,p}\\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};-x\right] \quad (4.3.13)$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,s/2),h\},(a_j,\alpha_j;A_j)_{a+1,p-a},\{\Delta(a,(s+1)/2),h\}\\ (b_j,\beta_j;B_j)_{1,q}\end{matrix};-A^{-2ha}x^2\right]$$

$$=\sum_{r=0}^{\infty}\frac{(2h)^r(-1)^{r/2}}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(a,(s-r)/2),h\},(a_j,\alpha_j;A_j)_{a+1,p-a},\{\Delta(a,(s-r+1)/2),h\}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-x^2\right] \tag{4.3.14}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p-a},\{\Delta(a,s),h\}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-A^{-ha}x\right]=\sum_{r=0}^{\infty}\frac{h^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}(a_j,\alpha_j;A_j)_{1,p-a},\{\Delta(a,s-r),h\}\\(b_j,\beta_j;B_j)_{1,q}\end{matrix};-x\right] \tag{4.3.15}$$

$$A^{s-1}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(\rho,s),h\},(a_j,\alpha_j;A_j)_{\rho+1,p-a},\{\Delta(a,s),h\}\\\{\Delta(\rho,s),h\},(b_j,\beta_j;B_j)_{\rho+1,q}\end{matrix};-A^{-ha}x\right]$$

$$=\sum_{r=0}^{\infty}\frac{h^r(-1)^r}{r!}\,{}_p\overline{\Psi}_q\left[\begin{matrix}\{\Delta(\rho,s-r),h\},(a_j,\alpha_j;A_j)_{\rho+1,p-a},\{\Delta(a,s-r),h\}\\\{\Delta(\rho,s-r),h\},(b_j,\beta_j;B_j)_{\rho+1,q}\end{matrix};-x\right] \tag{4.3.16}$$

In all the above results, it is assumed that all the parameters satisfy the conditions necessary for the existence of the $\overline{H}$-function involved.

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Fractional Integration Associated With the Transcendental Functions

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Abstract – The aim of the present paper is to discuss a number of interesting classes of Eulerian integrals and the theorem based upon the fractional calculus associated with general class of polynomials given by Srivastava [4, P.1, Eq.(1)], generalized polynomials given by Srivastava [8, P.185, Eq.(7)] and the multivariable H-function given by Srivastava and Panda [13, P.271, eq.(4.1)]. The results derived here are of a very general nature and hence encompass several cases of interest hitherto scattered in the literature.

Subject Classification: (MSC 2010) 33C99, 33C60

FRACTIONAL INTEGRATION ASSOCIATED WITH THE TRANSCENDENTAL FUNCTIONS

Strictly as per the compliance and regulations of :

Fractional Integration Associated With the Transcendental Functions

Rakeshwar Purohit [α] & Ashok Singh Shekhawat [σ]

Abstract - The aim of the present paper is to discuss a number of interesting classes of Eulerian integrals and the theorem based upon the fractional calculus associated with general class of polynomials given by Srivastava [4, P.1, Eq.(1)], generalized polynomials given by Srivastava [8, P.185, Eq.(7)] and the multivariable H-function given by Srivastava and Panda [13, P.271, eq.(4.1)]. The results derived here are of a very general nature and hence encompass several cases of interest hitherto scattered in the literature.

I. Introduction

In recent years, several authors namely Saigo and Saxena [5], Srivastava and Hussain [12], Saxena and Saigo [7], Saxena and Nishimoto [6], Srivastava and Owa [16] have established certain fractional integral formulae deduced from Eulerian integrals. The Riemann-Liouville operator of fractional integration $R^n f$ of order n is defined by,

$$ {}_xD_t^{-n}[f(t)] = \frac{1}{\Gamma(n)}\int_x^t (t-z)^{n-1} f(z)\, dz \tag{1.1}$$

for Re (n) > 0 and a constant x.
An equivalent form of the Beta function is [3, p.10, eq.(13)]

$$\int_x^y (p-x)^{m-1}(y-p)^{n-1}\, dp = (y-x)^{m+n-1} B(m,n), \tag{1.2}$$

where x, y ∈ R (x < y), Re (m) > 0, Re (n) > 0.

Using [3, p.62, eq. (15)], we have

$$(\alpha p+\beta)^{\nu} = (x\alpha+\beta)^{\nu}\left[1+\frac{\alpha(p-x)}{x\alpha+\beta}\right]^{\nu}$$

$$= \frac{(x\alpha+\beta)^{\nu}}{\Gamma(-\nu)}\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\Gamma(-\gamma)\Gamma(\gamma-\nu)\left[\frac{\alpha(p-x)}{x\alpha+\beta}\right]^{\nu} d\gamma. \tag{1.3}$$

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where α, β, ν ∈ C; x, p ∈ R; $\left|\arg\left(\frac{\alpha}{x\,\alpha-\beta}\right)\right| < \pi$ and the path of integration is indented, if necessary, in such a manner so as to separate the poles of Γ (-γ) from those of Γ (γ- ν). Srivastava [9] introduced the general class of polynomials (see also Srivastava and Singh [15])

$$S_N^M[w] = \sum_{k=0}^{[N/M]} \frac{(-N)_{Mk}}{k!} B_{N,k}\, w^k,\ k=0,1,2,\ldots \tag{1.4}$$

where M is an arbitrary positive integer and the coefficients $B_{N,k}$ (N,k ≥ 0) are arbitrary constants, real or complex.

By suitably specializing the coefficients $B_{N,k}$ the polynomials $S_N^M[w]$ can be reduced to the classical orthogonal polynomials such as Jacobi, Hermite, Legendre, Tchebycheff and Laguerre polynomials etc.

The generalized polynomials defined by Srivastava and Garg [8, p.686, eq.(1.4)], is given in the following manner:

$$S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}[w_1,\ldots,w_s] = S\begin{bmatrix} w_1 \\ \vdots \\ w_s \end{bmatrix}$$

$$= \sum_{k_1=0}^{[N_1/M_1]} \cdots \sum_{k_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1k_1}}{k_1!} \cdots \frac{(-N_s)_{M_sk_s}}{k_s!} A(N_1k_1;\ldots;N_sk_s)\, w_1^{k_1},\ldots,w_s^{k_s}, \tag{1.5}$$

where $M_1,\ldots,M_s$ are arbitrary positive integers and the coefficients $A(N_1k_1;\ldots;N_sk_s)$ are arbitrary constants, real or complex.

The H-function of several complex variables defined by Srivastava and Panda ([13] and [14]) by means of the multiple Mellin-Barnes type integral:

$$H\begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix} = H^{0,\lambda:(u',v');\ldots;(u^{(r)},v^{(r)})}_{A,C:[B',D'];\ldots;[B^{(r)},D^{(r)}]}\left[\begin{matrix} [(a):\theta',\ldots,\theta^{(r)}]:[(b'):\phi'];\ldots;[b^{(r)}:\phi^{(r)}]; \\ [c]:\psi',\ldots,\psi^{(r)}]:[(d'):\delta'];\ldots;[d^{(r)}:\delta^{(r)}]; \end{matrix}\ z_1,\ldots,z_r\right] \tag{1.6}$$

$$= \frac{1}{(2\pi i)^r}\int_{L_1}\cdots\int_{L_r} U_1(\xi_1)\ldots U_r(\xi_r)\, V(\xi_1,\ldots,\xi_r)\, z_1^{\xi_1}\ldots z_r^{\xi_r}\, d\xi_1\ldots d\xi_r, \tag{1.7}$$

$$(i = \sqrt{-1})$$

Ref.

9. H.M. Srivastava, A contour integral involving Fox's function, Indian J. Math. **14**, 1-6 (1972).

where

$$U_i(\xi_i) = \frac{\prod_{j=1}^{u^{(i)}} \Gamma(d_j^{(i)} - \delta_j^{(i)}\xi_i) \prod_{j=1}^{v^{(i)}} \Gamma(1 - b_j^{(i)} + \phi_j^{(i)}\xi_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)}\xi_i) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma(b_j^{(i)} - \phi_j^{(i)}\xi_i)} \ \forall\, i \in (1,\dots,r) \tag{1.8}$$

$$V(\xi_1,\dots,\xi_r) = \frac{\prod_{j=0}^{\lambda} \Gamma(1 - a_j + \sum_{i=1}^{r} \theta_j^{(i)}\xi_i)}{\prod_{j=\lambda+1}^{A} \Gamma(a_j - \sum_{i=1}^{r} \theta_j\xi_i) \prod_{j=1}^{C} \Gamma(1 - c_j + \sum_{i=1}^{r} \psi_j^{(i)}\xi_i)} \tag{1.9}$$

The multiple integral in (1.6) converges absolutely, if

$$T_i > 0 \ \text{and} \, |\arg z_i| < T_i\, \pi/2\,,\ \forall\, i\,(1,\dots,r) \tag{1.10}$$

where

$$T_i = -\sum_{j=\lambda+1}^{A} \alpha_j^{(i)} + \sum_{j=1}^{v^{(i)}} \phi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{C} \psi_j^{(i)} + \sum_{j=1}^{u^{(i)}} \delta_j^{(i)}$$

$$- \sum_{j=u^{(i)}+1}^{D^{(i)}} \delta_j^{(i)} > 0\,,\ \forall\, i\,(1,\dots,r) \tag{1.11}$$

The convergence conditions and other details of the H-function of several complex variables $H\begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix}$ are given by Srivastava, Gupta and Goyal [11, p.251].

The Lauricella function $F_D^{(h)}$ is defined in the integral form as

$$\frac{\Gamma(a)\,\Gamma(b_1)\dots\Gamma(b_h)}{\Gamma(c)} F_D^{(h)}[a, b_1,\dots,b_h; c; x_1,\dots,x_h]$$

$$= \frac{1}{(2\pi i)^h} \int_{-i\infty}^{i\infty} \dots \int_{-i\infty}^{i\infty} \frac{\Gamma(a+\xi_1+\dots+\xi_h)\Gamma(b_1+\xi_1)\dots\Gamma(b_h+\xi_h)}{\Gamma(c+\xi_1+\dots+\xi_h)}$$

Ref.

11. H.M. Srivastava, K.C. Gupta and S.P. Goyal, The H-functions of One and Two Variables with Applications, South Asian Publishers, New Delhi- Madras, 1982.

$$. \Gamma(-\xi_1)\ldots\Gamma(-\xi_h)(-x_1)^{\xi_1}\ldots(-x_h)^{\xi_h}\, d\xi_1\ldots d\xi_h\,, \qquad (1.12)$$

where $\max[|\arg(-x_1)|,\ldots,|\arg(-x_h)|] < \pi\,;\ C \neq 0,-1,-2,\ldots$

The following result will be used in establishing the Eulerian integral

$$\int_x^y (p-x)^{m-1}(y-p)^{n-1}(\alpha_1 p+\beta_1)^{\tau_1}\ldots(\alpha_h p+\beta_h)^{\tau_h}\,dp$$

$$=(y-x)^{m+n-1}B(m,n)(x\,\alpha_1+\beta_1)^{\tau_1}\ldots(x\,\alpha_h+\beta_h)^{\tau_h}$$

$$.F_D^{(h)}\left[m,-\tau_1,\ldots,-\tau_h;m+n;\ -\frac{(y-x)\alpha_1}{x\alpha_1+\beta_1},\ldots,-\frac{(y-x)\alpha_h}{x\,\alpha_h+\beta_h}\right],$$

where x, y ∈ R (x < y) ; α_j, β_j, τ_j ∈ C (j = 1,...,h);

$$\min[\mathrm{Re}(m),\mathrm{Re}(n)>0 \text{ and } \max\left[\left|\frac{(y-x)\alpha_1}{x\,\alpha_1+\beta_1}\right|,\ldots,\left|\frac{(y-x)\alpha_h}{x\,\alpha_h+\beta_h}\right|\right]<1$$

The formula (1.13) can be developed by making use of (1.2), (1.3) and (1.12).

The known result [4, p.301, entry (2.2.6.1)] and [12, p.81, Eq.(3.6)] are deducible for h=1 and h = 2 respectively.

In what follows h is a positive integer and 0,...,0 would mean h zeros.

II. Eulerian Integral

The main integral to be evaluated here is

$$\int_x^y (p-x)^{m-1}(y-p)^{n-1}\left\{\prod_{j=1}^{h}(\alpha_j p+\beta_j)^{\tau_j}\right\} S_N^M\left[w\,(p-x)^a(y-p)^b\prod_{j=1}^{h}(\alpha_j p+\beta_j)^{\mu_j}\right]$$

$$.S_{N_1,\ldots,N_s}^{M_1,\ldots,M_s}\begin{bmatrix} w_1(p-x)^{a_1}(y-p)^{b_1}\prod_{j=1}^{h}(\alpha_j p+\beta_j)^{\mu'_j} \\ \vdots \\ w_s(p-x)^{a_s}(y-p)^{b_s}\prod_{j=1}^{h}(\alpha_j p+\beta_j)^{\mu_j^{(s)}} \end{bmatrix}$$

Ref.

4. A.P. Prudnikov, Yu. A. Brychkov and O.I. Marichev, Integrals and series, Vol. I, Elementary functions, Gordon and Breach, New York- London- Paris- Montreux – Tokyo, 1986.

Notes

$$. H\left[\begin{array}{c} z_1 (p-x)^{\sigma_1} (y-p)^{\rho_1} \prod_{j=1}^{h} (\alpha_j p+\beta_j)^{-\lambda'_j} \\ \vdots \\ z_r (p-x)^{\sigma_r} (y-p)^{\rho_r} \prod_{j=1}^{h} (\alpha_j p+\beta_j)^{-\lambda_j^{(r)}} \end{array}\right] dp$$

$$= E_1 \sum_{k=0}^{[N/M]} \frac{(-N)_{Mk}}{k!} B_{N,k} \sum_{k_1=0}^{[N_1/M_1]} \cdots \sum_{k_s=0}^{[N_s/M_s]} \frac{(-N_1)_{M_1 k_1}}{k_1!} \cdots \frac{(-N_s)_{M_s k_s}}{k_s!}$$

$$. A (N_1, k_1; \ldots; N_s, k_s)(w)^k (w_1)^{k_1} \ldots (w_s)^{k_s} E_2$$

$$. H^{0,\lambda+h+2 \quad :(u',v');\ldots;(u^{(r)},v^{(r)});(1,0);\ldots;(1,0)}_{A+h+2,\, c+h+1:[B',D'];\ldots;[B^{(r)},D^{(r)}];\, [0,1];\ldots;(0,1]}$$

$$\left[\begin{array}{l} F_1, F_2, F_3, [(a):\theta',\ldots,\theta^{(r)},0,\ldots,0]:[(b'):\phi'];\ldots;[(b^{(r)}):\phi^{(r)}]; - \quad ;\ldots; - \quad ; G_1 \\ [(c):\psi',\ldots,\psi^{(r)};0,\ldots,0], F_4, F_5 \ :[(d'):\delta'];\ldots;[(d^{(r)}:\delta^{(r)}] \ ;[0,1];\ldots;[0,1]; G_2 \end{array}\right] \tag{2.1}$$

The following are the conditions of the validity of (2.1):

(1) $x, y \in R\ (x<y); \sigma_i, \rho_i, c_j^{(i)}, a_{i'} b'_{i'}\ u_j^{(i)}, a, b, \mu_i \in R^+, \tau_j \in R\ \alpha_j, \beta_j \in C, z_i \in C$

$(i = 1,\ldots,r\ ;\ i' = 1,\ldots,s;\ j = 1,\ldots,h)$;

(2) $\max_{1\le j\le h} \left[\left|\frac{(y-x)\alpha_j}{\alpha_j x+\beta_j}\right|\right] < 1$;

(3) $\text{Re}\left[m + \sum_{i=1}^{r} \frac{\sigma_j d_j^{(i)}}{\delta_j^{(i)}}\right] > 0\ (j=1,\ldots,u^{(i)})$,

$\text{Re}\left[n + \sum_{i=1}^{r} \frac{\rho_i d_j^{(i)}}{\delta_j^{(i)}}\right] > 0\ (j=1,\ldots,u^{(i)})$;

(4) $$R_i' = \sum_{j=1}^{A} \theta_j^{(i)} - \sum_{j=1}^{C} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} \le 0,$$

$$T_i = -\sum_{j=\lambda+1}^{A} \theta_j^{(i)} - \sum_{j=1}^{C} \psi_j^{(i)} + \sum_{j=1}^{v^{(i)}} \phi_j^{(i)} - \sum_{j=v^{(i)}+1}^{B^{(i)}} \phi_j^{(i)} + \sum_{j=1}^{u^{(i)}} \delta_j^{(i)} - \sum_{j=u^{(i)}+1}^{D^{(i)}}$$

$$-\sigma_i - \rho_i - \sum_{j=1}^{h} \lambda_j^{(i)} > 0 \quad (i=1,\ldots,r);$$

(5) $$\left| \arg\left(z_i \prod_{j=1}^{h} (\alpha_1 p + \beta_j) \right)^{-\lambda_i^{(i)}} \right| < \frac{T_i \pi}{2} \, (x \le p \le y \; ; \; i=1,\ldots,r);$$

(6) M and M_i $(i = 1,\ldots,s)$ are arbitrary positive integers and the coefficients $B_{N,k}$ $(N, k \ge 0)$ and $A\,(N_1,k_1\,;\ldots;\,N_s,k_s)$ are arbitrary constants, real or complex.

where

$$E_1 = (y-x)^{m+n-1} \left(\prod_{j=1}^{h} (\alpha_j x + \beta_j)^{\tau_j} \right),$$

$$E_2 = (y-x)^{\sum_{i'=1}^{s} (a_{i'}+b_{i'})k_{i'} + (a+b)k} \left(\prod_{j=1}^{h} (\alpha_j x + \beta_j)^{\sum_{i'=1}^{r} \mu_j^{(i')} k_{i'} + \mu_j k} \right),$$

$$F_1 = \left[1 - m - \sum_{i'=1}^{s} a_{i'} k_{i'} - a\,k : \sigma_1, \ldots, \sigma_r, 1, \ldots, 1 \right],$$

$$F_2 = \left[1 - n - \sum_{i'=1}^{s} b_{i'} k_{i'} - b\,k : \rho_1, \ldots, \rho_r, 0, \ldots, 0 \right],$$

$$F_3 = \left[1 + \tau_j + \sum_{i'=1}^{s} u_j^{(i')} k_{i'} + \mu_j k : \lambda_j', \ldots, \lambda_j^{(r)}, 0, \ldots, 1^j, \ldots, 0 \right]_{1,h},$$

Notes

$$F_4 = \left[1 + \tau_j + \sum_{i'=1}^{s} \mu_j^{(i')} k_{i'} + \mu_j k : \lambda'_j, \ldots, \lambda_j^{(r)}, 0, \ldots, 0 \right]_{1,h} ,$$

$$F_5 = \left[1 - m - n - \sum_{i'=1}^{s} (a_{i'} + b_{i'}) k_{i'} - (a+b)k : (\sigma_1 + \rho_1), \ldots, (\sigma_r + \rho_r), 1, \ldots, 1] \right] ;$$

Notes

$$G_1 = \begin{cases} z_1 (y-x)^{\sigma_1 + \rho_1} / \prod_{j=1}^{h} (\alpha_j x + \beta_j)^{\lambda'_j} \\ \vdots \\ z_r (y-x)^{\sigma_r + \rho_r} / \prod_{j=1}^{h} (\alpha_j x + \beta_j)^{\lambda_j^{(r)}} \end{cases} ,$$

$$G_2 = \begin{cases} (y-x)\alpha_i / (x\alpha_1 + \beta_1) \\ \vdots \\ (y-x)\alpha_h / (x\,\alpha_h + \beta_h) . \end{cases}$$

Proof. To establish (2.1) express the general class of polynomials, the generalized polynomials, the generalized polynomials with the help of equations (1.4) and (1.5) and the multivariable H-function in terms of Mellin-Barnes type contour integral by virtue of (1.7) and interchanging the order of summation and integration (which is permissible under the conditions of validity stated above). Appealing to the results in (1.3), (1.12) and (1.13), we arrive at the right hand side of (2.1).

III. Special Case

(i) If $\rho_1 = 0 = \ldots = \rho_r$ and $b_1 = 0 = \ldots = b_s$ and $\sigma_1 = 0 = \ldots = \sigma_r$ and $a = 0 = b$ equation (2.1) reduces to

$$\int_x^y (p-x)^{m-1} (y-p)^{n-1} \left\{ \prod_{j=1}^{h} (\alpha_j p + \beta_j)^{\tau_j} \right\} S_N^M \left[w \prod_{j=1}^{h} (\alpha_j p + \beta_j)^{\mu_j} \right]$$

$$. S_{N_1, \ldots, N_s}^{M_1, \ldots, M_s} \begin{bmatrix} w_1 \prod_{j=1}^{h} (\alpha_j p + \beta_j)^{\mu'_j} \\ \vdots \\ w_s \prod_{j=1}^{h} (\alpha_j p + \beta_j)^{\mu_j^{(s)}} \end{bmatrix}$$

$$.H\left[\begin{matrix} z_1 \prod_{j=1}^{h} (\alpha_j p+\beta_j)^{-\lambda'_j} \\ \vdots \\ z_r \prod_{j=1}^{h} (\alpha_j p+\beta_j)^{-\lambda_j^{(r)}} \end{matrix}\right] dp$$

$$= \Gamma(n)\, L_1 \sum_{k=0}^{[N/M]} \sum_{k_1=0}^{[N_1/M_1]} \cdots \sum_{k_s=0}^{[N_s/M_s]} \frac{(-N)_{Mk}}{k!} \frac{(-N_1)_{M_1k_1}}{k_1!} \cdots \frac{(-N_s)_{M_sk_s}}{k_s!}$$

$$.B_{N,k}\, A(N_1k_1;\ldots;N_s,k_s)\, w^{(k)}\, w_1^{(k_1)} \ldots w_s^{(k_s)} .L_2$$

$$.H^{0,\lambda+h+1 \quad :(u',v')\ ;\ldots;(u^{(r)},v^{(r)});(1,0);\ldots;(1,0)}_{A+h+1,C+h+1:[B',D'];\ldots;[B^{(r)},D^{(r)}];[0,1];\ldots;[0,1]}$$

$$\left[\begin{matrix} B_1,B_2,[(a):\theta',\ldots,\theta^{(r)},0,\ldots,0]:[b':\phi'];\ldots;[b^{(r)}:\phi^{(r)}] \quad ;-\quad ;\ldots;-\quad ;Y_1 \\ [(c):\psi',\ldots,\psi^{(r)};0,\ldots,0],B_3,B_4:[d':\delta'];\ldots;[d^{(r)}:\delta^{(r)}]\ ;[0,1];\ldots;[0,1];Y_2 \end{matrix}\right], \qquad (3.1)$$

valid under the same conditions as required in (2.1).

where

$$L_1 = (y-x)^{m+n-1}\left(\prod_{j=1}^{h} (\alpha_j x+\beta_j)^{\tau_j}\right),$$

$$L_2 = \left(\prod_{j=1}^{h} (\alpha_j x+\beta_j)^{\sum_{i'=1}^{r} \mu_j^{(i')} k_{i'} + \mu_j k}\right);$$

$$B_1 = \left[(1-m) : \overbrace{0,\ldots,0}^{r},1,\ldots,1\right],$$

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$$B_2 = \left[1+\tau_j+\sum_{i'=1}^{s} u_j^{(i')}k_{i'}+\mu_j k:\lambda_j',\dots,\lambda_j^{(r)},0,\dots,1^j,\dots,0\right]_{1,h},$$

$$B_3 = \left[1+\tau_j+\sum_{i'=1}^{s} u_j^{(i')}k_{i'}+\mu_j k:\lambda_j',\dots,\lambda_j^{(r)},0,\dots,0\right]_{1,h},$$

Notes

$$B_4 = \left[(1-m-n):\overbrace{0,\dots,0}^{r},1,\dots,1\right];$$

$$Y_1 = \begin{cases} z_1 / \prod_{j=1}^{h}(\alpha_j x+\beta_j)^{\lambda_j'} \\ \vdots \\ z_r / \prod_{j=1}^{h}(\alpha_j x+\beta_j)^{\lambda_j^{(r)}}, \end{cases}$$

$$Y_2 = \begin{cases} (y-x)\alpha_1/(x\alpha_1+\beta_1) \\ \vdots \\ (y-x)\alpha_h/(x\,\alpha_h+\beta_h). \end{cases}$$

IV. Main Theorem

Let

$$f(t) = (t-x)^{m-1}\left\{\prod_{j=1}^{h}(\alpha_j t+\beta_j)^{\tau_j}\right\} S_N^M\left[W(t-x)^a\prod_{j=1}^{h}(\alpha_j t+\beta_j)^{\mu_j}\right]$$

$$.S_{N_1,\dots,N_s}^{M_1,\dots,M_s}\begin{bmatrix} W_1(t-x)^{a_1}\prod_{j=1}^{h}(\alpha_j t+\beta_j)^{\mu_j'} \\ \vdots \\ W_1(t-x)^{a_s}\prod_{j=1}^{h}(\alpha_j t+\beta_j)^{\mu_j^{(s)}} \end{bmatrix}$$

$$.H\begin{bmatrix} y_1(t-x)^{\sigma_1}\prod_{j=1}^{h}(\alpha_j t+\beta_j)^{-\lambda_j'} \\ \vdots \\ y_r(t-x)^{\sigma_r}\prod_{j=1}^{h}(\alpha_j p+\beta_j)^{-\lambda_j^{(r)}} \end{bmatrix},$$

then

$$ {}_xD_t^{-n}[f(t)] = I_1 \sum_{k=0}^{[N/M]} \sum_{k_1=0}^{[N_1/M_1]} \cdots \sum_{k_s=0}^{[N_s/M_s]} \frac{(-N)_{Mk}}{k!} \frac{(-N_1)_{M_1k_1}}{k_1!} \cdots \frac{(-N_s)_{M_sk_s}}{k_s!}$$

$$.B_{N,k}\, A(N_1,k_1;\ldots;N_s,k_s)\, W^{(k)}\, W_1^{(k_1)} \ldots W_s^{(k_s)} .I_2$$

$$.H^{0,\lambda+h+1\ :(u',v')\ ;\ldots;(u^{(r)},v^{(r)});(1,0);\ldots;(1,0)}_{A+h+1,C+h+1:[B',D'];\ldots;[B^{(r)},D^{(r)}];[0,1];\ldots;[0,1]}$$

$$\begin{bmatrix} K_1,K_2,[(a):\theta',\ldots,\theta^{(r)},0,\ldots,0]:[(b'):\phi']\ ;\ldots;[(b^{(r)}):\phi^{(r)}];-\ ;\ldots;-\ ;Z_1 \\ [(c):\psi',\ldots,\psi^{(r)};0,\ldots,0],K_3,K_4:[(d'):\delta'];\ldots;[(d^{(r)}):\delta^{(r)}];[0,1];\ldots;[0,1];Z_2 \end{bmatrix}, \qquad (4.1)$$

holds true with the conditions associated with (2.1).

where

$$I_1 = (t-x)^{m+n-1}\left(\prod_{j=1}^{h}(\alpha_j x+\beta_j)^{\tau_j}\right),$$

$$I_2 = (t-x)^{\sum_{i'=1}^{s} a_{i'}k_{i'}+ak}\left(\prod_{j=1}^{h}(\alpha_j x+\beta_j)^{\sum_{i'=1}^{r}\mu_j^{(i')}k_{i'}+\mu_j k}\right);$$

$$K_1 = \left[1-m-\sum_{i'=1}^{s} a_{i'}k_{i'} - a\,k : \sigma_1,\ldots,\sigma_r,1,\ldots,1\right],$$

$$K_2 = \left[1+\tau_j+\sum_{i'=1}^{s} u_j^{(i')}k_{i'} + \mu_j k : \lambda_j',\ldots,\lambda_j^{(r)},0,\ldots,1^j,\ldots,0\right]_{1,h},$$

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$$K_3 = \left[1+\tau_j+\sum_{i'=1}^{s}\mu_j^{(i')}k_{i'}+\mu_j k:\lambda'_j,\dots,\lambda_j^{(r)},0,\dots,0\right]_{1,h},$$

$$K_4 = \left[1-m-n-\sum_{i'=1}^{s}a_{i'}k_{i'}-ak:\sigma_1,\dots,\sigma_r,1,\dots,1]\right];$$

$$Z_1 = \begin{cases} y_1(t-x)^{\sigma_1}/\prod_{j=1}^{h}(\alpha_j x+\beta_j)^{\lambda'_j} \\ \vdots \\ y_r(t-x)^{\sigma_r}/\prod_{j=1}^{h}(\alpha_j x+\beta_j)^{\lambda_j^{(r)}} \end{cases},$$

$$Z_2 = \begin{cases} (t-x)\alpha_1/(\alpha_1 x+\beta_1) \\ \vdots \\ (t-x)\alpha_h/(\alpha_h x+\beta_h) \end{cases}$$

V. Special Cases

(1) If $a_1 = 0 \dots = a_s$ and $\sigma_1 = \dots = \sigma_s = 0$, then equation (4.1) reduces to

$$ {}_xD_t^{-n}[f(t)] = I_1 \sum_{k=0}^{[N/M]} \sum_{k_1=0}^{[N_1/M_1]} \dots \sum_{k_s=0}^{[N_s/M_s]} \frac{(-N)_{Mk}}{k!} \frac{(-N_1)_{M_1k_1}}{k_1!} \dots \frac{(-N_s)_{M_sk_s}}{k_s!}$$

$$.B_{N,k}\,A(N_1,k_1;\dots;N_s,k_s)\,W^{(k)}\,W_1^{(k_1)}\dots W_s^{(k_s)}.I_2$$

$$.I_2\,H^{0,\lambda+h+1\ :(u',v')\ ;\dots;(u^{(r)},v^{(r)});(1,0);\dots;(1,0)}_{A+h+1,C+h+1:[B',D'];\dots;[B^{(r)},D^{(r)}];[0,1];\dots;[0,1]}$$

$$\left[\begin{matrix} K_1,K_2,[(a):\theta',\dots,\theta^{(r)},0,\dots,0]:[(b'):\phi']\ ;\dots;[(b^{(r)}):\phi^{(r)}];-\ ;\dots;-\ ;Z_1 \\ [(c):\psi',\dots,\psi^{(r)};0,\dots,0],K_3,K_4:[(d'):\delta']\ ;\dots;[(d^{(r)}):\delta^{(r)}];[0,1];\dots;[0,1];Z_2 \end{matrix}\right], \quad (5.1)$$

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valid under the same conditions as required for integral (2.1) and where I_1, I_2, K_1, K_2, K_3, K_4, Z_1 and Z_2 are the same as in integral (4.1) after eliminating $a_{i'}$ and σ_i, (i' = 1,...,k; i = 1,...,r).

2. Taking $M_i = 0$ (i = 2,...,s) and Ni = 0 and N= 0, the results given in (2.1) and (4.1) reduce to the known results recently obtained by Saigo and Saxena [5].
3. If we take $M_i = 0$ (i = 2,...,s), $N_i = 0 = N$, $\sigma_i = 0 = \rho_r$ (i = 1,...,r) and h = 2 in (2.1) and (4.1), then we arrive at the result given by Srivastava and Hussain [12] obtained in a different form.
4. For $N_i = 0$ (i = 2,...,s) and N = 0, the results in (2.1) and (4.1) can be reduced to the results recently obtained by Chaurasia and Godika [1].
5. For N = 0, the results in (2.1) and (4.1) reduce to known results given by Chaurasia and Singhal [2].

Notes

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New Theorems Involving the I-function and General Class of Polynomials

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Abstract – In the present paper we first establish three new theorems, which involves I-function and general class of polynomials. Next, we obtain certain new integrals and expansion formulas by the application of our theorems. By giving suitable values to the parameters, main integral reduces to Fox's H-function, G-function and generalized wright hypergeometric function, etc.

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Subject Classification : *(MSC 2010) 33C45, 33C60*

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Abstract - In the present paper we first establish three new theorems, which involves I-function and general class of polynomials. Next, we obtain certain new integrals and expansion formulas by the application of our theorems. By giving suitable values to the parameters, main integral reduces to Fox's H-function, G-function and generalized wright hypergeometric function, etc.

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I. Introduction

The I-function, introduced in 1982, is a byproduct of V.P. Saxena's work on higher transcendental function. Now I-function stands on fairly firm footing through the research contributions of various authors [1, 10, 11, 12]:
I-function is defined and represented in the following manner [11]:

$$I^{M,N}_{P_i,Q_i;R}[z] = I^{M,N}_{P_i,Q_i;R}\left[z\left|\begin{matrix}(a_j,\alpha_j)_{1,N},(a_{ji},\alpha_{ji})_{N+1,P}\\(b_j,\beta_j)_{1,M},(b_{ji},\beta_{ji})_{M+1,Q}\end{matrix}\right.\right] = \frac{1}{2\pi i}\int_L \phi(\xi)z^{\xi}d\xi \tag{1.1}$$

where

$$\phi(\xi) = \frac{\prod_{j=1}^{M}\Gamma(b_j-\beta_j\xi)\prod_{j=1}^{N}\Gamma(1-a_j+\alpha_j\xi)}{\sum_{i=1}^{R}\left[\prod_{j=M+1}^{Q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi)\prod_{j=N+1}^{P_i}\Gamma(a_{ji}-\alpha_{ji}\xi)\right]} \tag{1.2}$$

and M,N,P_i,Q_i are integers satisfying $1\le N\le P_i, 1\le M\le Q_i (i=1,\dots,R)$ and R is finite. $\alpha_j,\beta_j,\alpha_{ji},\beta_{ji}$ are positive integers and a_j,b_j,a_{ji},b_{ji} are complex numbers. I-function, which is a generalized form of the well known Fox's H-function [5, p.10, Eqn. (2.1.1)]. In the sequel the I-function is studied under the following conditions of existence:

(I) $\quad \Omega_i > 0, |\arg z| < \frac{\Omega_i\pi}{2}$ (1.3)

(II) $\quad \Omega_i \ge 0, |\arg z| \le \frac{\Omega_i\pi}{2}$ and $Re(B+1) < 0$ (1.4)

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Where

$$\Omega_i = \sum_{j=1}^{N} \alpha_j - \sum_{j=N+1}^{P_i} \alpha_{ji} + \sum_{j=1}^{M} \beta_j - \sum_{j=M+1}^{Q_i} \beta_{ji}, \forall i = (1,\ldots,R) \tag{1.5}$$

and

$$B = \sum_{j=1}^{M} b_j + \sum_{j=M+1}^{Q_i} b_{ji} - \sum_{j=1}^{N} a_j - \sum_{j=N+1}^{P_i} a_{ji} + \frac{1}{2}(P_i - Q_i), \forall i = (1,\ldots,R) \tag{1.6}$$

Notes

The general class of polynomials $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ will be defined and represented as follow [2, p.185, eqn. (7)]:

$$S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x] = \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} x^{l_i} \tag{1.7}$$

where $n_1,\ldots,n_r = 0,1,2,\ldots;m_1,\ldots m_r$ are arbitrary positive integers, the coefficients $A_{n_i,l_i}(n_i,l_i \geq 0)$ are arbitrary constants, real or complex. $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ yield a number of known polynomials as its special cases. These include, among other, the Jacobi polynomials, the Bessel Polynomials, the Lagurre Polynomials, the Brafman Polynomials and several others [6, p. 158-161].

The following formulas [8, p.77, Ens. (3.1), (3.2) & (3.3)] will be required in our investigation:

$$\int_0^\infty \left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2a(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)}, \; (a>0; b\geq 0; c+4ab>0; Re(p)+1/2>0) \tag{1.8}$$

$$\int_0^\infty \frac{1}{x^2}\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{2b(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)}, \; (a\geq 0; b>0; c+4ab>0; Re(p)+1/2>0) \tag{1.9}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)\left[\left(ax+\frac{b}{x}\right)^2+c\right]^{-p-1} dx = \frac{\sqrt{\pi}}{(4ab+c)^{p+1/2}} \frac{\Gamma(p+1/2)}{\Gamma(p+1)}, \; (a>0; b>0; c+4ab>0; Re(p)+1/2>0) \tag{1.10}$$

II. Main Theorems

Let X stands for $\left(ax+\frac{b}{x}\right)^2+c$

First Theorem:

If $$(1-y)^{\alpha+\beta-\gamma} \, {}_2F_1(2\alpha,2\beta;2\gamma;y) = \sum_{r=0}^{\infty} a_r y^r \tag{2.1}$$

then

$$\int_0^\infty X^{-\lambda-1} \, {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right] dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times \tag{2.2}$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right]$$

Notes

The above result will be converge under the following conditions:

(I) $a>0; b\geq 0; c+4ab>0$ and $\mu_i>0, \delta\geq 0$.

(II) $\operatorname{Re}\left[\lambda+\delta \min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

(III) $|\arg z|<\frac{1}{2}\Omega_i\pi$, where Ω_i is given by equation (1.5)

(IV) $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Second Theorem:

If $$(1-y)^{\alpha+\beta-\gamma}\ {}_2F_1(2\alpha,2\beta;2\gamma;y)=\sum_{r=0}^{\infty} a_r y^r \tag{2.3}$$

then

$$\int_0^{\infty} \frac{1}{x^2} X^{-\lambda-1}\ {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S^{m_1,\ldots,m_k}_{n_1,\ldots,n_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\ (b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{2.4}$$

The above result will be converge under the following conditions:

(I) $a\geq 0; b>0; c+4ab>0$ and $\mu_i>0, \delta\geq 0$

(II) $\operatorname{Re}\left[\lambda+\delta \min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

(III) $|\arg z|<\frac{1}{2}\Omega_i\pi$, where Ω_i is given by equation (1.5)

(IV) $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Third Theorem:

If $$(1-y)^{\alpha+\beta-\gamma} {}_2F_1(2\alpha,2\beta;2\gamma;y)=\sum_{r=0}^{\infty}a_r y^r \tag{2.5}$$

then

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta\right);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};\left(-\lambda+r-\sum_{i=1}^{k}\mu_il_i,\delta\right)\end{matrix}\right.\right] \tag{2.6}$$

The above result will be converge under the following conditions

(I) $a>0;b>0;c+4ab>0$ and $\mu_i>0,\delta\geq 0$

(II) $\mathrm{Re}\left[\lambda+\delta\min_{1\leq j\leq m}\left(\frac{b_j}{\beta_j}\right)\right]+\frac{1}{2}>0$

(III) $|\arg z|<\frac{1}{2}\Omega_i\pi$, where Ω_i is given by equation (1.5)

(IV) $-\frac{1}{2}<(\alpha-\beta-\gamma)<\frac{1}{2}$

Proof: In our investigation following result [7, p. 75] is also required:

$${}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)=\sum_{r=0}^{\infty}\frac{(\gamma)_r}{\left(\gamma+\frac{1}{2}\right)_r}a_rX^r \tag{2.7}$$

Where a_r is given by (2.1).

To prove the first theorem, using the result given by equation (2.7) and express I-function occurring on the L.H.S. of equation (2.2) in terms of contour integral given by equation (1.1) and the general class of polynomials $S_{n_1,\ldots,n_r}^{m_1,\ldots,m_r}[x]$ in series form with the help of equation (1.7) and then interchanging the order of integration and summation we get:

$$\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\frac{1}{2\pi i}\int_L\phi(\xi)z^{\xi}\left[\int_0^\infty\left[\left(ax+\frac{b}{x}\right)+c\right]^{-\lambda+r-\sum_{i=1}^{k}\mu_il_i-\delta\xi-1}dx\right]d\xi \tag{2.8}$$

Further using the formulae (1.8) the above integral becomes

$$\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

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$$\frac{1}{2\pi i}\int_L \phi(\xi) z^{\xi} \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1/2}} \frac{\Gamma\left(\lambda-r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1/2\right)}{\Gamma\left(\lambda-r+\sum_{i=1}^{k}\mu_i l_i+\delta\xi+1\right)} d\xi \tag{2.9}$$

Then interpreting with the help of (1.1) and (2.9) provides first integral.

Proceeding on the same parallel lines, theorems second and third given by equation (2.4) and (2.6) can be obtained by using the results (1.9) and (1.10) respectively.

Notes

III. Special Cases

(3.1) If we put $R=1$, I-function reduces to Fox's H-function [5, p. 10, Eqn. (2.1.1)], then the equation (2.2), (2.4) and (2.6) takes the following form:

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{3.1.1}$$

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{3.1.2}$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right]H_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$H_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \tag{3.1.3}$$

The Conditions of validity of (3.1.1), (3.1.2) and (3.1.3) easily follow from those given in (2.2), (2.4) and (2.6).

(3.2) By applying the our results given in (2.2), (2.4) and (2.6) to the case of Hermite polynomials [2, 3] by setting $S_n^2(x) \to x^{n/2} H_n\left[\frac{1}{2\sqrt{x}}\right]$ in which $m_1,\ldots,m_k = 2; n_1,\ldots,n_k = n; k=1; v_i = v, y_i = y, A_{n_i,l_i} = (-1)^l$, we have the following interesting results:

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right] I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^\delta}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,pi}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \quad (3.2.1)$$

$$\int_0^\infty \frac{1}{x^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right] I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^\delta}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \quad (3.2.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)\left(yX^{-\mu}\right)^{n/2} H_n\left[\frac{1}{2}\sqrt{\frac{X^\mu}{y}}\right] I^{M,N}_{P_i,Q_i;R}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}(-1)^l\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r\,a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I^{M,N+1}_{P_i+1,Q_i+1;R}\left[\frac{z}{(4ab+c)^\delta}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \quad (3.2.3)$$

The Conditions of validity of (3.2.1), (3.2.2) and (3.2.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.3) By applying the our results given in (2.2), (2.4) and (2.6) to the case of Lagurre polynomials [2, 3] by setting $S_n^2(x) \to L_n^{(\alpha')}[x]$ in which $m_1,\ldots,m_k = 1; n_1,\ldots,n_k = n; k=1, v_i = v, y_i = y, A_{n_i,l_i} = \binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}$, we have the following interesting results:

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$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \qquad (3.3.1)$$

$$\int_0^\infty \frac{1}{X^2}X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \qquad (3.3.2)$$

$$\int_0^\infty\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)L_n^{(\alpha)}\left[yX^{-\mu}\right]I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l=0}^{[n/2]}\frac{(-n)_{2l}}{l!}\binom{n+\alpha'}{n}\frac{1}{(\alpha'+1)_l}\frac{(y)^l}{(4ab+c)^{-r+\mu l}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}(1/2-\lambda+r-\mu l,\delta);(a_j,\alpha_j)_{1,n};(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m};(b_{ji},\beta_{ji})_{m+1,q_i};(-\lambda+r-\mu l,\delta)\end{matrix}\right.\right] \qquad (3.3.3)$$

The Conditions of validity of (3.3.1), (3.3.2) and (3.3.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.4) If we put $R=1;\alpha_j=\beta_j=1$, then the I-function reduces to general type of G-function [9] i.e. $I_{P_i,Q_i;1}^{M,N}\left[z\left|\begin{matrix}(a_j,1)_{1,n};(a_{j1},1)_{n+1,p_1}\\(b_j,1)_{1,m};(b_{j1},1)_{m+1,q_1}\end{matrix}\right.\right]=G\left[z\left|\begin{matrix}(a_j,1)_{1,p}\\(b_j,1)_{1,q}\end{matrix}\right.\right]$, the equation (2.2), (2.4) and (2.6) takes the following form:

$$\int_0^\infty X^{-\lambda-1}\,{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]G_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.4.1}$$

$$\int_0^{\infty}\frac{1}{x^2}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]G_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.4.2}$$

$$\int_0^{\infty}\left(a+\frac{b}{x^2}\right)X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]G_{p,q}^{m,n}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$$G_{p+1,q+1}^{m,n+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right),(a_j,1)_{1,p}\\(b_j,1)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta;1\right)\end{matrix}\right.\right] \tag{3.4.3}$$

The Conditions of validity of (3.4.1), (3.4.2) and (3.4.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.5) If we put $R=1, M=1, N=P_i=P, Q_i=Q+1, b_1=0, \beta_1=1, a_j=1-a_j, b_{ji}=1-b_j, \beta_{ji}=\beta_j$, then the I-function reduces to generalized wright hypergeometric function [12, p.33, Eq. (2.3.8)] i.e. $I_{P,Q+1;1}^{1,P}\left[z\left|\begin{matrix}(1-a_j,\alpha_j)_{1,p}\\(0,1),(1-b_j,\beta_j)_{1,q}\end{matrix}\right.\right]={}_p\Psi_q\left[\begin{matrix}(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q}\end{matrix};-z\right]$, the equation (2.2), (2.4) and (2.6) takes the following form:

$$\int_0^{\infty}X^{-\lambda-1}{}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right){}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right)S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k}y_iX^{-\mu_i}\right]{}_p\Psi_q\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}}\sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_il_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_il_i}}\frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r}\times$$

$${}_{p+1}\Psi_{q+1}\left[\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p}\\(b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix};\frac{-z}{(4ab+c)^{\delta}}\right] \tag{3.5.1}$$

Notes

Notes

$$\int_0^\infty \frac{1}{x^2} X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] {}_p\Psi_q\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$${}_{p+1}\Psi_{q+1}\left[\begin{matrix} \left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right);(a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q};\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right) \end{matrix}; \frac{-z}{(4ab+c)^{\delta}}\right] \quad (3.5.2)$$

$$\int_0^\infty \left(a+\frac{b}{x^2}\right) X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\gamma+\frac{1}{2};X\right) {}_2F_1\left(\gamma-\alpha,\gamma-\beta;\gamma+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] {}_p\Psi_q\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\gamma)_r a_r}{\left(\gamma+\frac{1}{2}\right)_r} \times$$

$${}_{p+1}\Psi_{q+1}\left[\begin{matrix} \left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right) \end{matrix}; \frac{-z}{(4ab+c)^{\delta}}\right] \quad (3.5.3)$$

The Conditions of validity of (3.5.1), (3.5.2) and (3.5.3) easily follow from those given in (2.2), (2.4) and (2.6)

(3.6) If we put $\alpha=\gamma$, in the main theorem, the value of a_r in (2.1) comes out to be equal to $\frac{\beta_r}{r!}$ and the result (2.2), (2.4) and (2.6) gives the following interesting integral:

$$\int_0^\infty X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\alpha)_r (\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!} \times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}} \middle| \begin{matrix} \left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right) \end{matrix}\right] \quad (3.6.1)$$

$$\int_0^\infty \frac{1}{X^2} X^{-\lambda-1} {}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\ldots,n_k}^{m_1,\ldots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$= \frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty} \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_k=0}^{[n_k/m_k]} \prod_{i=1}^{k} \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i,l_i} (y_i)^{l_i} \frac{1}{(4ab+c)^{-r+\mu_i l_i}} \frac{(\alpha)_r (\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!} \times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}} \middle| \begin{matrix} \left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i} \\ (b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right) \end{matrix}\right] \quad (3.6.2)$$

$$\int_0^\infty \left(a+\frac{b}{X^2}\right) X^{-\lambda-1}\, {}_2F_1\left(\alpha,\beta;\alpha+\frac{1}{2};X\right) S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(\alpha)_r(\beta)_r}{\left(\alpha+\frac{1}{2}\right)_r r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.6.3)$$

The Conditions of validity of (3.6.1), (3.6.2) and (3.6.3) easily follow from those given in (2.2), (2.4) and (2.6).

(3.7) If we put $\beta=\alpha+\frac{1}{2}$ and $\alpha=-f$ (f is non-negative integer) in (3.6.1), (3.6.2) and (3.6.3), we have:

$$\int_0^\infty X^{-\lambda-1}(1-X)^f S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2a(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{f}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.7.1)$$

$$\int_0^\infty \frac{1}{X^2} X^{-\lambda-1}(1-X)^f S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{2b(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.7.2)$$

$$\int_0^\infty \left(a+\frac{b}{X^2}\right) X^{-\lambda-1}(1-X)^f S_{n_1,\dots,n_k}^{m_1,\dots,m_k}\left[\prod_{i=1}^{k} y_i X^{-\mu_i}\right] I_{P_i,Q_i;R}^{M,N}\left[zX^{-\delta}\right]dx$$

$$=\frac{\sqrt{\pi}}{(4ab+c)^{\lambda+1/2}} \sum_{r=0}^{\infty}\sum_{l_1=0}^{[n_1/m_1]}\cdots\sum_{l_k=0}^{[n_k/m_k]}\prod_{i=1}^{k}\frac{(-n_i)_{m_i l_i}}{l_i!}A_{n_i,l_i}(y_i)^{l_i}\frac{1}{(4ab+c)^{-r+\mu_i l_i}}\frac{(-f)_r}{r!}\times$$

$$I_{P_i+1,Q_i+1;R}^{M,N+1}\left[\frac{z}{(4ab+c)^{\delta}}\left|\begin{matrix}\left(1/2-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right),(a_j,\alpha_j)_{1,n},(a_{ji},\alpha_{ji})_{n+1,p_i}\\(b_j,\beta_j)_{1,m},(b_{ji},\beta_{ji})_{m+1,q_i},\left(-\lambda+r-\sum_{i=1}^{k}\mu_i l_i,\delta\right)\end{matrix}\right.\right] \quad (3.7.3)$$

Notes

The Conditions of validity of (3.7.1), (3.7.2) and (3.7.3) easily follow from those given in (2.2), (2.4) and (2.6).

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Notes

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· Keep on paying attention on the research topic of the paper

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· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

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Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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