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Mathematics and Decision Sciences

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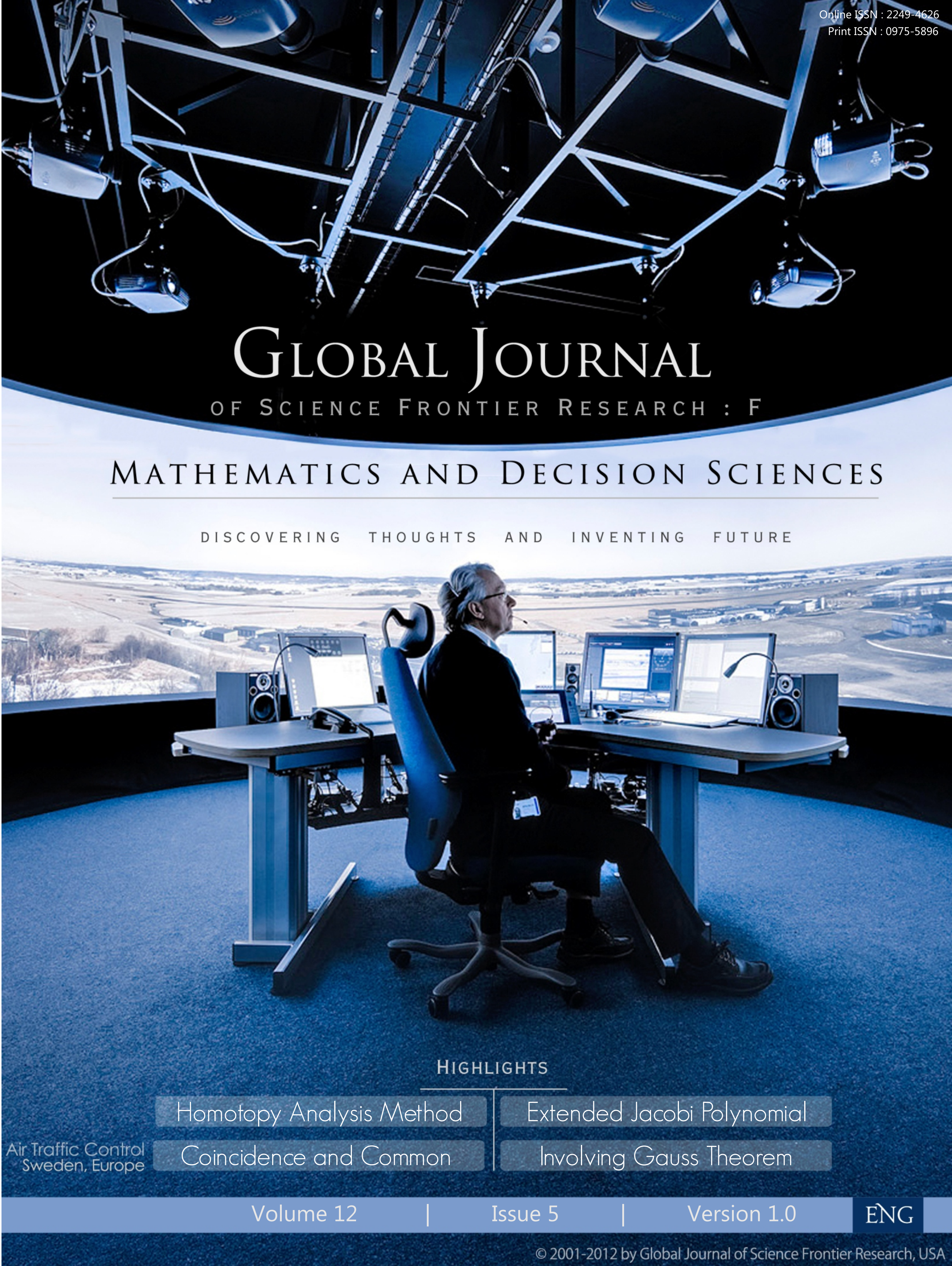

Air Traffic Control
Sweden, Europe

Highlights

- Homotopy Analysis Method
- Extended Jacobi Polynomial
- Coincidence and Common
- Involving Gauss Theorem

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New Representations in Terms of q-Product Identities for Ramanujan's Results

By M.P. Chaudhary
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Abstract - In this paper author has established six q-product identities, which are presumably new, and not available in the literature.

Keywords : Euler's pentagonal number theorem, triple product identities.

AMS Subject Classifications: Primary 05A17, 05A15; Secondary 11P83.

NEW REPRESENTATIONS IN TERMS OF Q-PRODUCT IDENTITIES FOR RAMANUJANS RESULTS

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New Representations in Terms of q-Product Identities for Ramanujan's Results

M.P. Chaudhary

Abstract - In this paper author has established six q-product identities, which are presumably new, and not available in the literature.

Keywords : Euler's pentagonal number theorem, triple product identities.

I. Introduction

For $|q| < 1$,

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n) \tag{1.1}$$

$$(a;q)_\infty = \prod_{n=1}^{\infty}(1-aq^{(n-1)}) \tag{1.2}$$

$$(a_1,a_2,a_3,...,a_k;q)_\infty = (a_1;q)_\infty(a_2;q)_\infty(a_3;q)_\infty...(a_k;q)_\infty \tag{1.3}$$

Ramanujan [2, p.1(1.2)]has defined general theta function, as

$$f(a,b) = \sum_{-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} \quad ; \quad |ab| < 1, \tag{1.4}$$

Jacobi's triple product identity [3,p.35] is given, as

$$f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty \tag{1.5}$$

Special cases of Jacobi's triple products identity are given, as

$$\phi(q) = f(q,q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)^2_\infty(q^2;q^2)_\infty \tag{1.6}$$

$$(q) = f(q,q^3) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.7}$$

$$f(-q) = f(-q,-q^2) = \sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.8}$$

Equation (1.8) is known as Euler's pentagonal number theorem. Euler's another well known identity is as

$$(q;q^2)^{-1}_\infty = (-q;q)_\infty \tag{1.9}$$

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Ref.

2. B.C. Berndt; *What is a q-series?*, preprint.
3. B.C. Berndt; *Ramanujan's notebook Part III*, Springer-Verlag, New York, 1991.

Throughout this paper we use the following representations

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty=(q^a,q^b,q^c\cdots q^t;q^n)_\infty \tag{1.10}$$

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty=(q^a,q^b,q^c\cdots q^t;q^n)_\infty \tag{1.11}$$

$$(-q^a;q^n)_\infty(-q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty=(-q^a,-q^b,q^c\cdots q^t;q^n)_\infty \tag{1.12}$$

Now we can have following q-products identities, as

$$(q^2;q^2)_\infty=\prod_{n=0}^{\infty}(1-q^{2n+2})$$

$$=\prod_{n=0}^{\infty}(1-q^{2(4n)+2})\times\prod_{n=0}^{\infty}(1-q^{2(4n+1)+2})\times\prod_{n=0}^{\infty}(1-q^{2(4n+2)+2})\times\prod_{n=0}^{\infty}(1-q^{2(4n+3)+2})$$

$$=\prod_{n=0}^{\infty}(1-q^{8n+2})\times\prod_{n=0}^{\infty}(1-q^{8n+4})\times\prod_{n=0}^{\infty}(1-q^{8n+6})\times\prod_{n=0}^{\infty}(1-q^{8n+8})$$

or,

$$(q^2;q^2)_\infty=(q^2;q^8)_\infty(q^4;q^8)_\infty(q^6;q^8)_\infty(q^8;q^8)_\infty=(q^2,q^4,q^6,q^8;q^8)_\infty \tag{1.13}$$

$$(q^4;q^4)_\infty=\prod_{n=0}^{\infty}(1-q^{4n+4})$$

$$=\prod_{n=0}^{\infty}(1-q^{4(3n)+4})\times\prod_{n=0}^{\infty}(1-q^{4(3n+1)+4})\times\prod_{n=0}^{\infty}(1-q^{4(3n+2)+4})$$

$$=\prod_{n=0}^{\infty}(1-q^{12n+4})\times\prod_{n=0}^{\infty}(1-q^{12n+8})\times\prod_{n=0}^{\infty}(1-q^{12n+12})$$

or,

$$(q^4;q^4)_\infty=(q^4;q^{12})_\infty(q^8;q^{12})_\infty(q^{12};q^{12})_\infty=(q^4,q^8,q^{12};q^{12})_\infty \tag{1.14}$$

$$(q^4;q^{12})_\infty=\prod_{n=0}^{\infty}(1-q^{12n+4})=\prod_{n=0}^{\infty}(1-q^{12(5n)+4})\times\prod_{n=0}^{\infty}(1-q^{12(5n+1)+4})\times$$

$$\times\prod_{n=0}^{\infty}(1-q^{12(5n+2)+4})\times\prod_{n=0}^{\infty}(1-q^{12(5n+3)+4})\times\prod_{n=0}^{\infty}(1-q^{12(5n+4)+4})$$

$$=\prod_{n=0}^{\infty}(1-q^{60n+4})\times\prod_{n=0}^{\infty}(1-q^{60n+16})\times\prod_{n=0}^{\infty}(1-q^{60n+28})\times\prod_{n=0}^{\infty}(1-q^{60n+40})\times\prod_{n=0}^{\infty}(1-q^{60n+52})$$

or,

$$(q^4;q^{12})_\infty=(q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$

$$=(q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \tag{1.15}$$

Notes

Similarly we can compute following as

$$(q^5;q^5)_\infty = (q^5;q^{15})_\infty(q^{10};q^{15})_\infty(q^{15};q^{15})_\infty \tag{1.16}$$

$$(q^6;q^6)_\infty = (q^6;q^{24})_\infty(q^{12};q^{24})_\infty(q^{18};q^{24})_\infty(q^{24};q^{24})_\infty = (q^6,q^{12},q^{18},q^{24};q^{24})_\infty \tag{1.17}$$

$$\begin{aligned}(q^6;q^{12})_\infty &= (q^6;q^{60})_\infty(q^{18};q^{60})_\infty(q^{30};q^{60})_\infty(q^{42};q^{60})_\infty(q^{54};q^{60})_\infty \\ &= (q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty\end{aligned} \tag{1.18}$$

The outline of this paper is as follows. In sections 2, some recent results obtained by the author in [1], and also some well known results are recorded, which are useful to the rest of the paper. In section 3, we state and prove six new q-product identities, which are not recorded in the literature.

II. Preliminaries

In [1], following identities are being established

$$(q^2,q^4,q^6;q^8)_\infty[(-q;q^2)^2_\infty + (q;q^2)^2_\infty] = 2(-q^4;q^8)^2_\infty \tag{2.1}$$

$$(q^2,q^4,q^6,q^8;q^8)_\infty[(-q;q^2)^2_\infty - (q;q^2)^2_\infty] = 4q\frac{(q^{16},q^{32},q^{48};q^{48})_\infty}{(q^8,q^{24},q^{40};q^{48})_\infty} \tag{2.2}$$

$$\frac{(-q;q^2)^2_\infty + (q;q^2)^2_\infty}{(-q;q^2)^2_\infty - (q;q^2)^2_\infty} = \frac{(-q^4;q^8)^2_\infty(q^8,q^8,q^{24},q^{24},q^{40},q^{40};q^{48})_\infty}{2q} \tag{2.3}$$

$$(-q;q^2)^2_\infty(q;q^2)^2_\infty(q^2;q^2)^2_\infty = (q^2,q^2,q^4;q^4)_\infty \tag{2.4}$$

$$\frac{(-q;q^2)_\infty(-q^3;q^6)_\infty - (q;q^2)_\infty(q^3;q^6)_\infty}{(-q;q^2)_\infty \times (-q^3;q^6)_\infty \times (q;q^2)_\infty \times (q^3;q^6)_\infty} = \frac{2q(-q^2;q^4)^2_\infty(q^4,q^8,q^{16},q^{20},q^{24};q^{24})_\infty}{(q^2,q^4,q^6,q^8;q^8)_\infty(q^6,q^{12},q^{18};q^{24})_\infty} \tag{2.5}$$

$$\begin{aligned}\frac{(-q^3;q^6)_\infty(-q^5;q^{10})_\infty - (q^3;q^6)_\infty(q^5;q^{10})_\infty}{(-q^3;q^6)_\infty \times (-q^5;q^{10})_\infty \times (q^3;q^6)_\infty \times (q^5;q^{10})_\infty} &= \frac{(q^4,q^8,q^{12};q^{12})_\infty}{(q^6,q^{12},q^{18},q^{24};q^{24})_\infty}\times \\ &\times\frac{2q^3}{(q^2,q^6,q^{10};q^{12})_\infty(q^{10},q^{20},q^{30},q^{30},q^{40},q^{50};q^{60})_\infty}\end{aligned} \tag{2.6}$$

$$\begin{aligned}\frac{[(q;q^2)_\infty(q^{15};q^{30})_\infty] + [(-q;q^2)_\infty(-q^{15};q^{30})_\infty]}{[(q;q^2)_\infty(q^{15};q^{30})_\infty][(-q;q^2)_\infty(-q^{15};q^{30})_\infty]} &= \frac{(q^{12},q^{20},q^{24},q^{36},q^{40},q^{48},q^{60},q^{60};q^{60})_\infty}{(q^{10},q^{30},q^{30},q^{50},q^{60};q^{60})_\infty}\times \\ &\times\frac{2}{(q^2,q^4,q^6,q^8,q^8;q^8)_\infty(q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty}\end{aligned} \tag{2.7}$$

In Ramanujan's notebooks [6, p.240], the following entries are recorded as

$$f(q^3,q^6) = \quad (q) - q\psi(q^9) \tag{2.8}$$

$$f(-q^2,-q^3)f(-q,-q^4) = f(-q,-q^2)f(-q^5,-q^{10}) \tag{2.9}$$

In Ramanujan's notebooks [6, p.243], the following entries are recorded as

$$f(q,q^7)f(q^3,q^5) = \quad (q) \quad (q^4) \tag{2.10}$$

Ref.

1. M.P. Chaudhary; *On q-product identities*, preprint.
6. S. Ramanujan; *Notebooks (Volume I)*, Tata Institute of Fundamental Research, Bombay, 1957.

$$2f(q^3, q^5) = \quad (q^{\frac{1}{2}}) + \quad (-q^{\frac{1}{2}}) \tag{2.11}$$

$$2q^{\frac{1}{2}} f(q, q^7) = \quad (q^{\frac{1}{2}}) - \quad (-q^{\frac{1}{2}}) \tag{2.12}$$

III. Main Results

In this paper, we established following new results, which are not recorded in the literature of special functions

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} = (-q^3, -q^6, q^9; q^9)_\infty + q\frac{(q^{18};q^{18})_\infty}{(q^9;q^{18})_\infty} \tag{3.1}$$

$$(q;q)_\infty = (q, q^2, q^3, q^4, q^5; q^5)_\infty \tag{3.2}$$

$$(q;q^2)_\infty = \frac{(q^2, q^6; q^8)_\infty}{(-q, -q^3, -q^5, -q^7; q^8)_\infty} \tag{3.3}$$

$$(-q^{\frac{1}{2}};q)_\infty + (q^{\frac{1}{2}};q)_\infty = \frac{2(-q^3, -q^5, q^8; q^8)_\infty (q^{\frac{1}{2}}, -q^{\frac{1}{2}}; q)_\infty}{(q, q^2, q^3, q^4, q^5; q^5)_\infty} \tag{3.4}$$

$$(-q^{\frac{1}{2}};q)_\infty - (q^{\frac{1}{2}};q)_\infty = \frac{2q^{\frac{1}{2}}(-q, -q^7, q^8; q^8)_\infty (q^{\frac{1}{2}}, -q^{\frac{1}{2}}; q)_\infty}{(q, q^2, q^3, q^4, q^5; q^5)_\infty} \tag{3.5}$$

$$\frac{(-q^{\frac{1}{2}};q)_\infty + (q^{\frac{1}{2}};q)_\infty}{(-q^{\frac{1}{2}};q)_\infty - (q^{\frac{1}{2}};q)_\infty} = \frac{(-q^3, -q^5; q^8)_\infty}{q^{\frac{1}{2}}(-q, -q^7; q^8)_\infty} \tag{3.6}$$

Proof of (3.1): By substituting $a = q^3$ and $b = q^6$ in (1.5), we have

$$f(q^3, q^6) = (-q^3; q^9)_\infty (-q^6; q^9)_\infty (q^9; q^9)_\infty$$

also, by substituting $q = q^9$ in (1.7), we get

$$(q^9) = \frac{(q^{18};q^{18})_\infty}{(q^9;q^{18})_\infty}$$

employing (1.7) and putting the values of $f(q^3, q^6)$ and (q^9) in (2.8), we get

$$(-q^3;q^9)_\infty (-q^6;q^9)_\infty (q^9;q^9)_\infty = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} - q\frac{(q^{18};q^{18})_\infty}{(q^9;q^{18})_\infty}$$

after simplification, we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} = (-q^3, -q^6, q^9; q^9)_\infty + q\frac{(q^{18};q^{18})_\infty}{(q^9;q^{18})_\infty}$$

which established (3.1).

Proof of (3.2): By substituting $a = -q^2$ and $b = -q^3$ in (1.5), we have

$$f(-q^2, -q^3) = (q^2; q^5)_\infty (q^3; q^5)_\infty (q^5; q^5)_\infty$$

Notes

Notes

by substituting $a=-q$ and $b=-q^4$ in (1.5), we have

$$f(-q,-q^4)=(q;q^5)_\infty(q^4;q^5)_\infty(q^5;q^5)_\infty$$

employing (1.8) and putting the values of $f(-q^2,-q^3)$ and $f(-q,-q^4)$ in (2.9), we get

$$(q^2;q^5)_\infty(q^3;q^5)_\infty(q^5;q^5)_\infty(q;q^5)_\infty(q^4;q^5)_\infty(q^5;q^5)_\infty=(q;q)_\infty(q^5;q^5)_\infty$$

after simplification, we get

$$(q;q)_\infty=(q,q^2,q^3,q^4,q^5;q^5)_\infty$$

which established (3.2).

Proof of (3.3): By substituting $a=q$ and $b=q^7$ in (1.5), we have

$$f(q,q^7)=(-q;q^8)_\infty(-q^7;q^8)_\infty(q^8;q^8)_\infty$$

by substituting $a=q^3$ and $b=q^5$ in (1.5), we have

$$f(q^3,q^5)=(-q^3;q^8)_\infty(-q^5;q^8)_\infty(q^8;q^8)_\infty$$

also by substituting $q=q^4$ in (1.7), we get

$$(q^4)=\frac{(q^8;q^8)_\infty}{(q^4;q^8)_\infty}$$

employing (1.7) and putting the values of $f(q,q^7)$, $f(q^3,q^5)$ and (q^4) in (2.10), we get

$$(-q;q^8)_\infty(-q^7;q^8)_\infty(q^8;q^8)_\infty(-q^3;q^8)_\infty(-q^5;q^8)_\infty(q^8;q^8)_\infty=\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\times\frac{(q^8;q^8)_\infty}{(q^4;q^8)_\infty}$$

further using (1.13), and after simplification, we get

$$(q;q^2)_\infty=\frac{(q^2,q^6;q^8)_\infty}{(-q,-q^3,-q^5,-q^7;q^8)_\infty}$$

which established (3.3).

Proof of (3.4): By putting $q=q^{\frac{1}{2}}$ and $q=-q^{\frac{1}{2}}$ in (1.7), we have

$$(q^{\frac{1}{2}})=\frac{(q;q)_\infty}{(q^{\frac{1}{2}};q)_\infty}\quad\text{and}\quad(-q^{\frac{1}{2}})=\frac{(q;q)_\infty}{(-q^{\frac{1}{2}};q)_\infty}$$

by substituting the values of $f(q^3,q^5)$ from proof of (3.3), $(q^{\frac{1}{2}})$ and $(-q^{\frac{1}{2}})$ in (2.11), we get

$$2(-q^3;q^8)_\infty(-q^5;q^8)_\infty(q^8;q^8)_\infty=\frac{(q;q)_\infty}{(q^{\frac{1}{2}};q)_\infty}+\frac{(q;q)_\infty}{(-q^{\frac{1}{2}};q)_\infty}$$

after simplification, and employing (3.2), we have

$$(-q^{\frac{1}{2}};q)_\infty + (q^{\frac{1}{2}};q)_\infty = \frac{2(-q^3,-q^5,q^8;q^8)_\infty(q^{\frac{1}{2}},-q^{\frac{1}{2}};q)_\infty}{(q,q^2,q^3,q^4,q^5;q^5)_\infty}$$

which established (3.4).

Proof of (3.5): Substituting the values of $f(q,q^7)$, $(q^{\frac{1}{2}})$ and $(-q^{\frac{1}{2}})$ in (2.12), we get

$$2q^{\frac{1}{2}}(-q;q^8)_\infty(-q^7;q^8)_\infty(q^8;q^8)_\infty = \frac{(q;q)_\infty}{(q^{\frac{1}{2}};q)_\infty} - \frac{(q;q)_\infty}{(-q^{\frac{1}{2}};q)_\infty}$$

after simplification, and employing (3.2), we have

$$(-q^{\frac{1}{2}};q)_\infty - (q^{\frac{1}{2}};q)_\infty = \frac{2q^{\frac{1}{2}}(-q,-q^7,q^8;q^8)_\infty(q^{\frac{1}{2}},-q^{\frac{1}{2}};q)_\infty}{(q,q^2,q^3,q^4,q^5;q^5)_\infty}$$

which established (3.5).

Proof of (3.6): Dividing (3.4) by (3.5), we get desired result.

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Notes

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Analytical Solutions for the Different Forms of Telegraph Equations by Homotopy Analysis Method

By R.Rajaraman

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Abstract - In this work Homotopy Analysis Method(HAM) is used for analytic treatment of the telegraph equations .This method can provide analytical solutions to the problems by just utilizing the initial conditions. The proposed iterative scheme finds the solution without any discretization, linearization or restrictive assumptions.The proposed method solves nonlinear problems without using Adomain polynomials which is the advantage of this method over Adomain Decomposition method.The results reveal that the HAM is very effective, fast, simple, convenient, flexible and accurate. Outcomes prove that HAM is in very good agreement with ADM,VIM HPM.

Keywords : Homotopy Analysis method, telegraph equations.

2000 AMS Subject Classification: 35L25

ANALYTICAL SOLUTIONS FOR THE DIFFERENT FORMS OF TELEGRAPH EQUATIONS BY HOMOTOPY ANALYSIS METHOD

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Analytical Solutions for the Different Forms of Telegraph Equations by Homotopy Analysis Method

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Keywords : Homotopy Analysis method, telegraph equations

I. Introduction

The telegraph equation also called telegraphist's equation was first introduced by kirchoff in his paper in 1857 and then appeared in the paper by Heavyside in 1876. But it first attracted serious attention when treated by Poincare in 1893.The original form of the telegraph equation is

$$KL\frac{\partial^2 u}{\partial t^2}+(RK+SL)\frac{\partial u}{\partial t}+RSu=\frac{\partial^2 u}{\partial x^2} \tag{1}$$

Where

u=electric potential or electric current

K=electrostatistic capacity

L=self inductance

R=resistance

S=leakage conductance

x=distance along the wire

t=time

The telegraph equation has characteristics of both wave motion and diffusion. If there is no resisistance and leakage, it reduces to

$$\frac{\partial^2 u}{\partial t^2}=\frac{1}{KL}\frac{\partial^2 u}{\partial x^2} \tag{2}$$

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Which is the equation of wave motion with phase speed of$(KL)^{-1/2}$ On the other hand when inductance is negligible compared with the resistance and there is no leakage we have

$$\frac{\partial u}{\partial t} = \frac{1}{RK}\frac{\partial^2 u}{\partial x^2} \tag{3}$$

Which is the equation of diffusion with diffusivity of $(RK)^{-1}$

Recently various iterative methods are applied for getting Numerical and analytical solutions of telegraph equations[1,2,3,4]. In this paper Homotopy Analysis Method is applied to solve the proposed equations. HAM introduced by Liao[5,67,8] has been used by many mathematicians and engineers to solve various equations based on homotopy , which is a basic concept in topology. In recent years this method has been successfully employed to solve many types of nonlinear homogeneous or nonhomogeneous equations and systems of equation as well as problems in Science and engineering[9-16]. The validity of the HAM is independent of whether or not there exists small parameters in the considered equation. HAM provides us with a simple way to adjust and control the convergence of solution series. .It provides us with freedom to use different base fuctions to approximate a nonlinear problem. Especially, it provides us with freedom of replace a nonlinear partial differential equation of first order n into an infinite number of linear Differential equations of order k, where the order k is even unnecessarily to be equal to order n.When the base functions are introduced the H(r,t) is properly chosen using the rule of solution expression, rule of coefficient of ergodicity and rule of solution existence.By plotting h curves the proper h is chosen. Thus as long as h,H(r,t), $U_0(r,t)$ and linear operator L are properly chosen the solution expression converges to exact solution in the convergence region. Homotopy analysis method provides us the great freedom to choose all of them.

II. Basic idea of Homotopy Analysis Method (ham)

In this section the basic ideas of the homotopy analysis method are introduced. Here a description of the method [7] is given to handle the general nonlinear problem.

$$Nu_0(t)=0, t>0, \tag{4}$$

Where N is a nonlinear operator and $u_0(t)$ is unknown function of the independent variable t.

Zero- order deformation equation

Let $u_0(t)$ denote the initial guess of the exact solution of Eq. (4), h$\neq 0$ an auxiliary parameter, $H(t) \neq 0$ an auxiliary function. and L an auxiliary linear operator with the property.

$$L\big(f(t)\big) = 0, \text{When} f(t) = 0. \tag{5}$$

The auxiliary parameter h, the auxiliary function $H(t)$, and the auxiliary linear operator L play important roles within the HAM to adjust and control the convergence region of solution series. Liao[8] constructs, using $q \in [0,1]$ as an embedding parameter, the so-called zero-order deformation equation.

$$(1-q)L[(\emptyset(t;q) - u_0(t)] = qhH(t)N[(\emptyset(t;q)], \tag{6}$$

Ref.

8. S.J.Liao, Beyond perturbation:Introduction to Homotopy Analysis Method, Chapman & Hall/CRC press.Boca Raton 2003.

Where $\emptyset(t;q)$ is the solution which depends on h, $H(t), L,$ $u_0(t)$ and q. when q=0,the zero-order deformation Eq.(6) becomes

$$\emptyset(t;0) = u_0(t), \quad (7)$$

And when q=1, since h$\neq 0$ and $H(t) \neq 0$, the zero-order deformation Eq.(6) reduces to,

$$N[\emptyset(t;1)] = 0, \quad (8)$$

So, $\emptyset(t;1)$ is exactly the solution of the nonlinear Eq.(4). Define the so-called mth order deformation derivatives.

$$u_m(t) = \frac{1}{m!}\frac{\partial^m \emptyset(t;q)}{\partial q^m} \quad (9)$$

If the power series (9) of $\emptyset(t;q)$ converges at q=1, then we gets the following series solution:

$$u(t) = u_0(t) + \sum_{m=1}^{\infty} u_m(t). \quad (10)$$

Where the terms $u_m(t)$ can be determined by the so-called high order deformation described below.
High- order deformation equation

Define the vector,

$$\overrightarrow{u_{n}}=\{u_0(t), u_1(t), u_2(t) \dots \dots u_n(t) \quad (11)$$

Differentialing Eq(6)m times with respect to embedding parameter q, the setting q=0 and dividingthem by $m!$, we have the so-called mth- oder deformation equation.

$$L[u_m(t) - \aleph_m u_{m-1}(t)] = hH(t)R_m(\overrightarrow{u_m}, t), \quad (12)$$

$$\text{Where } \aleph_m = \begin{cases} o, & m \leq 1 \\ 1, & otherwise \end{cases} \quad (13)$$

And

$$R_m(\overrightarrow{u_m}, t) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\emptyset(t;q)]}{\partial q^{m-1}} \quad (14)$$

For any given nonlinear operator N, the term $R_m(\overrightarrow{u_m}, t)$ can be easily expressed by (14). Thus, we can gain $u_1(t), u_2(t) \dots \dots$ by means of solving the linear high-order deformation Eq. (12) one after the other order in order. The mth –order approximation of u(t) is given by

$$u(t) = \sum_{k=0}^{m} u_k(t) \quad (15)$$

III. Illustrative Examples

Example1:

Let us consider the following telegraph equation

$$u_{tt} + u_t + u = u_{xx} \quad (16)$$

With initial conditions

$$U(x,0) = e^x \; u_t(x,0) = -e^{-x} \quad (17)$$

Ref.

7. Liao.S.J.(2004)On the Homotopy analysis method for non linear problems.Appl Math Comput,147, 499-513.

We apply homotopy analysis method to Eq. (16) and (17), as follows:

since m≥1, χ_m=1 set h= -1 and H(r,t)=1,L==$\partial^2 u/\partial t^2$ in(12)then (12) becomes

$$U_m(x,t)=u_{m-1}(x,t)-L^{-1}(R_m(u_{m-1,}x,t)) \quad (18)$$

Where

$$R_m(u_{m-1,}x,t) =\partial^2 u_{m-1}/\partial t^2+\partial u_{m-1}/\partial+u_{m-1}-\partial^2 u_{m-1}/\partial x^2$$

Notes

And solution for u_0:
Now we can select

$$u_0(x,t)= e^{-x}+te^{-x} \quad (19)$$

Applying (19) in (18) we get the following approximations

$U_1(x,t)= e^{-x}+te^{-x}$ $\quad u_2(x,t)= e^{-x}+te^{-x} +t^2/2!\, e^{-x}$ $\quad U_3(x,t)= e^{-x}+te^{-x} +t^2/2!e^{-x} \;+t^3/3!\, e^{-x}$

The Final solution is

$$u(x,t)= e^{t-x} \quad (20)$$

Example 2:
Consider the following form of telegraph equation

$$u_{tt} +\alpha u_t+\beta u=u_{xx}+f(x,t) \quad (21)$$

With α=1, β=1 and f(x,t)= e^x+t+1
With initial conditions

$$u(x,0)=e^x,u_t(x,0)=2 \quad (22)$$

We apply homotopy analysis method to Eq. (21) and (22), as follows:
since m≥1, χ_m=1 set h= -1 and H(r,t)=1,L==$\partial^2 u/\partial t^2$ in(12)then (12) becomes

$$U_m(x,t)=u_{m-1}(x,t)-L^{-1}(R_m(u_{m-1,}x,t)) \quad (23)$$

Where

$$R_m(u_{m-1,}x,t) =\partial^2 u_{m-1}/\partial t^2+\partial u_{m-1}/\partial t+u_{m-1}-\partial^2 u_{m-1}/\partial x^2-(e^x+t+1)$$

And solution for u_0:
Now we can select

$$u_0(x,t)= e^x\, t+t \quad (24)$$

Applying (24) in (23) we get the following approximations

$U_1(x,t)= e^x t+t$ $\;U_2(x,t)= e^x t+t$.........................

The Final solution is

$$U(x,t)= e^x t+t \quad (25)$$

Example3:
Let us consider the following telegraph equation

$$u_{tt} +(\alpha+\beta)u_t+\alpha\beta u=c^2u_{xx}$$

Notes

where

$$\alpha=2,\beta=2,c=1 \tag{26}$$

with initial condition

$$u(x,0)=1-e^{2x}\ u_t(x,0)=-2 \tag{27}$$

We apply homotopy analysis method to Eq. (26) and (27), as follows:
since m≥1, $\chi_m=1$ setting h= -1 and H(r,t)=1and here we setL==$\partial^2u/\partial x^2$ in(12)then (12) becomes

$$U_m(x,t)=u_{m-1}(x,t)-L^{-1}(R_m(u_{m-1,}x,t)) \tag{28}$$

Where

$$R_m(u_{m-1,}x,t)=\partial^2u_{m-1}/\partial x^2-\partial^2u_{m-1}/\partial t^2-4\partial u_{m-1}/\partial t-4u_{m-1}$$

And solution for u_0:
Now we can select

$$u_0(x,t)=e^{-2t}+1-2x \tag{29}$$

Applying (29) in (28) we get the following approximations

$$u_1(x,t)=e^{-2t}+1-2x+(2x)^2/2!-(2x)^3/3!$$

$$u_2(x,t)=e^{-2t}+1-2x+(2x)^2/2!-(2x)^3/3!+(2x)^4/4!-(2x)^5/5!$$

The Final solution is

$$u(x,t)=e^{-2t}+1-2x+(2x)^2/2!-(2x)^3/3!+(2x)^4/4!-(2x)^5/5!+\dots\dots$$

$$U(x,t)=e^{-2t}+e^{-2x} \tag{30}$$

IV. Conclusion

In this paper exact solutions for some of the telegraph equations have been established.The Homotopy Analysis method(HAM) is successfully used to develop these solutions. This work shows that HAM has significant advantages over the existing techniques.It avoids the need for calculating the Adomain polynomials which can be difficult in some cases. The reliability of the method and reduction in the size of computational domain give this method wider applicability.The results shows that HAM is a powerful mathematical tool for finding the exact and approximate solutions of the nonlinear equations.

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Notes

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An Integral Involving Extended Jacobi Polynomial

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Abstract - An attempt has been made to establish an integral concerning the product of two H-function of several complex variables (Srivastava and Panda [8] with extended Jacobi polynomial [5]). Mainly we are using the series representation of H-function given by Olkha and Chaurasia [6,7]. By assigning suitable values to the parameters, the results can be reduced to many new, known and unknown results.

Keywords : Multivariable H-function, H-function in series form, Jacobi polynomial, Lauricella function, Fox H-function.

Mathematics Subject Classification 2000: 26A33, 44A10, 33C60

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Strictly as per the compliance and regulations of :

6. Olkha, G.S. and Chaurasia, V.B.L. – Some integral transform involving the H-function of several complex variables, Kyungpook Math. J., 22 (1982), 309-315.

An Integral Involving Extended Jacobi Polynomial

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Abstract - An attempt has been made to establish an integral concerning the product of two H-function of several complex variables (Srivastava and Panda [8] with extended Jacobi polynomial [5]). Mainly we are using the series representation of H-function given by Olkha and Chaurasia [6,7]. By assigning suitable values to the parameters, the results can be reduced to many new, known and unknown results.

Keywords : Multivariable H-function, H-function in series form, Jacobi polynomial, Lauricella function, Fox H-function.

I. Introduction

The series representation of the H-function of several complex variable studied by Olkha and Chaurasia [6,7] is given as follows:

$$H[z_1,\dots,z_r] = H^{0,\lambda':(u',v');\dots;(u^{(r)},v^{(r)})}_{A',C':[B',D'];\dots;[B^{(r)},D^{(r)}]}\left[\begin{matrix}[(a):\theta',\dots,\theta^{(r)}]:\;[b':\phi'];\dots;[b^{(r)}:\phi^{(r)}];\\ [(c):\psi',\dots,\psi^{(r)}]:[d':\delta'];\dots;[d^{(r)}:\delta^{(r)}];\end{matrix}\; z_1,\dots,z_r\right]$$

$$=\sum_{m_i=1}^{u^{(i)}}\sum_{n_i=0}^{\infty}\Phi_1\Phi_2\frac{\prod_{i=1}^{r}(z_i)^{U_i}(-1)^{\sum_{i=1}^{r}(n_i)}}{\prod_{i=1}^{r}\left(\delta^{(i)}_{(m_i)}\,n_i!\right)}, \tag{1.1}$$

where

$$\Phi_1=\frac{\prod_{j=1}^{\lambda'}\Gamma\left[1-a_j+\sum_{i=1}^{r}\theta_j^{(i)}U_i\right]}{\prod_{j=\lambda'+1}^{A'}\Gamma\left[a_j-\sum_{i=1}^{r}\theta_j^{(i)}U_i\right]\prod_{j=1}^{C}\Gamma\left[1-c_j+\sum_{i=1}^{r}\psi_j^{(i)}U_i\right]}, \tag{1.2}$$

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$$\Phi_2 = \frac{\prod_{\substack{j=1\\ j\neq m_j}}^{u^{(i)}} \Gamma(d_j^{(i)} - \delta_j^{(i)} U_i) \prod_{j=1}^{v^{(i)}} \Gamma(1 - b_j^{(i)} + \phi_j^{(i)} U_i)}{\prod_{j=u^{(i)}+1}^{D^{(i)}} \Gamma(1 - d_j^{(i)} + \delta_j^{(i)} U_i) \prod_{j=v^{(i)}+1}^{B^{(i)}} \Gamma(b_j^{(i)} - \phi_j^{(i)} U_i)}, \tag{1.3}$$

$$U_i = \frac{d_{m_i}^{(i)} + n_i}{\delta_{m_i}^{(i)}},\ i = 1, \ldots, r \tag{1.4}$$

which is valid under the following conditions

$$\delta_{m_i}^{(i)} [d_j^{(i)} + p_i] \neq \delta_j^{(i)} [d_{m_i}^{(i)} + n_i] \tag{1.5}$$

for $j \neq m_i, m_i = 1, \ldots, u^{(i)}; p_i, n_i = 0, 1, 2, \ldots; z \neq 0$

$$\nabla_i = \sum_{j=1}^{\lambda'} \theta_j^{(i)} - \sum_{j=1}^{C'} \psi_j^{(i)} + \sum_{j=1}^{B^{(i)}} \phi_j^{(i)} - \sum_{j=1}^{D^{(i)}} \delta_j^{(i)} < 0 \quad \forall i = 1, \ldots, r \tag{1.6}$$

Srivastava and Panda [8] have introduced the multivariable H-function

$$H[y_1, \ldots, y_R] = H^{0,\lambda:(\alpha',\beta');\ldots;(\alpha^{(R)},\beta^{(R)})}_{A,C:[M',N'];\ldots;[M^{(R)},N^{(R)}]} \left[\begin{matrix} [(g):\gamma',\ldots,\gamma^{(R)}]:[q':n'];\ldots;(\alpha^{(R)},\beta^{(R)}): \\ [(f)L\xi',\ldots,\xi^{(R)}]:[p':\in'];\ldots;[p^{(R)},\in^{(R)}]: \end{matrix}\ y_1, \ldots, y_R \right] \tag{1.7}$$

For the sake of brevity

$$T_i = \sum_{j=1}^{\lambda} \gamma_j^{(i)} - \sum_{i=1}^{C} \xi^{(i)} + \sum_{j=1}^{M^{(i)}} \eta_j^{(i)} - \sum_{j=1}^{N^{(i)}} \in_j^{(i)} \leq 0, \tag{1.8}$$

$$\Omega_i = -\sum_{j=\lambda+1}^{A} \gamma_j^{(i)} - \sum_{i=1}^{C} \xi_j^{(i)} + \sum_{j=1}^{\beta^{(i)}} \eta_j^{(i)} - \sum_{j=\beta^{(i)}+1}^{M^{(i)}} \eta_j^{(i)} + \sum_{j=1}^{\alpha^{(i)}} \in_j^{(i)} - \sum_{j=\alpha^{(i)}+1}^{N^{(i)}} \in_j^{(i)} > 0, \tag{1.9}$$

$$|\arg(y_i)| < \frac{1}{2} T_i \pi,\ \forall\ i = 1, \ldots, R.$$

II. Main Theorem

The transformation valid under the following conditions:

(i) $\operatorname{Re}(\rho) > -1$,

Ref.

8. Srivastava, H.M. and Panda, R. – Expansion theorems for the H-function of several complex variables, J. Reine Angew. Math., 288 (1976), 129-145.

(ii) $\operatorname{Re}\left(\sigma+\sum_{i=1}^{R} h_i \frac{p_j^{(i)}}{\epsilon_j^{(i)}}+\sum_{i'=1}^{r} h'_{i'} \frac{d_j^{(i)}}{\delta_j^{(i)}}\right) > -1,$

(iii) $h_i, h'_{i'}, T_i, k > 0,\ t = k\,(q-p), i = 1,\dots,R,\ i' = 1,\dots,r,$

$|\arg(y_i)| < \frac{1}{2}\Omega_i, T_i, \Omega_i$ are given in (1.8) and (1.9).

(iv) $F_n(\rho,\omega;x)$ is Fujiwara's polynomial.

Thus, the following transformation holds

$$\int_p^q (x-p)^{\rho}(q-x)^{\sigma} F_n(\rho,\omega;x)\, H[z_1(q-x)^{h'},\dots,z_r(q-x)^{h_r}]$$

$$.\ H[y_1(q-x)^{h},\dots,y_R(q-x)^{h_R}]dx$$

$$= \sum_{m_i=1}^{u^{(i)}} \sum_{n_i=0}^{\infty} \frac{(-1)^{\sum_{i=1}^{r}(n_i)+n} \prod_{i=1}^{r} (z_i)^{U_i}\, t^n}{\prod_{i=1}^{r} ((\delta_{m_i}^{(i)} n_i!)\, n!} (q-p)^{\rho+\sigma+1+\sum_{i=1}^{r} h'_i U_i}\, \Gamma(1+\rho+n)\,\Phi_1\Phi_2$$

$$.\ H^{0,\lambda+2\ :(\alpha',\beta')\ ;\dots;(\alpha^{(R)},\beta^{(R)})}_{A+2,C+2:[M',N'];\dots;[M^{(R)},N^{(R)}]}\left[\begin{matrix}[\omega-\rho-\sum_{i=1}^{r} h'_i U_i : h_1,\dots,h_R],[-\sigma-\sum_{i=1}^{r} h'_i U_i : h_1,\dots,h_R],\\ [\omega+n-\sigma-\sum_{i=1}^{r} h'_i U_i : h_1,\dots,h_R],[-1-\rho-n-\sigma\sum_{i=1}^{r} h'_i U_i : h_1,\dots,h_R],\end{matrix}\right.$$

$$\left.\begin{matrix}[(g):\gamma',\dots,\gamma^{(R)}]:[q',\eta'];\dots;[q^{(R)},\eta^{(R)}];\\ [(f):\xi',\dots,\xi^{(R)}]:[p',\epsilon'];\dots;[p^{(R)},\epsilon^{(R)}];\end{matrix}\ y_1(q-p)^{h_1},\dots,y_R(q-p)^{h_R}\right]. \qquad (2.1)$$

III. Proof

To prove (2.1), we express the multivariable H-function in series form with the help of (1.1) and then changing the order of integration and summations which is valid under the conditions stated and evaluating the remaining integral with the help of a known result of Chaurasia and Sharma ([2], p.269, eqn. (2.1)), we get the required result.

IV. Special Cases

(a) Giving suitable values to the parameters and making use of a transformation formula given by Srivastava and Panda ([8], p.139, eqn. 4.11), after a little simplification, we have the following result

Ref.

2. Chaurasia, V.B.L. and Sharma, S.C. – An integral involving extended Jacobi polynomial and H-function of several complex variables, Vij. Pari. Anu. Pat., 27(3) (1984), 267-272.

Theorem (A)

The transformation valid under the following conditions

(i) $\mathrm{Re}(\rho) > -1, \mathrm{Re}(\sigma) > -1$

(ii) $h_i > 0, h^{i}_{i'} > 0, \Delta_{i'} \geq 0, i = 1,\ldots,R, i' = 1,\ldots,r, t = k(q-p), k > 0$

where

$$\Delta_j = 1 + \sum_{i=1}^{C} \xi^{(j)} + \sum_{i=1}^{B^{(j)}} \epsilon_i^{(j)} - \sum_{i=1}^{\lambda} \gamma_i^{(j)} - \sum_{i=1}^{\alpha^{(j)}} \eta_i^{(j)} \; (j = 1,\ldots,R)$$

(iii) The equality holds when $|y_j| < L_j$, j = 1,...,R, with the L_j defined by equation (5.3), p.157 in [10].

Thus, the following transformation holds

$$\int_p^q (x-p)^{\rho} (q-x)^{\sigma} F_n(\rho,\omega;x) \, F^{A:B';\ldots;B^{(r)}}_{C:D';\ldots;D^{(r)}} [z_1 (q-x)^{h'},\ldots, z_r (q-x)^{h'_r}]$$

$$. \, F^{\lambda:\alpha';\ldots;\alpha^{(R)}}_{\mu:\beta';\ldots;\beta^{(R)}} [y_1 (q-x)^{h},\ldots, y_R (q-x)^{h_R}] dx$$

$$= \sum_{m_1,\ldots,m_r=0}^{\infty} \frac{\prod_{i=1}^{A} (a_i)_{m_1\theta_1+\ldots+m_r\theta_i^{(r)}} \prod_{i=1}^{B'} (b'_i)_{m_1\phi'_i} \cdots \prod_{i=1}^{B^{(r)}} (b_i^{(r)})_{m_r\phi_i^{(r)}}}{\prod_{i=1}^{C} (c_i)_{m_1\psi_1+\ldots+m_r\psi_i^{(r)} m_r} \prod_{i=1}^{D'} (d'_{(i)})_{m_1\delta'_i} \cdots \prod_{i=1}^{D^{(r)}} (d_i^{(r)})_{m_r\delta_i^{(r)}}} \frac{z_1^{m_1}}{m_1!} \cdots \frac{z_r^{m_r}}{m_r!} \frac{t^n (-1)^n}{n!}$$

$$.(q-p)^{\rho+\sigma+1+\sum_{i=1}^{r} h' m_i} \frac{\Gamma(1+\rho+n)\,\Gamma(1+\sigma+\sum_{i=1}^{r} h' m_i)\Gamma(1+\sigma-\omega+\sum_{i=1}^{r} h' m_i)}{\Gamma(1+\sigma-\omega-n+\sum_{i=1}^{r} h' m_i)\Gamma(2+\omega+n+\sigma+\sum_{i=1}^{r} h' m_i)}$$

$$. \, F^{\lambda+2:\alpha';\ldots;\alpha^{(R)}}_{\mu+2:\beta';\ldots;\beta^{(R)}} \left[\begin{array}{l} [1+\sigma+\sum_{i=1}^{r} h' m_i : h_1,\ldots,h_R],[1+\sigma-\omega+\sum_{i=1}^{r} h' m_i : h_1,\ldots,h_R], \\ [1+\sigma-\omega-n+\sum_{i=1}^{r} h' m_i : h_1,\ldots,h_R],[2+\omega+n+\sigma+\sum_{i=1}^{r} h' m_i : h_1,\ldots,h_R], \end{array} \right.$$

$$\left. \begin{array}{l} [(g):\gamma',\ldots,\gamma^{(R)}]:[(q'):\eta'];\ldots;[(q^{(R)}):\eta^{(R)}]; \\ [(f):\xi',\ldots,\xi^{(R)}]:[(p'):\epsilon'];\ldots;[(p^{(R)}):\epsilon^{(R)}]; \end{array} y_1 (q-p)^{h_1},\ldots, y_R (q-p)^{h_R} \right]. \qquad (4.1)$$

Ref.

10. Srivastava, H.M. and Daoust, M.C. – A note on convergence of Kampé de Fériets double hypergeometric series, Math. Nachr. 53 (1972), 151-159.

(b) For r = 1 =R in (2.1), we obtain the following result

Theorem (B)

The transformation valid under the following conditions

(i) $\mathrm{Re}(\rho)>-1, h>0, h'>0, T>0, |\arg y|<\frac{1}{2}T\pi, t=k\,(q-p), k>0$

(ii) $\mathrm{Re}(\sigma+h'\frac{p_j}{\in_j}+h\frac{d_{j'}}{\delta_{j'}}+1>0, j=1,\ldots,u, j'=1,\ldots,\alpha.$

Notes

Thus, the following transformation holds

$$\int_p^q (x-p)^\rho (q-x)^\sigma F_n(\rho,\omega;x) H_{B,D}^{u,v}\left[\begin{matrix}[b:\phi]\\ [d:\delta]\end{matrix}\middle| z\,(q-x)^{h'}\right] H_{M,N}^{\alpha,\beta}\left[\begin{matrix}(q,\eta)\\ [p,\in]\end{matrix}\middle| y(q-x)^{h}\right]dx$$

$$=\sum_{m_1=1}^{u}\sum_{n_1=0}^{\infty}\frac{(-1)^{n_1} z^U t^n (q-p)^{\rho+\sigma+1+h'U}}{n!\;\; n_1!\;\; \delta n_1}\Gamma(1+\rho+n)$$

$$.\; H_{M+2,N+2}^{\alpha,\beta+2}\left[\begin{matrix}[\omega-\rho-h'U:h],[-\sigma-hU:h],[b:\phi]\\ [d:\delta],[\omega+n-\sigma-h'U:h],[-1-\rho-n-\sigma-h'U:h]\end{matrix}\right]. \qquad (4.2)$$

(c) Taking $p=1=q, \lambda=1, h_{i'}^{'}=1, i'=1,\ldots,r,$ we get a known result due to Srivastava and Panda [8].

(d) Letting $h_i^{'}=1, i=1,\ldots,r$ in (2.1), we have a result due to Chaurasia and Sharma [2].

(e) Putting $h_i^{'}=1, i=1,\ldots,r$ the result in (4.1) reduces to a known result obtained by Chaurasia and Sharma in [2].

(f) The result already established by the equation (3.2) and (3.3) in [2] can be deduced from our results.

A great number of interesting integral formulae as particular cases our main result can be deduced, but we omit them here for lack of space.

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Coincidence and Common Fixed Point Theorem in Cone Metric Spaces

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Abstract - In this paper we have proved some coincidence and common fixed point theorem in cone metric space by using Jungck type contractive condition and generalized the results of Huang Long Guang and Zhang Xian.

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COINCIDENCE AND COMMON FIXED POINT THEOREM IN CONE METRIC SPACES

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Coincidence and Common Fixed Point Theorem in Cone Metric Spaces

Piyush Kumar Tripathi[α] & A. Kumar[σ]

Abstract - In this paper we have proved some coincidence and common fixed point theorem in cone metric space by using Jungck type contractive condition and generalized the results of Huang Long Guang and Zhang Xian.

I. Introduction

Common fixed point theorems have been proved by many authors by using contractive conditions for single valued mapping [5], [6], [7]. The concept of weakly commuting mapping defined by S. Sessa [9] was generalized by Singh and Pant [10]. They proved some common fixed point theorem by using weak commutative condition of mappings. G. Jungck [3] has generalized this condition by defining compatible mapping and proved some common fixed point theorem.

Recently Huang Long [4] replaced the set of real numbers by an ordered Banach space and obtained some fixed point theorems for self mapping satisfying different contractive condition. Dejan Ilic and Rakocevic [1] and K. Jha [5] have proved some fixed point theorems in cone metric space by using commuting mapping. Abbas and Jungck [6] has proved some common fixed point theorem for non commuting mappings in the set of cone metric spaces. P. Raja [8] has proved new results by extending Banach contraction principle in complete cone metric spaces. Jungck type contraction [2] leads to a new development regarding the weaker forms of commuting mappings which is stated as:

Let Y be an arbitrary nonempty set and (X, d) is a metric space. Let f and I be maps from Y with the values in X. Consider the following properties,

a. $d(fx, fy) \leq kd(Ix, Iy), \quad xy \in X, \quad 0 \leq k < 1$

b. $d(fx, fy) < kd(Ix, Iy), \quad xy \in X,$

c. $d(fx, fy) \leq d(Ix, Iy), \quad xy \in X$

In the scenario of existing literature in metric fixed point theory the pair of mapping (f, I) satisfying properties a, b and c are called Jungck contraction, Jungck strictly contraction and Jungck non expensive contraction. Piyush tripathi [10] proved some common fixed point theorems and extended some well known results by using Jungck type contraction.

The papers of Jungck [2] and piyush tripathi [9] motivated us to establish coincidence and common fixed point theorem for two self mapping in a complete cone

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Ref.

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metric spaces by using Jungck type contractive condition which is the extended result of Huang Long [4].

The following definitions regarding cone metric space is defined in the paper of Huang Long [4].

Definition-1.1 Suppose E as the real Banach space and P is a subset of E. Then P is cone if and only if

(i) P is closed, non-empty and $P \neq \{0\}$

(ii) If $\alpha, \beta \in R$ and $\alpha, \beta \geq, 0$ and if $x, y \in P$ then $\alpha x + \beta y \in P$

(iii) If $x \in P$ and $- x \in P \Rightarrow x = 0$

For a cone P which is subset of E Huang Long [4] defined a partial ordering $<<$ with respect to P as $x << y$ if and only if $y - x \in \text{int } P$, where int P is interior of P

Definition-1.2 : P is called normal cone if there exist a number $K > 0$ such that $o \leq x \leq y \Rightarrow \|x\| \leq K \|y\|$

Definition-1.3: The cone P is said to be regular if every increasing sequence which is bounded above is convergent.

Definition-1.4 : Let X is a non - empty set. Let d is a mapping from $X \times X \to E$ and if d satisfies the following conditions.

(i) $d(x, y) > 0 \quad \forall x, y \in X$

(ii) $d(x, y) = 0$ *iff* $x = y$

(iii) $d(x, y) = d(y, x)$

(iv) $d(x, y) \leq d(x, z) + d(y, z) \qquad \forall\ x\ y\ z \in X$

Then d is called the cone metric on X and the pair (X, d) is called the cone metric space.

Definition-1.5: A sequence $\{x_n\}$ in a cone metric space (X, d) is said to be convergent and converges to $x \in X$ if for every $c \in E$ with o << c, $\exists\ N$ such that $n > N$, $d(x_n, x) << c$, and x is called the limit of $\{x_n\}$.

i.e. $\lim x_n = x$ or $x_n \to x$ for $n \to \infty$

Definition-1.6: A sequence $\{x_n\}$ in a cone metric space (X, d) is said to be convergent and converges to $x \in X$ if for every $c \in E$ with o $<< c\ \exists\ N$ such that $n > N$, $d(x_n, x) << c$.

Definition-1.7 : A cone metric space (X, d) is said to be complete cone metric space if every Cauchy sequence is convergent in (X, d).

Definition-1.8 : A cone metric space (X, d) is said to be sequentially compact if for any sequence $\{x_n\}$ in X, there exists a subsequence $\{x_{n_i}\}$ of $\{x_n\}$ such that $\{x_{n_i}\}$ is convergent in X. To prove our main result, the following Lemma [2] are very important.

Lemma – 1.1 [4] : Let (X, d) be a cone metric space, $\{x_n\}$ be a sequence in X. If $\{x_n\}$ converges to x then $\{x_n\}$ is a Cauchy sequence.

Lemma – 1.2 [4] : Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ be a sequence in X. Then $\{x_n\}$ is a Cauchy sequence if and only if

Ref.

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$$d\,(x_m, x_n) \to 0 \;\; (m, n \to \infty)$$

Lemma – 1.3 [4] : Let (X, d) be a cone metric space, P be a normal cone with normal constant K. Let $\{x_n\}$ and $\{y_n\}$ be two sequences in X and $x_n \to x, \;\; y_n \to y \;\; (n \to \infty)$

Then $$d\,(x_n, y_n) \to d\,(x, y) \text{ when } n \to \infty$$

Notes

II. Main Results

Theorem-2.1: Let (X, d) be a cone metric space, P be a normal cone with constant K. Suppose $T, I : X \to X$ are self mappings satisfies the condition

(i) $d(Tx\ Ty) \le kd(Ix\ Iy)$ for all $x, y \in Y$, where $k \in [0, 1)$.

(ii) $T(Y) \subseteq I(Y)$.

(iii) Either $T(Y)$ or $I(Y)$ is complete.

Then T and I have a coincidence point in X.

Proof: Since $T(Y) \subseteq I(Y)$ Choose $x_0, x_1 \in Y$, such that $Ix_1 = T(x_0)$ and $x_1, x_2 \in Y$, $Ix_2 = Tx_1$, hence we can construct a sequence $\{x_n\}$ such that $Ix_{n+1} = Tx_n$

$$\begin{aligned} d\,(Ix_{n+1}\ \ Ix_n) &= d\,(Tx_n,\ Tx_{n-1}) \\ &\le kd\,(Ix_n,\ Ix_{n-1}) \end{aligned}$$

$$\begin{aligned} d\,(Ix_n\ \ Ix_{n-1}) &= d\,(Tx_{n-1},\ Tx_{n-2}) \\ &\le kd\,(Ix_{n-1},\ Ix_{n-2}) \end{aligned}$$

$$d\,(I.x_{n+1}, Ix_n) \le k^2 d\,(Ix_{n-1},\ Ix_{n-2})$$

continuing this process we get

$$d\,(Ix_{n+1}, Ix_n) \le k^n d\,(Ix_1,\ Ix_0)$$

So for n > m

$$\begin{aligned} d\,(Ix_n, Ix_m) &\le d\,(Ix_n,\ Ix_{n-1}) + d\,(Ix_{n+1},\ Ix_{n-2}) + \ldots\ldots d\,(Ix_{m+1},\ Ix_m) \\ &\le (k^{n-1} + k^{n-2} + \ldots\ldots k^m)\ d\,(Ix_1,\ Ix_0) \\ &\le \frac{k^m}{1-k}\ d\,(Ix_1,\ Ix_0) \end{aligned}$$

We get

$$\|d(Ix_n, Ix_m)\| \le \frac{k^m}{1-k}\ K\ \|d\,(Ix_1,\ Ix_0)\|$$

$$\Rightarrow d\,(Ix_n,\ Ix_m) \to 0 \qquad as \qquad n,\ m \to \infty$$

Hence $\{Ix_n\} = \{Tx_{n-1}\}$ is a Cauchy sequence in $T(Y) \subseteq I(Y)$. Suppose $I(Y)$ is complete in X so there is $p \in I(Y)$, $z \in Y$ such that $Ix_n, Tx_n \to p$ as $n \to \infty$ and $Iz = p$.

Putting $x = x_n$ and $y = z$ in (i) .

$$d(Tx_n, Tz) \le kd(Ix_n, Iz)$$

$$d(Tx_n, Tz) \le kd(Ix_n, p)$$

as $n \to \infty$,

$$d(p, Tz) \le kd(p, p) \Rightarrow d(p, Tz) = 0$$

So we get $Tz = Iz = p$. Therefore z is a coincidence point of T and I.

Again if $T(Y)$ is complete then $Tx_n \to p \in T(Y) \subseteq I(Y)$ hence as above z is coincidence point of T and I.

Theorem-2.2: Let (X, d) be a cone metric space, P be a normal cone with constant K. Suppose $T, I : X \to X$ are self mappings satisfies the condition

(i) $d(Tx\ Ty) \le kd\ (Ix\ Iy)$ for all x, y ∈ Y, where k ∈ [0, 1).

(ii) $T(Y) \subseteq I(Y)$.

(iii) Either $T(X)$ or $I(X)$ is complete.

(iv) T and I are commuting at their coincidence point.

Then T and I have a unique common fixed point in X.

Proof: If we take $Y = X$ in Theorem 2.1 then we get sequence $\{Tx_n\}$ which is a Cauchy sequence. Suppoose g(X) is complete then $Tx_n \to p \in g(X)$, $z \in X$ such that $Iz = p$ and $Iz = Tz = p$. Since T and I are commuting at their coincidence point so $TIz = ITz$ and $Tp = Ip$.

Putting $x = z,\ y = Tz$, in (i),

$d(Tz, TTz) \le kd(Iz, ITz)$,

$d(p, Tp) \le kd(Iz, TIz)$

$d(p, Tp) \le k^2 d(p, Tp)$

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$d(p, Tp) \le k^n d(p, Tp)$

As $n \to \infty$,

$d(p, Tp) \to 0$

$Tp = Ip = p$

So p is a common fixed point of T and I. For uniqueness suppose p' and q' are two common fixed points of T and I, From (i) $d(Tp',Tq') \leq k(Ip',Iq') = k(Tp',Tq') \leq k^2(Ip',Iq'). \leq k^n(Ip',Iq')$, as $n \to \infty$ $d(Tp',Tq') = d(p',q') = 0 \Rightarrow p' = q'$. Therefore T and I have unique common fixed point. The following theorem was proved by Huang Long-Guang and Zhang Xian [4].

Theorem - 2.2.[4] : Let (X, d) be a complete cone metric space, P be a normal cone with normal constant K. Suppose the mapping $T : X \to X$ satisfies the contractive condition d $(Tx, Ty) \leq kd(x, y)$, for all $x, y \in X$, where $k \in [0, 1)$ is a constant. Then T has a unique fixed point in X and for any x ∈ X iterative sequence $(T^n x)$ converges to the fixed point.

Notes

Proof: In the our Theorem 2.2, mapping I is considered as identity mapping then the theorem stated by Huang Long [4] becomes special case of our proved Theorem 2.2

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Evaluation of a Summation Formula Associated with Gamma Function and Involving Gauss Theorem

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Abstract - The aim of this paper is to evaluate a summation formula based on half argument allied with Hypergeometric function and involving recurrence relation and Gauss summation theorem.

Keywords : Contiguous relation, Gauss second summation theorem ,Re-currence relation.

A.M.S. Subject Classification (2010): 33C60 , 33C70

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Abstract - The aim of this paper is to evaluate a summation formula based on half argument allied with Hypergeometric function and involving recurrence relation and Gauss summation theorem.

Keywords and Phrases : Contiguous relation, Gauss second summation theorem ,Re-currence relation.

I. Introduction

Generalized Gaussian Hypergeometric function

A generalized hypergeometric function ${}_pF_q(a_1,...a_p;b_1,...b_q;z)$ is a function which can be defined in the form of a hypergeometric series, i.e., a series for which the ratio of successive terms can be written

$$\frac{c_{k+1}}{c_k}=\frac{P(k)}{Q(k)}=\frac{(k+a_1)(k+a_2)...(k+a_p)}{(k+b_1)(K+b_2)...(k+b_q)(k+1)}\ z. \tag{1}$$

Where $k+1$ in the denominator is present for historical reasons of notation, and the resulting generalized hypergeometric function is written

$${}_pF_q\left[\begin{array}{ccc} a_1,a_2,\cdots,a_p & ; & \\ & & z \\ b_1,b_2,\cdots,b_q & ; & \end{array}\right]=\sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\cdots(a_p)_k z^k}{(b_1)_k(b_2)_k\cdots(b_q)_k k!} \tag{2}$$

or

$${}_pF_q\left[\begin{array}{ccc} (a_p) & ; & \\ & & z \\ (b_q) & ; & \end{array}\right]\equiv{}_pF_q\left[\begin{array}{ccc} (a_j)_{j=1}^p & ; & \\ & & z \\ (b_j)_{j=1}^q & ; & \end{array}\right]=\sum_{k=0}^{\infty}\frac{((a_p))_k z^k}{((b_q))_k k!} \tag{3}$$

where the parameters $b_1,b_2,\cdots,b_q$ are neither zero nor negative integers and $p,\ q$ are non-negative integers.

The ${}_pF_q$ series[23, p.156(3)] converges for all finite z if $p\leq q$, converges for $\mid z\mid<1$ if $p\neq q+1$, diverges for all z , $z\neq 0$ if $p>q+1$.

The ${}_pF_q$ series [23, p.156(4)]absolutely converges for $\mid z\mid=1$ if $R(\zeta)<0$, conditionally converges for $\mid z\mid=1, z\neq 0$ if $0\leq R(\zeta)<1$, diverges for $\mid z\mid=1$, if $1\leq R(\zeta)$, $\zeta=\sum_{i=1}^{p}a_i-\sum_{i=0}^{q}b_i$.

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The function ${}_2F_1(a,b;c;z)$ corresponding to $p=2, q=1$, is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function (Gauss 1812, Barnes 1908). To confuse matters even more, the term "hypergeometric function" is less commonly used to mean closed form, and "hypergeometric series" is sometimes used to mean hypergeometric function.
The hypergeometric functions are solutions to the hypergeometric differential equation

$$z(1-z)y'' + [c-(a+b+1)z]y' - aby = 0 \tag{4}$$

The solution of this equation is

$$y = A_0\Big[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\Big] \tag{5}$$

This is the so-called regular solution,denoted

$${}_2F_1(a,b;c;z) = \Big[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\Big] = \sum_{k=0}^{\infty}\frac{(a)_k\ (b)_k z^k}{(c)_k k!} \tag{6}$$

Which converges if c is not a negative integer for all of $|\ z\ |< 1$ and on the unit circle $|\ z\ |= 1$ if $R(c-a-b) > 0$. Here $(a)_k$ is a Pochhhammer symbol and is defined by

$$(a)_k = \frac{\Gamma(a+k)}{\Gamma(a)} = \begin{cases} a(a+1)(a+2)\cdots(a+k-1); & \text{if } \ k = 1,2,3,\cdots \\ 1 \qquad ; & \text{if } \ k = 0 \\ k! \qquad ; & \text{if } \ a = 1,\ k = 1,2,3,\cdots \end{cases} \tag{7}$$

Contiguous Relation is defined by
[Andrews p.363(9.16), E. D. p.51(10)]

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; \\ c\ \ ; \end{matrix}\ z\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b\ \ ; \\ c\ \ ; \end{matrix}\ z\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1\ \ ; \\ c\ ; \end{matrix}\ z\right] \tag{8}$$

Gauss second summation theorem is defined by [Prudnikov., 491(7.3.7.8)]

$${}_2F_1\left[\begin{matrix} a,\ b & ; \\ \frac{a+b+1}{2} & ; \end{matrix}\ \frac{1}{2}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{9}$$

$$= \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{10}$$

In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud., p.491(7.3.7.8)]

$${}_2F_1\left[\begin{matrix} a,\ b & ; \\ \frac{a+b-1}{2} & ; \end{matrix}\ \frac{1}{2}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{11}$$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b-1}{2} & ; & \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{12} $$

Recurrence relation is defined by

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{13} $$

II. Derivation of Main Result

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+36}{2} & ; & \end{matrix}\right] = \frac{2^b\,\Gamma(\frac{a+b+36}{2})}{(a-b)\,\Gamma(b)}\times $$

$$ \times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a}{2})}\left\{\frac{131072(137119595809968000a - 231782096547348480 0a^2 + 1687424939411374080a^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+\right.\right. $$

$$ +\frac{131072(-713340918791405568a^4 + 198107173820104704a^5 - 38628489336586240a^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+ $$

$$ +\frac{131072(5503512041431040a^7 - 587213457166336a^8 + 47604013453568a^9 - 2951316459520a^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+ $$

$$ +\frac{131072(139842010880a^{11} - 5022477824a^{12} + 134304352a^{13} - 2589440a^{14} + 34000a^{15} - 272a^{16} + a^{17})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+ $$

$$ +\frac{131072(137119595809968000b + 873188371313197056 0a^2b - 2360644428997066752a^3b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+ $$

$$ +\frac{131072(2267279959510548480a^4b - 332504193149960192a^5b + 113489856414113792a^6b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+ $$

$$ +\frac{131072(-9741187137183744a^7b + 1633123923118336a^8b - 84542302806016a^9b + 7814003087616a^{10}b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+ $$

$$ +\frac{131072(-239628553216a^{11}b + 12599892448a^{12}b - 209966592a^{13}b + 6046832a^{14}b - 41888a^{15}b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+ $$

$$+\frac{131072(561a^{16}b+2317820965473484800b^2+873188371313197056 0ab^2+6586791913742008320a^3b^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(-6415929250932326 40a^4b^2+6503087336912404 48a^5b^2-4484474106792345 6a^6b^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(16064617532609536a^7b^2-716669564497920a^8b^2+124598817697024a^9b^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(-3416752171008a^{10}b^2+322510879808a^{11}b^2-4919566080a^{12}b^2+258685328a^{13}b^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(-1669536a^{14}b^2+46376a^{15}b^2+1687424939411374080b^3+2360644428997066752ab^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(6586791913742008320a^2b^3+1475014418219950080a^4b^3-66433020866600960a^5b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(66036533182833664a^6b^3-2389236717666304a^7b^3+844644599515904a^8b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(-20324619617280a^9b^3+3472660479552a^{10}b^3-48420456448a^{11}b^3+4440873008a^{12}b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(-26898080a^{13}b^3+1344904a^{14}b^3+713340918791405568b^4+2267279959510548480ab^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(641592925093232640a^2b^4+1475014418219950080a^3b^4+130395697879248384a^5b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(-3018527499018240a^6b^4+2849386432882176a^7b^4-55437330212352a^8b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(18754939121952a^9b^4-230462749440a^{10}b^4+37583709072a^{11}b^4-209805024a^{12}b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(18156204a^{13}b^{4}+198107173820104704b^{5}+332504193149960192ab^{5})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(650308733691240448a^{2}b^{5}+664330208666600960a^{3}b^{5}+130395697879248384a^{4}b^{5})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(5152732631861760a^{6}b^{5}-63011227023360a^{7}b^{5}+55557257991840a^{8}b^{5}-548720832000a^{9}b^{5})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(174501294000a^{10}b^{5}-858293280a^{11}b^{5}+131128140a^{12}b^{5}+38628489336586240b^{6})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(113489856414113792ab^{6}+448447410679234 56a^{2}b^{6}+66036533182833664a^{3}b^{6})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(3018527499018240a^{4}b^{6}+5152732631861760a^{5}b^{6}+94632071909760a^{7}b^{6}-579061866240a^{8}b^{6})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(472343733840a^{9}b^{6}-1855967520a^{10}b^{6}+548354040a^{11}b^{6}+5503512041431040b^{7})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(9741187137183744ab^{7}+16064617532609536a^{2}b^{7}+2389236717666304a^{3}b^{7})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(2849386432882176a^{4}b^{7}+63011227023360a^{5}b^{7}+94632071909760a^{6}b^{7}+771154504560a^{8}b^{7})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(-1855967520a^{9}b^{7}+1391975640a^{10}b^{7}+587213457166336b^{8}+1633123923118336ab^{8})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(716669564497920a^{2}b^{8}+844644599515904a^{3}b^{8}+55437330212352a^{4}b^{8}+55557257991840a^{5}b^{8})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(579061866240a^{6}b^{8}+771154504560a^{7}b^{8}+2203961430a^{9}b^{8}+4760401345 3568b^{9})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(84542302806016ab^9+124598817697024a^2b^9+20324619617280a^3b^9+18754939121952a^4b^9)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(548720832000a^5b^9+472343733840a^6b^9+1855967520a^7b^9+2203961430a^8b^9)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(29513164459520b^{10}+7814003087616ab^{10}+3416752171008a^2b^{10}+3472660479552a^3b^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(230462749440a^4b^{10}+174501294000a^5b^{10}+1855967520a^6b^{10}+1391975640a^7b^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(1398420108880b^{11}+239628553216ab^{11}+322510879808a^2b^{11}+48420456448a^3b^{11})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(37583709072a^4b^{11}+858293280a^5b^{11}+548354040a^6b^{11}+5022477824b^{12}+12599892448ab^{12})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(4919566080a^2b^{12}+4440873008a^3b^{12}+209805024a^4b^{12}+131128140a^5b^{12}+134304352b^{13})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(209966592ab^{13}+258685328a^2b^{13}+26898080a^3b^{13}+18156204a^4b^{13}+2589440b^{14})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(6046832ab^{14}+1669536a^2b^{14}+1344904a^3b^{14}+34000b^{15}+41888ab^{15}+46376a^2b^{15})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(272b^{16}+561ab^{16}+b^{17})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144b(13711959580999680000+45446360098406400 0a+24085837498023936 00a^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(4097182965474263040a^3+3714503748312760320a^4+372095562202480640a^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(13119256415248384a^{6}+816573706268672a^{7}+143071677346048a^{8}+5580081723392a^{9})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(536072618752a^{10}+12670987264a^{11}+683148128a^{12}+8854144a^{13}+254864a^{14}+1360a^{15})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(17a^{16}-4544636009840640000b+5602141152938557440ab+5304437490749276 16a^{2}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(2236858534136119296a^{3}b+1819759877033820 16a^{4}b+150102662129467392a^{5}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(8324773808992256a^{6}b+2766373471713280a^{7}b+100364553873408a^{8}b+1671843229337 6a^{9}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(377613378560a^{10}b+34335380352a^{11}b+434701696a^{12}b+21799712a^{13}b+116688a^{14}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(2992a^{15}b+2408583749802393600b^{2}-530443749074927616ab^{2}+383556122913787084 8a^{2}b^{2})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(1541318760089518 08a^{3}b^{2}+546583153785495552a^{4}b^{2}+237724992805724 16a^{5}b^{2})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(17769653514624000a^{6}b^{2}+567373035896832a^{7}b^{2}+175071078614784a^{8}b^{2})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(3666336672768a^{9}b^{2}+570176595648a^{10}b^{2}+6908438784a^{11}b^{2}+582389808a^{12}b^{2})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(3060816a^{13}b^{2}+139128a^{14}b^{2}-4097182965474263 04b^{3}+2236858534136119296ab^{3})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-1541318760089518 08a^{2}b^{3}+824706375706312704a^{3}b^{3}+165738860474654 72a^{4}b^{3})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(50743149268748288a^5b^3+1238566414041088a^6b^3+837876594497536a^7b^3)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(15271496942592a^8b^3+4336637568384a^9b^3+48502617856a^{10}b^3+6955354432a^{11}b^3)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(34967504a^{12}b^3+2689808a^{13}b^3+371450374831276032b^4-181975987703382016ab^4)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(546583153785495552a^2b^4-16573886047465472a^3b^4+713330806664002 56a^4b^4)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(771002818375680a^5b^4+2068758086579712a^6b^4+28279271725056a^7b^4)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(17304861730464a^8b^4+166875688320a^9b^4+43340876304a^{10}b^4+200268432a^{11}b^4)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(26225628a^{12}b^4-37209556220248064b^5+150102662129467392ab^5-23772499280572416a^2b^5)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(50743149268748288a^3b^5-771002818375680a^4b^5+2781395797847040a^5b^5)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(16350893475840a^6b^5+388137775449 60a^7b^5+276657828480a^8b^5+153076973280a^9b^5)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(605206800a^{10}b^5+143048880a^{11}b^5+131192564152483 84b^6-8324773808992256ab^6)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(17769653514624000a^2b^6-1238566414041088a^3b^6+2068758086579712a^4b^6)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(-16350893475840a^5b^6+50631230643840a^6b^6+151970987520a^7b^6+321136968240a^8b^6)}{\left[\prod_{\zeta=0}^{17}\left\{a-b-2\zeta\right\}\right]\left[\prod_{\xi=1}^{16}\left\{a-b+2\xi\right\}\right]}+$$

$$+\frac{262144b(927983760a^9b^6+463991880a^10b^6-816573706268672b^7+2766373471713280ab^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-567373035896832a^2b^7+837876594497536a^3b^7-282792717250 56a^4b^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(38813777544960a^5b^7-151970987520a^6b^7+410059647360a^7b^7+491285520a^8b^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(927983760a^9b^7+1430716773460048b^8-100364553873408ab^8+175071078614784a^2b^8)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-15271496942592a^3b^8+17304861730464a^4b^8-276657828480a^5b^8+321136968240a^6b^8)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-491285520a^7b^8+1166803110a^8b^8-5580081723392b^9+16718432293376ab^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-3666336672768a^2b^9+4336637568384a^3b^9-166875688320a^4b^9+153076973280a^5b^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-927983760a^6b^9+927983760a^7b^9+536072618752b^{10}-377613378560ab^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(570176595648a^2b^{10}-48502617856a^3b^{10}+43340876304a^4b^{10}-605206800a^5b^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(463991880a^6b^{10}-126709872264b^{11}+34335380352ab^{11}-6908438784a^2b^{11}+6955354432a^3b^{11})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-200268432a^4b^{11}+143048880a^5b^{11}+6831481 28b^{12}-434701696ab^{12}+582389808a^2b^{12})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(-34967504a^3b^{12}+26225628a^4b^{12}-8854144b^{13}+21799712ab^{13}-3060816a^2b^{13})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144b(2689808a^3b^{13}+254864b^{14}-116688ab^{14}+139128a^2b^{14}-1360b^{15}+2992ab^{15}+17b^{16})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{262144a(1371195958099968000-454463600984064000a+240858374980239360 0a^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(-4097182965474263 04a^3+371450374831276032a^4-37209556220248064a^5)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(1311925641524838 4a^6-816573706268672a^7+14307167734604 8a^8-5580081723392a^9)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(536072618752a^{10}-12670987264a^{11}+683148128a^{12}-8854144a^{13}+254864a^{14}-1360a^{15})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(17a^{16}+454463600984064000b+560214115293855744 0ab-530443749074927616a^2b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(223685853413611929 6a^3b-1819759877033820 16a^4b+150102662129467392a^5b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(-8324773808992256a^6b+2766373471713280a^7b-100364553873408a^8b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(1671843229337 6a^9b-377613378560a^{10}b+34335380352a^{11}b-434701696a^{12}b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(21799712a^{13}b-116688a^{14}b+2992a^{15}b+240858374980239360 0b^2+530443749074927616ab^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(383556122913787084 8a^2b^2-154131876008951808a^3b^2+546583153785495552a^4b^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(-23772499280572416a^5b^2+1776965351462400 0a^6b^2-567373035896832a^7b^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(175071078614784a^8b^2-3666336672768a^9b^2+570176595648a^{10}b^2-6908438784a^{11}b^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(582389808a^{12}b^2-3060816a^{13}b^2+139128a^{14}b^2+409718296547426304b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(2236858534136119296ab^3+1541318760089518080a^2b^3+824706375706312704a^3b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(-165738860474654720a^4b^3+507431492687482880a^5b^3-12385664140410880a^6b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(837876594497536a^7b^3-15271496942592a^8b^3+4336637568384a^9b^3-48502617856a^{10}b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(6955354432a^{11}b^3-34967504a^{12}b^3+2689808a^{13}b^3+371450374831276032b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(1819759877033820160ab^4+546583153785495552a^2b^4+165738860474654720a^3b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(71333080666400256a^4b^4-771002818375680a^5b^4+20687580865797120a^6b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(-282792717250560a^7b^4+173048617304640a^8b^4-166875688320a^9b^4+43340876304a^{10}b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(-200268432a^{11}b^4+26225628a^{12}b^4+37209556220248064b^5+150102662129467392ab^5)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(237724992805724160a^2b^5+507431492687482880a^3b^5+771002818375680a^4b^5)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(2781395797847040a^5b^5-163508934758400a^6b^5+38813777544960a^7b^5-276657828480a^8b^5)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(153076973280a^9b^5-605206800a^{10}b^5+143048880a^{11}b^5+13119256415248384b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(8324773808992256ab^6+17769653514624000a^2b^6+12385664140410 88a^3b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(2068758086579712a^4b^6+16350893475840a^5b^6+50631230643840a^6b^6-151970987520a^7b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(321136968240a^8b^6-927983760a^9b^6+463991880a^{10}b^6+8165737062686 72b^7)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(2766373471713280ab^7+567373035896832a^2b^7+837876594497536a^3b^7)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(28279271725056a^4b^7+38813777544960a^5b^7+151970987520a^6b^7+410059647360a^7b^7)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(-491285520a^8b^7+927983760a^9b^7+143071677346048b^8+100364553873408ab^8)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(175071078614784a^2b^8+15271496942592a^3b^8+17304861730464a^4b^8+276657828480a^5b^8)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(321136968240a^6b^8+491285520a^7b^8+1166803110a^8b^8+5580081723392b^9)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(16718432293376ab^9+3666336672768a^2b^9+4336637568384a^3b^9+166875688320a^4b^9)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(153076973280a^5b^9+927983760a^6b^9+927983760a^7b^9+536072618752b^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(377613378560ab^{10}+570176595648a^2b^{10}+48502617856a^3b^{10}+43340876304a^4b^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(605206800a^5b^{10}+463991880a^6b^{10}+12670987264b^{11}+34335380352ab^{11}+6908438784a^2b^{11})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(6955354432a^3b^{11}+200268432a^4b^{11}+143048880a^5b^{11}+6831148128b^{12}+434701696ab^{12})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(582389808a^2b^{12}+34967504a^3b^{12}+26225628a^4b^{12}+8854144b^{13}+21799712ab^{13})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(3060816a^2b^{13}+2689808a^3b^{13}+254864b^{14}+116688ab^{14}+139128a^2b^{14}+1360b^{15})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144a(2992ab^{15}+17b^{16})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{131072(137119595809968000a+231782096547348480 0a^2+168742493941137408 0a^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(713340918791405568a^4+198107173820104704a^5+38628489336586240a^6)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(5503512041431040a^7+587213457166336a^8+47604013453568a^9+2951316459520a^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(139842010880a^{11}+5022477824a^{12}+134304352a^{13}+2589440a^{14}+34000a^{15}+272a^{16}+a^{17})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(137119595809968000b+873188371313197056 0a^2b+236064442899706675 2a^3b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(2267279959510548480a^4b+332504193149960192a^5b+1134898564141137 92a^6b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(9741187137183744a^7b+1633123923118336a^8b+84542302806016a^9b+7814003087616a^{10}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(239628553216a^{11}b+12599892448a^{12}b+209966592a^{13}b+6046832a^{14}b+41888a^{15}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(561a^{16}b-2317820965473484800b^2+8731883713131970560ab^2+6586791913742008320a^3b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(6415929250932326400a^4b^2+6503087336912404448a^5b^2+4484474106792345 6a^6b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(160646175326095 36a^7b^2+716669564497920a^8b^2+124598817697024a^9b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(3416752171008a^{10}b^2+322510879808a^{11}b^2+4919566080a^{12}b^2+258685328a^{13}b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(1669536a^{14}b^2+46376a^{15}b^2+1687424939411374080b^3-2360644428997066752ab^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(6586791913742008320a^2b^3+1475014418219950080a^4b^3+664330208666009 60a^5b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(66036533182833664a^6b^3+2389236717666304a^7b^3+844644599515904a^8b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(20324619617280a^9b^3+3472660479552a^{10}b^3+48420456448a^{11}b^3+4440873008a^{12}b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(26898080a^{13}b^3+1344904a^{14}b^3-713340918791405568b^4+2267279959510548480ab^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-6415929250932326400a^2b^4+1475014418219950080a^3b^4+130395697879248384a^5b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(3018527499018240a^6b^4+2849386432882176a^7b^4+55437330212352a^8b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(18754939121952a^9b^4+230462749440a^{10}b^4+37583709072a^{11}b^4+209805024a^{12}b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(18156204a^1 3b^4+198107173820104704b^5-332504193149960192ab^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(650308733691240448a^2b^5-664330208666009 60a^3b^5+130395697879248384a^4b^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(5152732631861760a^6b^5+63011227023360a^7b^5+55557257991840a^8b^5+548720832000a^9b^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(174501294000a^{10}b^5+858293280a^{11}b^5+131128140a^{12}b^5-386284893365862 40b^6)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(1134898564141137 92ab^6-448447410679234 56a^2b^6+660365331828336 64a^3b^6)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-3018527499018240a^4b^6+5152732631861760a^5b^6+94632071909760a^7b^6+579061866240a^8b^6)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(472343733840a^9b^6+1855967520a^1 0b^6+548354040a^1 1b^6+5503512041431040b^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-9741187137183744ab^7+16064617532609536a^2b^7-2389236717666304a^3b^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(2849386432882176a^4b^7-63011227023360a^5b^7+94632071909760a^6b^7+771154504560a^8b^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(1855967520a^9b^7+1391975640a^1 0b^7-5872134571663 36b^8+1633123923118336ab^8)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-716669564497920a^2b^8+844644599515904a^3b^8-55437330212352a^4b^8+55557257991840a^5b^8)}{\left[\prod_{\xi=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-579061866240a^6b^8+771154504560a^7b^8+2203961430a^9b^8+47604013453568b^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-84542302806016ab^9+124598817697024a^2b^9-20324619617280a^3b^9+18754939121952a^4b^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-548720832000a^5b^9+472343733840a^6b^9-1855967520a^7b^9+2203961430a^8b^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-2951316459520b^{10}+7814003087616ab^{10}-3416752171008a^2b^{10}+3472660479552a^3b^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-230462749440a^4b^{10}+174501294000a^5b^{10}-1855967520a^6b^{10}+1391975640a^7b^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(1398420108880b^{11}-239628553216ab^{11}+322510879808a^2b^{11}-48420456448a^3b^{11})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(37583709072a^4b^{11}-858293280a^5b^{11}+548354040a^6b^{11}-5022477824b^{12}+12599892448ab^{12})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-4919566080a^2b^{12}+4440873008a^3b^{12}-209805024a^4b^{12}+131128140a^5b^{12}+134304352b^{13})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-209966592ab^{13}+258685328a^2b^{13}-26898080a^3b^{13}+18156204a^4b^{13}-2589440b^{14})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(6046832ab^{14}-1669536a^2b^{14}+1344904a^3b^{14}+34000b^{15}-41888ab^{15}+46376a^2b^{15})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{131072(-272b^{16}+561ab^{16}+b^{17})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}\Bigg\}\Bigg] \tag{14}$$

III. Evaluation of Main Summation Formula (14)

Substituting $c=\frac{a+b+36}{2}$ and $z=\frac{1}{2}$ in equation (8), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix}a,\ b & ; & \frac{1}{2}\\ \frac{a+b+36}{2} & ; & \end{matrix}\right]=a\ {}_2F_1\left[\begin{matrix}a+1,\ b\ ; & \frac{1}{2}\\ \frac{a+b+36}{2}\ ; & \end{matrix}\right]-b\ {}_2F_1\left[\begin{matrix}a,\ b+1 & ; & \frac{1}{2}\\ \frac{a+b+36}{2} & ; & \end{matrix}\right]$$

Now involving the derived result from Gauss second summation theorem, we get

$$L.H.S=a\,\frac{2^b\ \Gamma(\frac{a+b+36}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+2}{2})}\left\{\frac{65536(1371195958099968000a-2317820965473484800a^2)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+\right.\right.$$

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$$+\frac{65536(1687424939411374080a^{3}-713340918791405568a^{4}+198107173820104704a^{5})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-38628489336586240a^{6}+5503512041431040a^{7}-587213457166336a^{8}+47604013453568a^{9})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-2951316459520a^{10}+139842010880a^{11}-5022477824a^{12}+134304352a^{13}-2589440a^{14})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(34000a^{15}-272a^{16}+a^{17}+1371195958099968000b+8731883713131970560a^{2}b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-2360644428997066752a^{3}b+2267279959510548480a^{4}b-332504193149960192a^{5}b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(113489856414113792a^{6}b-9741187137183744a^{7}b+1633123923118336a^{8}b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-84542302806016a^{9}b+7814003087616a^{10}b-239628553216a^{11}b+12599892448a^{12}b)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-209966592a^{13}b+6046832a^{14}b-41888a^{15}b+561a^{16}b+2317820965473484800b^{2})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(8731883713131970560ab^{2}+6586791913742008320a^{3}b^{2}-6415929250932326400a^{4}b^{2})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(650308733691240448a^{5}b^{2}-448447410679234560a^{6}b^{2}+16064617532609536a^{7}b^{2})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-716669564497920a^{8}b^{2}+124598817697024a^{9}b^{2}-3416752171008a^{10}b^{2}+322510879808a^{11}b^{2})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-4919566080a^{12}b^{2}+258685328a^{13}b^{2}-1669536a^{14}b^{2}+46376a^{15}b^{2}+1687424939411374080b^{3})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(2360644428997066752ab^{3}+6586791913742008320a^{2}b^{3}+14750144182199500800a^{4}b^{3})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-66433020866600960a^5b^3+66036533182833664a^6b^3-2389236717666304a^7b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(844644599515904a^8b^3-20324619617280a^9b^3+3472660479552a^{10}b^3-48420456448a^{11}b^3)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(4440873008a^{12}b^3-26898080a^{13}b^3+1344904a^{14}b^3+713340918791405568b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(2267279959510548480ab^4+641592925093232640a^2b^4+1475014418219950080a^3b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(130395697879248384a^5b^4-3018527499018240a^6b^4+2849386432882176a^7b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-55437330212352a^8b^4+18754939121952a^9b^4-230462749440a^{10}b^4+37583709072a^{11}b^4)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-209805024a^{12}b^4+18156204a^{13}b^4+1981071738201047040b^5+332504193149960192ab^5)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(650308733691240448a^2b^5+66433020866600960a^3b^5+130395697879248384a^4b^5)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(5152732631861760a^6b^5-63011227023360a^7b^5+55557257991840a^8b^5-548720832000a^9b^5)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(174501294000a^{10}b^5-858293280a^{11}b^5+131128140a^{12}b^5+386284893365862400b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(113489856414113792ab^6+448447410679234560a^2b^6+66036533182833664a^3b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(3018527499018240a^4b^6+5152732631861760a^5b^6+94632071909760a^7b^6-579061866240a^8b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(472343733840a^9b^6-1855967520a^{10}b^6+548354040a^{11}b^6+55035120414310400b^7)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(9741187137183744ab^{7}+16064617532609536a^{2}b^{7}+2389236717666304a^{3}b^{7})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(2849386432882176a^{4}b^{7}+63011227023360a^{5}b^{7}+94632071909760a^{6}b^{7}+771154504560a^{8}b^{7})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(-1855967520a^{9}b^{7}+1391975640a^{10}b^{7}+587213457166336b^{8}+1633123923118336ab^{8})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(716669564497920a^{2}b^{8}+844644599515904a^{3}b^{8}+55437330212352a^{4}b^{8}+55557257991840a^{5}b^{8})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(579061866240a^{6}b^{8}+771154504560a^{7}b^{8}+2203961430a^{9}b^{8}+47604013453568b^{9})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(84542302806016ab^{9}+124598817697024a^{2}b^{9}+20324619617280a^{3}b^{9}+18754939121952a^{4}b^{9})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(548720832000a^{5}b^{9}+472343733840a^{6}b^{9}+1855967520a^{7}b^{9}+2203961430a^{8}b^{9})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(2951316459520b^{10}+7814003087616ab^{10}+3416752171008a^{2}b^{10}+3472660479552a^{3}b^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(230462749440a^{4}b^{10}+174501294000a^{5}b^{10}+1855967520a^{6}b^{10}+1391975640a^{7}b^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(139842010880b^{11}+239628553216ab^{11}+322510879808a^{2}b^{11}+48420456448a^{3}b^{11})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(37583709072a^{4}b^{11}+858293280a^{5}b^{11}+548354040a^{6}b^{11}+5022477824b^{12}+12599892448ab^{12})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(4919566080a^{2}b^{12}+4440873008a^{3}b^{12}+209805024a^{4}b^{12}+131128140a^{5}b^{12}+134304352b^{13})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(209966592ab^{13}+258685328a^{2}b^{13}+26898080a^{3}b^{13}+18156204a^{4}b^{13}+2589440b^{14})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{65536(6046832ab^{14}+1669536a^2b^{14}+1344904a^3b^{14}+34000b^{15}+41888ab^{15}+46376a^2b^{15})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{65536(272b^{16}+561ab^{16}+b^{17})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{262144(13711959580999680000-4544636009840640000a+2408583749802393600a^2)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}$$

$$+\frac{262144(-409718296547426304a^3+371450374831276032a^4-37209556220248064a^5)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(13119256415248384a^6-816573706268672a^7+143071677346048a^8-5580081723392a^9)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(536072618752a^{10}-12670987264a^{11}+683148128a^{12}-8854144a^{13}+254864a^{14}-1360a^{15})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(17a^{16}+4544636009840640000b+5602141152938557440ab-5304437490749276160a^2b)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(2236858534136119296a^3b-1819759877033820160a^4b+150102662129467392a^5b)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-8324773808992256a^6b+2766373471713280a^7b-100364553873408a^8b+16718432293376a^9b)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-377613378560a^{10}b+34335380352a^{11}b-434701696a^{12}b+21799712a^{13}b-116688a^{14}b)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(2992a^{15}b+2408583749802393600b^2+5304437490749276160ab^2+3835561229137870848a^2b^2)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-1541318760089518080a^3b^2+546583153785495552a^4b^2-23772499280572416a^5b^2)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(17769653514624000a^6b^2-567373035896832a^7b^2+175071078614784a^8b^2)}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-3666336672768a^{9}b^{2}+570176595648a^{10}b^{2}-6908438784a^{11}b^{2}+582389808a^{12}b^{2})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-3060816a^{13}b^{2}+139128a^{14}b^{2}+4097182965474263046b^{3}+22368585341361192966ab^{3})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(15413187600895180 8a^{2}b^{3}+824706375706312704a^{3}b^{3}-16573886047465472a^{4}b^{3})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(50743149268748288a^{5}b^{3}-1238566414041088a^{6}b^{3}+837876594497536a^{7}b^{3})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-15271496942592a^{8}b^{3}+4336637568384a^{9}b^{3}-48502617856a^{10}b^{3}+6955354432a^{11}b^{3})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-34967504a^{12}b^{3}+2689808a^{13}b^{3}+371450374831276032b^{4}+181975987703382016ab^{4})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(546583153785495552a^{2}b^{4}+16573886047465472a^{3}b^{4}+71333080666400256a^{4}b^{4})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-771002818375680a^{5}b^{4}+2068758086579712a^{6}b^{4}-28279271725056a^{7}b^{4})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(17304861730464a^{8}b^{4}-166875688320a^{9}b^{4}+43340876304a^{10}b^{4}-200268432a^{11}b^{4})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(26225628a^{1}2b^{4}+3720955622024806 4b^{5}+150102662129467392ab^{5}+237724992805724 16a^{2}b^{5})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(50743149268748288a^{3}b^{5}+771002818375680a^{4}b^{5}+2781395797847040a^{5}b^{5})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-16350893475840a^{6}b^{5}+38813777544960a^{7}b^{5}-276657828480a^{8}b^{5}+153076973280a^{9}b^{5})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

$$+\frac{262144(-605206800a^{10}b^{5}+143048880a^{11}b^{5}+13119256415248384b^{6}+8324773808992256ab^{6})}{\left[\prod_{\rho=0}^{16}\left\{a-b-2\rho\right\}\right]\left[\prod_{\tau=1}^{17}\left\{a-b+2\tau\right\}\right]}+$$

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$$+\frac{262144(17769653514624000a^2b^6+1238566414041088a^3b^6+206875808657971 2a^4b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(16350893475840a^5b^6+50631230643840a^6b^6-151970987520a^7b^6+321136968240a^8b^6)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(-927983760a^9b^6+463991880a^{10}b^6+816573706268672b^7+2766373471713280ab^7)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(567373035896832a^2b^7+837876594497536a^3b^7+282792717250 56a^4b^7+38813777544960a^5b^7)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(151970987520a^6b^7+410059647360a^7b^7-491285520a^8b^7+927983760a^9b^7)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(1430716773460 48b^8+100364553873408ab^8+175071078614784a^2b^8+15271496942592a^3b^8)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(17304861730464a^4b^8+276657828480a^5b^8+321136968240a^6b^8+491285520a^7b^8)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(1166803110a^8b^8+5580081723392b^9+16718432293376ab^9+3666336672768a^2b^9)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(4336637568384a^3b^9+166875688320a^4b^9+153076973280a^5b^9+927983760a^6b^9)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(927983760a^7b^9+536072618752b^{10}+377613378560ab^{10}+570176595648a^2b^{10})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(48502617856a^3b^{10}+43340876304a^4b^{10}+605206800a^5b^{10}+463991880a^6b^{10}+12670987264b^{11})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(34335380352ab^{11}+6908438784a^2b^{11}+6955354432a^3b^{11}+200268432a^4b^{11}+143048880a^5b^{11})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(683148128b^{12}+434701696ab^{12}+582389808a^2b^{12}+34967504a^3b^{12}+26225628a^4b^{12})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(8854144b^{13}+21799712ab^{13}+3060816a^2b^{13}+2689808a^3b^{13}+254864b^{14}+116688ab^14)}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}+$$

$$+\frac{262144(139128a^2b^{14}+1360b^{15}+2992ab^{15}+17b^{16})}{\left[\prod_{\rho=0}^{16}\{a-b-2\rho\}\right]\left[\prod_{\tau=1}^{17}\{a-b+2\tau\}\right]}\Bigg\}\Bigg]-$$

$$-\frac{2^{b+1}\ \Gamma(\frac{a+b+36}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+1}{2})}\Bigg\{\frac{65536(1371195958099968000a+2317820965473484800a^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+\right.$$

$$+\frac{65536(1687424939411374080a^3+713340918791405568a^4+198107173820104704a^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(38628489336586240a^6+5503512041431040a^7+587213457166336a^8+47604013453568a^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(2951316459520a^{10}+139842010880a^{11}+5022477824a^{12}+134304352a^{13}+2589440a^{14})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(34000a^{15}+272a^{16}+a^{17}+1371195958099968000b+8731883713131970560a^2b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(2360644428997066752a^3b+2267279959510548480a^4b+332504193149960192a^5b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(1134898564141113792a^6b+9741187137183744a^7b+1633123923118336a^8b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(84542302806016a^9b+7814003087616a^{10}b+239628553216a^{11}b+12599892448a^{12}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(209966592a^{13}b+6046832a^{14}b+41888a^{15}b+561a^{16}b-2317820965473484800b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(8731883713131970560ab^2+6586791913742008320a^3b^2+6415929250932326\,40a^4b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(650308733691240448a^5b^2+44844741067923456a^6b^2+16064617532609536a^7b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(716669564497920a^8b^2+124598817697024a^9b^2+3416752171008a^{10}b^2+322510879808a^{11}b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(4919566080a^{12}b^2+258685328a^{13}b^2+1669536a^{14}b^2+46376a^{15}b^2+1687424939411374080b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-2360644428997066752ab^3+6586791913742008320a^2b^3+1475014418219950080a^4b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(66433020866600960a^5b^3+66036533182833664a^6b^3+2389236717666304a^7b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(844644599515904a^8b^3+20324619617280a^9b^3+3472660479552a^{10}b^3+48420456448a^{11}b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(4440873008a^{12}b^3+26898080a^{13}b^3+1344904a^{14}b^3-713340918791405568b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(2267279959510548480ab^4-641592925093232640a^2b^4+1475014418219950080a^3b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(130395697879248384a^5b^4+3018527499018240a^6b^4+2849386432882176a^7b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(55437330212352a^8b^4+18754939121952a^9b^4+230462749440a^{10}b^4+37583709072a^{11}b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(209805024a^{12}b^4+18156204a^{13}b^4+198107173820104704b^5-332504193149960192ab^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(650308733691240448a^2b^5-66433020866600960a^3b^5+130395697879248384a^4b^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(5152732631861760a^{6}b^{5}+63011227023360a^{7}b^{5}+55557257991840a^{8}b^{5}+548720832000a^{9}b^{5})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(174501294000a^{10}b^{5}+858293280a^{11}b^{5}+131128140a^{12}b^{5}-3862848933658624 0b^{6})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(11348985641411379 2ab^{6}-4484474106792345 6a^{2}b^{6}+6603653318283366 4a^{3}b^{6})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-3018527499018240a^{4}b^{6}+5152732631861760a^{5}b^{6}+94632071909760a^{7}b^{6}+579061866240a^{8}b^{6})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(472343733840a^{9}b^{6}+1855967520a^{10}b^{6}+548354040a^{11}b^{6}+5503512041431040b^{7})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-9741187137183744ab^{7}+16064617532609536a^{2}b^{7}-2389236717666304a^{3}b^{7})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(2849386432882176a^{4}b^{7}-63011227023360a^{5}b^{7}+94632071909760a^{6}b^{7}+771154504560a^{8}b^{7})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(1855967520a^{9}b^{7}+1391975640a^{1}0b^{7}-587213457166336b^{8}+1633123923118336ab^{8})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-716669564497920a^{2}b^{8}+844644599515904a^{3}b^{8}-55437330212352a^{4}b^{8}+55557257991840a^{5}b^{8})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-579061866240a^{6}b^{8}+771154504560a^{7}b^{8}+2203961430a^{9}b^{8}+47604013453568b^{9})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-84542302806016ab^{9}+124598817697024a^{2}b^{9}-20324619617280a^{3}b^{9}+18754939121952a^{4}b^{9})}{\left[\prod^{17}\{a_{\zeta=0}-b-2\zeta\}\right]\left[\prod^{16}\{a_{\xi=1}-b+2\xi\}\right]}+$$

$$+\frac{65536(-548720832000a^{5}b^{9}+472343733840a^{6}b^{9}-1855967520a^{7}b^{9}+2203961430a^{8}b^{9})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-2951316459520b^{10}+7814003087616ab^{10}-3416752171008a^{2}b^{10}+3472660479552a^{3}b^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{65536(-230462749440a^4b^{10}+174501294000a^5b^{10}-1855967520a^6b^{10}+1391975640a^7b^{10})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{65536(139842010880b^{11}-239628553216ab^{11}+322510879808a^2b^{11}-48420456448a^3b^{11})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{65536(37583709072a^4b^{11}-858293280a^5b^{11}+548354040a^6b^{11}-5022477824b^{12}+12599892448ab^{12})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{65536(-4919566080a^2b^{12}+4440873008a^3b^{12}-209805024a^4b^{12}+131128140a^5b^{12}+134304352b^{13})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{65536(-209966592ab^{13}+258685328a^2b^{13}-26898080a^3b^{13}+18156204a^4b^{13}-2589440b^{14})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{65536(6046832ab^{14}-1669536a^2b^{14}+1344904a^3b^{14}+34000b^{15}-41888ab^{15}+46376a^2b^{15})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{65536(-272b^{16}+561ab^{16}+b^{17})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{262144(13711959580999680000+45446360098406400a+24085837498023936000a^2)}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}$$

$$+\frac{262144(+4097182965474263040a^3+371450374831276032a^4+37209556220248064a^5)}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{262144(1311925641524838 4a^6+81657370626867 2a^7+143071677346048a^8+5580081723392a^9)}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{262144(536072618752a^{10}+12670987264a^{11}+683148128a^{12}+8854144a^{13}+254864a^{14}+1360a^{15})}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{262144(17a^{16}-45446360098406400 0b+56021411529385574 40ab+53044374907492761 6a^2b)}{\Big[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\Big]\Big[\prod_{\xi=1}^{16}\{a-b+2\xi\}\Big]}+$$

$$+\frac{262144(2236858534136119296a^3b+1819759877033820l6a^4b+150102662129467392a^5b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(8324773808992256a^6b+2766373471713280a^7b+100364553873408a^8b+16718432293376a^9b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(377613378560a^{10}b+34335380352a^{11}b+434701696a^{12}b+21799712a^{13}b+116688a^{14}b)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(2992a^{15}b+240858374980239360 0b^2-5304437490749276l6ab^2+3835561229137870848a^2b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(1541318760089518 08a^3b^2+546583153785495552a^4b^2+23772499280572416a^5b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(17769653514624000a^6b^2+567373035896832a^7b^2+175071078614784a^8b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(3666336672768a^9b^2+570176595648a^{10}b^2+6908438784a^{11}b^2+582389808a^{12}b^2)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(3060816a^{13}b^2+139128a^{14}b^2-4097182965474263040b^3+2236858534136119296ab^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-1541318760089518 08a^2b^3+824706375706312704a^3b^3+165738860474654 72a^4b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(50743149268748288a^5b^3+1238566414041088a^6b^3+837876594497536a^7b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(15271496942592a^8b^3+4336637568384a^9b^3+48502617856a^{10}b^3+6955354432a^{11}b^3)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(34967504a^{12}b^3+2689808a^{13}b^3+3714503748312760320b^4-1819759877033820l6ab^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(546583153785495552a^2b^4-16573886047465472a^3b^4+7133308066640025 6a^4b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

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$$+\frac{262144(771002818375680a^5b^4+2068758086579712a^6b^4+28279271725056a^7b^4+17304861730464a^8b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(166875688320a^9b^4+43340876304a^{10}b^4+200268432a^{11}b^4+26225628a^{12}b^4)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-3720955622024806 4b^5+150102662129467392ab^5-23772499280572416a^2b^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(5074314926874828 8a^3b^5-771002818375680a^4b^5+2781395797847040a^5b^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(16350893475840a^6b^5+38813777544960a^7b^5+276657828480a^8b^5+153076973280a^9b^5)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(605206800a^{10}b^5+143048880a^{11}b^5+13119256415248384b^6-8324773808992256ab^6)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(17769653514624000a^2b^6-1238566414041088a^3b^6+2068758086579712a^4b^6)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-16350893475840a^5b^6+50631230643840a^6b^6+151970987520a^7b^6+321136968240a^8b^6)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(927983760a^9b^6+463991880a^{10}b^6-816573706268672b^7+2766373471713280ab^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-567373035896832a^2b^7+837876594497536a^3b^7-28279271725056a^4b^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(38813777544960a^5b^7-151970987520a^6b^7+410059647360a^7b^7+491285520a^8b^7)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(927983760a^9b^7+143071677346048b^8-100364553873408ab^8+175071078614784a^2b^8)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-15271496942592a^3b^8+17304861730464a^4b^8-276657828480a^5b^8+321136968240a^6b^8)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-491285520a^7b^8+1166803110a^8b^8-5580081723392b^9+16718432293376ab^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-3666336672768a^2b^9+4336637568384a^3b^9-166875688320a^4b^9+153076973280a^5b^9)}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-927983760a^6b^9+927983760a^7b^9+536072618752b^{10}-377613378560ab^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(570176595648a^2b^{10}-48502617856a^3b^{10}+43340876304a^4b^{10}-605206800a^5b^{10})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(463991880a^6b^{10}-126709872264b^{11}+34335380352ab^{11}-6908438784a^2b^{11}+6955354432a^3b^{11})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-200268432a^4b^{11}+143048880a^5b^{11}+6831481128b^{12}-434701696ab^{12}+582389808a^2b^{12})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$+\frac{262144(-34967504a^3b^{12}+26225628a^4b^{12}-8854144b^{13}+21799712ab^{13}-3060816a^2b^{13})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}+$$

$$\left.\left.+\frac{262144(2689808a^3b^{13}+254864b^{14}-116688ab^{14}+139128a^2b^{14}-1360b^{15}+2992ab^{15}+17b^{16})}{\left[\prod_{\zeta=0}^{17}\{a-b-2\zeta\}\right]\left[\prod_{\xi=1}^{16}\{a-b+2\xi\}\right]}\right\}\right]$$

On simplification the result (14) is derived.

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Analytical Solutions for Some of the Nonlinear Hyperbolic-Like Equations with Variable Coefficients

By R. Rajaraman
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Abstract - In this work Homotopy Analysis Method(HAM) is used for analytic treatment of the nonlinear hyperbolic-like equations with variable coefficients. This method can provide analytical solutions to the problems by just utilizing the initial conditions. The proposed iterative scheme finds the solution without any discretization, linearization or restrictive assumptions. The proposed method solves nonlinear problems without using Adomain polynomials which is the advantage of this method over Adomain Decomposition method. The results reveal that the HAM is very effective, fast, simple, convenient, flexible and accurate. Outcomes prove that HAM is in very good agreement with ADM,VIM HPM.

Keywords : Homotopy Analysis method, Hyperbolic-like equation, Variable coefficients.

ANALYTICAL SOLUTIONS FOR SOME OF THE NONLINEAR HYPERBOLIC-LIKE EQUATIONS WITH VARIABLE COEFFICIENTS

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R. Rajaraman

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I. Introduction

Recently various iterative methods are applied for getting Numerical and analytical solutions of Nonlinear hyperbolic-like equations with variable coefficients. [1,2,3,4,5,6]. In this paper Homotopy Analysis Method is applied to solve the proposed equations. HAM introduced by Liao [7,8,9,10,11] has been used by many mathematicians and engineers to solve various equations based on homotopy, which is a basic concept in topology. In recent years this method has been successfully employed to solve many types of nonlinear homogeneous or nonhomogeneous equations and systems of equation as well as problems in Science and engineering [12,13,14,15,16]. The validity of the HAM is independent of whether or not there exists small parameters in the considered equation. HAM provides us with a simple way to adjust and control the convergence of solution series. It provides us with freedom to use different base functions to approximate a nonlinear problem. Especially, it provides us with freedom of replacing a nonlinear partial differential equation of first order n into an infinite number of linear Differential equations of order k, where the order k is even unnecessarily to be equal to order n. Thus as long as Auxillary parameter h, Auxillary function H(r,t), Initial approximation $U_0(r,t)$ and linear operator L are properly chosen the solution expression converges to exact solution in the convergence region. Homotopy analysis method provides us the great freedom to choose all of them.

II. Basic Idea of Homotopy Analysis Method (HAM)

In this section the basic ideas of the homotopy analysis method are introduced. Here a description of the method [9] is given to handle the general nonlinear problem.

Ref.

1. Mohammad Ghoreishi,Adomain Decomposition method(ADM) for Non linear wave-like equations with variable coefficient,Applied Mathematical sciences,vol4,2010,no 49,2431-2444.

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$$\boldsymbol{N u_0(t)=0, t>0,} \tag{1}$$

Where N is a nonlinear operator and $u_0(t)$ is unknown function of the independent variable t.

a) Zero- order deformation equation

Let $u_0(t)$ denote the initial guess of the exact solution of Eq. (1), h$\neq 0$ an auxiliary parameter, $H(t) \neq 0$ an auxiliary function. and L an auxiliary linear operator with the property.

$$L(f(t)) = 0 when f(t) = 0 \tag{2}$$

The auxiliary parameter h, the auxiliary function$H(t)$, and the auxiliary linear operator L play important roles within the HAM to adjust and control the convergence region of solution series. Liao[10] constructs, using $q \in [0,1]$ as an embedding parameter, the so-called zero-order deformation equation.

$$\boldsymbol{(1-q)L[(\emptyset(t;q) - u_0(t)] = qhH(t)N[(\emptyset(t;q)],} \tag{3}$$

Where $\emptyset(t;q)$ is the solution which depends on h, $H(t), L,\ u_0(t)$ and q. when q=0,the zero-order deformation Eq.(3) becomes

$$\boldsymbol{\emptyset(t;0) = u_0(t),} \tag{4}$$

And when q=1, since h$\neq 0$ and $H(t) \neq 0$, the zero-order deformation Eq.(3) reduces to,

$$\boldsymbol{N[\emptyset(t;1)] = 0,} \tag{5}$$

So, $\emptyset(t;1)$ is exactly the solution of the nonlinear Eq.(1). Define the so-called mth order deformation derivatives.

$$\boldsymbol{1u_m(t) = \frac{1}{m!}\frac{\partial^m \emptyset(t;q)}{\partial q^m}} \tag{6}$$

I the power series (6) of $\emptyset(t;q)$ converges at q=1, then we gets the following series solution:

$$\boldsymbol{u(t) = u_0(t) + \textstyle\sum_{m=1}^{\infty} u_m(t).} \tag{7}$$

Where the terms $u_m(t)$ can be determined by the so-called high order deformation described below.

b) High- order deformation equation

Define the vector,

$$\boldsymbol{\overrightarrow{u_{n}}= \{u_0(t), u_1(t), u_2(t) \ldots \ldots u_n(t)} \tag{8}$$

Differentialing Eq(4)m times with respect to embedding parameter q, the setting q=0 and dividingthem by $m!$, we have the so-called mth- oder deformation equation.

$$\boldsymbol{L[u_m(t) - \aleph_m u_{m-1}(t)] = hH(t)R_m(\overrightarrow{u_m}, t),} \tag{9}$$

Where
$$\boldsymbol{\aleph_m = \begin{cases} o, & m \leq 1 \\ 1, & otherwise \end{cases}} \tag{10}$$

Ref.

10. S.J.Liao,Beyond perturbation:Introduction to Homotopy Analysis Method, Chapman &Hall/CRC press.Boca Raton 2003.

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And

$$R_m(\overrightarrow{u_m}, t) = \frac{1}{(m-1)!}\frac{\partial^{m-1} N[\emptyset(t;q)]}{\partial q^{m-1}} \quad (11)$$

For any given nonlinear operator N, the term $R_m(\overrightarrow{u_m}, t)$ can be easily expressed by (11). Thus, we can gain $u_1(t), u_2(t) \ldots \ldots$ by means of solving the linear high-order deformation Eq. (9) one after the other order in order. The mth − order approximation of u(t) is given by

$$u(t) = \sum_{k=0}^{m} u_k(t). \quad (12)$$

III. Illustrative Examples

Example1:
Let us consider the nonlinear hyperbolic-like equation

$$u_{tt}=u_{xx}+u_x u-e^x t-e^{2x}t^2 \quad (13)$$

With initial conditions

$$U(x,y,0)=0,\ u_t(x,y,0)=e^x \quad (14)$$

We apply homotopy analysis method to Eq. (13) and (14), as follows:
since m≥1, χ_m=1 set h= -1 and H(r,t)=1,L==$\partial^2 u/\partial t^2$ in(9)then (9) becomes

$$u_m(x,t) = u_{m-1}(x,t) - L^{-1}(R_m(u_{m-1}, x, t)$$

Where

$$R_m(u_{m-1}, x, t) = \frac{\partial^2 u_{m-1}}{\partial t^2} - \frac{\partial^2 u_{m-1}}{\partial x^2} - \frac{\partial u_{m-1}}{\partial x} + e^x\, t + e^{2x} t^2 \quad (15)$$

$$R_m(u_{m-1},x,t) = \partial^2 u_{m-1}/\partial t^2 - \partial^2 u_{m-1}/\partial x^2 - \partial u_{m-1}/\partial x\, u_{m-1} + e^x t + e^{2x} t^2$$

And solution for u_0:
Now we can select

$$u_0(x,t) = e^x t \quad (16)$$

Applying (16) in (15) we obtain the following successive approximations:

$$U_1(x,t)= e^x t \quad (17)$$

$$U_2(x,t)= e^x t \quad (18)$$

$$U_3(x,t)= e^x t \quad (19)$$

The Final solution is

$$U(x,t)= e^x t \quad (20)$$

Example2:

Let us consider the following nonlinear two dimensional Hyperbolic-like equation with variable coefficients

$$u_{tt} = \frac{\partial^2}{\partial x \partial y}\left(u_{xx} u_{yy}\right) - \frac{\partial^2}{\partial x \partial y}\left(xy u_x u_y\right) - u \quad (21)$$

With initial conditions

$$U(x,y,0)=0 \quad u_t=(x,y,0)=e^{xy} \quad (22)$$

Ref.

12. A.Molabahrami,F.Khani,The homotopy analysis method to solve Burgers-Huxley equation,Non linear analysis:Real world Applications10(2009)589-600.

We apply Homotopy Analysis method to (21) and (22) as follows
since $m \geq 1$, $\chi_m=1$ set h= -1 and H(r,t)=1,$L==\partial^2 u/\partial t^2$ in (9)then (9) becomes

$$U_m(x,y,t)=u_{m-1}(x,t)-L^{-1}(R_m(u_{m-1},x,y,t)) \qquad (23)$$

Where

$$R(u_{m-1},x,y,t)=\partial^2 u_{m-1}/\partial t^2-\partial^2/\partial x\partial y(\partial^2 u_{m-1}/\partial x^2\partial^2 u_{m-1}/\partial y^2)+\partial^2/\partial x\partial y(xy\partial u_{m-1}/\partial x\partial u_{m-1}/\partial y)+u_{m-1}$$

And solution for u_0:
Now we can select

$$u_0(x,y,t)= e^{xy}t \qquad (24)$$

Applying (24) in (23) we obtain the following successive approximations:

$$u_1(x,y,t)=e^{xy}(\frac{-t^3}{3!}),$$
$$u_2(x,y,t)=e^{xy}(\frac{t^5}{5!}) \qquad (25)$$

The Final solution is

$$U(x,y,t)= e^{xy}(t-t^3/3!+t^5/5!-t^7/7!- \ldots\ldots\ldots\ldots) =e^{xy}\sin t \qquad (26)$$

Example3:
Let us consider the following Nonlinear Hyperbolic-like equation

$$U_{tt}=u^2\partial^2/\partial x^2(u_x u_{xx} u_{xxx})+u^2{}_x\partial^2/\partial x^2(u^3{}_{xx})-18u^5+u, \quad 0<x<1,t>0 \qquad (27)$$

With initial conditions

$$U(x,0)=e^x \quad u_t(x,0)=e^x \qquad (28)$$

We apply Homotopy Analysis method to (27) and (28) as follows
since $m \geq 1$, $\chi_m=1$ set h= -1 and H(r,t)=1,$L==\partial^2 u/\partial t^2$ in(9)then (9) becomes

$$U_m(x,t)=u_{m-1}(x,t)-L^{-1}(R_m(u_{m-1},x,t)) \qquad (29)$$

Where

$$R(u_{m-1},x,y,t)=\partial^2 u_{m-1}/\partial t^2-(u_{m-1})^2\partial^2/\partial x^2(\partial u_{m-1}/\partial x\partial^2 u_{m-1}/\partial x^2\partial^3 u_{m-1}/\partial x^3)-(\partial u_{m-1}/\partial x)^2\partial^2/\partial x^2(\partial^2 u_{m-1}/\partial x^2)^3+18(u_{m-1})^5-u_{m-1}$$

And solution for u_0:
Now we can select

$$u_0(x,t)= e^x(1+t) \qquad (30)$$

Applying (30) in (29) we obtain the following successive approximations:

$$U_1(x,t)= e^x(t^2/2!+t^3/3!) \qquad (31)$$

$$U_2(x,t)= e^x(t^4/4!+t^5/5!) \qquad (32)$$

$$U_3(x,t)= e^x(t^6/6!+t^7/7!) \qquad (33)$$

Notes

Notes

The Final solution is

$$U(x,t)= e^{x} (1+t^2/2!+t^3/3!+t^4/4!+.........) = e^{x+t} \qquad (34)$$

Example4:

Finally we solve another nonlinear Hyperbolic-like equation

$$U_{tt}=x^2\partial/\partial x(u_x u_{xx})- x^2(u_{XX})^2+u, \quad 0<x<1,t>0 \qquad (35)$$

With initial conditions

$$u(x,t)=x^2, \frac{\partial u(x,t)}{\partial t}=x^2 t \qquad (36)$$

We apply Homotopy Analysis method to (35) and (36) as follows since m≥1, χ_m=1 set h= -1 and H(r,t)=1,L==$\partial^2u/\partial t^2$ in(9)then (9) becomes

$$U_m(x,t)=u_{m-1}(x,t)-L^{-1}(R_m(u_{m-1},x,t)) \qquad (37)$$

Where

$$R_m(u_{m-1},x,t) =\partial^2u_{m-1}/\partial t^2-x^2\partial/\partial x(\partial u_{m-1}/\partial x\partial^2u_{m-1}/\partial x^2)+x^2(\partial^2u_{m-1}/\partial x^2)^2-u_{m-1}$$

And solution for u_0:
Now we can select

$$u_0(x,t)= x^2 (1+t) \qquad (38)$$

Applying (38) in (37) we obtain the following successive approximations:

$$U_1(x,t)= x^2 (1+t+t^2/2!+t^3/3!) \qquad (39)$$

$$U_2(x,t)= x^2 (1+t+t^2/2!+t^3/3!+t^4/4!+t^5/5!) \qquad (40)$$

The Final solution is

$$U(x,t)=x^2 (1+t+t^2/2!+t^3/3!+t^4/4!+t^5/5!+..........) = x^2e^t \qquad (41)$$

IV. Conclusion

In this paper exact solutions for some of the hyperbolic-like equations have been established. The Homotopy Analysis method(HAM) is successfully used to develop these solutions. This work shows that HAM has significant advantages over the existing techniques. It avoids the need for calculating the Adomain polynomials which can be difficult in some cases. The reliability of the method and reduction in the size of computational domain give this method wider applicability. The results shows that HAM is a powerful mathematical tool for finding the exact and approximate solutions of the nonlinear equations.

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Approach

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- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
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Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
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Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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