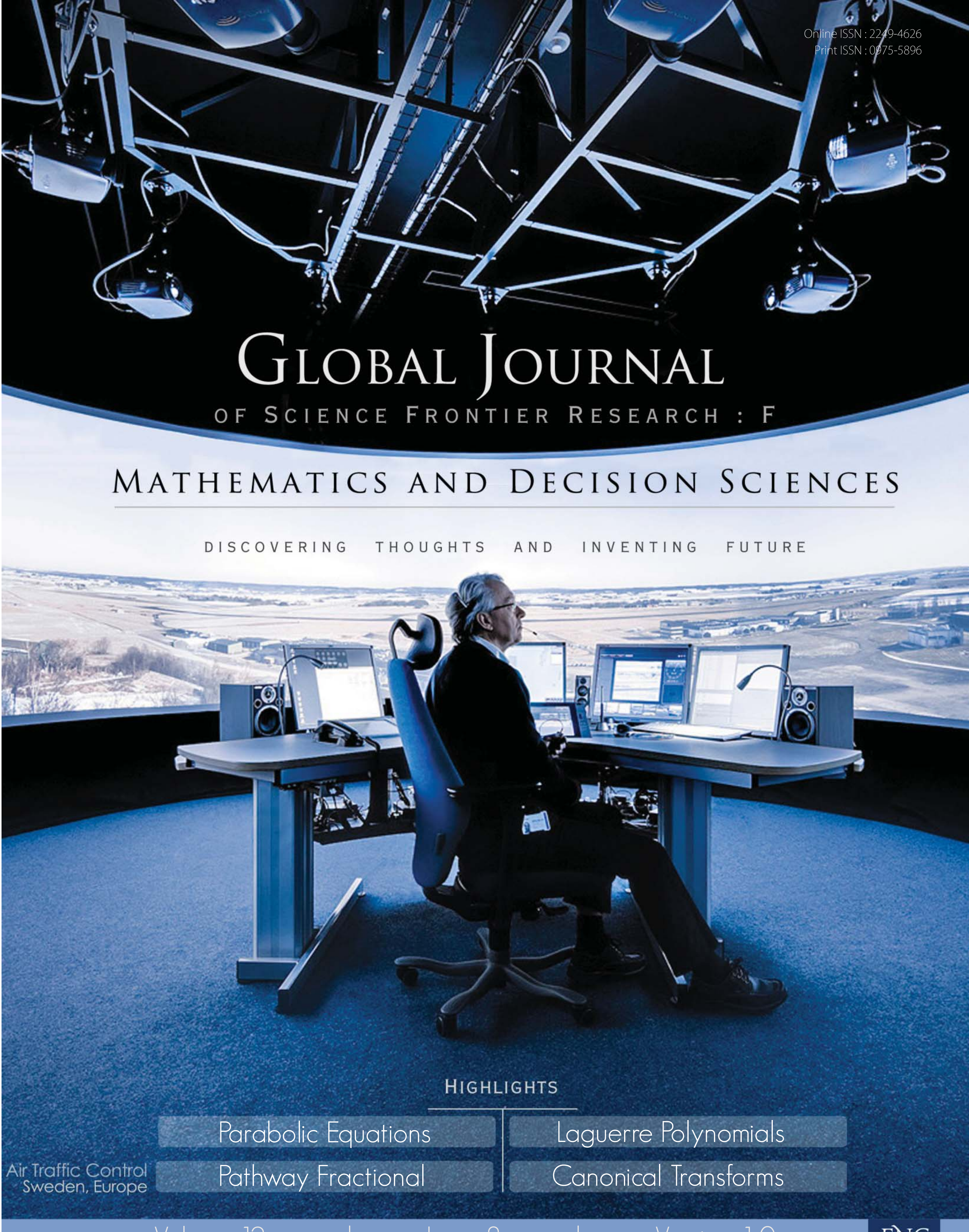

Online ISSN : 2249-4626
Print ISSN : 0975-5896

Global Journal
of Science Frontier Research : F

Mathematics and Decision Sciences

Discovering Thoughts and Inventing Future

Highlights

Parabolic Equations | Laguerre Polynomials

Pathway Fractional | Canonical Transforms

Air Traffic Control
Sweden, Europe

Volume 12 | Issue 8 | Version 1.0

ENG

Global Journal of Science Frontier Research: F
Mathematics & Decision Science

Global Journal of Science Frontier Research: F
Mathematics & Decision Science

Volume 12 Issue 8 (Ver. 1.0)

Open Association of Research Society

Incorporation No.: 0423089
License No.: 42125/022010/1186
Registration No.: 430374
Import-Export Code: 1109007027
Employer Identification Number (EIN):
USA Tax ID: 98-0673427

Global Journals Inc.

*(A Delaware USA Incorporation with "Good Standing"; **Reg. Number: 0423089**)*

Sponsors: *Open Association of Research Society*
Open Scientific Standards

Publisher's Headquarters office

Global Journals Inc., Headquarters Corporate Office, Cambridge Office Center, II Canal Park, Floor No. 5th, ***Cambridge (Massachusetts)***, Pin: MA 02141 United States

USA Toll Free: +001-888-839-7392
USA Toll Free Fax: +001-888-839-7392

Offset Typesetting

Open Association of Research Society, Marsh Road, Rainham, Essex, London RM13 8EU
United Kingdom.

Packaging & Continental Dispatching

Global Journals, India

Find a correspondence nodal officer near you

To find nodal officer of your country, please email us at *local@globaljournals.org*

eContacts

Press Inquiries: *press@globaljournals.org*
Investor Inquiries: *investers@globaljournals.org*
Technical Support: *technology@globaljournals.org*
Media & Releases: *media@globaljournals.org*

Pricing (Including by Air Parcel Charges):

For Authors:

22 USD (B/W) & 50 USD (Color)

Yearly Subscription (Personal & Institutional):

200 USD (B/W) & 250 USD (Color)

Editorial Board Members (HON.)

Dr. Bart Lambrecht
Director of Research in Accounting and FinanceProfessor of Finance
Lancaster University Management School
BA (Antwerp); MPhil, MA, PhD (Cambridge)

Dr. Carlos García Pont
Associate Professor of Marketing
IESE Business School, University of Navarra
Doctor of Philosophy (Management), Massachusetts Institute of Technology (MIT)
Master in Business Administration, IESE, University of Navarra
Degree in Industrial Engineering, Universitat Politècnica de Catalunya

Dr. Fotini Labropulu
Mathematics - Luther College
University of ReginaPh.D., M.Sc. in Mathematics
B.A. (Honors) in Mathematics
University of Windso

Dr. Lynn Lim
Reader in Business and Marketing
Roehampton University, London
BCom, PGDip, MBA (Distinction), PhD, FHEA

Dr. Mihaly Mezei
ASSOCIATE PROFESSOR
Department of Structural and Chemical Biology, Mount Sinai School of Medical Center
Ph.D., Etvs Lornd University
Postdoctoral Training,
New York University

Dr. Söhnke M. Bartram
Department of Accounting and FinanceLancaster University Management SchoolPh.D. (WHU Koblenz)
MBA/BBA (University of Saarbrücken)

Dr. Miguel Angel Ariño
Professor of Decision Sciences
IESE Business School
Barcelona, Spain (Universidad de Navarra)
CEIBS (China Europe International Business School).
Beijing, Shanghai and Shenzhen
Ph.D. in Mathematics
University of Barcelona
BA in Mathematics (Licenciatura)
University of Barcelona

Philip G. Moscoso
Technology and Operations Management
IESE Business School, University of Navarra
Ph.D in Industrial Engineering and Management, ETH Zurich
M.Sc. in Chemical Engineering, ETH Zurich

Dr. Sanjay Dixit, M.D.
Director, EP Laboratories, Philadelphia VA Medical Center
Cardiovascular Medicine - Cardiac Arrhythmia
Univ of Penn School of Medicine

Dr. Han-Xiang Deng
MD., Ph.D
Associate Professor and Research Department Division of Neuromuscular Medicine
Davee Department of Neurology and Clinical NeuroscienceNorthwestern University
Feinberg School of Medicine

Contents of the Volume

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 8 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Global Existence of Classical Solutions for a Class Nonlinear Parabolic Equations

By Svetlin Georgiev Georgiev
University of Sofia

Abstract - In this work we propose a new approach for investigating the local and global existence of classical solutions of nonlinear parabolic equations. This approach gives new results.

Keywords: *nonlinear parabolic equation, classical solutions, local existence, global existence.*

GJSFR-F Classification : *AMS Subject Classification: 35K55*

GLOBAL EXISTENCE OF CLASSICAL SOLUTIONS FOR A CLASS NONLINEAR PARABOLIC EQUATIONS

Strictly as per the compliance and regulations of :

Global Existence of Classical Solutions for a Class Nonlinear Parabolic Equations

Svetlin Georgiev Georgiev

Abstract - In this work we propose a new approach for investigating the local and global existence of classical solutions of nonlinear parabolic equations This approach gives new results

Keywords and phrases : nonlinear parabolic equation, classical solutions, local existence, global existence.

I. Introduction

In this paper we investigate the Cauchy problem

$$u_t - (\Delta + m)u = \lambda |u|^{p-1}u, \quad t \in [0, \infty), x \in \mathbb{R}^n, \tag{1.1}$$

$$u(0, x) = u_0(x), \quad x \in \mathbb{R}^n, \tag{1.2}$$

where $n \geq 2$ is fixed, $\Delta = \partial^2_{x_1} + \cdots + \partial^2_{x_n}$, $\lambda, m \in \mathbb{R}$, $p \geq 1$, $u_0(x) \in \mathcal{C}^2(\mathbb{R}^n)$, $|u_0(x)| \leq P$, $|(u_0)_{x_i}(x)| \leq P$ for every $i = 1, 2, \ldots, n$ and for every $x \in \mathbb{R}^n$, $P > 0$ is given constant. Here we propose new approach for invetsigating of this problem which gives new results.

The Cauchy problem for the equation (1.1) is investigated by many authors. For instance see [1] and references therein. Obviously the problem for existence of solutions to the Cauchy problem (1.1), (1.2) is connected with the Fujita's exponent, which depends of the dimension n and the values of the parameter p. Here we propose new conception about this problem. Our thesis is that this problem depends only of the integral representation for the solutions which is used. In this work we propose new integral representation. Our conception tell us that there are cases in which there is a global existence of solutions under specific set and local existence and blow up under another specific set. We will illustrate our new conception with the following example.

Example. Let $\lambda = -1$, $p = 3$, $m = 0$. Then

$$u(t, x) = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{t+1}}$$

is a solution to the problem

$$\begin{aligned} u_t - \Delta u = -|u|^2 u, \quad t \in [0, \infty), \quad x \in \mathbb{R}^n, \\ u(0, x) = \tfrac{1}{\sqrt{2}}, \quad x \in \mathbb{R}^n. \end{aligned} \tag{1.3}$$

Really, for $u(t, x) = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{t+1}}$ we have

$$u_t = -\frac{1}{2\sqrt{2}(t+1)^{\frac{3}{2}}},$$

$$u_t - \Delta u = -|u|^2 u \Longleftrightarrow$$

$$-\frac{1}{2\sqrt{2}(t+1)^{\frac{3}{2}}} = -\frac{1}{2(t+1)} \frac{1}{\sqrt{2}\sqrt{t+1}}.$$

Ref.

1. Fujita, H. On the blowing up of solutions of the Cauchy problem for ut = u + ul+, J. Fac. Sci. Univ. Tokyo Sect. I 13 (1966), 109124.

Author : University of Sofia, Faculty of Mathematics and Informatics, Department of Differential Equations, Blvd "Tzar Osvoboditel" 15, Sofia 1000, Bulgaria. E-mail : sgg2000bg@yahoo.com

This example and our main result we can consider as counter - example of the well known theory.

Our new nice result is due to our new integral representation.

This approach is used for hyperbolic equations in [2].

Our main result is

Theorem 1.1. *Let $n \geq 2$ be fixed, $p \geq 1$ be fixed, $P > 0$ be fixed, $\lambda, m \in \mathbb{R}$ be fixed, $u_0(x) \in \mathcal{C}^2(\mathbb{R}^n)$, $|u_0(x)| \leq P$, $|(u_0)_{x_i}(x)| \leq P$ for every $i = 1, 2, \dots, n$ and for every $x \in \mathbb{R}^n$. Then the Cauchy problem* (1.1), (1.2) *has a solution $u \in \mathcal{C}^1([0, \infty), \mathcal{C}^2(\mathbb{R}^n))$.*

II. Proof of Theorem 1.1

Let ϵ is fixed so that $1 > \epsilon > 0$. For fixed $P > 0$ we choose the constants A_i, $i = 1, 2, \dots, n$, so that

$$|\lambda|(A_1A_2\dots A_n)^2P^{p-1} + (3 + |m|)(A_1A_2\dots A_n)^2$$

$$+(A_1A_2\dots A_{n-1})^2 + (A_1A_2\dots A_{n-2}A_n)^2 + \cdots + (A_2A_3\dots A_n)^2 \leq (1 - \epsilon),$$

$$\Big(|\lambda|(A_1A_2\cdots A_{j-1}A_{j+1}\cdots A_n)^2P^{p-1} + (4 + |m|)(A_1A_2\cdots A_{j-1}A_{j+1}\cdots A_n)^2 \tag{2.1}$$

$$+(A_1A_2\cdots A_{j-1}A_{j+1}\cdots A_{n-1})^2 + \cdots + (A_2\cdots A_{j-1}A_{j+1}\cdots A_n)^2\Big)A_j \leq (1 - \epsilon)$$

$$\forall j = 1, 2, \dots, n,$$

where $A_0 = A_{n+1} = 1$. There exist such constants A_1, A_2, $\dots$, A_n, for which the inequalities (2.1) are possible. For instance when $1 > A_i > 0$ are enough small, $i = 1, 2, \dots, n$.

With B_1 we will denote the set

$$B_1 = \Big\{x \in \mathbb{R}^n : 0 \leq x_i \leq A_i, \quad i = 1, 2, \dots, n\Big\}.$$

Firstly we will prove that the Cauchy problem

$$u_t - (\Delta + m)u = \lambda|u|^{p-1}u, \quad t \in [0, 1], x \in B_1, \tag{2.2}$$

$$u(0, x) = u_0(x), \quad x \in B_1, \tag{2.3}$$

has a solution u for which $u \in \mathcal{C}^1([0, 1], \mathcal{C}^2(B_1))$. For this purpose we will use fixed point arguments. Therefore we have a need to define an operator whose fixed points satisfy the above Cauchy problem.

Our observation is

Lemma 2.1. *If $u \in \mathcal{C}^1([0, 1], \mathcal{C}^2(B_1))$ satisfies the integral equation*

$$\lambda \int_0^t \int_x^A \int_z^A |u|^{p-1}udsdzdy + m \int_0^t \int_x^A \int_z^A u(y, s)dsdzdy + \sum_{i=1}^n \int_0^t \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A u(y, \hat{s}_i)d\overline{s}_id\overline{z}_idy \tag{2.4}$$

$$-\int_x^A \int_z^A u(t, s)dsdz + \int_x^A \int_z^A u_0(s)dsdz = 0, t \in [0, 1], x \in B_1,$$

then u is a solution to the Cauchy problem (2.2), (2.3).

Here

$$\overline{s}_i = (s_1, \dots, s_{i-1}, s_{i+1}, \dots, s_n), \quad \hat{s}_i = (s_1, \dots, s_{i-1}, x_i, s_{i+1}, \dots, s_n),$$

$$\int_x^A = \int_{x_1}^{A_1} \cdots \int_{x_n}^{A_n}, \quad \int_{\overline{x}_i}^A = \int_{x_1}^{A_1} \cdots \int_{x_{i-1}}^{A_{i-1}} \int_{x_{i+1}}^{A_{i+1}} \cdots \int_{x_n}^{A_n}.$$

Proof. We differentiate in t the equality (2.4) and obtain

$$\lambda \int_x^A \int_z^A |u|^{p-1}udsdz + m \int_x^A \int_z^A u(t, s)dsdz + \sum_{i=1}^n \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A u(t, \hat{s}_i)d\overline{s}_id\overline{z}_i$$

$$-\int_x^A \int_z^A u_t(t, s)dsdz = 0.$$

Ref.

2. Georgiev, S. Global existence for nonlinear wave equation. Dyn. PDE, Vol. 7, No. 3, pp. 207-216.

Now we differentiate twice in x_1, after which twice in x_2 and etc. twice in x_n and we obtain

$$u_t - (\Delta + m)u = \lambda |u|^{p-1}u.$$

We put $t = 0$ in (2.4) and we obtain

$$-\int_x^A \int_z^A u(0,s)dsdz + \int_x^A \int_z^A u_0(s)dsdz = 0.$$

After we differentiate the last equality twice in x_1, after which twice in x_2 and etc. twice in x_n we obtain

$$u_0(x) = u(0,x).$$

Consequently $u(t,x)$ is a solution to the Cauchy problem (2.2), (2.3).

The above lemma motivate us to define the integral operator

$$L_{11}(u) = u(t,x) + m\int_0^t \int_x^A \int_z^A u(y,s)dsdzdy + \lambda \int_0^t \int_x^A \int_z^A |u|^{p-1}udsdzdy$$

$$+\sum_{i=1}^n \int_0^t \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A u(y,\hat{s}_i)d\overline{s}_i d\overline{z}_i dy - \int_x^A \int_z^A \Big(u(t,s) - u_0(s)\Big)dsdz, \quad t \in [0,1], \quad x \in B_1.$$

Our aim is to prove that the operator L_{11} has a fixed point. We will use the following fixed point theorem.

Theorem 2.2. *(see [3], Corrolary 2.4, pp. 3231) Let X be a nonempty closed convex subset of a Banach space Y. Suppose that T and S map X into Y such that*

(i) *S is continuous, $S(X)$ resides in a compact subset of Y;*

(ii) *$T : X \longrightarrow Y$ is expansive and onto.*

Then there exists a point $x^\star \in X$ with $Sx^\star + Tx^\star = x^\star$.

Here we will use the following definition for expansive operator.

Definition. (see [3], pp. 3230) Let (X,d) be a metric space and M be a subset of X. The mapping $T : M \longrightarrow X$ is said to be expansive, if there exists a constant $h > 1$ such that

$$d(Tx,Ty) \geq hd(x,y) \quad \forall x,y \in M.$$

For this purpose we will use the representation of the operator L_{11} as follows

$$L_{11}(u) = T_{11}(u) + S_{11}(u),$$

where

$$T_{11}(u) = (1+\epsilon)u(t,x),$$

$$S_{11}(u) = -\epsilon u(t,x) + m\int_0^t \int_x^A \int_z^A u(y,s)dsdzdy + \lambda \int_0^t \int_x^A \int_z^A |u|^{p-1}udsdzdy$$

$$+\sum_{i=1}^n \int_0^t \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A u(y,\hat{s}_i)d\overline{s}_i d\overline{z}_i dy - \int_x^A \int_z^A \Big(u(t,s) - u_0(s)\Big)dsdz.$$

Also, we define the sets

$$M_{11} = \Big\{u(t,x) \in \mathcal{C}^1([0,1],\mathcal{C}^2(B_1)), \quad \max_{t\in[0,1]} \max_{x\in B_1} \Big|u(t,x)\Big| \leq P,$$

$$\max_{t\in[0,1]} \max_{x\in B_1} \Big|u_{x_i}(t,x)\Big| \leq P, \quad i = 0,1,2,\dots,n\Big\},$$

$$N_{11} = \Big\{u(t,x) \in \mathcal{C}^1([0,1],\mathcal{C}^2(B_1)), \quad \max_{t\in[0,1]} \max_{x\in B_1} \Big|u(t,x)\Big| \leq (1+\epsilon)P,$$

$$\max_{t\in[0,1]} \max_{x\in B_1} \Big|u_{x_i}(t,x)\Big| \leq (1+\epsilon)P, \quad i = 0,1,2,\dots,n\Big\},$$

Ref.

3. Xiang, T., Rong Yuan. A class of expansive - type Krasnosel'skii ˙xed point theorems. Nonlinear Analysis, 71(2009), 3229-3239.

where $u_{x_0} = u_t$. In these sets we define a norm as follows

$$||u||_2 = \sup\Big\{\Big|u(t,x)\Big| : (t,x) \in [0,1] \times B_1\Big\}.$$

Lemma 2.3. *The sets M_{11} and N_{11} are closed, compact and convex spaces in $\mathcal{C}([0,1] \times B_1)$ in the sense of norm $||\cdot||_2$.*

Proof. We will prove our assertion for M_{11}.

Let $\{u_n\}$ is a sequence of elements of M_{11} and $u_n \longrightarrow_{n\longrightarrow\infty} u$ in the sense of the norm $||\cdot||_2$. Evidently $u \in \mathcal{C}([0,1] \times B_1)$ and $|u(t,x)| \leq P$ for every $(t,x) \in [0,1] \times B_1$.

We suppose that $u \notin \mathcal{C}^1([0,1] \times B_1)$. Then there exists $j \in \{0,1,2,\ldots,n\}$ and $\epsilon > 0$ so that for every $\delta_1 = \delta_1(\epsilon) > 0$ and $|h| < \delta_1$, $h \neq 0$, $(x_0, x_1, \ldots, x_{j-1}, x_j + h, x_{j+1}, \ldots, x_n) \in [0,1] \times B_1$, we have

$$\Big|\frac{u(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n) - u(x_0,x_1,\cdots,x_n)}{h}\Big| > \epsilon. \tag{2.5}$$

On the other hand since $u_n \in \mathcal{C}^1([0,1],\mathcal{C}^2(B_1))$ we have that there exists $\delta_2 = \delta_2(\epsilon) > 0$ so that from $|h| < \delta_2$, $h \neq 0$, $(x_0, x_1, \ldots, x_{j-1}, x_j + h, x_{j+1}, \ldots, x_n) \in [0,1] \times B_1$, we have

$$\Big|\frac{u_n(x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n) - u_n(x_1,\cdots,x_n)}{h}\Big| < \frac{\epsilon}{3}. \tag{2.6}$$

Also, from $u_n \longrightarrow_{n\longrightarrow\infty} u$ in the sense of the norm $||\cdot||_2$ we have for enough large n and $|h| < \min\{\delta_1,\delta_2\}$, $h \neq 0$, $(x_0, x_1, x_2, \cdots, x_{j-1}, x_j + h, x_{j+1}, \cdots, x_n) \in [0,1] \times B_1$ that

$$\Big|\tfrac{u_n(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u_n(x_0,x_1,\cdots,x_n)}{h} - \tfrac{u(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u(x_0,x_1,\cdots,x_n)}{h}\Big| < \tfrac{\epsilon}{3}. \tag{2.7}$$

Then from (2.7), (2.6), (2.5) we obtain for $|h| < \min\{\delta_1,\delta_2\}$, $h \neq 0$, $(x_0, x_1, x_2, \cdots, x_{j-1}, x_j + h, x_{j+1}, \cdots, x_n) \in [0,1] \times B_1$, for enough large n,

$$\epsilon < \Big|\tfrac{u(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u(x_0,x_1,\cdots,x_n)}{h}\Big|$$

$$\leq \Big|\tfrac{u_n(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u_n(x_0,x_1,\cdots,x_n)}{h}\Big|$$

$$+\Big|\tfrac{u_n(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u_n(x_0,x_1,\cdots,x_n)}{h} - \tfrac{u(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u(x_0,x_1,\cdots,x_n)}{h}\Big| < 2\tfrac{\epsilon}{3},$$

which is a contradiction with our assumption that $u \notin \mathcal{C}^1([0,1] \times B_1)$.

Therefore $u \in \mathcal{C}^1([0,1] \times B_1)$

We suppose that $u \notin \mathcal{C}^1([0,1],\mathcal{C}^2(B_1))$. Then there exists $j \in \{1,2,\ldots,n\}$ and $\epsilon_1 > 0$ so that for every $\delta_3 = \delta_3(\epsilon_1) > 0$ and $|h| < \delta_3$, $h \neq 0$, $(x_0, x_1, \ldots, x_{j-1}, x_j + h, x_{j+1}, \ldots, x_n) \in [0,1] \times B_1$ we have

$$\Big|\frac{u_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n) - u_{x_j}(x_0,x_1,\cdots,x_n)}{h}\Big| > \epsilon_1. \tag{2.8}$$

On the other hand since $u_n \in \mathcal{C}^1([0,1],\mathcal{C}^2(B_1))$ we have that there exists $\delta_4 = \delta_4(\epsilon_1) > 0$ so that from $|h| < \delta_2$, $h \neq 0$, $(x_0, x_1, \ldots, x_{j-1}, x_j + h, x_{j+1}, \ldots, x_n) \in [0,1] \times B_1$ we have

$$\Big|\frac{(u_n)_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n) - (u_n)_{x_j}(x_0,x_1,\cdots,x_n)}{h}\Big| < \frac{\epsilon}{3}. \tag{2.9}$$

Notes

Also, from $u_n \longrightarrow_{n\longrightarrow\infty} u$ in the sense of the norm $||\cdot||_2$ we have for enough large n and $|h| < \min\{\delta_3, \delta_4\}$, $h \neq 0$, $(x_1, x_2, \cdots, x_{j-1}, x_j + h, x_{j+1}, \cdots, x_n) \in B_1$ that

$$\left|\frac{(u_n)_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-(u_n)_{x_j}(x_0,x_1,\cdots,x_n)}{h} - \frac{u_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u_{x_j}(x_0,x_1,\cdots,x_n)}{h}\right| < \frac{\epsilon_1}{3}. \tag{2.10}$$

Then from (2.10), (2.9), (2.8) we obtain for $|h| < \min\{\delta_3, \delta_4\}$, $h \neq 0$, $(x_1, x_2, \cdots, x_{j-1}, x_j + h, x_{j+1}, \cdots, x_n) \in B_1$, for enough large n,

Notes

$$\epsilon_1 < \left|\frac{u_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u_{x_j}(x_0,x_1,\cdots,x_n)}{h}\right|$$

$$\leq \left|\frac{(u_n)_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-(u_n)_{x_j}(x_0,x_1,\cdots,x_n)}{h}\right|$$

$$+\left|\frac{(u_n)_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-(u_n)_{x_j}(x_0,x_1,\cdots,x_n)}{h} - \frac{u_{x_j}(x_0,x_1,\cdots,x_{j-1},x_j+h,x_{j+1},\cdots,x_n)-u_{x_j}(x_0,x_1,\cdots,x_n)}{h}\right| < 2\frac{\epsilon_1}{3},$$

which is a contradiction with our assumption that $u \notin \mathcal{C}^1([0,1], \mathcal{C}^2(B_1))$.

Therefore $u \in \mathcal{C}^1([0,1], \mathcal{C}^2(B_1))$.

Now we suppose that there exists $j \in \{0, 1, \ldots, n\}$ and $(\tilde{t}, \tilde{x}) \in [0,1] \times B_1$ so that

$$|u_{x_j}(\tilde{t}, \tilde{x})| > P.$$

Then there exists $\epsilon_2 > 0$ so that

$$|u_{x_j}(\tilde{t}, \tilde{x})| \geq P + \epsilon_2.$$

From here there exists $\delta_5 = \delta_5(\epsilon_2) > 0$ such that from $|h| < \delta_5$, $h \neq 0$, $(\tilde{t}, \tilde{x}_1, \ldots, \tilde{x}_{j-1}, \tilde{x}_j + h, \tilde{x}_{j+1}, \ldots \tilde{x}_n) \in [0,1] \times B_1$ we have

$$\left|\frac{u(\tilde{t}, \tilde{x}_1, \ldots, \tilde{x}_{j-1}, \tilde{x}_j + h, \tilde{x}_{j+1}, \ldots \tilde{x}_n) - u(\tilde{t}, \tilde{x})}{h}\right| \geq P + \epsilon_2.$$

On the other hand, since $u_n(\tilde{t}, \tilde{x}) \longrightarrow u(\tilde{t}, \tilde{x})$ in sense of $||\cdot||_2$, as $n \longrightarrow \infty$, follows that there exists $\delta_6 = \delta_6(\epsilon_2) > 0$ so that we have from $|h| < \delta_6$, $h \neq 0$, $(\tilde{t}, \tilde{x}_1, \ldots, \tilde{x}_{j-1}, \tilde{x}_j + h, \tilde{x}_{j+1}, \ldots \tilde{x}_n) \in [0,1] \times B_1$

$$\left|\frac{u_n(\tilde{t},\tilde{x}_1,\ldots,\tilde{x}_{j-1},\tilde{x}_j+h,\tilde{x}_{j+1},\ldots\tilde{x}_n)-u_n(\tilde{t},\tilde{x})}{h} - \frac{u(\tilde{t},\tilde{x}_1,\ldots,\tilde{x}_{j-1},\tilde{x}_j+h,\tilde{x}_{j+1},\ldots\tilde{x}_n)-u(\tilde{t},\tilde{x})}{h}\right| < \epsilon_2$$

and since $|(u_n)_{x_j}| \leq P$ in $[0,1] \times B_1$

$$\left|\frac{u_n(\tilde{t}, \tilde{x}_1, \ldots, \tilde{x}_{j-1}, \tilde{x}_j + h, \tilde{x}_{j+1}, \ldots \tilde{x}_n) - u_n(\tilde{t}, \tilde{x})}{h}\right| \leq P$$

for enough large n. From here, for enough large n and for $|h| < \min\{\delta_5, \delta_6\}$, $h \neq 0$, $(\tilde{t}, \tilde{x}_1, \ldots, \tilde{x}_{j-1}, \tilde{x}_j + h, \tilde{x}_{j+1}, \ldots \tilde{x}_n) \in [0,1] \times B_1$ we have

$$\epsilon_2 = P + \epsilon_2 - P$$

$$\leq \left|\frac{u(\tilde{t},\tilde{x}_1,\ldots,\tilde{x}_{j-1},\tilde{x}_j+h,\tilde{x}_{j+1},\ldots\tilde{x}_n)-u(\tilde{t},\tilde{x})}{h}\right| - \left|\frac{u_n(\tilde{t},\tilde{x}_1,\ldots,\tilde{x}_{j-1},\tilde{x}_j+h,\tilde{x}_{j+1},\ldots\tilde{x}_n)-u_n(\tilde{t},\tilde{x})}{h}\right|$$

$$\leq \left|\frac{u(\tilde{t},\tilde{x}_1,\ldots,\tilde{x}_{j-1},\tilde{x}_j+h,\tilde{x}_{j+1},\ldots\tilde{x}_n)-u(\tilde{t},\tilde{x})}{h} - \frac{u_n(\tilde{t},\tilde{x}_1,\ldots,\tilde{x}_{j-1},\tilde{x}_j+h,\tilde{x}_{j+1},\ldots\tilde{x}_n)-u_n(\tilde{t},\tilde{x})}{h}\right| < \epsilon_2,$$

which is a contradiction. Therefore $|u_{x_j}| \leq P$ in $[0,1] \times B_1$ for every $j = 0, 1, \ldots, n$. Consequently $u \in M_{11}$ and M_{11} is closed in $\mathcal{C}([0,1] \times B_1)$ in sense of $||\cdot||_2$. Using Arzela - Ascoli Theorem the set M_{11} is a compact set in $\mathcal{C}([0,1] \times B_1)$ in sense of $||\cdot||_2$.

Let now $\lambda \in [0,1]$ is arbitrary chosen and fixed and $u_1, u_2 \in M_{11}$. Then for $(t,x) \in [0,1] \times B_1$ we have $\lambda u_1(t,x) + (1-\lambda)u_2(t,x) \in \mathcal{C}^1([0,1], \mathcal{C}^2(B_1))$ and

$$|u_i(t,x)| \leq P, |u_{ix_j}(t,x)| \leq P \quad \text{for} \quad j = 0,1,\dots,n, i = 1,2,$$

$$|\lambda u_1(t,x) + (1-\lambda)u_2(t,x)| \leq \lambda|u_1(t,x)| + (1-\lambda)|u_2(t,x)| \leq \lambda P + (1-\lambda)P = P,$$

$$|\lambda u_{1x_j}(t,x) + (1-\lambda)u_{2x_j}(t,x)| \leq \lambda|u_{1x_j}(t,x)| + (1-\lambda)|u_{2x_j}(t,x)| \leq \lambda P + (1-\lambda)P = P, \quad j = 0,1,\dots,n.$$

Therefore M_{11} is convex.

As in above we can prove that N_{11} is closed, compact and convex in $\mathcal{C}([0,1] \times B_1)$ in sense of $||\cdot||_2$.

Notes

Lemma 2.4. *The operator $T_{11} : M_{11} \longrightarrow N_{11}$ is an expansive operator and onto.*

Proof. Let $u \in M_{11}$. Then $u \in \mathcal{C}^1([0,1], \mathcal{C}^2(B_1))$, from here $(1+\epsilon)u \in \mathcal{C}^1([0,1], \mathcal{C}^2(B_1))$, i.e. $T_{11}(u) \in \mathcal{C}^1([0,1], \mathcal{C}^2(B_1))$ and $|T_{11}(u)| \leq (1+\epsilon)P$, $|(T_{11}(u))_{x_i}| \leq (1+\epsilon)P$, $i = 0,1,2,\dots,n$. Let now $u, v \in M_{11}$. Then we have

$$||T_{11}(u) - T_{11}(v)||_2 = (1+\epsilon)||u-v||_2.$$

From here and above follows that $T_{11} : M_{11} \longrightarrow N_{11}$ is an expansive operator.

Now we will see that $T_{11} : M_{11} \longrightarrow N_{11}$ is onto. Really, let $v \in N_{11}$, $v \neq 0$. Let also $u = \frac{v}{1+\epsilon}$. From here $\max_{t\in[0,1]} \max_{x\in B_1} |u| \leq P$, $\max_{t\in[0,1]} \max_{x\in B_1} |u_{x_i}| \leq P$ for $i = 0,1,2,\dots,n$. Therefore $u \in M_{11}$ and the operator $T_{11} : M_{11} \longrightarrow N_{11}$is onto.

Lemma 2.5. *We have*

$$S_{11} : M_{11} \longrightarrow M_{11}$$

is continuous.

Proof. Let $u \in M_{11}$ is arbitrary chosen element. Then, using the definition of the operator S_{11}, we have

$$|S_{11}(u)| \leq \epsilon|u(t,x)| + |m| \int_0^t \int_x^A \int_z^A |u(y,s)|dsdzdy + |\lambda| \int_0^t \int_x^A \int_z^A |u|^p dsdzdy$$

$$+ \sum_{i=1}^n \int_0^t \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A |u(y,\hat{s}_i)|d\overline{s}_i d\overline{z}_i dy + \int_x^A \int_z^A \Big(|u(t,s)| + |u_0(s)|\Big) dsdz$$

$$\leq \epsilon P + |m|(A_1A_2\dots A_n)^2 P + |\lambda|(A_1A_2\dots A_n)^2 P^p + 3(A_1A_2\dots A_n)^2 P$$

$$+P\Big((A_1A_2\dots A_{n-1})^2 + (A_1A_2\dots A_{n-2}A_n)^2 + \dots + (A_2A_3\dots A_n)^2\Big) \leq \epsilon P + (1-\epsilon)P = P.$$

Also,

$$(S_{11}(u))_t = -\epsilon u_t(t,x) + m \int_x^A \int_z^A u(t,s)dsdz + \lambda \int_x^A \int_z^A |u|^{p-1}udsdz + \sum_{i=1}^n \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A u(t,\hat{s}_i)d\overline{s}_i d\overline{z}_i$$

$$- \int_x^A \int_z^A u_t(t,s)dsdz.$$

Then

$$|(S_{11}(u))_t| \leq \epsilon|u_t(t,x)| + |m| \int_x^A \int_z^A |u(t,s)|dsdz + |\lambda| \int_x^A \int_z^A |u|^p dsdz$$

$$+ \sum_{i=1}^n \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A |u(t,\hat{s}_i)|d\overline{s}_i d\overline{z}_i + \int_x^A \int_z^A |u_t(t,s)|dsdz$$

$$\leq \epsilon P + |\lambda|(A_1A_2\dots A_n)^2 P^p + (2+|m|)(A_1A_2\dots A_n)^2 P$$

$$+P\Big((A_1A_2\dots A_{n-1})^2 + (A_1A_2\dots A_{n-2}A_n)^2 + \dots + (A_2A_3\dots A_n)^2\Big)$$

$$\leq \epsilon P + (1-\epsilon)P = P.$$

Let now $j = 1, 2, \cdots, n$, is arbitrary chosen and fixed. Then

$$\Big(S_{11}(u)\Big)_{x_j} = -\epsilon u_{x_j}(t,x) - m\int_0^t \int_{\overline{x}_j}^A \int_{\hat{z}_j}^A u(y,s)dsd\overline{z}_jdy - \lambda\int_0^t \int_{\overline{x}_j}^A \int_{\hat{z}_j}^A |u|^{p-1}udsd\overline{z}_jdy$$

$$-\sum_{i=1,i\neq j}^n \int_0^t \int_{\overline{\overline{x}}_{ij}}^A \int_{\hat{\overline{z}}_{ij}}^A u(y,\hat{s}_i)d\overline{s}_id\overline{z}_idy + \int_0^t \int_{\overline{x}_j}^A \int_{\overline{z}_j}^A u_{x_j}(y,\hat{s}_j)d\overline{s}_jd\overline{z}_jdy$$

$$+\int_{\overline{x}_j}^A \int_{\hat{z}_j}^A \Big(u(t,\hat{s}_j) - u_0(\hat{s}_j)\Big)d\hat{s}_jd\overline{z}_j,$$

where

$$\int_{\overline{\overline{x}}_{ij}}^A = \int_{x_1}^{A_1}\cdots\int_{x_{i-1}}^{A_{i-1}}\int_{x_{i+1}}^{A_{i+1}}\cdots\int_{x_{j-1}}^{A_{j-1}}\int_{x_{j+1}}^{A_{j+1}}\cdots\int_{x_n}^{A_n},$$

$$\int_{\hat{z}_j}^A = \int_{z_1}^{A_1}\cdots\int_{z_{j-1}}^{A_{j-1}}\int_{x_j}^{A_j}\int_{z_{j+1}}^{A_{j+1}}\cdots\int_{z_n}^{A_n},$$

and from here

$$\Big|\Big(S_{11}(u)\Big)_{x_j}\Big| \leq \epsilon|u_{x_j}(t,x)| + |m|\int_0^t \int_{\overline{x}_j}^A \int_{\hat{z}_j}^A |u(y,s)|dsd\overline{z}_jdy + |\lambda|\int_0^t \int_{\overline{x}_j}^A \int_{\hat{z}_j}^A |u|^pdsd\overline{z}_jdy$$

$$+\sum_{i=1,i\neq j}^n \int_0^t \int_{\overline{\overline{x}}_{ij}}^A \int_{\hat{\overline{z}}_{ij}}^A |u(y,\hat{s}_i)|d\overline{s}_id\overline{z}_idy + \int_0^t \int_{\overline{x}_j}^A \int_{\overline{z}_j}^A |u_{x_j}(y,\hat{s}_j)|d\overline{s}_jd\overline{z}_jdy$$

$$+\int_{\overline{x}_j}^A \int_{\hat{z}_j}^A \Big(|u(t,\hat{s}_j)| + |u_0(\hat{s}_j)|\Big)d\hat{s}_jd\overline{z}_j$$

$$\leq \epsilon P + |\lambda|(A_1A_2\cdots A_{j-1}A_{j+1}\cdots A_n)^2A_jP^p + (4+|m|)(A_1A_2\cdots A_{j-1}A_{j+1}\cdots A_n)^2A_jP$$

$$+\Big((A_1A_2\cdots A_{j-1}A_{j+1}\cdots A_{n-1})^2 + \cdots + (A_2\cdots A_{j-1}A_{j+1}\cdots A_n)^2\Big)A_jP \leq \epsilon P + (1-\epsilon)P = P.$$

Consequently

$$S_{11} : M_{11} \longrightarrow M_{11}.$$

From the above estimates for $|S_{11}(u)|$, $|(S_{11}(u))_{x_j}|$, $j = 0, 1, 2, \ldots, n$, follows that if $v_n \longrightarrow_{n\longrightarrow\infty} v$ in the sense of the topology of the set M_{11}, $v_n, v \in M_{11}$, we have that $S_{11}(v_n) \longrightarrow_{n\longrightarrow\infty} S_{11}(v)$ in the sense of topology of the set M_{11}. Therefore the operator $S_{11} : M_{11} \longrightarrow M_{11}$ is a continuous operator.

Using Lemma 2.3, Lemma 2.4, Lemma 2.5 we apply Theorem 2.2 as the operator S in Theorem 2.2 corresponds of S_{11}, the operator T in Theorem 2.2 corresponds of T_{11}, the set X in Theorem 2.2 corresponds of M_{11} and Y in Theorem 2.2 corresponds of N_{11} and therefore follows that there exists $u^{11} \in M_{11}$ so that $u^{11} = S(u^{11}) + T(u^{11})$, i.e. u^{11} is a fixed point of the operator L_{11}. From here and Lemma 2.1 follows that u^{11} is a solution to the Cauchy problem (2.2), (2.3), for which $u^{11} \in \mathcal{C}^1([0,1], \mathcal{C}^2(B_1))$.

Now we define the set

$$B_2 = \Big\{x \in \mathbb{R}^n : A_1 \leq x_1 \leq 2A_1, 0 \leq x_i \leq A_i, \quad i = 2, \ldots, n\Big\},$$

the operators

$$L_{12}(u) = u(t,x) + m\int_0^t \int_x^A \int_z^A u(y,s)dsdzdy + \lambda\int_0^t \int_x^A \int_z^A |u|^{p-1}udsdzdy$$

$$+\sum_{i=2}^n \int_0^t \int_{\overline{x}_i}^A \int_{\overline{z}_i}^A u(y,\hat{s}_i)d\overline{s}_id\overline{z}_idy$$

$$-\int_x^A \int_z^A \Big(u(t,s) - u^{11}(0,s)\Big)dsdz$$

$$+\int_0^t \int_{\overline{x}_1}^A \int_{\overline{z}_1}^A (u(y,x_1,s_2,\ldots,s_n) - u^{11}(y,A_1,s_2,\ldots,s_n) + (A_1 - x_1)u^{11}_{x_1}(y,A_1,s_2,\ldots,s_n))d\overline{s}_1d\overline{z}_1dy,$$

$t \in [0,1], \quad x \in B_2,$

$$L_{12}(u) = T_{12}(u) + S_{12}(u),$$

where

$T_{12}(u)=(1+\epsilon)u(t,x), \quad t\in[0,1], x\in B_2,$

$S_{12}(u)=-\epsilon u(t,x)+m\int_0^t\int_x^A\int_z^A u(y,s)dsdzdy+\lambda\int_0^t\int_x^A\int_z^A |u|^{p-1}udsdzdy$

$+\sum_{i=2}^n\int_0^t\int_{\overline{x}_i}^A\int_{\overline{z}_i}^A u(y,\hat{s}_i)d\overline{s}_i d\overline{z}_i dy$

$-\int_x^A\int_z^A\Big(u(t,s)-u^{11}(0,s)\Big)dsdz$

$+\int_0^t\int_{\overline{x}_1}^A\int_{\overline{z}_1}^A(u(y,x_1,s_2,\dots,s_n)-u^{11}(y,A_1,s_2,\dots,s_n)+(A_1-x_1)u^{11}_{x_1}(y,A_1,s_2,\dots,s_n))d\overline{s}_1 d\overline{z}_1 dy,$

$t\in[0,1], x\in B_2,$

the sets

$$M_{12}=\Big\{u(t,x)\in\mathcal{C}^1([0,1],\mathcal{C}^2(B_2)), \quad \max_{t\in[0,1]}\max_{x\in B_2}\Big|u(t,x)\Big|\le P,$$

$$\max_{t\in[0,1]}\max_{x\in B_2}\Big|u_{x_i}(t,x)\Big|\le P, \quad i=0,1,2,\dots,n\Big\},$$

$$N_{12}=\Big\{u(t,x)\in\mathcal{C}^1([0,1],\mathcal{C}^2(B_2)), \quad \max_{t\in[0,1]}\max_{x\in B_2}\Big|u(t,x)\Big|\le(1+\epsilon)P,$$

$$\max_{t\in[0,1]}\max_{x\in B_2}\Big|u_{x_i}(t,x)\Big|\le(1+\epsilon)P, \quad i=0,1,2,\dots,n\Big\},$$

in these sets we define a norm as follows

$$||u||_2=\sup\Big\{\Big|u(t,x)\Big|:(t,x)\in[0,1]\times B_2\Big\}.$$

The sets M_{12} and N_{12} are closed, convex and compact in $\mathcal{C}([0,1]\times B_2)$ in sense of $||\cdot||_2$.

As in above we conclude that there exists $u^{12}\in M_{12}$ so that $L_{12}u^{12}=u^{12}$, i.e. u^{12} is a solution to the Cauchy problem

$$u_t-(\Delta+m)u=\lambda|u|^{p-1}u, \quad t\in[0,1], x\in B_2,$$
$$u(0,x)=u^{11}(0,x), x\in B_2.$$

For u^{12} we have

$m\int_0^t\int_x^A\int_z^A u^{12}(y,s)dsdzdy+\lambda\int_0^t\int_x^A\int_z^A |u^{12}|^{p-1}u^{12}dsdzdy$

$+\sum_{i=2}^n\int_0^t\int_{\overline{x}_i}^A\int_{\overline{z}_i}^A u^{12}(y,\hat{s}_i)d\overline{s}_i d\overline{z}_i dy$

$-\int_x^A\int_z^A\Big(u^{12}(t,s)-u^{11}(0,s)\Big)dsdz$

$$+\int_0^t\int_{\overline{x}_1}^A\int_{\overline{z}_1}^A(u^{12}(y,x_1,s_2,\dots,s_n)-u^{11}(y,A_1,s_2,\dots,s_n)+(A_1-x_1)u^{11}_{x_1}(y,A_1,s_2,\dots,s_n))d\overline{s}_1 d\overline{z}_1 dy=0. \quad (2.11)$$

Now we put $x_1=A_1$ in the last equality and we obtain

$$\int_0^t\int_{\overline{x}_1}^A\int_{\overline{z}_1}^A(u^{12}(y,A_1,s_2,\dots,s_n)-u^{11}(y,A_1,s_2,\dots,s_n))d\overline{s}_1 d\overline{z}_1 dy=0$$

and we differentiate it in t, twice in x_2 and etc. twice in x_n and we obtain

$$u^{12}_{|_{x_1=A_1}}=u^{11}_{|_{x_1=A_1}}.$$

Now we differentiate in x_1 the equality (2.11), after which we put $x_1=A_1$ and we obtain

$$\int_0^t\int_{\overline{x}_1}^A\int_{\overline{z}_1}^A(u^{12}_{x_1}(y,A_1,s_2,\dots,s_n)-u^{11}_{x_1}(y,A_1,s_2,\dots,s_n))d\overline{s}_1 d\overline{z}_1 dy=0$$

Notes

Notes

and we differentiate it in t, twice in x_2 and etc. twice in x_n and we obtain

$$(u^{12}_{x_1})_{|x_1=A_1} = (u^{11}_{x_1})_{|x_1=A_1}.$$

Using the equalities

$$u^{11}_t - (\Delta+m)u^{11} = \lambda|u^{11}|^{p-1}u^{11}, \quad u^{12}_t - (\Delta+m)u^{12} = \lambda|u^{12}|^{p-1}u^{12},$$

$$u^{11}_{|x_1=A_1} = u^{12}_{|x_1=A_1}, (u^{11}_{x_1})_{|x_1=A_1} = (u^{12}_{x_1})_{|x_1=A_1}$$

we conclude that

$$(u^{11}_{x_1x_1})_{|x_1=A_1} = (u^{12}_{x_1x_1})_{|x_1=A_1}.$$

In this way we obtain that the function

$$u = \begin{cases} u^{11} & t\in[0,1], 0\le x_1\le A_1, 0\le x_2\le A_2,\dots,0\le x_n\le A_n,\\ u^{12} & t\in[0,1], A_1\le x_1\le 2A_1, 0\le x_2\le A_2,\dots,0\le x_n\le A_n\end{cases}$$

is a solution to the Cauchy problem

$$u_t - (\Delta+m)u = \lambda|u|^{p-1}u, \quad t\in[0,1], 0\le x_1\le 2A_1, 0\le x_2\le A_2,\dots,0\le x_n\le A_n,$$

$$u(0,x) = u_0(x), 0\le x_1\le 2A_1, 0\le x_2\le A_2,\dots,0\le x_n\le A_n.$$

Repeat the above steps in x_1, x_2 and etc. x_n we obtain a solution u_1 to the Cauchy problem (1.1), (1.2) which belongs to the space $\mathcal{C}^1([0,1]), \mathcal{C}^2(\mathbb{R}^n))$.

Now we consider the Cauchy problem

$$\begin{aligned} &u_t - (\Delta+m)u = \lambda|u|^{p-1}u, \quad t\in[1,2], \quad x\in B_1,\\ &u(1,x) = u_1(1,x), \quad x\in B_1.\end{aligned} \tag{2.12}$$

For this purpose we consider the operator

$$L^1_{11}(u) = T^1_{11}(u) + S^1_{11}(u),$$

where

$$T^1_{11}(u) = (1+\epsilon)u(t,x),$$

$$S^1_{11}(u) = -\epsilon u(t,x) + m\int_1^t\int_x^A\int_z^A u(y,s)dsdzdy + \lambda\int_1^t\int_x^A\int_z^A |u|^{p-1}udsdzdy$$

$$+\sum_{i=1}^n\int_1^t\int_{\overline{x}_i}^A\int_{\overline{z}_i}^A u(y,\hat{s}_i)d\overline{s}_id\overline{z}_idy - \int_x^A\int_z^A\Big(u(t,s)-u_1(1,s)\Big)dsdz.$$

Also, we define the sets

$$M^1_{11} = \Big\{u(t,x)\in\mathcal{C}^1([1,2],\mathcal{C}^2(B_1)), \quad \max_{t\in[1,2]}\max_{x\in B_1}\Big|u(t,x)\Big| \le P,$$

$$\max_{t\in[1,2]}\max_{x\in B_1}\Big|u_{x_i}(t,x)\Big| \le P, \quad i=0,1,2,\dots,n\Big\},$$

$$N^1_{11} = \Big\{u(t,x)\in\mathcal{C}^1([1,2],\mathcal{C}^2(B_1)), \quad \max_{t\in[1,2]}\max_{x\in B_1}\Big|u(t,x)\Big| \le (1+\epsilon)P,$$

$$\max_{t\in[1,2]}\max_{x\in B_1}\Big|u_{x_i}(t,x)\Big| \le (1+\epsilon)P, \quad i=0,1,2,\dots,n\Big\}.$$

In these sets we define a norm as follows

$$||u||_2 = \sup\Big\{\Big|u(t,x)\Big| : (t,x)\in[1,2]\times B_1\Big\},$$

these sets are closed, convex and compact in $\mathcal{C}([1,2]\times B_1)$ in sense of $||\cdot||$.

As in above we conclude that the problem (2.12) has a solution $u_1^{11}\in\mathcal{C}^1([1,2],\mathcal{C}^2(B_1))$.

Now we consider the Cauchy problem

$$u_t-(\Delta+m)u=\lambda|u|^{p-1}u,\quad t\in[1,2],\quad x\in B_2,$$
$$u(1,x)=u_1(1,x) \tag{2.13}$$

$$L_{12}^1(u)=u(t,x)+m\int_1^t\int_x^A\int_z^A u(y,s)dsdzdy+\lambda\int_1^t\int_x^A\int_z^A|u|^{p-1}udsdzdy$$
$$+\sum_{i=2}^n\int_1^t\int_{\overline{x}_i}^A\int_{\overline{z}_i}^A u(y,\hat{s}_i)d\overline{s}_id\overline{z}_idy$$
$$-\int_x^A\int_z^A\Big(u(t,s)-u_1(1,s)\Big)dsdz$$
$$+\int_1^t\int_{\overline{x}_1}^A\int_{\overline{z}_1}^A(u(y,x_1,s_2,\dots,s_n)-u_1^{11}(y,A_1,s_2,\dots,s_n)+(A_1-x_1)u_{x_1}^{11}(y,A_1,s_2,\dots,s_n))d\overline{s}_1d\overline{z}_1dy,$$
$$t\in[1,2],\quad x\in B_2,$$

$$L_{12}^1(u)=T_{12}^1(u)+S_{12}^1(u),$$

where

$$T_{12}^1(u)=(1+\epsilon)u(t,x),\quad t\in[1,2],x\in B_2,$$

$$S_{12}^1(u)=-\epsilon u(t,x)+m\int_1^t\int_x^A\int_z^A u(y,s)dsdzdy+\lambda\int_1^t\int_x^A\int_z^A|u|^{p-1}udsdzdy$$
$$+\sum_{i=2}^n\int_1^t\int_{\overline{x}_i}^A\int_{\overline{z}_i}^A u(y,\hat{s}_i)d\overline{s}_id\overline{z}_idy$$
$$-\int_x^A\int_z^A\Big(u(t,s)-u_1(1,s)\Big)dsdz$$
$$+\int_1^t\int_{\overline{x}_1}^A\int_{\overline{z}_1}^A(u(y,x_1,s_2,\dots,s_n)-u_1^{11}(y,A_1,s_2,\dots,s_n)+(A_1-x_1)u_{1_{x_1}}^{11}(y,A_1,s_2,\dots,s_n))d\overline{s}_1d\overline{z}_1dy,$$
$$t\in[1,2],x\in B_2,$$

the sets

$$M_{12}^1=\Big\{u(t,x)\in\mathcal{C}^1([1,2],\mathcal{C}^2(B_2)),\quad \max_{t\in[1,2]}\max_{x\in B_2}\Big|u(t,x)\Big|\le P,$$
$$\max_{t\in[1,2]}\max_{x\in B_2}\Big|u_{x_i}(t,x)\Big|\le P,\quad i=0,1,2,\dots,n\Big\},$$

$$N_{12}^1=\Big\{u(t,x)\in\mathcal{C}^1([1,2],\mathcal{C}^2(B_2)),\quad \max_{t\in[1,2]}\max_{x\in B_2}\Big|u(t,x)\Big|\le(1+\epsilon)P,$$
$$\max_{t\in[1,2]}\max_{x\in B_2}\Big|u_{x_i}(t,x)\Big|\le(1+\epsilon)P,\quad i=0,1,2,\dots,n\Big\},$$

in these sets we define a norm as follows

$$||u||_2=\sup\Big\{\Big|u(t,x)\Big|:(t,x)\in[1,2]\times B_2\Big\}.$$

The sets M_{12}^1 and N_{12}^1 are closed, convex and compact in $\mathcal{C}([1,2]\times B_2)$ in sense of $||\cdot||_2$. As in the step 1 we conclude that the problem (2.13) has a solution $u_1^{12}\in\mathcal{C}^1([1,2],\mathcal{C}^2(B_2))$.

Since $u_1^{12}(1,x)=u_1(1,x)$ for $x\in B_2$ and $u_1^{11}(1,x)=u_1(1,x)$ for $x\in B_1$ we have that

$$u_1^{12}{}_{|_{t=1,x_1=A_1}}=u_1{}_{|_{t=1,x_1=A_1}},$$

$$(u_1^{12})_{x_1}{}_{|_{t=1,x_1=A_1}}=(u_1)_{x_1}{}_{|_{t=1,x_1=A_1}},$$

Notes

$$(u_1^{12})_{x_1x_1}{}_{|_{t=1,x_1=A_1}} = (u_1)_{x_1x_1}{}_{|_{t=1,x_1=A_1}},$$
$$u_1^{11}{}_{|_{t=1,x_1=A_1}} = u_1{}_{|_{t=1,x_1=A_1}},$$
$$(u_1^{11})_{x_1}{}_{|_{t=1,x_1=A_1}} = (u_1)_{x_1}{}_{|_{t=1,x_1=A_1}},$$
$$(u_1^{11})_{x_1x_1}{}_{|_{t=1,x_1=A_1}} = (u_1)_{x_1x_1}{}_{|_{t=1,x_1=A_1}}, \tag{2.14}$$

Notes

as in the case $t \in [0,1]$ we have

$$u_1^{12}{}_{|_{x_1=A_1}} = u_1^{11}{}_{|_{x_1=A_1}},$$
$$(u_1^{12})_{x_1}{}_{|_{x_1=A_1}} = (u_1^{11})_{x_1}{}_{|_{x_1=A_1}}, \tag{2.15}$$
$$(u_1^{12})_{x_1x_1}{}_{|_{x_1=A_1}} = (u_1^{11})_{x_1x_1}{}_{|_{x_1=A_1}}$$

for $t \in [1,2]$ and etc. In this way we obtain a solution $u_2 \in \mathcal{C}^1([1,2], \mathcal{C}^2(\mathbb{R}^n))$ to the Cauchy problem

$$u_t - (\Delta + m)u = \lambda |u|^{p-1}u, \quad t \in [1,2], x \in \mathbb{R}^n,$$
$$u(1,x) = u_1(1,x), \quad x \in \mathbb{R}^n.$$

Using reasonings as (2.14), (2.15) we have that

$$\begin{cases} u_1 & t \in [0,1], x \in \mathbb{R}^n, \\ u_2 & t \in [1,2], x \in \mathbb{R}^n, \end{cases}$$

is a solution to the Cauchy problem

$$u_t - (\Delta + m)u = \lambda |u|^{p-1}u \quad t \in [0,2], x \in \mathbb{R}^n,$$
$$u(0,x) = u_0(x) \quad x \in \mathbb{R}^n$$

which belongs to the space $\mathcal{C}^1([0,2], \mathcal{C}^2(\mathbb{R}^n))$ and etc. we obtain a solution to the problem (1.1), (1.2) which belongs to the space $\mathcal{C}^1([0,\infty), \mathcal{C}^2(\mathbb{R}^n))$.

III. Acknowledgements

This article is supported by the grant DD-VU 02/90, Bulgaria.

References Références Referencias

1. Fujita, H. On the blowing up of solutions of the Cauchy problem for ut = u + u1+, J. Fac. Sci. Univ. Tokyo Sect. I 13 (1966), 109124.
2. Georgiev, S. Global existence for nonlinear wave equation. Dyn. PDE, Vol. 7, No. 3, pp. 207-216.
3. Xiang, T., Rong Yuan. A class of expansive - type Krasnosel'skii ˙xed point theorems. Nonlinear Analysis, 71(2009), 3229-3239.

Notes

This page is intentionally left blank

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 8 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

The Collatz 3n+1 Conjecture is Unprovable

By Craig Alan Feinstein

Abstract - In this paper, we show that any proof of the Collatz 3n + 1 Conjecture must have an infinite number of lines; therefore, no formal proof is possible.

GJSFR-F Classification : MSC 2010: 13L99

Strictly as per the compliance and regulations of :

The Collatz 3n+1 Conjecture is Unprovable

Craig Alan Feinstein

Abstract - In this paper, we show that any proof of the Collatz 3n + 1 Conjecture must have an infinite number of lines; therefore, no formal proof is possible.

In 2005, the famous mathematician Freeman Dyson was asked, "What do you believe is true even though you cannot prove it?" He answered:

"Since I am a mathematician, I give a precise answer to this question. Thanks to Kurt Gödel, we know that there are true mathematical statements that cannot be proved. But I want a little more than this. I want a statement that is true, unprovable, and simple enough to be understood by people who are not mathematicians. Here it is.

"Numbers that are exact powers of two are 2, 4, 8, 16, 32, 64, 128 and so on. Numbers that are exact powers of five are 5, 25, 125, 625 and so on. Given any number such as 131072 (which happens to be a power of two), the reverse of it is 270131, with the same digits taken in the opposite order. Now my statement is: it never happens that the reverse of a power of two is a power of five.

"The digits in a big power of two seem to occur in a random way without any regular pattern. If it ever happened that the reverse of a power of two was a power of five, this would be an unlikely accident, and the chance of it happening grows rapidly smaller as the numbers grow bigger. If we assume that the digits occur at random, then the chance of the accident happening for any power of two greater than a billion is less than one in a billion. It is easy to check that it does not happen for powers of two smaller than a billion. So the chance that it ever happens at all is less than one in a billion. That is why I believe the statement is true.

"But the assumption that digits in a big power of two occur at random also implies that the statement is unprovable. Any proof of the statement would have to be based on some non-random property of the digits. The assumption of randomness means that the statement is true just because the odds are in its favor. It cannot be proved because there is no deep mathematical reason why it has to be true. (Note for experts: this argument does not work if we use powers of three instead of powers of five. In that case the statement is easy to prove because the reverse of a number divisible by three is also divisible by three. Divisibility by three happens to be a non-random property of the digits).

"It is easy to find other examples of statements that are likely to be true but unprovable. The essential trick is to find an infinite sequence of events, each of which might happen by accident, but with a small total probability for even one of them happening. Then the statement that none of the events ever happens is probably true but cannot be proved." [1]

In the spirit of Dyson's observation, we shall give an example of a statement that is likely to be true and then take things one step further by presenting a formal proof that the statement is unprovable. Consider the following function:

Author : 2712 Willow Glen Drive, Baltimore, Maryland 21209. E-mail : cafeinst@msn.com

1. Dyson, F., "What do you believe is true even though you cannot prove it?", http://www.edge.org/q2005/q05.9.html

Definition 1: Let $T:\mathbb{N}\rightarrow\mathbb{N}$ be a function such that $T(n)=\frac{3n+1}{2}$ if n is odd and $T(n)=\frac{n}{2}$ if n is even.

The Collatz $3n+1$ Conjecture states that for each $n\in\mathbb{N}$, there exists a $k\in\mathbb{N}$ such that $T^{(k)}(n)=1$, where $T^{(k)}(n)$ is the function T iteratively applied k times to n [2]. As of May 10, 2011, this conjecture has been verified for all positive integers up to about 2^{60} [3]. Furthermore, one can give a heuristic probabilistic argument [4] that since every iterate of the function T decreases on average by a multiplicative factor of about $(\frac{3}{2})^{1/2}(\frac{1}{2})^{1/2}=(\frac{3}{4})^{1/2}$, all iterates will eventually converge into the infinite cycle $\{1, 2, 1, 2, ...\}$, assuming that each $T^{(k)}$ sufficiently mixes up n as if each $T^{(k)}(n)$ (mod 2) were drawn at random from the set $\{0, 1\}$.

However, the Collatz $3n+1$ Conjecture has never been formally proven. We shall prove that the Collatz $3n+1$ Conjecture can, in fact, never be formally proven, even though there is a lot of evidence for its truth. The underlying assumption in our argument is that any proof of a theorem can be written in a computer text-file, which is composed of bits (zeroes and ones). First, let us present a definition of "random".

Definition 2 : We shall say that vector $x\in\{0,1\}^k$ is random if x cannot be specified in less than k bits in a computer text-file [5].

For example, the vector of one million concatenations of the vector (0, 1) is not random, since we can specify it in less than two million bits in a computer text-file by just writing, "the vector of one million concatenations of the vector (0, 1)" in the text-file. However, the vector of outcomes of one million coin-tosses has a good chance of fitting our definition of "random", since much of the time the most compact way of specifying such a vector is to simply make a list of the results of each coin-toss, in which one million bits are necessary. We now prove three theorems.

Theorem 1 : For any vector $x\in\{0,1\}^k$, there exists an $n\in\mathbb{N}$ such that $x=(n,T(n),\ldots,T^{(k-1)}(n))$ (mod 2).

Proof : A proof of this can be found in "The $3x+1$ problem and its generalizations" [2].

Theorem 2 : If $k,n\in\mathbb{N}$ and $T^{(k)}(n)=1$, then in order to prove that $T^{(k)}(n)=1$, it is necessary to specify the values of $(n,T(n),\ldots,T^{(k-1)}(n))$ (mod 2) in the proof.

Proof : Let the vector $(x_0(n),x_1(n),\ldots,x_{k-1}(n))$ equal $(n,T(n),\ldots,T^{(k-1)}(n))$(mod 2). Then notice that the formula, $T^{(k)}(n)=\lambda_k(n)n+\rho_k(n)$ [2], where

$$\lambda_k(n)=\frac{3^{x_0(n)+\ldots+x_{k-1}(n)}}{2^k}$$

and

$$\rho_k(n)=\sum_{i=0}^{k-1}x_i(n)\frac{3^{x_{i+1}(n)+\ldots+x_{k-1}(n)}}{2^{k-i}},$$

is determined by the values of $(n,T(n),\ldots,T^{(k-1)}(n))$ (mod 2) and there is a one-to-one correspondence between all of the possible formulas for $T^{(k)}(n)$ and all of the possible values of $(n,T(n),\ldots,T^{(k-1)}(n))$ (mod 2);therefore, in order to prove that $T^{(k)}(n)=1$, it

Ref

2. Lagarias, J.C., "The 3x + 1 problem and its generalizations", Amer. Math. Monthly 92 (1985) 3-23. [Reprinted in: Conference on Organic Mathematics, Canadian Math. Society Conference Proceedings vol 20, 1997, pp. 305 - 331], http://www.cecm.sfu.ca/organics/papers.

is necessary to specify the values of $(n,\ T(n),\ ...,\ T^{(k-1)}(n))$ (mod 2) in the proof, since in order to prove that $T^{(k)}(n) = 1$, it is necessary to specify the formula for $T^{(k)}(n)$ in the proof.

Theorem 3 : It is impossible to prove the Collatz $3n$+1 Conjecture.

Proof : Suppose that there exists a proof of the Collatz $3n+1$ Conjecture, and let L be the number of bits in such a proof. Now, let $x \in \{0, 1\}^{L+1}$ be a random vector, as defined above. (It is not difficult to prove that at least half of all vectors in $\{0, 1\}^{L+1}$ are random [5].) By Theorem 1, there exists an $n \in \mathbb{N}$ such that $\mathbf{x} = (n, T(n), ..., T^{(L)}(n))$ (mod 2) and $T^{(L+1)}(n) = T^{(L)}(n)$ (mod 2). Then $T^{(L)}(n) > 2$, so if $T^{(k)}(n) = 1$, then $k > L$. Hence, by Theorem 2 it is necessary to specify the values of $(n,\ T(n),\ ...,\ T^{(L)}(n))$ (mod 2) in order to prove that there exists a $k \in \mathbb{N}$ such that $T^{(k)}(n) = 1$. But since $(n,\ T(n),\ ...,\ T^{(L)}(n))$ (mod 2) is a random vector, at least $L + 1$ bits are necessary to specify $(n, T(n), ..., T^{(L)}(n))$ (mod 2), contradicting our assumption that the proof contains only L bits; therefore, a formal proof of the Collatz $3n + 1$ Conjecture cannot exist.

References Références Referencias

1. Dyson, F., “What do you believe is true even though you cannot prove it?”, http://www.edge.org/q2005/q05_9.html
2. Lagarias, J.C., “The 3x + 1 problem and its generalizations”, *Amer. Math. Monthly 92* (1985) 3-23. [Reprinted in: *Conference on Organic Mathematics*, Canadian Math. Society Conference Proceedings vol 20, 1997, pp.305-331], http://www.cecm.sfu.ca/organics/papers
3. Roosendaal, E. (2011+), “On the 3x + 1 problem”, http://www.ericr.nl/wondrous/index.html
4. Crandall, R. E., “On the 3x+1 problem”, *Math. Comp.*, 32 (1978) 1281-1292.
5. Chaitin, G.J., *Algorithmic Information Theory*, revised third printing, Cambridge University Press, 1990.

Ref.

5. Chaitin, G.J., Algorithmic Information Theory, revised third printing, Cambridge University Press, 1990.

This page is intentionally left blank

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 8 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Linear Canonical Transforms On the Zemanian Spaces

By Gudadhe A S
Govt. Vidarbha Institute of Science and Humanities,India

Abstract - The objective of this paper is to make the theory of Linear canonical transform mathematically rigorous. Here we define the linear canonical transform on the Zemanian space $\mathcal{SS}$, the space of functions of rapid descent, and prove some results. We have also discussed some operation formulae for linear canonical transform on this space.

Keywords : *Fractional Fourier transform, Linear canonical transform, space $\mathcal{SS}$.*

GJSFR - F Classification : *MSC 2010: 46F12, 44*

LINEAR CANONICAL TRANSFORMS ON THE ZEMANIAN SPACES

Strictly as per the compliance and regulations of :

Linear Canonical Transforms On the Zemanian Spaces

Gudadhe A S

Abstract - The objective of this paper is to make the theory of Linear canonical transform mathematically rigorous. Here we define the linear canonical transform on the Zemanian space $\mathcal{S}$, the space of functions of rapid descent, and prove some results. We have also discussed some operation formulae for linear canonical transform on this space.
Keywords : Fractional Fourier transform, Linear canonical transform, space $\mathcal{S}$.

I. Introduction

The linear canonical transform is a much more general integral transform which has the Fourier transform, Fractional Fourier transform, Fresnel transform etc as its special cases. As it is well known that fractional Fourier transform (FrFT) is the powerful mathematical tool and is widely used for spectral analysis, signal processing, optical system analysis etc.[6], the LCT which is the generalization of FrFT has obviously more ability and potential due to its three parameters. Especially it is used to analyze optical systems with prisms or shifted lenses.

Since Namias [5] had introduced Fractional Fourier transform, many mathematicians had studied it thoroughly e.g. Almeida [2] studied its convolution and product, Akay [1] discussed unitary operators on it, where as Kerr [4] extended it to the spaces of distributions.

As compared to this, Linear Canonical transform is less attended integral transform. We have studied its analytic properties [3] and here we want to study it in some Zemanian spaces.

Definition : The linear canonical transform (LCT) is a four parameter (a, b, c, d), class of integral transform given by,

$$F_A(u) = [LCT f(t)](u) = \frac{1}{\sqrt{2\pi i b}} e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} e^{\frac{ia}{2b}t^2} e^{\frac{-i}{b}ut} f(t)dt \quad \text{for } b \neq 0$$
$$= \sqrt{d}\, e^{\frac{i}{2}cdu^2} f(du) \qquad \text{for } b = 0 \qquad (1)$$

It is characterized by a general 2×2 matrix, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc = 1$, moreover the entries a, b, c, d are real or complex but for this paper we assume that they are real parameters. Inverse Linear canonical transform is given by

$$[F_A^{-1}(u)]\,(t) = f(t) = \int_{-\infty}^{\infty} F_A(u)\, K_A^{-1}\,(t,u)du \qquad (2)$$

where $K_A^{-1}(t,u)$ is the Hermitian conjugate of $K_A(t,u)$ that is $K_A^{-1}(t,u) = K_A^{*}\,(u,t)$.

This paper attempts to provide the necessary mathematical framework for extending LCT to the spaces of generalized functions. We begin by recalling the definition of the space $\mathcal{S}$, the space of functions of rapid descent and some properties of the Fourier transform on it.

Section 2 is devoted to define LCT on the space $\mathcal{S}$ and prove the basic properties of the concerned transform. In section 3 some operational formulae are developed. Application of the

Author : Dept. of mathematics, govt vidarbha institute of science and humanities, Amravati, India. E-mail : alka.gudadhe@gmail.com

theory is discussed by solving one example in section 4 and finally we conclude in the last section.

This work is under the Major Research Project sanctioned by UGC (F No. 40-241/2011)

Notations as per Zemanian [7].

II. Preliminaries

In this section we note some of the results which will be used later on.

Let $\mathcal{S}$ be the vector space of all smooth functions φ such that,

$\gamma_{m,n}(\varphi) = \sup_{t\epsilon R} \left|t^m \varphi^{(n)}(t)\right| < \infty$, for all $m,n \in \{0,1,2,\dots\}$.

Clearly $\mathcal{S}$ is equipped with the topology generated by the collection of seminorms $\{\gamma_{m,n}\}_{m,n=0}^{\infty}$ also $\mathcal{S}$ is a Frechet space.

Fourier transform defined on $\mathcal{S}$

$$\mathcal{F}\big(\varphi(t)\big) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-itw} \varphi(t)\, dt, \qquad \varphi \epsilon \mathcal{S}, w \in R \tag{3}$$

Some useful results as in [3] :

a) $\mathcal{F} : \mathcal{S} \to \mathcal{S}$ is a homeomorphism.

b) $(\mathcal{F}^{-1}\big(\varphi(t)\big)(w) = \big(\mathcal{F}\varphi(t)\big)(-x) \qquad \varphi \epsilon \mathcal{S}, w \in R.$

c) If $f(t) = e^{\frac{1}{2}At^2}, A \epsilon C$ then for $n = 0,1,2,\dots$,

$$f^{(2n)}(t) = e^{\frac{1}{2}At^2} \sum_{r=0}^{n} \alpha_r\, A^{2n-r}\, x^{2n-2r}$$

$$f^{(2n+1)}(t) = e^{\frac{1}{2}At^2} \sum_{r=0}^{n} \beta_r\, A^{2n+1-r}\, x^{2n+1-2r}$$

where the constants α_r and β_r are independent of A.

III. Linear Canonical Transform on $\mathcal{S}$ Spaces

For each $f \epsilon \mathcal{S}$, the Linear canonical transform of f as given by,

$$F_A(u) = [LCT_A f(t)]\,(u) = \int_{-\infty}^{\infty} f(t)\; K_A(t,u)\, dt$$

where $K_A(t,u) = \frac{1}{\sqrt{2\pi i b}}\, e^{\frac{i}{2b}(at^2+du^2)}\; e^{\frac{-i}{b}ut}$ for $b \neq 0$

$= \sqrt{d}\, e^{\frac{i}{2}cdu^2} f(du)$ for $b = 0$

where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and a, b, c, d are real.

a) Proposition : $F_A(u) = \frac{1}{\sqrt{ib}} e^{\frac{id}{2b}u^2} [\mathcal{F}\left(e^{\frac{ia}{2b}t^2} f(t)\right)](\frac{u}{b})$ where $\mathcal{F}$ is the Fourier transform as explained above.

Proof : The result is very clear from the definition (1) and (2).

Also we note that

b) Proposition : $F_A(u)$, Linear canonical transform of $f(t)$, is the homeomorphism on $\mathcal{S}$ with inverse as given by (3).

Proof : We know that $\mathcal{F}$, Fourier transform is homeomorphism on $\mathcal{S}$ [7]. Also the function $h(u) = e^{\frac{id}{2b}u^2} g(u)$, for $g(u) \epsilon \mathcal{S}$, is a homeomorphism on $\mathcal{S}$. Therefore by 2.1, Linear canonical transform of $f(t)$, is the homeomorphism on $\mathcal{S}$.

Now it remains to show that (2) represents the inverse under the space $\mathcal{S}$.

$$[LCT_A^{-1}\, LCT_A\, f(t)](v) = \int_{-\infty}^{\infty} f(t)\ K_A(t,u) \int_{-\infty}^{\infty} K_A^{-1}(u,v)\, dt\, du$$

$$= C_1 e^{\frac{-ia}{2b}v^2} \int_{-\infty}^{\infty} e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} C_1^* \, e^{\frac{-ivu}{b}}\ e^{\frac{ia}{2b}t^2} e^{\frac{-id}{2b}u^2} e^{\frac{iut}{b}}\, dt\, du$$

where $C_1 = \frac{1}{\sqrt{2\pi i b}}$ and C_1^* is its conjugate.

$$= \frac{1}{2\pi|b|}\ e^{\frac{-ia}{2b}v^2} \ \int_{-\infty}^{\infty}\{\int_{-\infty}^{\infty}[e^{\frac{ia}{2b}t^2}\ f(t)]\, e^{\frac{iut}{b}}\, dt\} e^{\frac{-iuv}{b}}\, du$$

$$= \ e^{\frac{-ia}{2b}v^2}[\,\mathcal{F}^{-1}\mathcal{F}\{e^{\frac{ia}{2b}t^2}\ f(t)\}]\,(v)$$

$$= f(v)$$

Hence the result follows.

c) Theorem : For each $f \,\epsilon\, \mathcal{S}$, $\{LCT_A f(t)\}(u)$ converges to $f(t)$ with respect to the topology of $\mathcal{S}$ as $b \to 0^+$ and $a \to 1$.

Proof : Since Fourier transform is a homeomorphism on $\mathcal{S}$, equivalently we prove that, $LCT_A\mathcal{F}f$ converges to $\mathcal{F}f$ as $b \to 0^+$ in $\mathcal{S}$ where $\mathcal{F}$ denotes the Fourier transform of f.

$$(F_A\mathcal{F}f)(v) = [LCT_A\mathcal{F}f](v) = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi i b}}\ e^{\frac{i}{2b}(au^2+dv^2)}\ e^{\frac{-i}{b}uv}\ \{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-itu}\ f(t)dt\}\, du$$

$$= \frac{1}{\sqrt{2\pi i b}}\ \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{\frac{idv^2}{2b}} \left\{\sqrt{\frac{2\pi i b}{a}}\ e^{\frac{i}{2a}\left(\frac{v}{b}+t\right)^2}\right\} f(t)dt$$

Using $\int_{-\infty}^{\infty} e^{iau^2}\, e^{ibu}\, du = \sqrt{\frac{\pi i}{a}}\ e^{\frac{-ib^2}{4a}}$ if $a > 0$.

$$= \frac{1}{\sqrt{2\pi a}}\int_{-\infty}^{\infty} e^{\frac{idv^2}{2b}}\ e^{\frac{-ib}{2a}(\frac{v^2}{b^2}+\frac{2}{b}vt+t^2)}\ f(t)dt$$

$$= \frac{1}{\sqrt{2\pi a}} e^{\frac{i(d-\frac{1}{a})v^2}{2b}} \int_{-\infty}^{\infty} e^{\frac{-ib}{2a}t^2}\ f(t)\ e^{-\frac{i}{a}vt}\ dt \qquad (1)$$

$$\therefore\ v^m\ (D_v^n\ F_A\mathcal{F}f)(v) = \ \frac{v^m}{\sqrt{2\pi a}}\sum_{k=0}^{n}\binom{n}{k}\ D_v^k\ e^{\frac{i(d-\frac{1}{a})v^2}{2b}}\ D_v^{n-k} \int_{-\infty}^{\infty} e^{\frac{-ib}{2a}t^2}\ f(t)\ e^{-\frac{i}{a}vt}\ dt$$

$$= \frac{1}{\sqrt{2\pi a}}\ \sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\binom{n}{k} a_r\ \left(\frac{i}{2b}(d-\frac{1}{a})\right)^{k-r} e^{\frac{i(d-\frac{1}{a})v^2}{2b}}\, v^{m+k-2r} \int_{-\infty}^{\infty} e^{\frac{-ib}{2a}t^2}\ f(t)(\frac{-it}{a})^{n-k}\ e^{-\frac{i}{a}vt} dt$$

$$=\frac{1}{\sqrt{2\pi a}}\sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\binom{n}{k}a_r\ (\frac{i}{2b}(d-\frac{1}{a}))^{k-r}e^{\frac{i(d-\frac{1}{a})v^2}{2b}}\int_{-\infty}^{\infty}e^{\frac{-ib}{2a}t^2}f(t)(\frac{-it}{a})^{n-k}v^{m+k-2r}e^{-\frac{i}{a}vt}dt \qquad (2)$$

Now,

$$D_t^{m+k-2r}\left(e^{\frac{-ivt}{a}}\right)=(\frac{-iv}{a})^{m+k-2r}\ e^{\frac{-ivt}{a}}$$

Hence (3) becomes,

$$=\frac{1}{\sqrt{2\pi a}}\sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\binom{n}{k}a_r\ (\frac{i}{2b}(d\frac{1}{a}))^{k-r}e^{\frac{i(d-\frac{1}{a})v^2}{2b}}\int_{-\infty}^{\infty}e^{\frac{-ib}{2a}t^2}f(t)(\frac{-it}{a})^{n-k}$$

$$(ia)^{m+k-2r}D_t^{m+k-2r}e^{-\frac{i}{a}vt}dt$$

$$=\frac{1}{\sqrt{2\pi a}}\sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\binom{n}{k}a_r\ (\frac{i}{2b}(d\frac{1}{a}))^{k-r}(ia)^{m+2k-2r-n}e^{\frac{i(d-\frac{1}{a})v^2}{2b}}\int_{-\infty}^{\infty}e^{-\frac{i}{a}vt}D_t^{m+k-2r}\{e^{\frac{-ib}{2a}t^2}f(t)(t)^{n-k}\}\,dt$$

by [7 p 49]

$$=\frac{1}{\sqrt{2\pi a}}\sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\binom{n}{k}a_r\ (\frac{i}{2b}(d\frac{1}{a}))^{k-r}(ia)^{m+2k-2r-n}e^{\frac{i(d-\frac{1}{a})v^2}{2b}}\int_{-\infty}^{\infty}e^{-\frac{i}{a}vt}$$

$$\sum_{j=0}^{m+k-2r}\binom{m+k-2r}{j}D_t^j\ e^{\frac{-ib}{2a}t^2}D_t^{m+k-2r-j}\left(t^{n-k}f(t)\right)dt$$

$$=\frac{1}{\sqrt{2\pi a}}\sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\binom{n}{k}a_r\ (\frac{i}{2b}(d-\frac{1}{a}))^{k-r}(ia)^{m+2k-2r-n}e^{\frac{i(d-\frac{1}{a})v^2}{2b}}\int_{-\infty}^{\infty}e^{-\frac{i}{a}vt}$$

$$\sum_{j=0}^{m+k-2r}\sum_{l=0}^{[\frac{j}{2}]}b_r\binom{m+k-2r}{j}(\frac{b}{ia})^{j-l}\ t^{j-2l}D_t^{m+k-2r-j}\left(t^{n-k}f(t)\right)dt$$

$$=\sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\sum_{j=0}^{m+k-2r}\sum_{l=0}^{[\frac{j}{2}]}\binom{n}{k}\binom{m+k-2r}{j}a_r b_r\ (\frac{i}{2b}(d-\frac{1}{a}))^{k-r}(ia)^{m+2k-2r-n}\ (\frac{b}{ia})^{j-l}$$

$$\int_{-\infty}^{\infty}e^{-\frac{i}{a}vt}\ t^{j-2l}D_t^{m+k-2r-j}\left(t^{n-k}f(t)\right)dt$$

$$=\sum_{k=0}^{n}\sum_{r=0}^{[\frac{k}{2}]}\sum_{j=0}^{m+k-2r}\sum_{l=0}^{[\frac{j}{2}]}\ G_{j,k,r,l}$$

where, $G_{j,k,r,l}=\binom{n}{k}\binom{m+k-2r}{j}a_r b_r\left(\frac{i}{2b}\left(d-\frac{1}{a}\right)\right)^{k-r}\ (ia)^{m+2k-2r-n}\ \left(\frac{b}{ia}\right)^{j-l}$

$$\int_{-\infty}^{\infty}e^{-\frac{i}{a}vt}\ t^{j-2l}D_t^{m+k-2r-j}\left(t^{n-k}f(t)\right)dt$$

$\therefore \left| \ G_{j,k,r,l} \right|$

$$= \left| \binom{n}{k} \binom{m+k-2r}{j} a_r b_r \left(\frac{i}{2b} \left(-\frac{1}{a} \right) \right)^{k-r} (ia)^{m+2k-2r-n} \left(\frac{b}{ia} \right)^{j-l} \int_{-\infty}^{\infty} e^{-\frac{i}{a}vt}\, t^{j-2l} D_t^{m+k-2r-j} \left(t^{n-k} f(t) \right) dt \right|$$

Since $\left| e^{-\frac{i}{a}vt} \right| = 1$

$\left| \ G_{j,k,r,l} \right| =$

$$\left| \binom{n}{k} \binom{m+k-2r}{j} a_r b_r \left(\frac{i}{2b} \left(d - \frac{1}{a} \right) \right)^{k-r} (ia)^{m+2k-2r-n} \left(\frac{b}{ia} \right)^{j-l} \int_{-\infty}^{\infty} t^{j-2l} D_t^{m+k-2r-j} \left(t^{n-k} f(t) \right) dt \right|$$

$$= \left| \binom{n}{k} \binom{m+k-2r}{j} a_r b_r \left(\frac{i}{2b} \left(\frac{bc}{a} \right) \right)^{k-r} (ia)^{m+2k-2r-n} \left(\frac{b}{ia} \right)^{j-l} \int_{-\infty}^{\infty} t^{j-2l} D_t^{m+k-2r-j} \left(t^{n-k} f(t) \right) dt \right|$$

$$= \left| \binom{n}{k} \binom{m+k-2r}{j} a_r b_r \left(\frac{i}{2} \left(\frac{c}{a} \right) \right)^{k-r} (ia)^{m+2k-2r-n} \left(\frac{b}{ia} \right)^{j-l} \int_{-\infty}^{\infty} t^{j-2l} D_t^{m+k-2r-j} \left(t^{n-k} f(t) \right) dt \right|$$

Now $\int_{-\infty}^{\infty} t^{j-2l} D_t^{m+k-2r-j} \left(t^{n-k} f(t) \right) dt$, is independent of the parameters $a, b, c, d,$ hence $\left| \ G_{j,k,r,l} \right| \to 0$ as $b \to 0$ and $a \to 1$ provided $j - l > 0$, which is always true.

$\therefore \left| v^m \left(D_v^n\, F_A \mathcal{F} f \right)(v) - v^m \frac{D_v^n}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t) e^{-ivt}\ dt \right| < \infty$, since $f \in \mathcal{S}$.

Hence the theorem is proved.

IV. Some Properties of Linear Canonical Transform in $\mathcal{S}$

a) **Theorem** : For each $f \in \mathcal{S}$, $F_A F_B = F_C$ where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$ and $C = B\,A$.

Proof : Consider,

$$[F_B F_A f(t)](v) = LCT_B \frac{1}{\sqrt{2\pi ib}} \int_{-\infty}^{\infty} e^{\frac{i}{2b}(at^2+du^2)}\ e^{\frac{-i}{b}ut}\ f(t)\, dt$$

$$= \frac{1}{\sqrt{2\pi ib}} \frac{1}{\sqrt{2\pi iq}} \int_{-\infty}^{\infty} e^{\frac{i}{2q}(pu^2+sv^2)}\ e^{\frac{-i}{q}uv} \left\{ \int_{-\infty}^{\infty} e^{\frac{i}{2b}(at^2+du^2)}\ e^{\frac{-i}{b}ut}\ f(t)\, dt \right\} du$$

$$= \frac{1}{\sqrt{2\pi ib}} \frac{1}{\sqrt{2\pi iq}} \left[\int_{-\infty}^{\infty} e^{\frac{i}{2}\left(\frac{p}{q}+\frac{d}{b}\right)u^2 - i\left(\frac{v}{q}+\frac{t}{b}\right)u}\ du \right] e^{\frac{iat^2}{2b}} e^{\frac{isv^2}{2q}} f(t) dt$$

$$= \frac{1}{\sqrt{2\pi ib}} \frac{1}{\sqrt{2\pi iq}} \int_{-\infty}^{\infty} \frac{\sqrt{i}\sqrt{2\pi bq}}{\sqrt{bp+dq}}\ e^{\frac{-i(bv+qt)^2}{2(bp+dq)}} e^{\frac{iat^2}{2b}} e^{\frac{isv^2}{2q}}\ f(t) dt$$

$$= \frac{1}{\sqrt{2\pi i(bp+dq)}} \int_{-\infty}^{\infty} e^{\frac{i}{2b}\left(a-\frac{q}{(bp+dq)}\right)t^2}\ e^{\frac{i}{2q}\left(s-\frac{b}{(bp+dq)}\right)v^2}\ e^{\frac{-ivt}{(bp+dq)}}\ f(t) dt$$

Now by simple calculations and using $ad - bc = 1$ and $ps - rq = 1$, it can be shown that the right hand side is nothing but,

$$= [F_C f(t)](v).$$

b) **Theorem :** For $f \in \mathcal{S}$, and $\tau \in R$,

$$F_A\, f(t-\tau) = e^{\frac{-iac\tau^2}{2}}\, e^{icu\tau}\, [F_A f(t)]\,(u-a\tau).$$

Proof : As $f \in \mathcal{S}$ implies $f(t-\tau) \in \mathcal{S}$, (by ex. 3, p 102 [2]) so that both sides of above equation are defined as elements of $\mathcal{S}$. We shall prove the result for $f \in \mathcal{D}$, the space of smooth functions of compact support, then the result will be clear from the continuity and denseness.

$$F_A\, f(t-\tau) = \frac{1}{\sqrt{2\pi i b}}\, e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} e^{\frac{ia}{2b}t^2}\, e^{\frac{-i}{b}ut}\, f(t-\tau)dt$$

$$= \frac{1}{\sqrt{2\pi i b}}\, e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} e^{\frac{ia}{2b}(t+\tau)^2}\, e^{\frac{-i}{b}u(t+\tau)}\, f(t)dt$$

Simple calculations will show that right hand side is equal to,

$$= e^{\frac{-iac\tau^2}{2}}\, e^{icu\tau}\, [F_A f(t)]\,(u-a\tau)$$

c) **Theorem :** For $f \in \mathcal{S}$, and $n \in \mathbb{N}$,

$$[F_A\,(D^n f(t))]\,(u) = (-icu + a\frac{d}{du})^n\, F_A\,(u).$$

Proof : As $(D^n f(t)) \in \mathcal{S}$ if $f \in \mathcal{S}$, both sides of above equation are defined in $\mathcal{S}$.

We shall prove for $n = 1$ and it follows for all $n > 1$by induction.

Consider,

$$[F_A\,\big(Df(t)\big)(u) = \frac{1}{\sqrt{2\pi i b}}\, e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} e^{\frac{ia}{2b}t^2}\, e^{\frac{-i}{b}ut}\, Df(t)dt$$

$$= \frac{-ia}{b}\,\big[F_A\,\big(tf(t)\big)\big](u) + \frac{iu}{b}\,[F_A\, f(t)](u)$$

and

$$\left(-icu + a\frac{d}{du}\right)F_A(u) = -icuF_A(u) + a\frac{d}{du}\frac{1}{\sqrt{2\pi i b}}\, e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} e^{\frac{ia}{2b}t^2}\, e^{\frac{-i}{b}ut}\, f(t)dt$$

$$= -icuF_A(u) + \frac{iad}{b}u\, F_A\,(u) - \frac{ia}{b}\,[F_A(t\, f(t))](u)$$

$$= \frac{iu}{b}\,[F_A\, f(t)](u) + \frac{-ia}{b}\,\big[F_A\,\big(tf(t)\big)\big](u)$$

Hence the theorem.

d) **Theorem :** For $f \in \mathcal{S}$, and $k \in R$,

$$[F_A\,\left(e^{ikt} f(t)\right)(u) = e^{\frac{idk\,(2u-bk)}{2}}[F_A\big(f(t)\big)(u-bk).$$

Proof : The proof of this is simple and hence omitted.

V. Application

a) **Example** : If $f(t) = e^{\frac{-t^2}{2}}$ then $[F_A f(t)](u) = \frac{1}{\sqrt{a+ib}} e^{\frac{(c+id)u^2}{2(b-ai)}}$.

Sol. Clearly $e^{\frac{-t^2}{2}} \in \mathcal{S}$,

Now, $[F_A f(t)](u) = \frac{1}{\sqrt{2\pi ib}} e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} e^{\frac{ia}{2b}t^2} e^{\frac{-i}{b}ut} e^{\frac{-t^2}{2}} dt$

$$= \frac{1}{\sqrt{2\pi ib}} e^{\frac{id}{2b}u^2} \int_{-\infty}^{\infty} e^{\frac{-(b-ia)}{2b}t^2} e^{\frac{-i}{b}ut} \, dt$$

$$= \frac{1}{\sqrt{a+ib}} e^{\frac{id}{2b}u^2} e^{\frac{-u^2}{2b(b-ai)}} = \frac{1}{\sqrt{a+ib}} e^{\frac{(c+id)u^2}{2(b-ai)}}.$$

b) **Example** : If $f(t) = \delta(t-\tau)$ then , $[F_A f(t)](u) = \frac{1}{\sqrt{2\pi ib}} e^{\frac{i}{2b}(a\tau^2+du^2)} e^{\frac{-i}{b}u\tau}$

Sol: Since $\delta(t-\tau) \in \mathcal{S}$ we can apply the above theory and by the definition of delta function the result is clear.

VI. Conclusion

In this paper we have confined ourselves to the space $\mathcal{S}$ next we propose to extend this theory to the spaces of generalized functions $\mathcal{S}'$ the space of distributions of slow growth. Another extension we plan is to study the linear canonical transform with complex entries. Then we shall use this theory to solve some partial differential equations.

References Références Referencias

1. Akay O: Unitary and Hermitian fractional Operators and Their relation to Fractional Fourier Transform, IEEE, Signal Processing Letters, Vol 5, No12, Dec 1998, 312-14.
2. Almeida L B: Product and Convolution Theorems for the Fractional Fourier Transform, IEEE Signal Processing Letters, Vol. 4, No.1, January 1997, 15-17.
3. Gudadhe A S , Thakare P P: Analytic Study of Linear Canonical Transform, communicated.
4. Kerr F H: A distributional approach to Namias Fractional Fourier transform, Proc. Of Royal society of Edinburgh, 108A, (1988), 133-143.
5. Namias V: The Fractional Fourier Transform and Its application in Quantum Mechanics, J. Inst. Maths.Applics., Vol 25, (1980), 241-265.
6. Ozaktas H M, Zalevsky Z, Kutay M A: The Fractional Fourier Transform with Applications in Optics and Signal Proessing, John Wiley and Sons, NY, 2001.
7. Zemanian A H : Distribution Theory and Transform Analysis, Mc Graw Hill, NY, 1965.

This page is intentionally left blank

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 8 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Invention of a Summation Formula Accumulated with Hypergeometric Function

By Salahuddin , M. P. Chaudhary & Vinesh Kumar
P.D.M College of Engineering, India.

Abstract - The main aim of this paper is to create a summation formula amassed with Hypergeometric function.

Keywords : *Contiguous relation, Recurrence relation, Gauss second summation theorem.*

GJSFR -F Classification : *2010 MSC: 33C05 , 33C20 , 33D15 , 33D50, 33D60*

INVENTION OF A SUMMATION FORMULA ACCUMULATED WITH HYPERGEOMETRIC FUNCTION

Strictly as per the compliance and regulations of :

Notes

Invention of a Summation Formula Accumulated with Hypergeometric Function

Salahuddin [α], M. P. Chaudhary [σ] & Vinesh Kumar [ρ]

Abstract - The main aim of this paper is to create a summation formula amassed with Hypergeometric function.

Keywords : *Contiguous relation, Recurrence relation, Gauss second summation theorem.*

I. Introduction

Generalized Gaussian Hypergeometric function of one variable :

$$ {}_AF_B\left[\begin{array}{ccc} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} \tag{1} $$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

Contiguous Relations :

[Andrews p.367(8)]

$$ c\,(1-z)\,{}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = c\,{}_2F_1\left[\begin{array}{cc} a-1,\ b\ ; & \\ c\ \ ; & z \end{array}\right] - (c-b)\,z\,{}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \\ c+1\ ; & z \end{array}\right] \tag{2} $$

[Abramowitz p.558(15.2.19)]

$$ (a-b)\,(1-z)\,{}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \\ c\ \ ; & z \end{array}\right] = (c-b)\,{}_2F_1\left[\begin{array}{cc} a,\ b-1\ ; & \\ c\ \ ; & z \end{array}\right] + (a-c)\,{}_2F_1\left[\begin{array}{cc} a-1,\ b\ ; & \\ c\ ; & z \end{array}\right] \tag{3} $$

Recurrence relation :

$$ \Gamma(z+1) = z\,\Gamma(z) \tag{4} $$

Gauss second summation theorem [Prud.,p. 491(7.3.7.8)]

$$ {}_2F_1\left[\begin{array}{cc} a,\ b\ ; & \\ \frac{a+b+1}{2}\ ; & \frac{1}{2} \end{array}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{5} $$

$$ = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{6} $$

Author α : P.D.M College of Engineering, Bahadurgarh, Haryana, India. E-mails : sludn@yahoo.com ; vsludn@gmail.com
Author σ : International Scientific Research and Welfare Organization, New Delhi, India. E-mail : mpchaudhary_2000@yahoo.com
Author ρ : J.N.U., New Delhi, India

A new summation formula [Ref.[3], p.337(10)]

$$ {}_2F_1\left[\begin{matrix} a, \quad b \;; & \frac{1}{2} \\ \frac{a+b-1}{2} \;; & \frac{1}{2} \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\} + \frac{2\,\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \tag{7} $$

II. Main Summation Formula

For the main formula $a \neq b$

For $a < 1$ and $a > 35$

$$ {}_2F_1\left[\begin{matrix} a, \quad b \;; & \frac{1}{2} \\ \frac{a+b-35}{2} \;; & \frac{1}{2} \end{matrix}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-35}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-35}{2})}\left\{\frac{(221643095476699771875a)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+\right.\right. $$

$$ +\frac{(-537928935889764226500a^2 + 517596339235489288425a^3 - 277705505168550027360a^4)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(95853765344939263692a^5 - 23003823190786749936a^6 + 4025053173951400564a^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(-529465109186351520a^8 + 53416477786629594a^9 - 4184718381424152a^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(256190750871198a^{11} - 12267387269280a^{12} + 457279214236a^{13} - 13119887088a^{14})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(283927812a^{15} - 4479840a^{16} + 48603a^{17} - 324a^{18} + a^{19} - 221643095476699771875b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(794460568958575915725a^2b - 954106896831586263360a^3b + 583473807108908300484a^4b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(-200209016127040580160a^5b + 52111887197927490740a^6b - 8592784683577819200a^7b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(1229578887992586390a^8b - 112651380427012416a^9b + 9758821555276134a^{10}b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

$$ +\frac{(-521382728433600a^{11}b + 28326660226388a^{12}b - 873242129088a^{13}b + 29555243172a^{14}b)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+ $$

Ref.

3. Arora, Asish, Singh, Rahul, Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems , Journal of Rajasthan Academy of Physical Sciences., 7(2008), 335-342.

Notes

$$+\frac{(-483675840a^{15}b+9621813a^{16}b-63936a^{17}b+629a^{18}b+537928935889764226500b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-794460568958575915725ab^{2}+539930822961781218744a^{3}b^{2}-386145326367083741616a^{4}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(167890730348008826916a^{5}b^{2}-38716437757273456320a^{6}b^{2}+7935347639494594536a^{7}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-908895245233633416a^{8}b^{2}+108166740015493698a^{9}b^{2}-6820153444748160a^{10}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(508087027139016a^{11}b^{2}-17810342469360a^{12}b^{2}+849592365636a^{13}b^{2}-15388862400a^{14}b^{2})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(461771544a^{15}b^{2}-3328668a^{16}b^{2}+58275a^{17}b^{2}-517596339235489288425b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(954106896831586263360ab^{3}-539930822961781218744a^{2}b^{3}+123336195405232240356a^{4}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-56552210538568153920a^{5}b^{3}+18623361536764442808a^{6}b^{3}-2926284502864922496a^{7}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(486797755152510186a^{8}b^{3}-38116127387131200a^{9}b^{3}+3830565140713080a^{10}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-158626558911744a^{11}b^{3}+10223320539684a^{12}b^{3}-212083704000a^{13}b^{3}+8868220680a^{14}b^{3})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-71736192a^{15}b^{3}+1888887a^{16}b^{3}+27770550516855002736 0b^{4}-583473807108908300484ab^{4})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(386145326367083741616a^{2}b^{4}-123336195405232240356a^{3}b^{4}+11976026816214506892a^{5}b^{4})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-36216277568953907 04a^{6}b^{4}+942055925530878444a^{7}b^{4}-9965736432 2023200a^{8}b^{4})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(13764795916582260a^9b^4-703215442424880a^{10}b^4+60448094196756a^{11}b^4)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-1480020897600a^{12}b^4+83034151620a^{13}b^4-772160400a^{14}b^4+28312548a^{15}b^4)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-958537653449392663692b^5+200209016127040580160ab^5-167890730348008826916a^2b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(56552210538568153920a^3b^5-11976026816214506892a^4b^5+555797963695121532a^6b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-110524284078975360a^7b^5+23405963382833724a^8b^5-1596308168613312a^9b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(186344810270388a^{10}b^5-5597348621760a^{11}b^5+415953681948a^{12}b^5-4559958144a^{13}b^5)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(223926516a^{14}b^5+230038231907867499936b^6-52111887197927490740ab^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(38716437757273456320a^2b^6-18623361536764442808a^3b^6+3621627756895390704a^4b^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-555797963695121532a^5b^6+13009040083811376a^7b^6-1638821697792048a^8b^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(289107691712532a^9b^6-11485059284160a^{10}b^6+1149394449672a^{11}b^6-15389858736a^{12}b^6)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(997490844a^{13}b^6-4025053173951400564b^7+8592784683577819200ab^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-7935347639494594536a^2b^7+2926284502864922496a^3b^7-942055925530878444a^4b^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(110524284078975360a^5b^7-13009040083811376a^6b^7+153898964562420a^8b^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

Notes

$$+\frac{(-11143637208000a^9b^7 + 1667462215320a^{10}b^7 - 29314016640a^{11}b^7 + 2544619500a^{12}b^7)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(529465109186351520b^8 - 1229578887992586390ab^8 + 908895245233633416a^2b^8)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

Notes

$$+\frac{(-486797755152510186a^3b^8 + 99657364322023200a^4b^8 - 23405963382833724a^5b^8)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1638821697792048a^6b^8 - 153898964562420a^7b^8 + 859647308850a^9b^8 - 27266346360a^{10}b^8)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(3511574910a^{11}b^8 - 534164777866295946b^9 + 1126513804270124166ab^9)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-1081667400154936 98a^2b^9 + 381161273871312 00a^3b^9 - 13764795916582260a^4b^9)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1596308168613312a^5b^9 - 289107691712532a^6b^9 + 11143637208000a^7b^9 - 859647308850a^8b^9)}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1767263190a^{10}b^9 + 41847183814241 52b^{10} - 9758821555276134ab^{10} + 6820153444748160a^2b^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-3830565140713080a^3b^{10} + 703215442424880a^4b^{10} - 186344810270388a^5b^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(11485059284160a^6b^{10} - 1667462215320a^7b^{10} + 27266346360a^8b^{10} - 1767263190a^9b^{10})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-2561907508711198b^{11} + 5213827284336 00ab^{11} - 508087027139016a^2b^{11})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(158626558911744a^3b^{11} - 60448094196756a^4b^{11} + 5597348621760a^5b^{11} - 1149394449672a^6b^{11})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(29314016640a^7b^{11} - 3511574910a^8b^{11} + 122673872692 80b^{12} - 28326660226388ab^{12})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(17810342469360a^2b^{12}-10223320539684a^3b^{12}+1480020897600a^4b^{12}-415953681948a^5b^{12})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(15389858736a^6b^{12}-2544619500a^7b^{12}-457279214236b^{13}+873242129088ab^{13})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-849592365636a^2b^{13}+212083704000a^3b^{13}-83034151620a^4b^{13}+4559958144a^5b^{13})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-997490844a^6b^{13}+131198870886b^{14}-29555243172ab^{14}+15388862400a^2b^{14})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-8868220680a^3b^{14}+772160400a^4b^{14}-223926516a^5b^{14}-283927812b^{15}+483675840ab^{15})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-461771544a^2b^{15}+71736192a^3b^{15}-28312548a^4b^{15}+4479840b^{16}-9621813ab^{16})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(3328668a^2b^{16}-1888887a^3b^{16}-48603b^{17}+63936ab^{17}-58275a^2b^{17}+324b^{18})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-629ab^{18}-b^{19})}{\prod_{\zeta=1}^{18}\{a-(2\zeta-1)\}}\Bigg\}+\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-34}{2})}\Bigg\{\frac{(-3420041400046021905000a+618589042130777249700a^2)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-439776869912495100000a^3+207660990003844961376a^4-55646018917045943664a^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(126927794975986374244a^6-1727329171783934880a^7+227673953705453472a^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-17503765732175448a^9+1437382773094488a^{10}-64498668078240a^{11}+3385607303328a^{12})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-86814694512a^{13}+2872970352a^{14}-38294880a^{15}+745824a^{16}-3876a^{17}+36a^{18})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(342004140004602190500b-565735909026242999520a^2b+605915064265530229056a^3b)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

Notes

$$+\frac{(-26613151552677738728a^4b+92005049057668566080a^5b-16857776376518415520a^6b)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(3063661048628106304a^7b-293176469928015080a^8b+32203327729514304a^9b)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

Notes

$$+\frac{(-1723576078229280a^{10}b+121070246033344a^{11}b-3609181157456a^{12}b+164273746112a^{13}b)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-2510666080a^{14}b+72098496a^{15}b-430236a^{16}b+7104a^{17}b-618589042130777249700b^2)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(565735909026242999520ab^2-222917145657651459168a^3b^2+162758440229415575344a^4b^2)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-46807570830991840800a^5b^2+12487467579332170432a^6b^2-1576581661708571232a^7b^2)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(23455052407533498 4a^8b^2-15467084885720160a^9b^2+1437907007343488a^{10}b^2)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-50905841320992a^{11}b^2+3084537853040a^{12}b^2-54812066400a^{13}b^2+2168792000a^{14}b^2)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-14914848a^{15}b^2+369852a^{16}b^2+439776869912495100000b^3-605915064265530229056ab^3)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(222917145657651459168a^2b^3-321884444171137401120a^4b^3+173331763902678792 32a^5b^3)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-3333735232568168736a^6b^3+711712084009024384a^7b^3-60978459051109600a^8b^3)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(7580367458452800a^9b^3-328001164876000a^{10}b^3+26128258084096a^{11}b^3)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-549526101600a^{12}b^3+28918904000a^{13}b^3-231371360a^{14}b^3+7970688a^{15}b^3)}{\prod\limits_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-2076609900038449613 76b^4 + 2661315155267773877 28ab^4 - 1627584402294155753 44a^2b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(321884444171374011 20a^3b^4 - 204326578571272942 4a^5b^4 + 853645273887046256a^6b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-1090397107978358 40a^7b^4 + 191324490908936 00a^8b^4 - 1063083728026320a^9b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(112220361169840a^{10}b^4 - 2866811616480a^{11}b^4 + 197641864000a^{12}b^4 - 1867224240a^{13}b^4)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(85795600a^{14}b^4 + 5564601891704594366 4b^5 - 9200504905766856608 0ab^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(468075708309918408 00a^2b^5 - 173331763902678792 32a^3b^5 + 2043265785712729424a^4b^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-619832405087772 80a^6b^5 + 20817488019036544a^7b^5 - 1696964964093168a^8b^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(249794015368128a^9b^5 - 8121009770720a^{10}b^5 + 737446564288a^{11}b^5 - 8423256016a^{12}b^5)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(506662016a^{13}b^5 - 1269277949759863742 4b^6 + 1685777637651841552 0ab^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-124874675793321704 32a^2b^6 + 333373523256816873 6a^3b^6 - 853645273887046256a^4b^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(61983240508777280a^5b^6 - 915171126674112a^7b^6 + 253701418913712a^8b^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-11953460867040a^9b^6 + 1500360345792a^{10}b^6 - 21659801184a^{11}b^6 + 1709984304a^{12}b^6)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(172732917178393488 0b^7 - 306366104862810630 4ab^7 + 1576581661708571232a^2b^7)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

Notes

Notes

$$+\frac{(-711712084009024384a^3b^7+109039710797835840a^4b^7-20817488019036544a^5b^7)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(915171126674112a^6b^7-6204081708000a^8b^7+1448936280000a^9b^7-30032497440a^{10}b^7)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(3257112960a^{11}b^7-227673953705453472b^8+293176469928015080ab^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-234550524075334984a^2b^8+60978459051109600a^3b^8-19132449090893600a^4b^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(1696964964093168a^5b^8-253701418913712a^6b^8+6204081708000a^7b^8-15147970200a^9b^8)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(3029594040a^{1}0b^8+17503765732175448b^9-32203327729514304ab^9)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(15467084885720160a^2b^9-7580367458452800a^3b^9+1063083728026320a^4b^9)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-249794015368128a^5b^9+11953460867040a^6b^9-1448936280000a^7b^9+15147970200a^8b^9)}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-1437382773094488b^{10}+1723576078229280ab^{10}-1437907007343488a^2b^{10})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(328001164876000a^3b^{10}-112220361169840a^4b^{10}+8121009770720a^5b^{10})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-1500360345792a^6b^{10}+30032497440a^7b^{10}-3029594040a^8b^{10}+64498668078240b^{11})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-121070246033344ab^{11}+50905841320992a^2b^{11}-26128258084096a^3b^{11})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(2866811616480a^4b^{11}-737446564288a^5b^{11}+21659801184a^6b^{11}-3257112960a^7b^{11})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-33856073303328b^{12}+3609181157456ab^{12}-3084537853040a^2b^{12}+549526101600a^3b^{12})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-197641864000a^4b^{12}+8423256016a^5b^{12}-1709984304a^6b^{12}+868146945 12b^{13})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-164273746112ab^{13}+54812066400a^2b^{13}-28918904000a^3b^{13}+1867224240a^4b^{13})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-506662016a^5b^{13}-28729703352b^14+2510666080ab^{14}-2168792000a^2b^{14}+231371360a^3b^{14})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-85795600a^4b^{14}+38294880b^{15}-72098496ab^{15}+14914848a^2b^{15}-7970688a^3b^{15})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}+$$

$$+\frac{(-745824b^{16}+430236ab^{16}-369852a^2b^{16}+3876b^{17}-7104ab^{17}-36b^{18})}{\prod_{\sigma=1}^{17}\{a-2\sigma\}}\Bigg\}\Bigg] \qquad (9)$$

III. Derivation of Main Summation Formula

Substituting $c=\frac{a+b-35}{2}$ and $z=\frac{1}{2}$ in equation (3), we get

$$(a-b)\,{}_2F_1\left[\begin{matrix}a,\ b & ; & \frac{1}{2}\\ \frac{a+b-35}{2} & ; & \end{matrix}\right]=(a-b-35)\,{}_2F_1\left[\begin{matrix}a,\ b-1 & ; & \frac{1}{2}\\ \frac{a+b-35}{2} & ; & \end{matrix}\right]+(a-b+35)\,{}_2F_1\left[\begin{matrix}a-1,\ b & ; & \frac{1}{2}\\ \frac{a+b-35}{2} & ; & \end{matrix}\right]$$

Now using the same method of Ref[9] the main result is derived.

References Références Referencias

1. Abramowitz, Milton. , A and Stegun, Irene; Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables. National Bureau of Standards, 1970.
2. Andrews, L.C.; Special Function of mathematics for Engineers, second Edition, McGraw-Hill Co Inc., New York, 1992.
3. Arora, Asish, Singh, Rahul, Salahuddin. ; Development of a family of summation formulae of half argument using Gauss and Bailey theorems , Journal of Rajasthan Academy of Physical Sciences., 7(2008), 335-342.
4. Luke, Y.L, Mathematical Functions and their Approximations, Academic Press Inc, 1975.
5. Prudnikov, A. P., Brychkov, Yu. A. and Marichev, O.I.; Integrals and Series Vol. 3: More Special Functions. Nauka, Moscow, 1986. Translated from the Russian by G.G. Gould, Gordon and Breach Science Publishers, New York, Philadelphia, London, Paris, Montreux, Tokyo, Melbourne, 1990.
6. Salahuddin. ; Evaluation of a Summation Formula Involving Recurrence Relation , Gen. Math. Notes., 2(2010), 42-59.

Notes

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 8 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Modified H-Tran sform and Pathway Fractional Integral Operator

By Neeti Ghiya
R.V. College of Engineering , India

Abstract - In this paper we have established a theorem wherein we have obtained the image of modified H-transform under the pathway fractional integral operator defined by Nair [8]. Three corollaries of the main theorem have been derived. Our findings provide interesting unification and extension of number of (new and known) results.

Keywords : Pathway fractional integral operator, modified H-function transform, H-function of one variable, Whittaker function, Wright's generalized Bessel function.

GJSFR - F Classification : MSC 2010: 26A33

MODIFIED H-TRAN SFORM AND PATHWAY FRACTIONAL INTEGRAL OPERATOR

Strictly as per the compliance and regulations of :

Modified H-Transform and Pathway Fractional Integral Operator

Neeti Ghiya

Abstract - In this paper we have established a theorem wherein we have obtained the image of modified H- transform under the pathway fractional integral operator defined by Nair [8]. Three corollaries of the main theorem have been derived. Our findings provide interesting unification and extension of number of (new and known) results.

Keywords : Pathway fractional integral operator, modified H-function transform, H- function of one variable, Whittaker function, Wright's generalized Bessel function.

I. Introduction

The modified H-function transform was introduced by Saigo, Saxena and Ram [7] and is defined in the following manner:

$$h(s) \;=\; h_{P,\,Q}^{M,\,N}\left[F(x)\,;\rho\,,s\right]$$

$$= \int_d^{\infty} (sx)^{\rho-1}\; H_{P,Q}^{M,N}\left[(sx)^k \left| \begin{matrix} (c_j,\gamma_j)_{1,\,P} \\ (d_j,\delta_j)_{1,\,Q} \end{matrix} \right.\right] F(x)\,dx\,, \quad \text{for } k>0, \tag{1}$$

$$F(x) \;=\; f\,(\,a'\sqrt{x^2-d^2}\,)\,U(x-d), \quad x>d>0\,, \tag{2}$$

where U(x-d) is the well- known Heaviside unit function.

Further we assume that h(s) exists and belongs to U. where U is the class of functions f(x) on $R_+ = (0,\infty)$, which is infinitely differentiable with partial derivatives of any order such that

$$f(x) \;=\; \begin{bmatrix} 0 \;\; (|x|^{w_1}) \;\; \text{as } x\to 0 \\ 0 \;\; (|x|^{-w_2}) \;\; \text{as } x\to\infty \end{bmatrix}. \tag{3}$$

The transform defined by (1) exists provided that following (sufficient) conditions are satisfied:

(i) $\left|\arg s\right| < \dfrac{1}{2}\pi\,\Omega/k\,,$

Author : Department of Mathematics, R.V. College of Engineering, Bangalore-560059, Karnataka. India.
E-mail : neeti.varadam@gmail.com

Ref.

7. M. Saigo, R. K. Saxena and J.Ram, On the two dimensional generalized Weyl fractional calculus associated with two dimensional H-transform, J.Fractional Calculus, 8(1995), 63-73.

where $\Omega = \sum_{j=1}^{N} \gamma_j - \sum_{j=N+1}^{P} \gamma_j + \sum_{j=1}^{M} d_j - \sum_{j=M+1}^{Q} d_j$

(ii) $\mathrm{Re}(w_1) + 1 > 0$,

(iii) $\mathrm{Re}(\rho - w_2) + k \max_{1 \le j \le N} \left[\mathrm{Re}\left(\frac{c_j - 1}{\gamma_j} \right) \right] < 0$.

The Fox's H-function or simply H-function was introduced by Charles Fox [5]. This function is defined and represented by means of the following Mellin-Barnes type of contour integral:

$$H_{P,Q}^{M,N}[z] = H_{P,Q}^{M,N}\left[z \,\middle|\, \begin{matrix} (a_j, \alpha_j)_{1,P} \\ (b_j, \beta_j)_{1,Q} \end{matrix} \right] = \frac{1}{2\pi i} \int_L \theta(s_1) z^{s_1} ds_1 , \tag{4}$$

where $i = (-1)^{1/2}$, $z \neq 0$ and

$$\theta(s_1) = \frac{\prod_{j=1}^{M} \Gamma(b_j - \beta_j s_1) \prod_{j=1}^{N} \Gamma(1 - a_j + \alpha_j s_1)}{\prod_{j=M+1}^{Q} \Gamma(1 - b_j + \beta_j s_1) \prod_{j=N+1}^{P} \Gamma(a_j - \alpha_j s_1)} . \tag{5}$$

The nature of the contour L in (4), the conditions of convergence of the integral (4), the asymptotic expansion of the H-function and some of its special cases can be referred to the work of Srivastava, Gupta and Goyal [6] and Mathai and Saxena [4].

The Pathway fractional integral operator introduced by Nair [8] and is defined in the following manner:

$$\left(P_{0_+}^{(\eta,\alpha)} f \right)(x) = x^{\eta} \int_0^{\left[\frac{x}{a(1-\alpha)}\right]} \left[1 - \frac{a(1-\alpha)t}{x} \right]^{\frac{\eta}{(1-\alpha)}} f(t)\, dt , \tag{6}$$

where $f(x) \in L(a,b)$, $\eta \in C$, $R(\eta) > 0$, $a > 0$ and 'pathway parameter' $\alpha < 1$.

The pathway model is introduced by Mathai [1] and studied further by Mathai and Haubold ([2], [3]). For real scalar α, the pathway model for scalar random variables is represented by the following probability density function (p.d.f.):

$$f(x) = c|x|^{\gamma-1} \left[1 - a(1-\alpha)|x|^{\delta} \right]^{\frac{\beta}{1-\alpha}} , \tag{7}$$

$-\infty < x < \infty$, $\delta > 0$, $\beta \ge 0$, $(1 - a(1-\alpha)|x|^{\delta}) > 0$, $\gamma > 0$, where c is the normalizing constant and α is called the pathway parameter. For real α, the normalizing constant is as follows:

$$c = \frac{1}{2} \frac{\delta [a(1-\alpha)]^{\frac{\gamma}{\delta}} \Gamma\left(\frac{\gamma}{\delta} + \frac{\beta}{1-\alpha} + 1 \right)}{\Gamma\left(\frac{\gamma}{\delta} \right) \Gamma\left(\frac{\beta}{1-\alpha} + 1 \right)}, \quad \text{for } \alpha < 1, \tag{8}$$

Ref.

5. Fox, The G and H-functions as symmetrical Fourier kernels, Trans.Amer.Math. Soc., 98(1961), 395- 429.

$$=\frac{1}{2}\frac{\delta\,[a(\alpha-1)]^{\frac{\gamma}{\delta}}\,\Gamma\left(\frac{\beta}{\alpha-1}\right)}{\Gamma\left(\frac{\gamma}{\delta}\right)\,\Gamma\left(\frac{\beta}{\alpha-1}-\frac{\gamma}{\delta}\right)}\;,\;\text{for}\;\;\frac{1}{\alpha-1}-\frac{\gamma}{\delta}>0\;\;,\alpha>1, \tag{9}$$

Notes

$$=\frac{1}{2}\frac{\delta\,(a\,\beta)^{\frac{\gamma}{\delta}}}{\Gamma\left(\frac{\gamma}{\delta}\right)}\quad\text{for}\quad\alpha\to 1. \tag{10}$$

For $\alpha<1$, it is a finite range density with $1-a(1-\alpha)|x|^{\delta}>0$ and (7) remains in the extended generalized type-1 beta family. The pathway density in (7), for $\alpha<1$, includes the extended type-1 beta density, the triangular density, the uniform density and many other p.d.f.

For $\alpha>1$, we have

$$f(x)=c\;|x|^{\gamma-1}\left[1+a\,(\alpha-1)\,|x|^{\delta}\right]^{-\frac{\beta}{\alpha-1}}, \tag{11}$$

$-\infty<x<\infty$, $\delta>0$, $\beta\geq 0$, $\alpha>1$, which is the extended generalized type-2 beta model for real x. It includes the type-2 beta density, the F density, the Student-t density, the Cauchy density and many more.

Here it is considered only the case of pathway parameter $\alpha<1$. For $\alpha\to 1$ (7) and (11) take the exponential form, since

$$\lim_{\alpha\to 1}\;c\,|x|^{\gamma-1}\left[1-a\,(1-\alpha)\,|x|^{\delta}\right]^{\frac{\eta}{1-\alpha}}=\lim_{\alpha\to 1}\;c\,|x|^{\gamma-1}\left[1+a\,(\alpha-1)\,|x|^{\delta}\right]^{-\frac{\eta}{\alpha-1}}$$

$$=\;c\;|x|^{\gamma-1}\,e^{-a\,\eta\,|x|^{\delta}} \tag{12}$$

This includes the generalized Gamma-, the Weibull-, the Chi-square, the Laplace-, the Maxwell- Boltzmann and other related densities.

When $\alpha\to 1_{-}$, $\left[1-\frac{a\,(1-\alpha)\,t}{x}\right]^{\frac{\eta}{1-\alpha}}\to e^{-\frac{a\,\eta}{x}t}$, then operator (6) reduces to the Laplace integral transform of f with parameter $\frac{a\eta}{x}$:

$$(\,P_{0_{+}}^{(\eta\,,\,1)}f\,)(x)\;=x^{\eta}\int_{0}^{\infty}e^{-\frac{a\,\eta}{x}t}\,f(t)\,dt\;=x^{\eta}\;L_{f}\left(\frac{a\eta}{x}\right). \tag{13}$$

When $\alpha=0$, $a=1$, then replacing η by $\eta-1$ in (6) the integral operator reduces to the Riemann-Liouville fractional integral operator.

II. Main Theorem

If

$$h(s) = \int_d^\infty (sx)^{\rho-1} H_{P,Q}^{M,N}\left[(sx)^k \middle| \begin{matrix}(c_j,\gamma_j)_{1,P}\\(d_j,\delta_j)_{1,Q}\end{matrix}\right] F(x)\,dx\,, \tag{14}$$

and

$$(P_{0_+}^{(\eta,\alpha)} f)(x) = x^\eta \int_0^{\left[\frac{x}{a(1-\alpha)}\right]} \left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)\,dt\,, \tag{15}$$

then

$$P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}}{[a(1-\alpha)]^\rho}\Gamma\left(\frac{\eta}{1-\alpha}+1\right)\int_d^\infty x^{\rho-1} H_{P+1,Q+1}^{M,N+1}\left[\left(\frac{sx}{a(1-\alpha)}\right)^k \middle| \begin{matrix}(c_j,\gamma_j)_{1,P},(1-\rho,k)\\(d_j,\delta_j)_{1,Q},(\frac{\eta}{\alpha-1}-\rho,k)\end{matrix}\right] F(x)\,dx\,, \tag{16}$$

where $F(x) = f(a'\sqrt{x^2-d^2})\,U(x-d),\ x>d>0$ as defined in (2) provided that

(i) $\Omega>0,\ |\arg s| < \frac{1}{2}\pi\Omega/k$,

(ii) $\mathrm{Re}(w_1)+1>0,\ \mathrm{Re}(\rho-w_2)+k\max\limits_{1\le j\le N}\left[\mathrm{Re}\left(\frac{c_j-1}{\gamma_j}\right)\right]<0$,

(iii) $f(x)\in L(a,b),\ \eta\in C,\ R(\eta)>0,\ a>0,\ \alpha<1$,

(iv) $R(1+\frac{\eta}{1-\alpha})>0$.

Proof: In order to prove the main theorem, substituting the value of h(s) from (14) in the left hand side of (15), we find that

$$P_{0_+}^{(\eta,\alpha)}[h(s)] = s^\eta \int_{t=0}^{\left[\frac{s}{a(1-\alpha)}\right]} \left[1-\frac{a(1-\alpha)t}{s}\right]^{\frac{\eta}{(1-\alpha)}} \left\{\int_{x=d}^\infty (tx)^{\rho-1} H_{P,Q}^{M,N}\left[(tx)^k \middle| \begin{matrix}(c_j,\gamma_j)_{1,P}\\(d_j,\delta_j)_{1,Q}\end{matrix}\right] F(x)\,dx\right\} dt \tag{17}$$

Now interchanging the orders of x and t integrals which is permissible under given conditions, we get

Ref.

2. A. M. Mathai, H. J. Haubold, Pathway model, Super statistics, Tsalle's statistics and a generalized measure of entropy, Physica A, 375(2007), 110-122.

$$P_{0_+}^{(\eta,\alpha)}[h(s)] = s^{\eta}\int_{x=d}^{\infty} F(x)\, x^{\rho-1}\left\{\int_{t=0}^{\left[\frac{s}{a(1-\alpha)}\right]}\left[1-\frac{a(1-\alpha)t}{s}\right]^{\frac{\eta}{(1-\alpha)}} t^{\rho-1}\, H_{P,Q}^{M,N}\left[(tx)^{k}\left|\begin{matrix}(c_j,\gamma_j)_{1,P}\\ (d_j,\delta_j)_{1,Q}\end{matrix}\right.\right]dt\right\}dx \tag{18}$$

To evaluate the t-integral, we express the H- function in terms of Mellin-Barnes contour integrals with the help of (4) and change the order of contour integrations and t-integral. After evaluating the t- integral and reinterpreting the result thus obtained in terms of H-function, we easily arrive at the right hand side of (16) after a little simplification.

When $\alpha\to 1_-$, then (16) tends to

$$\lim_{\alpha\to 1_-} P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}}{(a\eta)^{\rho}}\int_{d}^{\infty} x^{\rho-1}\, H_{P+1,Q}^{M,N+1}\left[\left(\frac{sx}{a\eta}\right)^{k}\left|\begin{matrix}(c_j,\gamma_j)_{1,P},\,(1-\rho,k)\\ (d_j,\delta_j)_{1,Q}\end{matrix}\right.\right]F(x)\,dx \tag{19}$$

III. Special Cases

(I) If we reduce the Fox's H-function involved in (14) to Whittaker function by using a known result [4, p.155], a little simplification will give the following

Corollary 1. If

$$h(s) = \int_{d}^{\infty}(sx)^{\rho-1} e^{-\frac{1}{2}sx}\, W_{b,c}(sx)\,F(x)\,dx, \tag{20}$$

and

$$(P_{0_+}^{(\eta,\alpha)} f)(x) = x^{\eta}\int_{0}^{\left[\frac{x}{a(1-\alpha)}\right]}\left[1-\frac{a(1-\alpha)t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)\,dt, \tag{21}$$

then

$$P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}}{[a(1-\alpha)]^{\rho}}\,\Gamma\left(\frac{\eta}{1-\alpha}+1\right)\int_{d}^{\infty} x^{\rho-1}\, H_{2,3}^{2,1}\left[\frac{sx}{a(1-\alpha)}\left|\begin{matrix}(1-\rho,1),\ (1-b,1)\\ (\frac{\eta}{\alpha-1}-\rho,1),(\frac{1}{2}-c,1),\ (\frac{1}{2}+c,1)\end{matrix}\right.\right]F(x)\,dx. \tag{22}$$

The conditions of validity of the aforementioned corollary can be easily derived from our main theorem.

When $\alpha\to 1_-$ then (22) tends to

$$\lim_{\alpha\to 1_-} P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}}{[a\eta]^{\rho}}\int_{d}^{\infty} x^{\rho-1}\, H_{2,2}^{2,1}\left[\frac{sx}{a\eta}\left|\begin{matrix}(1-\rho,1),\ (1-b,1)\\ (\frac{1}{2}-c,1),\ (\frac{1}{2}+c,1)\end{matrix}\right.\right]F(x)\,dx. \tag{23}$$

Ref.

4. A.M. Mathai and R. K. Saxena, The H-functions with Applications in Statistics and other Disciplines, John Wiley and Sons (1974).

(II) Reducing Fox's H-function in (14) to exponential function using known result [6, p.18] then we have the following

If

$$h(s) = \int_d^\infty (sx)^{\rho-1}\; e^{-sx}\; F(x)\,dx\,, \tag{24}$$

and

$$\left(P_{0_+}^{(\eta,\alpha)} f\right)(x) = x^\eta \int_0^{\left[\frac{x}{a(1-\alpha)}\right]} \left[1-\frac{a(1-\alpha)\,t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)\,dt\,, \tag{25}$$

then

$$P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}}{[a(1-\alpha)]^\rho} B\left(\rho, \frac{\eta}{1-\alpha}+1\right) \int_d^\infty x^{\rho-1}\,{}_1F_1\left[(\rho);\ (\rho+\frac{\eta}{1-\alpha}+1); -\frac{sx}{a(1-\alpha)}\right] F(x)\,dx\,.. \tag{26}$$

The conditions of validity of the aforementioned corollary can be easily derived from our main theorem.
When $\alpha \to 1_-$ then (26) tends to

$$\lim_{\alpha\to 1_-} P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}\,\Gamma(\rho)}{[a\eta]^\rho} \int_d^\infty x^{\rho-1}\left(1+\frac{sx}{a\eta}\right)^{-\rho} F(x)\,dx\,. \tag{27}$$

(III) If we reduce the Fox's H-function involved in (14) to Wright's generalized Bessel function by using a known result [6, p.19], after a little simplification we have

If

$$h(s) = \int_d^\infty (sx)^{\rho-1}\; J_\lambda^\nu(x)\, F(x)\,dx\,, \tag{28}$$

and

$$\left(P_{0_+}^{(\eta,\alpha)} f\right)(x) = x^\eta \int_0^{\left[\frac{x}{a(1-\alpha)}\right]} \left[1-\frac{a(1-\alpha)\,t}{x}\right]^{\frac{\eta}{(1-\alpha)}} f(t)\,dt\,, \tag{29}$$

then

$$P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}}{[a(1-\alpha)]^\rho} \Gamma\left(\frac{\eta}{1-\alpha}+1\right) \int_d^\infty x^{\rho-1}\,{}_1\Psi_2\left[\begin{matrix} (\rho,1) & ; \\ (1+\lambda,\nu),(\rho+\frac{\eta}{1-\alpha}+1,1); & -\frac{sx}{a(1-\alpha)} \end{matrix}\right] F(x)\,dx\,, \tag{30}$$

Ref.
6. H. M. Srivastava, K. C. Gupta, S. P. Goyal, The H- function of one and two variables with applications, South Asian publisher, New Delhi (1982).

Notes

where ${}_1\psi_2$ is wright's hypergeometric function and the conditions of validity of the aforementioned corollary can be easily derived from our main theorem.

When $\alpha \to 1\_$ then (30) tends to

$$\lim_{\alpha\to 1\_} P_{0_+}^{(\eta,\alpha)}[h(s)] = \frac{s^{\eta+\rho}}{[a\eta]^{\rho}} \int_d^{\infty} x^{\rho-1} \; {}_1\psi_1\left[\begin{matrix}(\rho,1)\;; \\ (1+\lambda,\nu);\end{matrix} \; -\frac{s\,x}{a\,\eta}\right] F(x)\,dx \; . \tag{31}$$

References Références Referencias

1. M. Mathai, A pathway to matrix – variate gamma and normal densities, Linear Algebra and its Applications, 396(2005), 317-328.
2. A. M. Mathai, H. J. Haubold, Pathway model, Super statistics, Tsalle's statistics and a generalized measure of entropy, Physica A, 375(2007), 110-122.
3. A. M. Mathai, H. J. Haubold, On generalized distributions and pathways, Physics Letters, 372(2008), 2109-2113.
4. A.M. Mathai and R. K. Saxena, The H-functions with Applications in Statistics and other Disciplines, John Wiley and Sons (1974).
5. Fox, The G and H-functions as symmetrical Fourier kernels,Trans.Amer.Math. Soc., 98(1961), 395- 429.
6. H. M. Srivastava, K. C. Gupta, S. P. Goyal, The H- function of one and two variables with applications, South Asian publisher, New Delhi (1982).
7. M. Saigo, R. K. Saxena and J.Ram, On the two dimensional generalized Weyl fractional calculus associated with two dimensional H-transform, J.Fractional Calculus, 8(1995), 63-73.
8. Seema S. Nair, Pathway fractional integration operator, Fractional calculus & Applied analysis, 12(3)(2009), 237-252.

This page is intentionally left blank

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 8 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Certain Indefinite Integrals Involving Laguerre Polynomials

By Salahuddin
Department of Mathematics, University of Rajasthan, India.

Abstract - In this paper we have established certain indefinite integrals involving Polylogarithm and Laguerre Polynomials. The results represent here are assume to be new.

Keywords : Polylogarithm; Laguerre polynomials; Gaussian Hypergeometric Function.

GJSFR- F Classification : 2010 MSC NO: 33C05,33C45,33C15,33D50,33D60

CERTAIN INDEFINITE INTEGRALS INVOLVING LAGUERRE POLYNOMIALS

Strictly as per the compliance and regulations of :

Notes

Certain Indefinite Integrals Involving Laguerre Polynomials

Salahuddin

Abstract - In this paper we have established certain indefinite integrals involving Polylogarithm and Laguerre Polynomials. The results represent here are assume to be new.

Keywords and Phrases : Polylogarithm; Laguerre polynomials; Gaussian Hypergeometric Function.

I. Introduction and Preliminaries

Laguerre polynomials

In mathematics, the Laguerre polynomials, named after Edmond Laguerre (1834 - 1886), are solutions of Laguerre's equation:

$$xy'' + (1-x)y' + ny = 0 \tag{1.1}$$

which is a second-order linear differential equation. This equation has nonsingular solutions only if n is a non-negative integer. The associated Laguerre polynomials (also named Sonin polynomials after Nikolay Yakovlevich Sonin in some older books) are solutions of

$$xy'' + (\alpha + 1 - x)y' + ny = 0 \tag{1.2}$$

The Laguerre polynomials are also used for Gaussian quadrature to numerically compute integrals of the form

$$\int_0^\infty f(x)e^{-x}dx$$

These polynomials, usually denoted $L0, L1, ...$, are a polynomial sequence which may be defined by the Rodrigues formula

$$L_n(x) = \frac{e^x}{n!}\frac{d^n}{dx^n}(e^{-x}x^n) \tag{1.3}$$

They are orthonormal to each other with respect to the inner product given by

$$< f, g >= \int_0^\infty f(x)g(x)e^{-x}dx \tag{1.4}$$

The sequence of Laguerre polynomials is a Sheffer sequence.

Author : P.D.M College of Engineering, Bahadurgarh, Haryana, India. E-mails: vsludn@gmail., sludn@yahoo.com

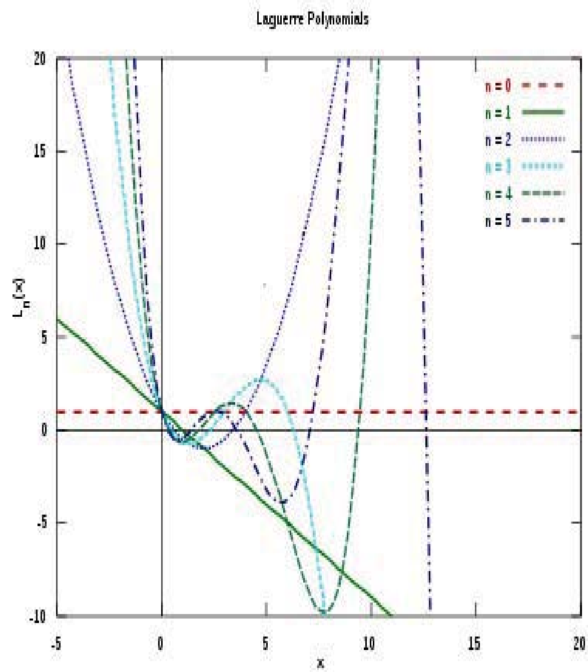

Figure 1: The first six Laguerre polynomials

The first six Laguerre polynomials are(also in fig.)

$$L_0(x) = 1$$

$$L_1(x) = -x + 1$$

$$L_2(x) = \frac{1}{2}(x^2 - 4x + 2)$$

$$L_3(x) = \frac{1}{6}(-x^3 + 9x^2 - 18x + 6)$$

$$L_4(x) = \frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$$

$$L_5(x) = \frac{1}{720}(x^6 - 36x^5 + 450x^4 - 2400x^3 + 5400x^2 - 4320x + 720)$$

Polylogarithm

The polylogarithm (also known as Jonquire's function) is a special function $Li_s(z)$ that is defined by the infinite sum, or power series:

$$Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s} \tag{1.5}$$

It is in general not an elementary function, unlike the related logarithm function. The above definition is valid for all complex values of the order s and the argument z where$\mid z \mid < 1$.The polylogarithm is defined over a larger range of z than the above definition allows by the process of analytic continuation.

The special case s = 1 involves the ordinary natural logarithm ($Li_1(z) = -ln(1-z)$) while the special cases s = 2 and s = 3 are called the dilogarithm (also referred to as Spence's function) and trilogarithm respectively. The name of the function comes from the fact that it may alternatively be defined as the repeated integral of itself, namely that

$$Li_{s+1}(z) = \int_0^z \frac{Li_s(t)}{t} dt \tag{1.6}$$

Notes

Thus the dilogarithm is an integral of the logarithm, and so on. For nonpositive integer orders s, the polylogarithm is a rational function.
The polylogarithm also arises in the closed form of the integral of the FermiDirac distribution and the BoseEinstein distribution and is sometimes known as the Fermi- Dirac integral or the BoseEinstein integral. Polylogarithms should not be confused with polylogarithmic functions nor with the offset logarithmic integral which has a similar notation.

Generalized Gaussian Hypergeometric Function

Generalized ordinary hypergeometric function of one variable is defined by

$$ {}_AF_B\left[\begin{array}{lll} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!} $$

or

$$ {}_AF_B\left[\begin{array}{lll} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array}\right] \equiv {}_AF_B\left[\begin{array}{lll} (a_j)_{j=1}^{A} & ; & \\ & & z \\ (b_j)_{j=1}^{B} & ; & \end{array}\right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1.7} $$

where denominator parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers.

II. Main Indefinite Integrals

$$ \int \frac{\cosh x\ L_1(x)}{\sqrt{1-\cos x}}\,\mathrm{dx} = -\frac{1}{\sqrt{1-\cos x}}\left(\frac{2}{25}-\frac{4\iota}{25}\right)e^{(-1-\frac{\iota}{2})x}\sin\frac{x}{2}\Big[-(4+2\iota)e^{2x}\times $$
$$ \times{}_3F_2\Big(-\frac{1}{2}-\iota,-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota,\frac{1}{2}-\iota;e^{\iota x}\Big)-(4+2\iota)e^{\iota x}{}_3F_2\Big(\frac{1}{2}+\iota,\frac{1}{2}+\iota,1;\frac{3}{2}+\iota,\frac{3}{2}+\iota;e^{\iota x}\Big)+ $$
$$ +5xe^{2x}{}_2F_1\Big(-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota;e^{\iota x}\Big)-5xe^{\iota x}{}_2F_1\Big(\frac{1}{2}+\iota,1;\frac{3}{2}+\iota;e^{\iota x}\Big)- $$
$$ -(3-4\iota)e^{(2+\iota)x}{}_2F_1\Big(\frac{1}{2}-\iota,1;\frac{3}{2}-\iota;e^{\iota x}\Big)+5e^{\iota x}{}_2F_1\Big(\frac{1}{2}+\iota,1;\frac{3}{2}+\iota;e^{\iota x}\Big)- $$
$$ -5xe^{2x}+(4+2\iota)e^{2x}\Big]+Constant \tag{2.1} $$

$$ \int \frac{\cosh x\ L_2(x)}{\sqrt{1-\cosh x}}\,\mathrm{dx} = \frac{1}{\sqrt{1-\cosh x}}\sinh\frac{x}{2}\Big[4(x-2)Li_2(-e^{-\frac{x}{2}})-4(x-2)Li_2(e^{-\frac{x}{2}})+8Li_3(-e^{-\frac{x}{2}})- $$
$$ -8Li_3(e^{-\frac{x}{2}})+x^2\log(1-e^{-\frac{x}{2}})-x^2\log(1+e^{-\frac{x}{2}})+2x^2\cosh\frac{x}{2}-4x\log(1-e^{-\frac{x}{2}})+ $$
$$ +4x\log(1+e^{-\frac{x}{2}})-8x\sinh\frac{x}{2}+16\sinh\frac{x}{2}-8x\cosh\frac{x}{2}+20\cosh\frac{x}{2}+2\log(\tanh\frac{x}{4})\Big]+Constant \tag{2.2} $$

$$ \int \frac{\sin x\ L_2(x)}{\sqrt{1-\sin x}}\,\mathrm{dx} = \frac{1}{\sqrt{1-\sin x}}\Big(\frac{1}{2}+\frac{\iota}{2}\Big)(\cos\frac{x}{2}-\sin\frac{x}{2})\Big[(-1)^{\frac{3}{4}}\Big\{-4\iota(x-2)Li_2\Big(-(-1)^{\frac{3}{4}}e^{\frac{\iota x}{2}}\Big)+ $$
$$ +4\iota(x-2)Li_2\Big((-1)^{\frac{3}{4}}e^{\frac{\iota x}{2}}\Big)+8Li_3\Big(-(-1)^{\frac{3}{4}}e^{\frac{\iota x}{2}}\Big)-8Li_3\Big((-1)^{\frac{3}{4}}e^{\frac{\iota x}{2}}\Big)+x^2\Big(-\log\Big(1-(-1)^{\frac{3}{4}}e^{\iota x}\Big)\Big)+ $$
$$ +x^2\Big(\log\Big(1+(-1)^{\frac{3}{4}}e^{\iota x}\Big)\Big)+4x\Big(\log\Big(1-(-1)^{\frac{3}{4}}e^{\iota x}\Big)\Big)-4x\Big(\log\Big(1+(-1)^{\frac{3}{4}}e^{\iota x}\Big)\Big)+ $$
$$ +4\iota\tan^{-1}\Big((-1)^{\frac{1}{4}}e^{\frac{\iota x}{2}}\Big)\Big\}-(1-\iota)(x^2-8x+2)\sin\frac{x}{2}+(-1+\iota)(x^2-14)\cos\frac{x}{2}\Big]+Constant \tag{2.3} $$

$$ \int \frac{\cosh x\ L_3(x)}{\sqrt{1-\cosh x}}\,\mathrm{dx} = -\frac{1}{24\sqrt{1-\cosh x}}\sinh\frac{x}{2}\Big[48x^2Li_2(e^{\frac{x}{2}})+48(x^2-6x+6)Li_2(-e^{-\frac{x}{2}})+ $$
$$ +192xLi_3(-e^{-\frac{x}{2}})-192xLi_3(e^{\frac{x}{2}})+288(x-1)Li_2(e^{-\frac{x}{2}})-576Li_3(-e^{-\frac{x}{2}})+576Li_3(e^{-\frac{x}{2}})+ $$

$$+384Li_4(-e^{-\frac{x}{2}}) + 384Li_4(e^{\frac{x}{2}}) - x^4 - 8x^3\log(1+e^{-\frac{x}{2}}) + 8x^3\log(1-e^{\frac{x}{2}}) + 16x^3\cosh\frac{x}{2} -$$

$$-72x^2\log(1-e^{-\frac{x}{2}}) + 72x^2\log(1+e^{-\frac{x}{2}}) - 96x^2\sinh\frac{x}{2} - 144x^2\cosh\frac{x}{2} + 144x\log(1-e^{-\frac{x}{2}}) -$$

$$-144x\log(1+e^{-\frac{x}{2}}) + 576x\sinh\frac{x}{2} - 1344\sinh\frac{x}{2} + 672x\cosh\frac{x}{2} -$$

$$-1248\cosh\frac{x}{2} - 48\log(\tanh\frac{x}{4}) + 8\pi^4\Big] + Constant \qquad (2.4)$$

III. Derivation of The Integrals

Involving the same method of ref[8] , one can derive the integrals.

IV. Conclusion

In our work we have established certain indefinite integrals involving Laguerre polynomials,Jonquire's function and Hypergeometric function .We hope that the development presented in this work will stimulate further interest and research in this important area of Computational Mathematics.

References Références Referencias

1. Abramowitz, Milton., A and Stegun, Irene ; *Handbook of Mathematical Functions with Formulas , Graphs , and Mathematical Tables*, National Bureau of Standards, 1970.
2. Arfken, G.,Hans Weber ; *Mathematical Methods for Physicists*, Academic Press,2000.
3. B Spain, M G Smith; *Functions of mathematical physics*, Van Nostrand Reinhold Company, London, 1970.
4. Guillera, J., Sondow, J. ;Double integrals and infinite products for some classical constants via analytic continuations of Lerch's transcendent, *The Ramanujan Journal*,16(2008), 247-270.
5. Jahnke, E., Emde, F. ; *Tables of Functions with Formulae and Curves (4th ed.)*, Dover Publications,New York,1945.
6. Olver, Frank, W. J., Lozier, Daniel, M., Boisvert, Ronald F.,Clark, C. W.; *NIST Handbook of Mathematical Functions*,Cambridge University Press,2010.
7. Ricci, Paolo Emilio ;Generalized Lucas polynomials and Fibonacci polynomials, *Rivista di Matematica della Universit di Parma*, 4(1995),137-146.
8. Salahuddin; Hypergeometric Form of Certain Indefinite Integrals, *Global Journal of Science Frontier Research(F)*, 12(2012), 37-41.
9. Stein, Elias, Weiss, Guido ; *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton, N.J.: Princeton University Press,1971.
10. Temme, Nico ; *Special Functions: An Introduction to the Classical Functions of Mathematical Physics*, Wiley, New York, 1996
11. Whittaker, E. T.; Watson, G. N. ; *A Course of Modern Analysis*, London: Cambridge University Press,1962.
12. Wiener, Norbert ; *The Fourier Integral and Certain of its Applications*, New York: Dover Publications,1958.
13. Zhi-Wei Sun; Hao Pan .;Identities concerning Bernoulli and Euler polynomials, *Acta Arithmetica*, 125(2006),21-39.

Ref.

8. Salahuddin; Hypergeometric Form of Certain Indefinite Integrals, *Global Journal of Science Frontier Research(F)*, 12(2012), 37-41.

Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 12 Issue 8 Version 1.0 Year 2012
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Generalizations of the Distance and Dependent Function in Extenics to 2D, 3D, and N-D

By Florentin Smarandache
University of New Mexico

Abstract - Dr. Cai Wen defined in his 1983 paper :

- the distance formula between a point *x0* and a one-dimensional *(1D)* interval *[a, b]* ; - and the dependence function which gives the degree of dependence of a point with respect to a pair of included *1D* - intervals.

This paper inspired us to generalize the Extension Set to two-dimensions, i.e. in plane of real numbers *R2* where one has a rectangle (instead of a segment of line), determined by two arbitrary points *A (a1, a2)* and *B (b1, b2)*. And similarly in *R3*, where one has a prism determined by two arbitrary points *A (a1, a2, a3)* and *B(b1, b2, b3).* We geometrically define the linear and non-linear distance between a point and the *2D*- and *3D*-extension set and the dependent function for a nest of two included *2D* - and *3D* - extension sets. Linearly and non-linearly attraction point principles towards the optimal point are presented as well.

The same procedure can be then used considering, instead of a rectangle, any bounded *2D*-surface and similarly any bounded *3D* - solid, and any bounded *n – D* - body in *Rn*.

These generalizations are very important since the Extension Set is generalized from one-dimension to *2, 3* and even n-dimensions, therefore more classes of applications will result in consequence.

GJSFR -F Classification : MSC 2010: 00A73

GENERALIZATIONS OF THE DISTANCE AND DEPENDENT FUNCTION IN EXTENICS TO 2D, 3D, AND N-D

Strictly as per the compliance and regulations of :

eJournal Print Journal epaper open standard 3d virtual journal

Notes

Generalizations of the Distance and Dependent Function in Extenics to *2D, 3D*, and *N-D*

Florentin Smarandache

Abstract - Dr. Cai Wen defined in his 1983 paper:
- the distance formula between a point x_0 and a one-dimensional ***(1D)*** interval ***[a, b]***;
- and the dependence function which gives the degree of dependence of a point with respect to a pair of included ***1D*** - intervals.

This paper inspired us to generalize the Extension Set to two-dimensions, i.e. in plane of real numbers R^2 where one has a rectangle (instead of a segment of line), determined by two arbitrary points $A\ (a_1, a_2)$ and $B\ (b_1, b_2)$. And similarly in R^3, where one has a prism determined by two arbitrary points $A\ (a_1, a_2, a_3)$ and $B(b_1, b_2, b_3)$. We geometrically define the linear and non-linear distance between a point and the ***2D***- and ***3D***-extension set and the dependent function for a nest of two included ***2D*** - and ***3D*** - extension sets. Linearly and non-linearly attraction point principles towards the optimal point are presented as well.

The same procedure can be then used considering, instead of a rectangle, any bounded ***2D***-surface and similarly any bounded ***3D*** - solid, and any bounded $n-D$ - body in R^n.

These generalizations are very important since the Extension Set is generalized from one-dimension to ***2, 3*** and even *n*-dimensions, therefore more classes of applications will result in consequence.

I. Introduction

Extension Theory (or Extenics) was developed by Professor Cai Wen in 1983 by publishing a paper called "Extension Set and Non-Compatible Problems". Its goal is to solve contradictory problems and also nonconventional, nontraditional ideas in many fields.

Extenics is at the confluence of three disciplines: philosophy, mathematics, and engineering.

A contradictory problem is converted by a transformation function into a non-contradictory one.

The functions of transformation are: extension, decomposition, combination, etc.

Extenics has many practical applications in Management, Decision-Making, Strategic Planning, Methodology, Data Mining, Artificial Intelligence, Information Systems, Control Theory, etc.

Extenics is based on matter-element, affair-element, and relation-element.

II. Extension Distance in 1D-Space

Prof. Cai Wen has defined the extension distance between a point x_0 and a real interval $X = [a,b]$ by

$$\rho(x_0, X) = \left| x_0 - \frac{a+b}{2} \right| - \frac{b-a}{2}$$

Author : Mathematics and Science Department, 705 Gurey Ave, Gallup, University of New Mexico-87301, USA.
E-mail : smarand@unm.edu

where in general $\rho : (R, R^2) \rightarrow (-\infty, +\infty)$.

Algebraically studying this extension distance, we find that actually the range of it is:

$$\rho(x_0, X) \in [-\frac{b-a}{2}, +\infty)$$

or its minimum range value $-\frac{b-a}{2}$ depends on the interval X extremities a and b, and it occurs when the point x_0 coincides with the midpoint of the interval X, i.e. $x_0 = \frac{a+b}{2}$.

The closer is the interior point x_0 to the midpoint $\frac{a+b}{2}$ of the interval $[a, b]$, the negatively larger is $\rho(x_0, X)$.

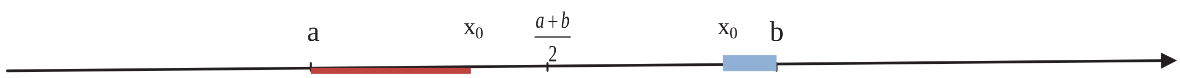

Fig. 1

In Fig. 1, for interior point x_o between a and $\frac{a+b}{2}$, the extension distance $\rho(x_0, X)$ $= a - x_0$ the negative length of the brown line segment[left side].Whereas for interior point x_o between $\frac{a+b}{2}$ and b, the extension distance $\rho(x_0, X) = x_0 - b$ = the negative length of the blue line segment [right side].

Similarly, the further is exterior point x_o with respect to the closest extremity of the interval $[a, b]$ to it (i.e. to either a or b), the positively larger is $\rho(x_0, X)$.

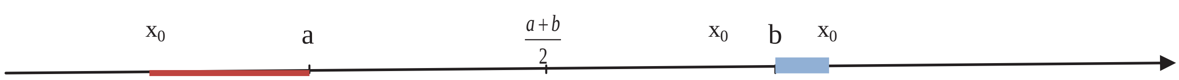

Fig. 2

In Fig. 2, for exterior point $x_o < a$, the extension distance $\rho(x_0, X) = a - x_0$ = *the positive length of the brown line segment* [left side]. Whereas for exterior point $x_o > b$, the extension distance $\rho(x_0, X) = x_0 - b$ = *the positive length of the blue line segment* [right side].

III. Principle of the Extension 1D-Distance

Geometrically studying this extension distance, we find the following principle that Prof. Cai has used in 1983 defining it:

$\rho(x_0, X)$ = the geometric distance between the point x_o and the closest extremity point of the interval $[a, b]$ to it (going in the direction that connects x_o with the optimal point), distance taken as negative if $x_0 \in [a, b]$, and as positive if $x_0 \subset [a, b]$.

This principle is very important in order to generalize the extension distance from $1D$ to $2D$ (twodimensional real space), $3D$ (three-dimensional real space), and n-D (n-dimensional real space).

The extremity points of interval $[a, b]$ are the point a and b, which are also the boundary (frontier) of the interval $[a, b]$.

IV. Dependent Function in *1D-Space*

Prof. Cai Wen defined in 1983 in *1D* the Dependent Function *K(y)*.

If one considers two intervals X_0 and X, that have no common end point, and $X_0 \subset X$, then:

Global Journal of Science Frontier Research (F) Volume XII Issue VIII Version I Year 2012

Notes

$$K(y)=\frac{\rho(y,X)}{\rho(y,X)-\rho(y,X_0)}.$$

Since *K(y)* was constructed in 1D in terms of the extension distance (.,.) , we simply generalize it to higher dimensions by replacing (.,.) with the generalized (.,.) in a higher dimension.

Notes

V. Extension Distance in 2D-Space

Instead of considering a segment of line *AB* representing the interval *[a, b]* in *1R*, we consider a rectangle *AMBN* representing all points of its surface in *2D*.

Let's consider two arbitrary points $A(a_1, a_2)$ and $B(b_1, b_2)$. Through the points *A* and *B* one draws parallels to the axes of the Cartesian system *XY* and one thus one forms a rectangle *AMBN* whose one of the diagonals is just *AB*.

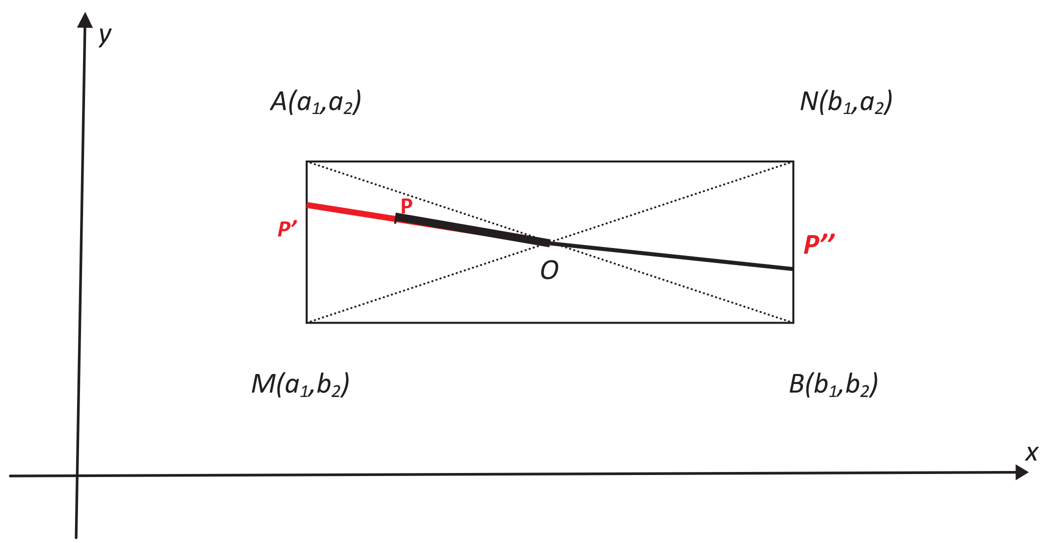

Fig. 3 : P is an interior point to the rectangle AMBN and the optimal point O is in the center of symmetry of the rectangle.

Let's note by *O* the midpoint of the diagonal *AB*, but *O* is also the center of symmetry (intersection of the diagonals) of the rectangle *AMBN*. Then one computes the distance between a point $P(x_o, y_o)$ and the rectangle *AMBN*.

One can do that following the same principle as Dr. Cai Wen did: - compute the distance in *2D* (two dimensions) between the point *P* and the center *O* of the rectangle (intersection of rectangle's diagonals); - next compute the distance between the point *P* and the closest point (let's note it by *P'*) to it on the frontier (the rectangle's four edges) of the rectangle *AMBN*;

this step can be done in the following way:

considering *P'* as the intersection point between the line *PO* and the frontier of the rectangle, and taken among the intersection points that point *P'* which is the closest to *P*;

this case is entirely consistent with Dr. Cai's approach in the sense that when reducing from *2D*- space to *1D*- space, *i.e.* the points $A(a_1, a_2)$ and $B(b_1, b_2)$ reduced to *A(a)* and respectively *B(b)*, which is equivalent to the rectangle *AMBN* reduced to its diagonal *AB*, one exactly gets his result.

The Extension 2D - Distance, for P O, will be:

$$\rho((x_0, y_0), AMBM) = d(\text{point } P, \text{ rectangle } AMBN) = |PO| - |P'O| = |PP'|$$

i) which is equal to the negative length of the red segment |PP'| in Fig. 3 when P is interior to the rectangle AMBN;

ii) or equal to zero when P lies on the frontier of the rectangle AMBN (i.e. on edges AM, MB, BN, or NA) since P coincides with P';

iii) or equal to the positive length of the blue segment |PP'| in Fig. 4 when P is exterior to the rectangle AMBN.

where |*PO*| means the classical *2D*-distance between the point *P* and *O*, and similarly for |*P'O*| and |*PP'*|.

The Extension *2D*-Distance, for the optimal point (i.e. *P*=*O*), will be

$\rho(O, AMBM)$ = *d(point O, rectangle AMBN)* =*-max d(point O, point M on the frontier of AMBN).*

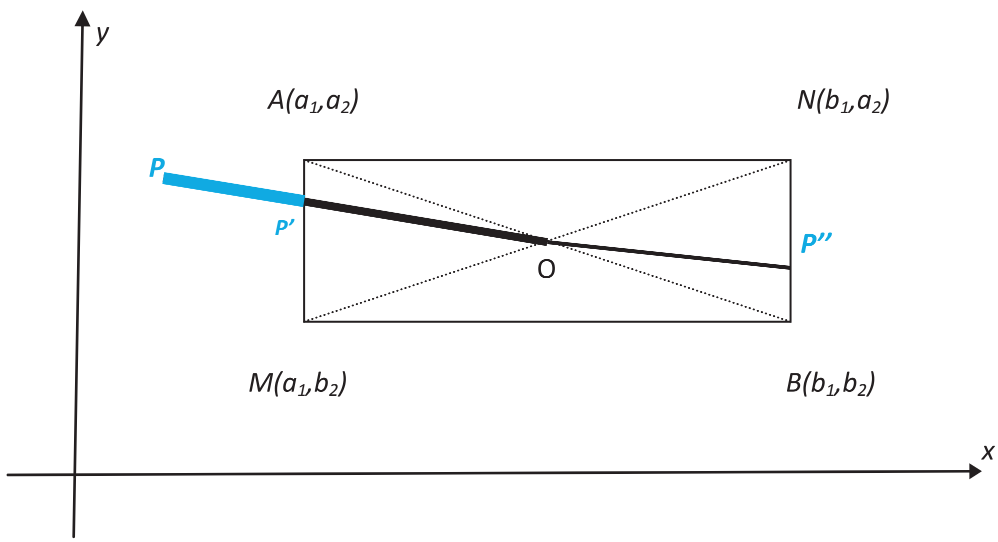

Fig. 4 : *P* is an exterior point to the rectangle *AMBN* and the optimal point *O* is in the center of symmetry of the rectangle.

The last step is to devise the Dependent Function in *2D*- space similarly as Dr. Cai's defined the dependent function in *1D*.

The midpoint (or center of symmetry) *O* has the coordinates $O(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2})$.

Let's compute the |*PO*| - |*P'O*|.

In this case, we extend the line *OP* to intersect the frontier of the rectangle *AMBN*. *P'* is closer to *P* than *P''*, therefore we consider *P'*.

The equation of the line *PO*, that of course passes through the points $P(x_o, y_o)$ and $O(\frac{a_1+b_1}{2}, \frac{a_2+b_2}{2})$, is:

$$y - y_0 = \frac{\frac{a_2+b_2}{2} - y_0}{\frac{a_1+b_1}{2} - x_0}(x - x_0)$$

Since the *x*-coordinate of point *P'* is a_1 because *P'* lies on the rectangle's edge *AM*, one gets the *y*-coordinate of point *P'* by a simple substitution of $x_{P'} = a_1$ into the above equality:

Global Journal of Science Frontier Research (F) Volume XII Issue VIII Version I Year 2012

Notes

$$y_{P'} = y_0 + \frac{a_2 + b_2 - 2y_0}{a_1 + b_1 - 2x_0}(a_1 - x_0).$$

Therefore P' has the coordinates P' $\left(x_{P'} = a_1,\ y_{P'} = y_0 + \frac{a_2 + b_2 - 2y_0}{a_1 + b_1 - 2x_0}(a_1 - x_0)\right)$.

The distance $d(P,O) = |PO| = \sqrt{(x_0 - \frac{a_1+b_1}{2})^2 + (y_0 - \frac{a_2+b_2}{2})^2}$

while the distance

$$d(P',O) = |P'O| = \sqrt{(a_1 - \frac{a_1+b_1}{2})^2 + (y_{P'} - \frac{a_2+b_2}{2})^2} = \sqrt{(\frac{a_1-b_1}{2})^2 + (y_{P'} - \frac{a_2+b_2}{2})^2}$$

Also, the distance $d(P,P') = |PP'| = \sqrt{(a_1 - x_0)^2 + (y_{P'} - y_0)^2}$.

Whence the Extension *2D*-Distance formula:

$$\rho((x_0, y_0), AMBM) = d(\,P(x_0,y_0),\ A(a_1,a_2)MB(b_1,b_2)N\,) = |PO| - |P'O|$$

$$= \sqrt{(x_0 - \frac{a_1+b_1}{2})^2 + (y_0 - \frac{a_2+b_2}{2})^2} - \sqrt{(\frac{a_1-b_1}{2})^2 + (y_{P'} - \frac{a_2+b_2}{2})^2}$$

$$= \pm\, |PP'|$$

$$= \pm \sqrt{(a_1 - x_0)^2 + (y_{P'} - y_0)^2}$$

where $y_{P'} = y_0 + \frac{a_2 + b_2 - 2y_0}{a_1 + b_1 - 2x_0}(a_1 - x_0)$.

Properties:

As for *1D* - distance, the following properties hold in *2D*:

Property 1.

a) $(x,y) \in \mathrm{Int}(AMBN)$ *iff* $\rho((x, y), AMBN) < 0$, *where Int(AMBN) means interior of AMBN;*

b) $(x,y) \in \mathrm{Fr}(AMBN)$ *iff* $\rho((x, y), AMBN) = 0$, *where Fr(AMBN) means frontier of AMBN;*

c) $(x,y) \notin AMBN$ *iff* $\rho((x, y), AMBN) > 0$.

Property 2.

Let $A_0M_0B_0N_0$ and $AMBN$ be two rectangles whose sides are parallel to the axes of the Cartesian system of coordinates, such that they have no common end points, and $A_0M_0B_0N_0 \subset AMBN$. We assume they have the same optimal points $O_1 \equiv O_2 \equiv O$ located in the center of symmetry of the two rectangles.

Then for any point $(x,y) \in R^2$ *one has* $\rho((x,y), A_0M_0B_0N_0) \geq \rho((x,y), AMBN)$.

Notes

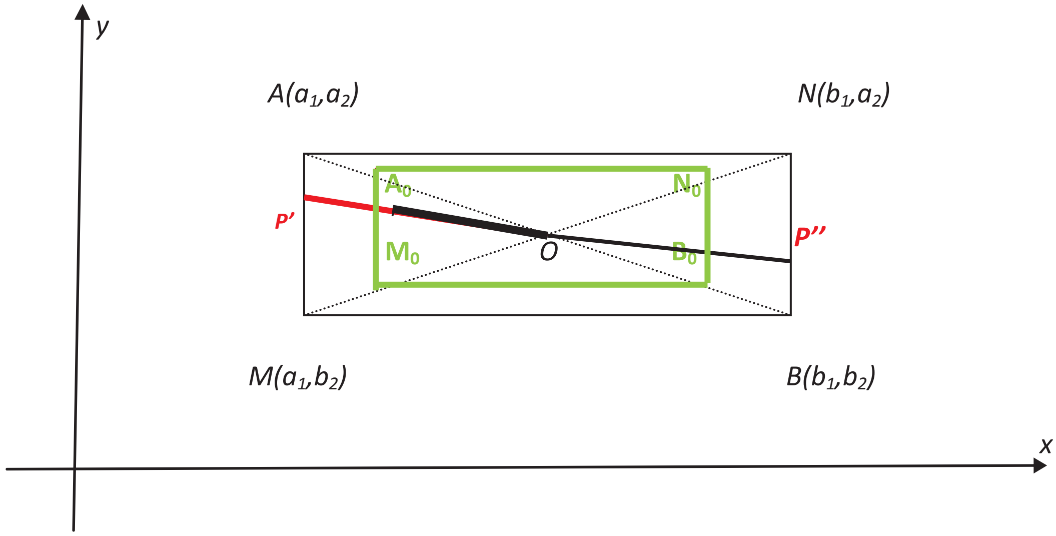

Fig. 5 : Two included rectangles with the same optimal points $O_1 \equiv O_2 \equiv O$ located in their common center of symmetry.

VI. Dependent 2D-Function

Let $A_0M_0B_0N_0$ and $AMBN$ be two rectangles whose sides are parallel to the axes of the Cartesian system of coordinates, such that they have no common end points, and $A_0M_0B_0N_0 \subset AMBN$.

The Dependent 2D-Function formula is :

$$K_{2D}(x,y) = \frac{\rho((x,y), AMBN)}{\rho((x,y), AMBN) - \rho((x,y), A_0M_0B_0N_0)}$$

Property 3.

Again, similarly to the Dependent Function in 1D-space, one has:

a) If $(x, y) \in \text{Int}\ (A_0M_0B_0N_0)$, then $K_{2D}(x, y) > 1$;
b) If $(x, y) \in \text{Fr}\ (A_0M_0B_0N_0)$, then $K_{2D}(x, y) = 1$;
c) If $(x, y) \in \text{Int}(AMBN - A_0M_0B_0N_0)$, then $0 < K_{2D}(x, y) < 1$;
d) If $(x, y) \in \text{Fr}(AMBN)$, then $K_{2D}(x, y) = 0$;
e) If $(x, y) \notin AMBN$, then $K_{2D}(x, y) < 0$.

VII. General Case in *2D*-Space

One can replace the rectangles by any finite surfaces, bounded by closed curves in *2D* - space, and one can consider any optimal point O (not necessarily the symmetry center). Again, we assume the optimal points are the same for this nest of two surfaces.

Notes

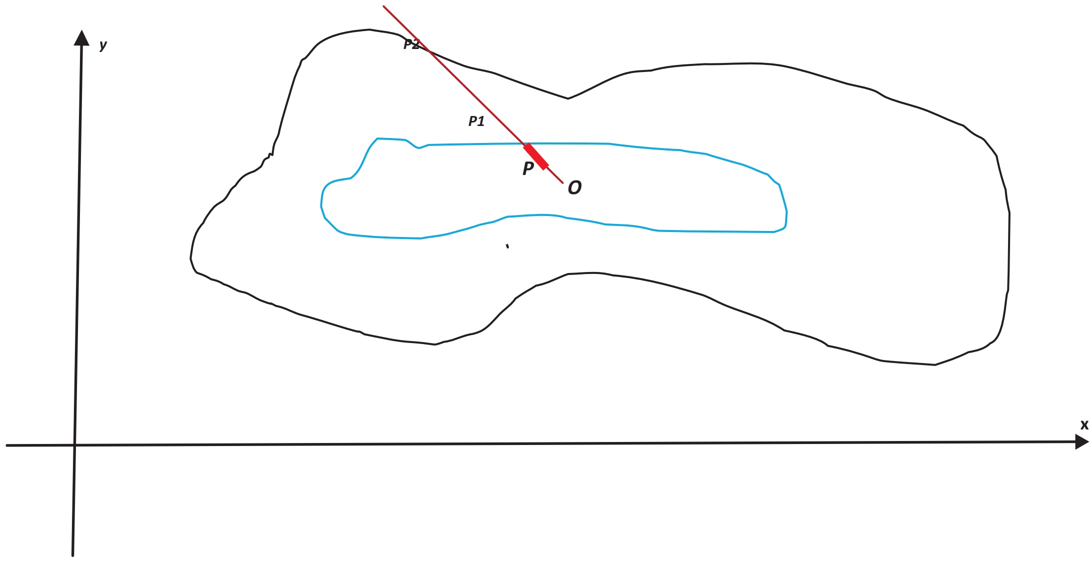

Fig. 6 : Two included arbitrary bounded surfaces with the same optimal points situated in their common center of symmetry.

VIII. Linear Attraction Point Principle

We introduce the Attraction Point Principle, which is the following:

Let $\mathcal{S}$ be a given set in the universe of discourse U, and the optimal point $O \in \mathcal{S}$. Then each point $P(x_1, x_2, \ldots, x_n)$ from the universe of discourse tends towards, or is attracted by, the optimal point O, because the optimal point O is an ideal of each point.

That's why one computes the extension n-D-distance between the point P and the set $\mathcal{S}$ as $\rho\ ((x_1, x_2, \ldots, x_n), \mathcal{S})$ on the direction determined by the point P and the optimal point O, or on the line PO, i.e.:

a) $\rho((x_1, x_2, \ldots, x_n), \mathcal{S})$ = the *negative distance* between P and the set frontier, if P is inside the set $\mathcal{S}$;
b) $\rho\ ((x_1, x_2, \ldots, x_n), \mathcal{S}) = 0$, if P lies on the frontier of the set $\mathcal{S}$;
c) $\rho\ ((x_1, x_2, \ldots, x_n), \mathcal{S})$ = the *positive distance* between P and the set frontier, if P is outside the set.

It is a king of convergence/attraction of each point towards the optimal point. There are classes of examples where such attraction point principle works.

If this principle is good in all cases, then there is no need to take into consideration the center of symmetry of the set $\mathcal{S}$, since for example if we have a *2D* piece which has heterogeneous material density, then its center of weight (barycenter) is different from the center of symmetry.

Let's see below such example in the *2D* - space:

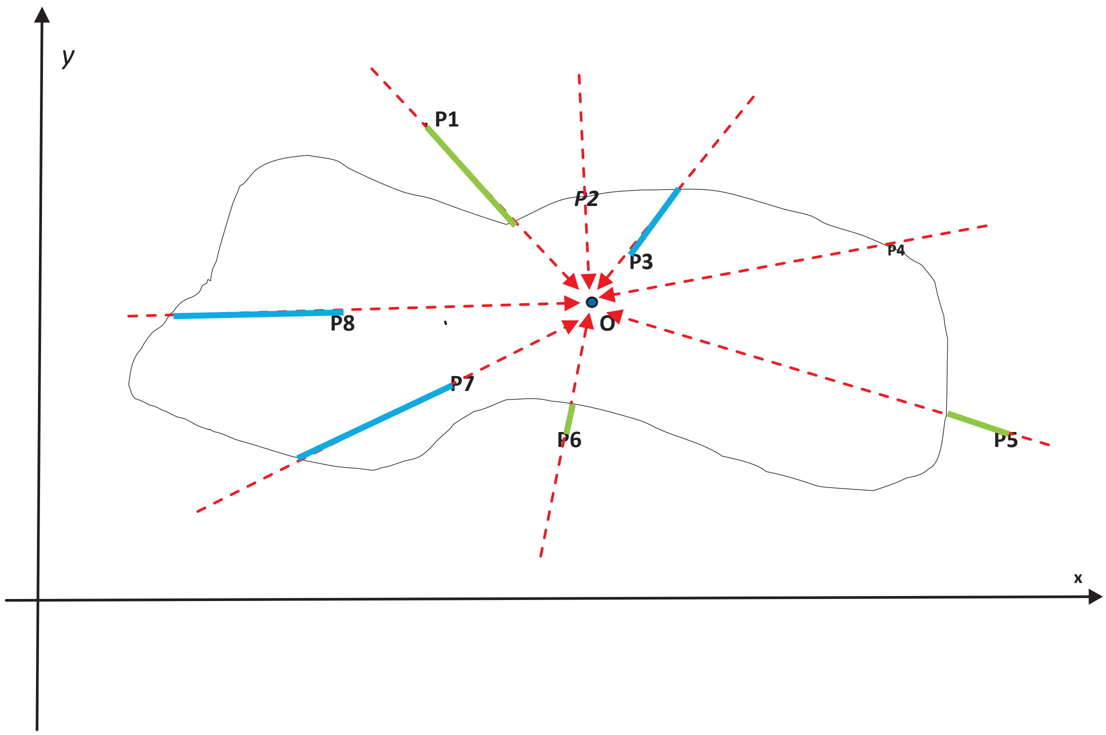

Fig. 7 : The optimal point O as an attraction point for all other points P_1, P_2, …, P_8 in the universe of discourse R^2.

Remark 1.

Another possible way, for computing the distance between the point *P* and the closest point *P'* to it on the frontier (the rectangle's four edges) of the rectangle *AMBN*, would be by drawing a perpendicular from *P* onto the closest rectangle's edge, and denoting by *P'* the intersection between the perpendicular and the rectangle's edge.

And similarly if one has an arbitrary set *S* in the *2D*-space, bounded by a closed curve. One computes

$$d(P, S) = \inf_{Q \in S} |PQ|$$

as in the classical mathematics.

IX. Extension Distance in 3D-Space

We further generalize to *3D* - space the Extension Set and the Dependent Function. Assume we have two points *A(a1, a2, a3)* and *B(b1, b2, b3)* in *3D*. Drawing through *A* and *B* parallel planes to the planes' axes *(XY, XZ, YZ)* in the Cartesian system *XYZ* we get a prism $AM_1M_2M_3BN_1N_2N_3$ (with eight vertices) whose one of the transversal diagonals is just the line segment *AB*. Let's note by *O* the midpoint of the transverse diagonal *AB*, but *O* is also the center of symmetry of the prism.

Therefore, from the line segment *AB* in *1D*-space, to a rectangle *AMBN* in *2D*-space, and now to a prism $AM_1M_2M_3BN_1N_2N_3$ in *3D*- space.

Then one computes the distance between a point $P(x_0, y_0, z_0)$ and the prism $AM_1M_2M_3BN_1N_2N_3$.

One can do that following the same principle as Dr. Cai's:

- compute the distance in *3D* (two dimensions) between the point *P* and the center *O* of the prism (intersection of prism's transverse diagonals);
- next compute the distance between the point *P*and the closest point (let's note it by *P'*) to it on the frontier (the prism's lateral surface) of the prism $AM_1M_2M_3BN_1N_2N_3$;

considering *P'* as the intersection point between the line *OP* and the frontier of the prism,

Notes

and taken among the intersection points that point P' which is the closest to P; this case is entirely consistent with Dr. Cai's approach in the sense that when reducing from *3D* - space to *1D* - space one gets exactly Dr. Cai's result;

- the Extension *3D* - Distance will be: $d(P, AM_1M_2M_3BN_1N_2N_3) = |PO| - |P'O| = \pm |PP'|$, where $|PO|$ means the classical distance in *3D* - space between the point P and O, and similarly for $|P'O|$ *and* $|PP'|$.

Notes

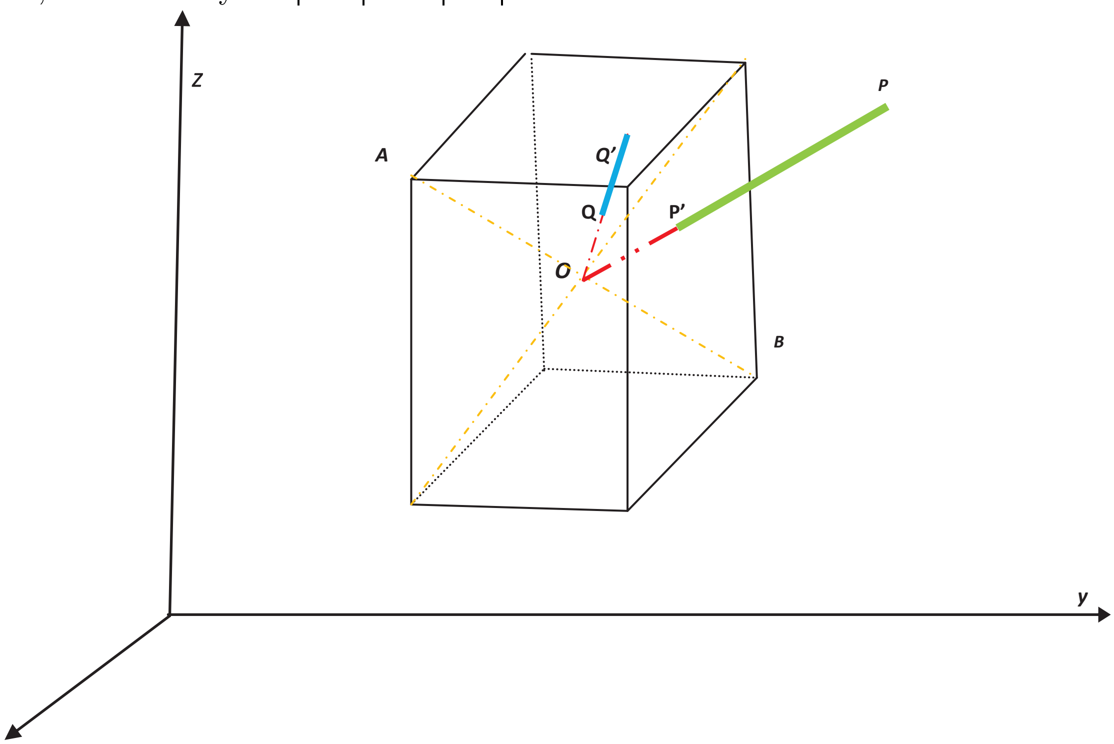

Fig. 8 : Extension 3D-Distance between a point and a prism, where O is the optimal point coinciding with the center of symmetry.

Property 4.

a) $(x,y,z) \in \mathrm{Int}\ (AM_1M_2M_3BN_1N_2N_3)$ *iff* $\rho((x, y, z), AM_1M_2M_3BN_1N_2N_3) < 0$, *where* $Int(AM_1M_2M_3BN_1N_2N_3)$ *means interior of* $AM_1M_2M_3BN_1N_2N_3$*;*

b) $(x,y,z) \in \mathrm{Fr}\ (AM_1M_2M_3BN_1N_2N_3)$ *iff* $\rho((x, y, z), AM_1M_2M_3BN_1N_2N_3) = 0$, *where* $Fr\ (AM_1M_2M_3BN_1N_2N_3)$ *means frontier of* $AM_1M_2M_3BN_1N_2N_3$*;*

c) $(x,y,z) \notin AM_1M_2M_3BN_1N_2N_3$ *iff* $\rho((x, y, z), AM_1M_2M_3BN_1N_2N_3) > 0$.

Property 5.

Let $A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03}$ and $AM_1M_2M_3BN_1N_2N_3$ be two prisms whose sides are parallel to the axes of the Cartesian system of coordinates, such that they have no common end points, and $A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03} \subset AM_1M_2M_3BN_1N_2N_3$. We assume they have the same optimal points $O_1 \equiv O_2 \equiv O$ located in the center of symmetry of the two prisms.

Then for any point $(x,y,z) \in R^3$ *one has*

$$\rho((x,y,z), A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03}) \geq \rho((x,y,z), AM_1M_2M_3BN_1N_2N_3).$$

X. Dependent *2D* - Function

The last step is to devise the Dependent Function in *3D* - space similarly to Dr. Cai's definition of the dependent function in *1D* - space.

Let $A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03}$ and $AM_1M_2M_3BN_1N_2N_3$ be two prisms whose faces are parallel to the axes of the Cartesian system of coordinates XYZ, such that they have no common end points, such that $A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03} \subset AM_1M_2M_3BN_1N_2N_3$. We assume they have the same optimal points $O_1 \equiv O_2 \equiv O$ located in the center of symmetry of these two prisms.

The **Dependent *3D* - Function** formula is:

$$K_{3D}(x,y,z) = \frac{\rho((x,y,z), AM_1M_2M_3BN_1N_2N_3)}{\rho((x,y,z), AM_1M_2M_3BN_1N_2N_3) - \rho((x,y,z), A_0M_{01}M_{02}M_{03}BN_{01}N_{02}N_{03})}$$

Notes

Property 6.

Again, similarly to the Dependent Function in *1D* - and *2D*- spaces, one has:

a) If $(x, y, z) \in Int\ (A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03})$, then $K_{3D}(x, y, z) > 1$;

b) If $(x, y, z) \in Fr\ (A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03})$, then $K_{3D}(x, y, z) = 1$;

c) If $(x, y, z) \in Int\ (AM_1M_2M_3BN_1N_2N_3 - A_0M_{01}M_{02}M_{03}B_0N_{01}N_{02}N_{03})$, then $0 < K_{3D}(x, y, z) < 1$;

d) If $(x, y, z) \in Fr\ (AM_1M_2M_3BN_1N_2N_3)$, then $K_{3D}(x, y, z) = 0$;

e) If $(x, y, z) \notin AM_1M_2M_3BN_1N_2N_3$, then $K_{3D}(x,y,z) < 0$.

XI. General Case in *3D*-Space

One can replace the prisms by any finite *3D* - bodies, bounded by closed surfaces, and one considers any optimal point O (not necessarily the centers of surfaces' symmetry). Again, we assume the optimal points are the same for this nest of two *3D* - bodies.

Remark 2.

Another possible way, for computing the distance between the point P and the closest point P' to it on the frontier (lateral surface) of the prism $AM_1M_2M_3BN_1N_2N_3$ is by drawing a perpendicular from P onto the closest prism's face, and denoting by P' the intersection between the perpendicular and the prism's face.

And similarly if one has an arbitrary finite body B in the *3D* - space, bounded by surfaces. One computes as in classical mathematics:

Linear Attraction Point Principle in 3D-space.

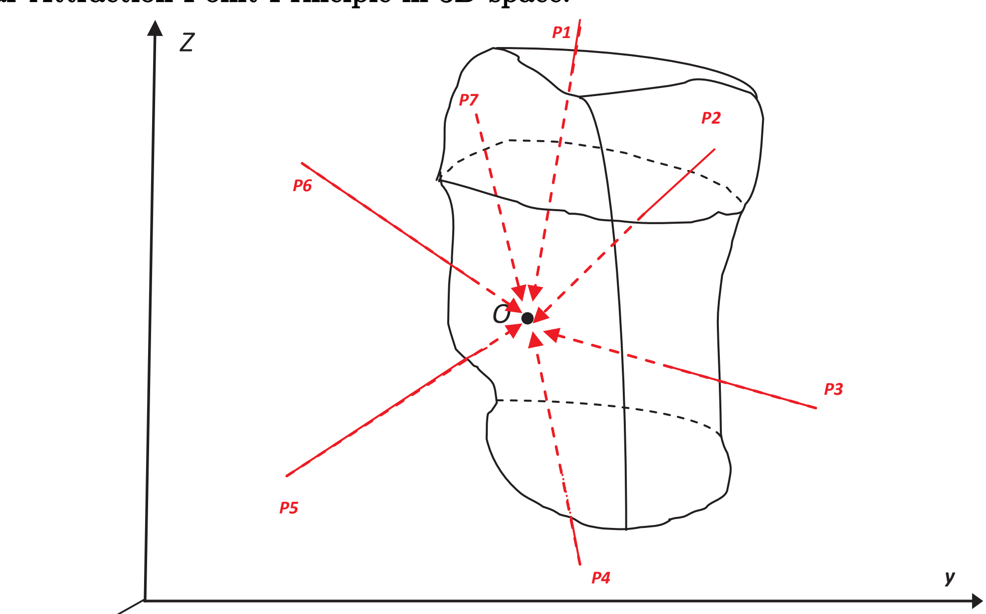

Fig. 9 : Linear Attraction Point Principle for any bounded 3D-body.

Non-Linear Attraction Point Principle in 3D - Space (and in n-D-Space).

There might be spaces where the attraction phenomena undergo not linearly by upon some specific non-linear curves. Let's see below such example for points P_i whose trajectories of attraction towards the optimal point follow some non-linear *3D*- curves.

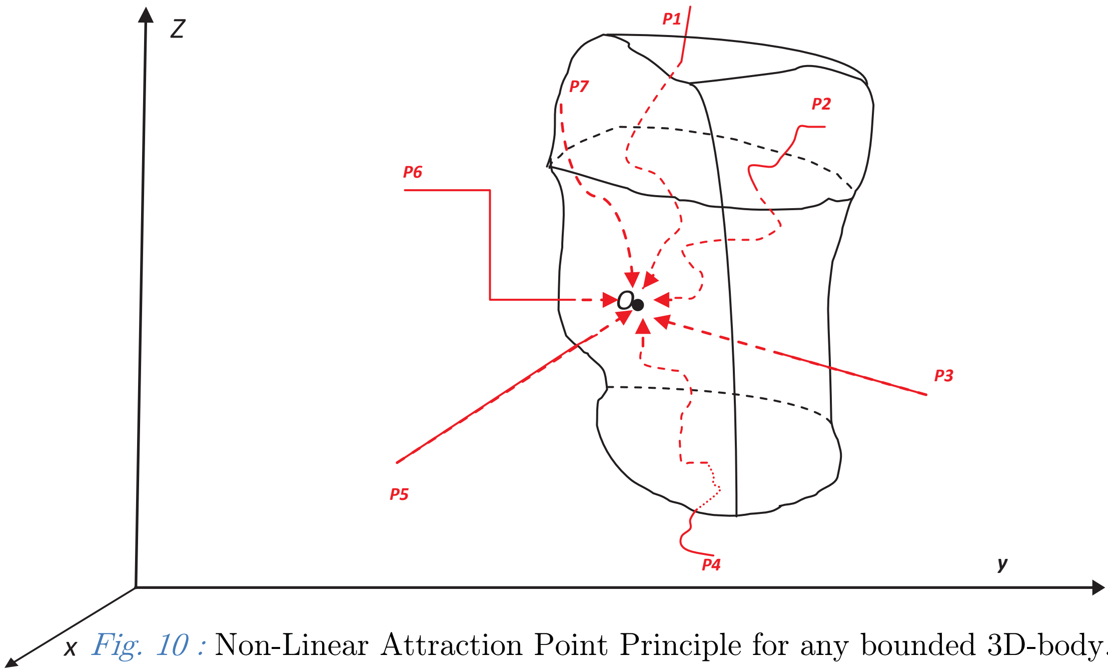

Fig. 10 : Non-Linear Attraction Point Principle for any bounded 3D-body.

n-D-Space.

In general, in a universe of discourse U, let's have an $n-D$ - set $\mathcal{S}$ and a point P.
Then the **Extension Linear n - D - Distance** between point P and set $\mathcal{S}$,is:

$$\rho(P,S)=\begin{cases} -\underset{P'\in Fr(S)}{d(P,P')}, & P\neq O, P\in|OP'|; \\ \underset{P'\in Fr(S)}{d(P,P')}, & P\neq O, P'\in|OP|; \\ -\underset{M\in Fr(S)}{\max}\, d(P,M), & P=O. \end{cases}$$

where O is the optimal point (or linearly attraction point);
$d(P,P')$ means the classical linearly $n-D$ - distance between two points P and P';
$Fr(\mathcal{S})$ means the frontier of set $\mathcal{S}$;
and $|OP'|$ means the line segment between the points O and P' (the extremity points O and P' included), therefore P $|OP'|$ means that P lies on the line OP', in between the points O and P'.

For P coinciding with O, one defined the distance between the optimal point O and the set S as the negatively maximum distance (to be in concordance with the *1D* - definition).

And the **Extension Non-Linear *n-D*-Distance** between point P and set $\mathcal{S}$, is:

$$\rho_c(P,S)=\begin{cases} -\underset{P'\in Fr(S)}{d_c(P,P')}, & P\neq O, P\in c(OP'); \\ \underset{P'\in Fr(S)}{d_c(P,P')}, & P\neq O, P'\in c(OP); \\ -\underset{M\in Fr(S), M\in c(O)}{\max}\, d_c(P,M), & P=O. \end{cases}$$

where $\rho_c(P, S)$ means the extension distance as measured along the curve c;
O is the optimal point (or non-linearly attraction point);
the points are attracting by the optimal point on trajectories described by an injective curve c;
$d_c(P,P')$ means the non-linearly n-D-distance between two points P and P', or the arclength of the curve c between the points P and P';
$Fr(\mathcal{S})$ means the frontier of set $\mathcal{S}$;
and $c(OP')$ means the curve segment between the points O and P' (the extremity points O and P' included), therefore $P \in c(OP')$ means that P lies on the curve c in between the points O and P'.

For P coinciding with O, one defined the distance between the optimal point O and the set S as the negatively maximum curvilinear distance (to be in concordance with the *1D*-definition).

In general, in a universe of discourse U, let's have a nest of two n-D-sets, $S_1 \subset S_2$, with no common end points, and a point P.
Then the **Extension Linear Dependent n-D-Function** referring to the point $P(x_1, x_2, \ldots, x_n)$ is:

$$K_{nD}(P) = \frac{\rho(P, S_2)}{\rho(P, S_2) - \rho(P, S_1)}$$

where is the previous extension linear n - D - distance between the point P and the n – D - set $\mathcal{S}_2$.
And the **Extension Non-Linear Dependent n-D-Function** referring to point $P(x_1, x_2, \ldots, x_n)$ along the curve c is:

$$K_{nD}(P) = \frac{\rho_c(P, S_2)}{\rho_c(P, S_2) - \rho_c(P, S_1)}$$

where $\rho_c(P, S_2)$ is the previous extension non-linear $n - D$ - distance between the point P and the $n - D$- set $\mathcal{S}_2$ along the curve c.

Remark 3.

Particular cases of curves c could be interesting to studying, for example if c are parabolas, or have elliptic forms, or arcs of circle, etc. Especially considering the geodesics would be for many practical applications.

Tremendous number of applications of Extenics could follow in all domains where attraction points would exist; these attraction points could be in physics (for example, the earth center is an attraction point), economics (attraction towards a specific product), sociology (for example attraction towards a specific life style), etc.

XII. Conclusion

In this paper we introduced the **Linear and Non-Linear Attraction Point Principle**, which is the following:

Let $\mathcal{S}$ be an arbitrary set in the universe of discourse U of any dimension, and the optimal point $O \in \mathcal{S}$.

Then each point $P\ (x_1, x_2, \ldots, x_n)$, $n \geq 1$, from the universe of discourse (linearly or non-linearly) tends towards, or is attracted by, the optimal point O, because the optimal point O is an ideal of each point.

Notes

It is a king of convergence/attraction of each point towards the optimal point. There are classes of examples and applications where such attraction point principle may apply.

If this principle is good in all cases, then there is no need to take into consideration the center of symmetry of the set $\mathcal{S}$, since for example if we have a *2D* factory piece which has heterogeneous material density, then its center of weight (barycenter) is different from the center of symmetry.

Notes

Then we generalized in the track of Cai Wen's idea the extension *1D* - set to an extension *n - D* - set, and defined the Linear (or Non-Linear) **Extension *n – D* - Distance** between a point $P(x_1, x_2, \ldots, x_n)$ and the *n - D* set $\mathcal{S}$ as $\rho(\ (x_1, x_2, \ldots, x_n), \mathcal{S}\ )$ on the linear (or non-linear) direction determined by the point *P* and the optimal point *O* (the line *PO*, or respectively the curvilinear *PO)* in the following way:

d) $\rho((x_1, x_2, \ldots, x_n), \mathcal{S})$ = the *negative distance* between *P* and the set frontier, if *P* is inside the set S;

e) $\rho((x_1, x_2, \ldots, x_n), \mathcal{S}) = 0$, if *P* lies on the frontier of the set $\mathcal{S}$;

f) $\rho((x_1, x_2, \ldots, x_n), \mathcal{S})$ = the *positive distance* between P and the set frontier, if *P* is outside the set.

We got the following **properties**:

a) It is obvious from the above definition of the extension *n – D* - distance between a point *P* in the universe of discourse and the extension *n – D* - set $\mathcal{S}$ that:

i) Point $P(x_1, x_2, \ldots, x_n) \in \mathrm{Int}(\mathcal{S})$ iff $\rho\ ((x_1, x_2, \ldots, x_n), \mathcal{S}) < 0;$

ii) Point $P(x_1, x_2, \ldots, x_n) \in \mathrm{Fr}(\mathcal{S})$ iff $\rho((x_1, x_2, \ldots, x_n), \mathcal{S}) = 0;$

iii) Point $P(x_1, x_2, \ldots, x_n) \notin \mathcal{S}$ iff $\rho\ ((x_1, x_2, \ldots, x_n), \mathcal{S}) > 0.$

b) Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be two extension sets, in the universe of discourse *U*, such that they have no common end points, and $\mathcal{S}_1 \subset \mathcal{S}_2$. We assume they have the same optimal points $O_1 \equiv O_2 \equiv O$ located in their center of symmetry. Then for any point $P(x_1, x_2, \ldots, x_n)$ *U* one has:

$$\rho(\ (x_1, x_2, \ldots, x_n), \mathcal{S}_1\ ) \geq \rho(\ (x_1, x_2, \ldots, x_n), \mathcal{S}_2\ ).$$

Then we proceed to the generalization of the dependent function from *1D* - space to Linear (or Non- Linear) *n – D* - space Dependent Function, using the previous notations.

The **Linear** (or **Non-Linear**) **Dependent *n – D* - Function** of point $P(x_1, x_2, \ldots, x_n)$ along the curve *c*, is:

$$K_{nD}(x_1, x_2, \ldots, x_n) = \frac{\rho_c((x_1, x_2, \ldots, x_n), S_2)}{\rho_c((x_1, x_2, \ldots, x_n), S_2) - \rho_c((x_1, x_2, \ldots, x_n), S_1)}$$

(where *c* may be a curve or even a line)
which has the following **property**:

d) If point $P(x_1, x_2, \ldots, x_n) \in \mathrm{Int}(\mathcal{S}_1)$, then $K_{nD}(x_1, x_2, \ldots, x_n) > 1$;

e) If point $P(x_1, x_2, \ldots, x_n) \in \mathrm{Fr}(\mathcal{S}_1)$, then $K_{nD}(x_1, x_2, \ldots, x_n) = 1$;

f) If point $P(x_1, x_2, \ldots, x_n) \in \mathrm{Int}(\mathcal{S}_2\text{-}\mathcal{S}_1)$, then $K_{nD}(x_1, x_2, \ldots, x_n)$ $(0, 1)$;

g) If point $P(x_1, x_2, \ldots, x_n) \in \mathrm{Int}(\mathcal{S}_2)$, then $K_{nD}(x_1, x_2, \ldots, x_n) = 0$;

h) If point $P(x_1, x_2, \ldots, x_n) \notin \mathrm{Int}(\mathcal{S}_2)$, then $K_{nD}(x_1, x_2, \ldots, x_n) < 0$.

References Références Referencias

1. Cai Wen. Extension Set and Non-Compatible Problems [J]. Journal of Scientific Exploration, 1983, (1): 83-97; also Cai Wen. Extension Set and Non-Compatible Problems [A]. Advances in Applied Mathematics and Mechanics in China [C]. Peking: International Academic Publishers, 1990.1-21.
2. Cai Wen. Extension theory and its application, [J]. Chinese Science Bulletin, 1999, 44 (7): 673-682. Cai Wen. Extension theory and its application, [J]. Chinese Science Bulletin, 1999, 44 (17): 1538- 1548.
3. Yang Chunyan, Cai Wen. Extension Engineering [M]. Beijing: Public Library of Science, 2007.
4. Wu Wenjun et al. "Research on Extension theory and its application" Expert Opinion. 2004, 2; http://web.gdut.edu.cn/˜extenics/jianding.htm.
5. Xiangshan Science Conferences Office. Scientific Significance and Future Development of Extenics – No. 271 Academic Discussion of Xiangshan Science Conferences, Brief Report of Xiangshan Science Conferences, Period 260, 2006, 1.

Notes

Global Journals Inc. (US) Guidelines Handbook 2012

www.GlobalJournals.org

FELLOW OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (FARSS)

- 'FARSS' title will be awarded to the person after approval of Editor-in-Chief and Editorial Board. The title 'FARSS" can be added to name in the following manner. eg. Dr. John E. Hall, Ph.D., FARSS or William Walldroff Ph. D., M.S., FARSS
- Being FARSS is a respectful honor. It authenticates your research activities. After becoming FARSS, you can use 'FARSS' title as you use your degree in suffix of your name. This will definitely will enhance and add up your name. You can use it on your Career Counseling Materials/CV/Resume/Visiting Card/Name Plate etc.
- 60% Discount will be provided to FARSS members for publishing research papers in Global Journals Inc., if our Editorial Board and Peer Reviewers accept the paper. For the life time, if you are author/co-author of any paper bill sent to you will automatically be discounted one by 60%
- FARSS will be given a renowned, secure, free professional email address with 100 GB of space eg.johnhall@globaljournals.org. You will be facilitated with Webmail, SpamAssassin, Email Forwarders, Auto-Responders, Email Delivery Route tracing, etc.
- FARSS member is eligible to become paid peer reviewer at Global Journals Inc. to earn up to 15% of realized author charges taken from author of respective paper. After reviewing 5 or more papers you can request to transfer the amount to your bank account or to your PayPal account.
- Eg. If we had taken 420 USD from author, we can send 63 USD to your account.
- FARSS member can apply for free approval, grading and certification of some of their Educational and Institutional Degrees from Global Journals Inc. (US) and Open Association of Research,Society U.S.A.
- After you are FARSS. You can send us scanned copy of all of your documents. We will verify, grade and certify them within a month. It will be based on your academic records, quality of research papers published by you, and 50 more criteria. This is beneficial for your job interviews as recruiting organization need not just rely on you for authenticity and your unknown qualities, you would have authentic ranks of all of your documents. Our scale is unique worldwide.
- FARSS member can proceed to get benefits of free research podcasting in Global Research Radio with their research documents, slides and online movies.
- After your publication anywhere in the world, you can upload you research paper with your recorded voice or you can use our professional RJs to record your paper their voice. We can also stream your conference videos and display your slides online.
- FARSS will be eligible for free application of Standardization of their Researches by Open Scientific Standards. Standardization is next step and level after publishing in a journal. A team of research and professional will work with you to take your research to its next level, which is worldwide open standardization.

- FARSS is eligible to earn from their researches: While publishing his paper with Global Journals Inc. (US), FARSS can decide whether he/she would like to publish his/her research in closed manner. When readers will buy that individual research paper for reading, 80% of its earning by Global Journals Inc. (US) will be transferred to FARSS member's bank account after certain threshold balance. There is no time limit for collection. FARSS member can decide its price and we can help in decision.

MEMBER OF ASSOCIATION OF RESEARCH SOCIETY IN SCIENCE (MARSS)

- 'MARSS' title will be awarded to the person after approval of Editor-in-Chief and Editorial Board. The title 'MARSS" can be added to name in the following manner. eg. Dr. John E. Hall, Ph.D., MARSS or William Walldroff Ph. D., M.S., MARSS
- Being MARSS is a respectful honor. It authenticates your research activities. After becoming MARSS, you can use 'MARSS' title as you use your degree in suffix of your name. This will definitely will enhance and add up your name. You can use it on your Career Counseling Materials/CV/Resume/Visiting Card/Name Plate etc.
- 40% Discount will be provided to MARSS members for publishing research papers in Global Journals Inc., if our Editorial Board and Peer Reviewers accept the paper. For the life time, if you are author/co-author of any paper bill sent to you will automatically be discounted one by 60%
- MARSS will be given a renowned, secure, free professional email address with 30 GB of space eg.johnhall@globaljournals.org. You will be facilitated with Webmail, SpamAssassin, Email Forwarders, Auto-Responders, Email Delivery Route tracing, etc.
- MARSS member is eligible to become paid peer reviewer at Global Journals Inc. to earn up to 10% of realized author charges taken from author of respective paper. After reviewing 5 or more papers you can request to transfer the amount to your bank account or to your PayPal account.
- MARSS member can apply for free approval, grading and certification of some of their Educational and Institutional Degrees from Global Journals Inc. (US) and Open Association of Research,Society U.S.A.
- MARSS is eligible to earn from their researches: While publishing his paper with Global Journals Inc. (US), MARSS can decide whether he/she would like to publish his/her research in closed manner. When readers will buy that individual research paper for reading, 40% of its earning by Global Journals Inc. (US) will be transferred to MARSS member's bank account after certain threshold balance. There is no time limit for collection. MARSS member can decide its price and we can help in decision.

ANNUAL MEMBER

- Annual Member will be authorized to receive e-Journal GJSFR for one year (subscription for one year).
- The member will be allotted free 1 GB Web-space along with subDomain to contribute and participate in our activities.
- A professional email address will be allotted free 500 MB email space.

PAPER PUBLICATION

- The members can publish paper once. The paper will be sent to two-peer reviewer. The paper will be published after the acceptance of peer reviewers and Editorial Board.

Process of submission of Research Paper

The Area or field of specialization may or may not be of any category as mentioned in 'Scope of Journal' menu of the GlobalJournals.org website. There are 37 Research Journal categorized with Six parental Journals GJCST, GJMR, GJRE, GJMBR, GJSFR, GJHSS. For Authors should prefer the mentioned categories. There are three widely used systems UDC, DDC and LCC. The details are available as 'Knowledge Abstract' at Home page. The major advantage of this coding is that, the research work will be exposed to and shared with all over the world as we are being abstracted and indexed worldwide.

The paper should be in proper format. The format can be downloaded from first page of 'Author Guideline' Menu. The Author is expected to follow the general rules as mentioned in this menu. The paper should be written in MS-Word Format (*.DOC,*.DOCX).

The Author can submit the paper either online or offline. The authors should prefer online submission.Online Submission: There are three ways to submit your paper:

(A) (I) First, register yourself using top right corner of Home page then Login. If you are already registered, then login using your username and password.

(II) Choose corresponding Journal.

(III) Click 'Submit Manuscript'. Fill required information and Upload the paper.

(B) If you are using Internet Explorer, then Direct Submission through Homepage is also available.

(C) If these two are not conveninet , and then email the paper directly to dean@globaljournals.org.

Offline Submission: Author can send the typed form of paper by Post. However, online submission should be preferred.

MANUSCRIPT STYLE INSTRUCTION **<u>(Must be strictly followed)</u>**

Page Size: 8.27" X 11'"

- Left Margin: 0.65
- Right Margin: 0.65
- Top Margin: 0.75
- Bottom Margin: 0.75
- Font type of all text should be Swis 721 Lt BT.
- Paper Title should be of Font Size 24 with one Column section.
- Author Name in Font Size of 11 with one column as of Title.
- Abstract Font size of 9 Bold, “Abstract” word in Italic Bold.
- Main Text: Font size 10 with justified two columns section
- Two Column with Equal Column with of 3.38 and Gaping of .2
- First Character must be three lines Drop capped.
- Paragraph before Spacing of 1 pt and After of 0 pt.
- Line Spacing of 1 pt
- Large Images must be in One Column
- Numbering of First Main Headings (Heading 1) must be in Roman Letters, Capital Letter, and Font Size of 10.
- Numbering of Second Main Headings (Heading 2) must be in Alphabets, Italic, and Font Size of 10.

You can use your own standard format also.
Author Guidelines:

1. General,

2. Ethical Guidelines,

3. Submission of Manuscripts,

4. Manuscript’s Category,

5. Structure and Format of Manuscript,

6. After Acceptance.

1. GENERAL

Before submitting your research paper, one is advised to go through the details as mentioned in following heads. It will be beneficial, while peer reviewer justify your paper for publication.

Scope

The Global Journals Inc. (US) welcome the submission of original paper, review paper, survey article relevant to the all the streams of Philosophy and knowledge. The Global Journals Inc. (US) is parental platform for Global Journal of Computer Science and Technology, Researches in Engineering, Medical Research, Science Frontier Research, Human Social Science, Management, and Business organization. The choice of specific field can be done otherwise as following in Abstracting and Indexing Page on this Website. As the all Global

Journals Inc. (US) are being abstracted and indexed (in process) by most of the reputed organizations. Topics of only narrow interest will not be accepted unless they have wider potential or consequences.

2. ETHICAL GUIDELINES

Authors should follow the ethical guidelines as mentioned below for publication of research paper and research activities.

Papers are accepted on strict understanding that the material in whole or in part has not been, nor is being, considered for publication elsewhere. If the paper once accepted by Global Journals Inc. (US) and Editorial Board, will become the copyright of the Global Journals Inc. (US).

Authorship: The authors and coauthors should have active contribution to conception design, analysis and interpretation of findings. They should critically review the contents and drafting of the paper. All should approve the final version of the paper before submission

The Global Journals Inc. (US) follows the definition of authorship set up by the Global Academy of Research and Development. According to the Global Academy of R&D authorship, criteria must be based on:

1) Substantial contributions to conception and acquisition of data, analysis and interpretation of the findings.

2) Drafting the paper and revising it critically regarding important academic content.

3) Final approval of the version of the paper to be published.

All authors should have been credited according to their appropriate contribution in research activity and preparing paper. Contributors who do not match the criteria as authors may be mentioned under Acknowledgement.

Acknowledgements: Contributors to the research other than authors credited should be mentioned under acknowledgement. The specifications of the source of funding for the research if appropriate can be included. Suppliers of resources may be mentioned along with address.

Appeal of Decision: The Editorial Board’s decision on publication of the paper is final and cannot be appealed elsewhere.

Permissions: It is the author's responsibility to have prior permission if all or parts of earlier published illustrations are used in this paper.

Please mention proper reference and appropriate acknowledgements wherever expected.

If all or parts of previously published illustrations are used, permission must be taken from the copyright holder concerned. It is the author's responsibility to take these in writing.

Approval for reproduction/modification of any information (including figures and tables) published elsewhere must be obtained by the authors/copyright holders before submission of the manuscript. Contributors (Authors) are responsible for any copyright fee involved.

3. SUBMISSION OF MANUSCRIPTS

Manuscripts should be uploaded via this online submission page. The online submission is most efficient method for submission of papers, as it enables rapid distribution of manuscripts and consequently speeds up the review procedure. It also enables authors to know the status of their own manuscripts by emailing us. Complete instructions for submitting a paper is available below.

Manuscript submission is a systematic procedure and little preparation is required beyond having all parts of your manuscript in a given format and a computer with an Internet connection and a Web browser. Full help and instructions are provided on-screen. As an author, you will be prompted for login and manuscript details as Field of Paper and then to upload your manuscript file(s) according to the instructions.

To avoid postal delays, all transaction is preferred by e-mail. A finished manuscript submission is confirmed by e-mail immediately and your paper enters the editorial process with no postal delays. When a conclusion is made about the publication of your paper by our Editorial Board, revisions can be submitted online with the same procedure, with an occasion to view and respond to all comments.

Complete support for both authors and co-author is provided.

4. MANUSCRIPT'S CATEGORY

Based on potential and nature, the manuscript can be categorized under the following heads:

Original research paper: Such papers are reports of high-level significant original research work.

Review papers: These are concise, significant but helpful and decisive topics for young researchers.

Research articles: These are handled with small investigation and applications

Research letters: The letters are small and concise comments on previously published matters.

5.STRUCTURE AND FORMAT OF MANUSCRIPT

The recommended size of original research paper is less than seven thousand words, review papers fewer than seven thousands words also.Preparation of research paper or how to write research paper, are major hurdle, while writing manuscript. The research articles and research letters should be fewer than three thousand words, the structure original research paper; sometime review paper should be as follows:

Papers: These are reports of significant research (typically less than 7000 words equivalent, including tables, figures, references), and comprise:

(a)Title should be relevant and commensurate with the theme of the paper.

(b) A brief Summary, "Abstract" (less than 150 words) containing the major results and conclusions.

(c) Up to ten keywords, that precisely identifies the paper's subject, purpose, and focus.

(d) An Introduction, giving necessary background excluding subheadings; objectives must be clearly declared.

(e) Resources and techniques with sufficient complete experimental details (wherever possible by reference) to permit repetition; sources of information must be given and numerical methods must be specified by reference, unless non-standard.

(f) Results should be presented concisely, by well-designed tables and/or figures; the same data may not be used in both; suitable statistical data should be given. All data must be obtained with attention to numerical detail in the planning stage. As reproduced design has been recognized to be important to experiments for a considerable time, the Editor has decided that any paper that appears not to have adequate numerical treatments of the data will be returned un-refereed;

(g) Discussion should cover the implications and consequences, not just recapitulating the results; conclusions should be summarizing.

(h) Brief Acknowledgements.

(i) References in the proper form.

Authors should very cautiously consider the preparation of papers to ensure that they communicate efficiently. Papers are much more likely to be accepted, if they are cautiously designed and laid out, contain few or no errors, are summarizing, and be conventional to the approach and instructions. They will in addition, be published with much less delays than those that require much technical and editorial correction.

The Editorial Board reserves the right to make literary corrections and to make suggestions to improve briefness.

It is vital, that authors take care in submitting a manuscript that is written in simple language and adheres to published guidelines.

Format

Language: The language of publication is UK English. Authors, for whom English is a second language, must have their manuscript efficiently edited by an English-speaking person before submission to make sure that, the English is of high excellence. It is preferable, that manuscripts should be professionally edited.

Standard Usage, Abbreviations, and Units: Spelling and hyphenation should be conventional to The Concise Oxford English Dictionary. Statistics and measurements should at all times be given in figures, e.g. 16 min, except for when the number begins a sentence. When the number does not refer to a unit of measurement it should be spelt in full unless, it is 160 or greater.

Abbreviations supposed to be used carefully. The abbreviated name or expression is supposed to be cited in full at first usage, followed by the conventional abbreviation in parentheses.

Metric SI units are supposed to generally be used excluding where they conflict with current practice or are confusing. For illustration, 1.4 l rather than 1.4 × 10-3 m3, or 4 mm somewhat than 4 × 10-3 m. Chemical formula and solutions must identify the form used, e.g. anhydrous or hydrated, and the concentration must be in clearly defined units. Common species names should be followed by underlines at the first mention. For following use the generic name should be constricted to a single letter, if it is clear.

Structure

All manuscripts submitted to Global Journals Inc. (US), ought to include:

Title: The title page must carry an instructive title that reflects the content, a running title (less than 45 characters together with spaces), names of the authors and co-authors, and the place(s) wherever the work was carried out. The full postal address in addition with the e-mail address of related author must be given. Up to eleven keywords or very brief phrases have to be given to help data retrieval, mining and indexing.

Abstract, used in Original Papers and Reviews:

Optimizing Abstract for Search Engines

Many researchers searching for information online will use search engines such as Google, Yahoo or similar. By optimizing your paper for search engines, you will amplify the chance of someone finding it. This in turn will make it more likely to be viewed and/or cited in a further work. Global Journals Inc. (US) have compiled these guidelines to facilitate you to maximize the web-friendliness of the most public part of your paper.

Key Words

A major linchpin in research work for the writing research paper is the keyword search, which one will employ to find both library and Internet resources.

One must be persistent and creative in using keywords. An effective keyword search requires a strategy and planning a list of possible keywords and phrases to try.

Search engines for most searches, use Boolean searching, which is somewhat different from Internet searches. The Boolean search uses "operators," words (and, or, not, and near) that enable you to expand or narrow your affords. Tips for research paper while preparing research paper are very helpful guideline of research paper.

Choice of key words is first tool of tips to write research paper. Research paper writing is an art.A few tips for deciding as strategically as possible about keyword search:

- One should start brainstorming lists of possible keywords before even begin searching. Think about the most important concepts related to research work. Ask, "What words would a source have to include to be truly valuable in research paper?" Then consider synonyms for the important words.
- It may take the discovery of only one relevant paper to let steer in the right keyword direction because in most databases, the keywords under which a research paper is abstracted are listed with the paper.
- One should avoid outdated words.

Keywords are the key that opens a door to research work sources. Keyword searching is an art in which researcher's skills are bound to improve with experience and time.

Numerical Methods: Numerical methods used should be clear and, where appropriate, supported by references.

Acknowledgements: Please make these as concise as possible.

References

References follow the Harvard scheme of referencing. References in the text should cite the authors' names followed by the time of their publication, unless there are three or more authors when simply the first author's name is quoted followed by et al. unpublished work has to only be cited where necessary, and only in the text. Copies of references in press in other journals have to be supplied with submitted typescripts. It is necessary that all citations and references be carefully checked before submission, as mistakes or omissions will cause delays.

References to information on the World Wide Web can be given, but only if the information is available without charge to readers on an official site. Wikipedia and Similar websites are not allowed where anyone can change the information. Authors will be asked to make available electronic copies of the cited information for inclusion on the Global Journals Inc. (US) homepage at the judgment of the Editorial Board.

The Editorial Board and Global Journals Inc. (US) recommend that, citation of online-published papers and other material should be done via a DOI (digital object identifier). If an author cites anything, which does not have a DOI, they run the risk of the cited material not being noticeable.

The Editorial Board and Global Journals Inc. (US) recommend the use of a tool such as Reference Manager for reference management and formatting.

Tables, Figures and Figure Legends

Tables: Tables should be few in number, cautiously designed, uncrowned, and include only essential data. Each must have an Arabic number, e.g. Table 4, a self-explanatory caption and be on a separate sheet. Vertical lines should not be used.

Figures: Figures are supposed to be submitted as separate files. Always take in a citation in the text for each figure using Arabic numbers, e.g. Fig. 4. Artwork must be submitted online in electronic form by e-mailing them.

Preparation of Electronic Figures for Publication

Even though low quality images are sufficient for review purposes, print publication requires high quality images to prevent the final product being blurred or fuzzy. Submit (or e-mail) EPS (line art) or TIFF (halftone/photographs) files only. MS PowerPoint and Word Graphics are unsuitable for printed pictures. Do not use pixel-oriented software. Scans (TIFF only) should have a resolution of at least 350 dpi (halftone) or 700 to 1100 dpi (line drawings) in relation to the imitation size. Please give the data for figures in black and white or submit a Color Work Agreement Form. EPS files must be saved with fonts embedded (and with a TIFF preview, if possible).

For scanned images, the scanning resolution (at final image size) ought to be as follows to ensure good reproduction: line art: >650 dpi; halftones (including gel photographs) : >350 dpi; figures containing both halftone and line images: >650 dpi.

Color Charges: It is the rule of the Global Journals Inc. (US) for authors to pay the full cost for the reproduction of their color artwork. Hence, please note that, if there is color artwork in your manuscript when it is accepted for publication, we would require you to complete and return a color work agreement form before your paper can be published.

Figure Legends: Self-explanatory legends of all figures should be incorporated separately under the heading 'Legends to Figures'. In the full-text online edition of the journal, figure legends may possibly be truncated in abbreviated links to the full screen version. Therefore, the first 100 characters of any legend should notify the reader, about the key aspects of the figure.

6. AFTER ACCEPTANCE

Upon approval of a paper for publication, the manuscript will be forwarded to the dean, who is responsible for the publication of the Global Journals Inc. (US).

6.1 Proof Corrections

The corresponding author will receive an e-mail alert containing a link to a website or will be attached. A working e-mail address must therefore be provided for the related author.

Acrobat Reader will be required in order to read this file. This software can be downloaded

(Free of charge) from the following website:

www.adobe.com/products/acrobat/readstep2.html. This will facilitate the file to be opened, read on screen, and printed out in order for any corrections to be added. Further instructions will be sent with the proof.

Proofs must be returned to the dean at dean@globaljournals.org within three days of receipt.

As changes to proofs are costly, we inquire that you only correct typesetting errors. All illustrations are retained by the publisher. Please note that the authors are responsible for all statements made in their work, including changes made by the copy editor.

6.2 Early View of Global Journals Inc. (US) (Publication Prior to Print)

The Global Journals Inc. (US) are enclosed by our publishing's Early View service. Early View articles are complete full-text articles sent in advance of their publication. Early View articles are absolute and final. They have been completely reviewed, revised and edited for publication, and the authors' final corrections have been incorporated. Because they are in final form, no changes can be made after sending them. The nature of Early View articles means that they do not yet have volume, issue or page numbers, so Early View articles cannot be cited in the conventional way.

6.3 Author Services

Online production tracking is available for your article through Author Services. Author Services enables authors to track their article - once it has been accepted - through the production process to publication online and in print. Authors can check the status of their articles online and choose to receive automated e-mails at key stages of production. The authors will receive an e-mail with a unique link that enables them to register and have their article automatically added to the system. Please ensure that a complete e-mail address is provided when submitting the manuscript.

6.4 Author Material Archive Policy

Please note that if not specifically requested, publisher will dispose off hardcopy & electronic information submitted, after the two months of publication. If you require the return of any information submitted, please inform the Editorial Board or dean as soon as possible.

6.5 Offprint and Extra Copies

A PDF offprint of the online-published article will be provided free of charge to the related author, and may be distributed according to the Publisher's terms and conditions. Additional paper offprint may be ordered by emailing us at: editor@globaljournals.org .

the search? Will I be able to find all information in this field area? If the answer of these types of questions will be "Yes" then you can choose that topic. In most of the cases, you may have to conduct the surveys and have to visit several places because this field is related to Computer Science and Information Technology. Also, you may have to do a lot of work to find all rise and falls regarding the various data of that subject. Sometimes, detailed information plays a vital role, instead of short information.

2. Evaluators are human: First thing to remember that evaluators are also human being. They are not only meant for rejecting a paper. They are here to evaluate your paper. So, present your Best.

3. Think Like Evaluators: If you are in a confusion or getting demotivated that your paper will be accepted by evaluators or not, then think and try to evaluate your paper like an Evaluator. Try to understand that what an evaluator wants in your research paper and automatically you will have your answer.

4. Make blueprints of paper: The outline is the plan or framework that will help you to arrange your thoughts. It will make your paper logical. But remember that all points of your outline must be related to the topic you have chosen.

5. Ask your Guides: If you are having any difficulty in your research, then do not hesitate to share your difficulty to your guide (if you have any). They will surely help you out and resolve your doubts. If you can't clarify what exactly you require for your work then ask the supervisor to help you with the alternative. He might also provide you the list of essential readings.

6. Use of computer is recommended: As you are doing research in the field of Computer Science, then this point is quite obvious.

7. Use right software: Always use good quality software packages. If you are not capable to judge good software then you can lose quality of your paper unknowingly. There are various software programs available to help you, which you can get through Internet.

8. Use the Internet for help: An excellent start for your paper can be by using the Google. It is an excellent search engine, where you can have your doubts resolved. You may also read some answers for the frequent question how to write my research paper or find model research paper. From the internet library you can download books. If you have all required books make important reading selecting and analyzing the specified information. Then put together research paper sketch out.

9. Use and get big pictures: Always use encyclopedias, Wikipedia to get pictures so that you can go into the depth.

10. Bookmarks are useful: When you read any book or magazine, you generally use bookmarks, right! It is a good habit, which helps to not to lose your continuity. You should always use bookmarks while searching on Internet also, which will make your search easier.

11. Revise what you wrote: When you write anything, always read it, summarize it and then finalize it.

12. Make all efforts: Make all efforts to mention what you are going to write in your paper. That means always have a good start. Try to mention everything in introduction, that what is the need of a particular research paper. Polish your work by good skill of writing and always give an evaluator, what he wants.

13. Have backups: When you are going to do any important thing like making research paper, you should always have backup copies of it either in your computer or in paper. This will help you to not to lose any of your important.

14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. **Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page

- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.
- Do not give permission to anyone else to "PROOFREAD" your manuscript.
- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

CRITERION FOR GRADING A RESEARCH PAPER (COMPILATION)
BY GLOBAL JOURNALS INC. (US)

Please note that following table is only a Grading of "Paper Compilation" and not on "Performed/Stated Research" whose grading solely depends on Individual Assigned Peer Reviewer and Editorial Board Member. These can be available only on request and after decision of Paper. This report will be the property of Global Journals Inc. (US).

Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

INDEX

save our planet

Global Journal of Science Frontier Research

Visit us on the Web at www.GlobalJournals.org | www.JournalofScience.org
or email us at helpdesk@globaljournals.org

ISSN 9755896