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Mathematics and Decision Sciences

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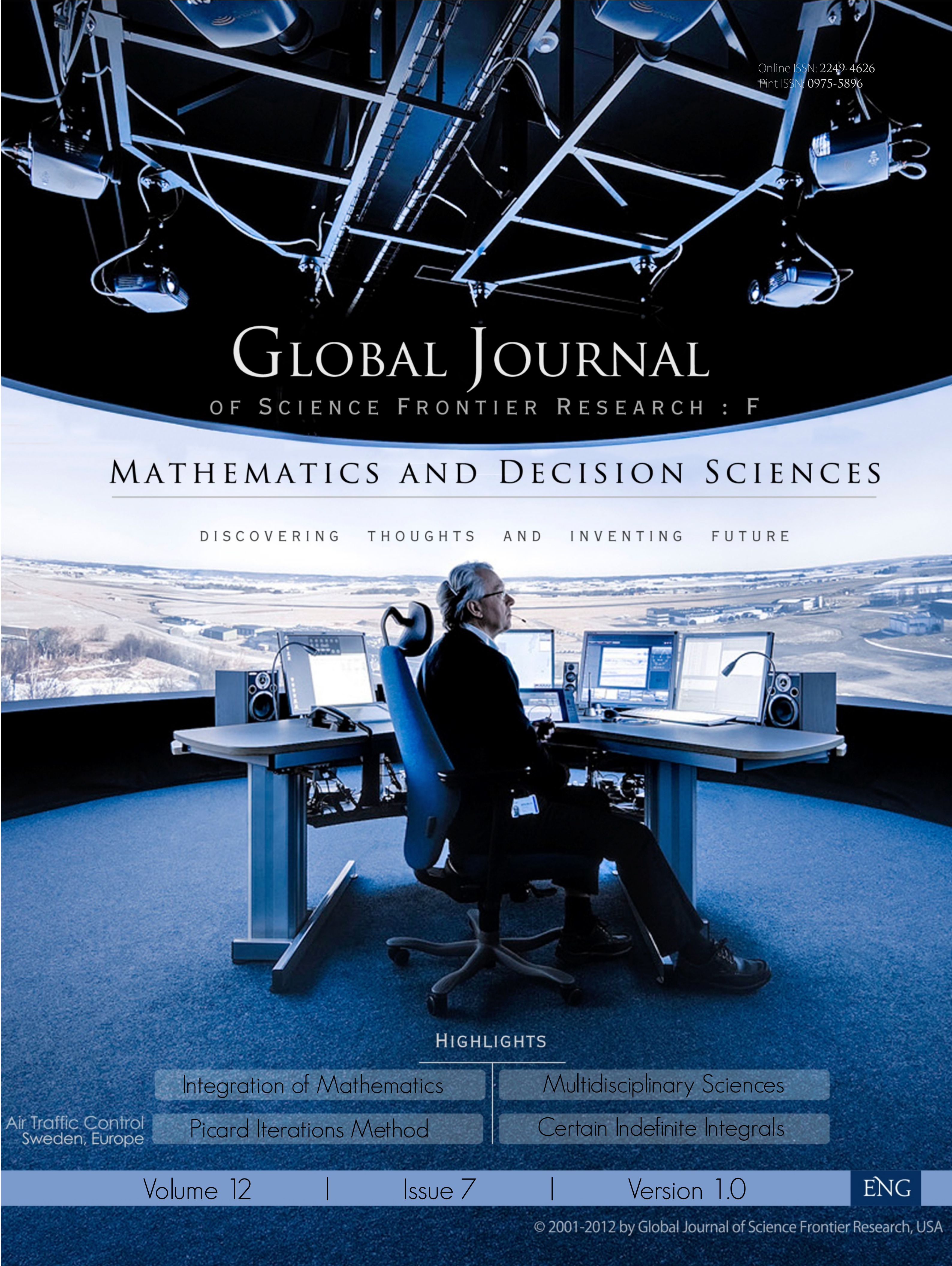

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Highlights

- Integration of Mathematics
- Multidisciplinary Sciences
- Picard Iterations Method
- Certain Indefinite Integrals

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Evaluation of A Summation Formula Based on Half Argument and Allied with Recurrence Relation

By Salahuddin
P.D.M College of Engineering, Bahadurgarh, Haryana, India

Abstract - The aim of this paper is to evaluate a summation formula based on half argument allied with Hypergeometric function and involving recurrence relation.

Keywords and Phrases : Contiguous relation, Gauss second summation theorem ,Re-currence relation.

A.M.S. Subject Classification (2000): 33C05, 33C45, 33C60, 33C70.

EVALUATION OF A SUMMATION FORMULA BASED ON HALF ARGUMENT AND ALLIED WITH RECURRENCE RELATION

Strictly as per the compliance and regulations of :

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Abstract - The aim of this paper is to evaluate a summation formula based on half argument allied with Hypergeometric function and involving recurrence relation.

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I. Introduction

Special functions and their applications are now awe-inspiring in their scope, variety and depth. Not only in their rapid growth in pure Mathematics and its applications to the traditional fields of Physics, Engineering and Statistics but in new fields of applications like Behavioral Science, Optimization, Biology, Environmental Science and Economics, etc. they are emerging. The term hypergeometric series was first used by John Wallis in his 1655 book Arithmetica Infinitorum. Hypergeometric series were studied by Euler, but the first full systematic treatment was given by Gauss (1813), Studies in the nineteenth century included those of Ernst Kummer (1836), and the fundamental characterisation by Bernhard Riemann of the hypergeometric function by means of the differential equation it satisfies. Multiple hypergeometric functions constitute a natural generalization of the Gauss hypergeometric functions of one variable. Since the introduction of double hypergeometric function by Appell and hypergeometric series by Lauricella, numerous papers by many workers have been published and the theory has been considerably extended. An extensive study has been made in Europe, America and India for these multiple functions, which produced explosion of knowledge of the subject.

The **Pochhammer symbol** or **generalized factorial function** or **shifted factorial** or **falling factorial** is defined by

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} a(a+1)(a+2)\cdots(a+n-1)\ ; & \text{if } \ n = 1,2,3,\dots \\ 1 \ ; & \text{if } \ n = 0 \\ n! \ ; & \text{if } \ a = 1,\ n = 1,2,3,\dots \end{cases} \tag{1}$$

where $a \neq 0, -1, -2, \dots$ and the notation Γ stands for Gamma function. Note that $(0)_0 = 1$.

Author : P.D.M College of Engineering, Bahadurgarh, Haryana, India. E-mails : sludn@yahoo.com; vsludn@gmail.com

If $m = 1, 2, 3, 4, \ldots$ and $n = 0, 1, 2, 3, 4, \ldots$ then

$$(b)_{mn} = m^{mn}\left(\frac{b}{m}\right)_n\left(\frac{b+1}{m}\right)_n\cdots\left(\frac{b+m-2}{m}\right)_n\left(\frac{b+m-1}{m}\right)_n \tag{2}$$

$$(\alpha)_{p+q} = (\alpha)_p(\alpha+p)_q = (\alpha)_q(\alpha+q)_p \tag{3}$$

$$\text{If } 0 \le k \le \left[\frac{n}{m}\right], \text{ then } (b)_{n-mk} = \frac{(\frac{-1}{m})^{mk}(b)_n}{\prod_{j=1}^{m}(\frac{j-b-n}{m})_k} \tag{4}$$

$$(n-mk)! = \frac{(\frac{-1}{m})^{mk}\, n!}{\prod_{j=1}^{m-1}(\frac{j-n}{m})_k} \tag{5}$$

where $[x]$ returns the largest integer less than or equal to x. In Slater's book, $(a)_n$ is denoted by $(a,\, n)$.

$$(b)_{-m} = \frac{\Gamma(b-m)}{\Gamma(b)} = \frac{(-1)^m}{(1-b)_m} \tag{6}$$

where, $b \neq 0, \pm1, \pm2, \pm3, \pm4, \ldots$ and $m = 1, 2, 3, 4, \ldots$

Generalized Gaussian Hypergeometric function of one variable is defined by

$${}_AF_B\left[\begin{matrix} a_1, a_2, \cdots, a_A & ; \\ & & z \\ b_1, b_2, \cdots, b_B & ; \end{matrix}\right] = \sum_{k=0}^{\infty}\frac{(a_1)_k(a_2)_k\cdots(a_A)_k z^k}{(b_1)_k(b_2)_k\cdots(b_B)_k k!}$$

or

$${}_AF_B\left[\begin{matrix} (a_A) & ; \\ & & z \\ (b_B) & ; \end{matrix}\right] \equiv {}_AF_B\left[\begin{matrix} (a_j)_{j=1}^A & ; \\ & & z \\ (b_j)_{j=1}^B & ; \end{matrix}\right] = \sum_{k=0}^{\infty}\frac{((a_A))_k z^k}{((b_B))_k k!} \tag{7}$$

where the parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and $A,\ B$ are non-negative integers and $\mid z \mid = 1$.

Contiguous Relation is defined by

[Andrews p.363(9.16), E. D. p.51(10)]

$$(a-b)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; \\ c\ \ ; \end{matrix}\ z\right] = a\ {}_2F_1\left[\begin{matrix} a+1,\ b & ; \\ c & ; \end{matrix}\ z\right] - b\ {}_2F_1\left[\begin{matrix} a,\ b+1 & ; \\ c\ ; \end{matrix}\ z\right] \tag{8}$$

Gauss second summation theorem is defined by [Prud., 491(7.3.7.8)]

$${}_2F_1\left[\begin{matrix} a,\ b & ; & \frac{1}{2} \\ \frac{a+b+1}{2} & ; & 2 \end{matrix}\right] = \frac{\Gamma(\frac{a+b+1}{2})\ \Gamma(\frac{1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} \tag{9}$$

$$= \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{a+b+1}{2})}{\Gamma(b)\ \Gamma(\frac{a+1}{2})} \tag{10}$$

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In a monograph of Prudnikov et al., a summation theorem is given in the form [Prud., p.491(7.3.7.8)]

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & 1 \\ \frac{a+b-1}{2} & ; & 2 \end{matrix}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\ \Gamma(\frac{b+1}{2})} + \frac{2\ \Gamma(\frac{a+b-1}{2})}{\Gamma(a)\ \Gamma(b)}\right] \tag{11} $$

Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above theorem can be written in the form

$$ {}_2F_1\left[\begin{matrix} a,\ b & ; & 1 \\ \frac{a+b-1}{2} & ; & 2 \end{matrix}\right] = \frac{2^{(b-1)}\ \Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\ \Gamma(\frac{a}{2})\ \Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{12} $$

Recurrence relation is defined by

$$ \Gamma(z+1) = z\ \Gamma(z) \tag{13} $$

II. Main Result

$$ {}_2F_1\left[\begin{matrix} a, \quad b & ; & 1 \\ \frac{a+b+35}{2} & ; & 2 \end{matrix}\right] = \frac{2^b\ \Gamma(\frac{a+b+35}{2})}{(a-b)\ \Gamma(b)}\times $$

$$ \times\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\ \left\{\frac{65536a(191898783962510625 - 454441401368236800a + 421214220916438680a^2)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+\right.\right. $$

$$ +\frac{65536a(-215245451727154944a^3 + 69953125893139644a^4 - 15611917903312640a^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+ $$

$$ +\frac{65536a(2505185827387880a^6 - 297403077939968a^7 + 26569595376038a^8 - 1801370405120a^9)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+ $$

$$ +\frac{65536a(92751487400a^{10} - 3600792832a^{11} + 103613692a^{12} - 2141440a^{13} + 30040a^{14} - 256a^{15} + a^{16})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+ $$

$$ +\frac{65536a(-158998505377975200b + 160158645208764792 0ab - 624388051013229216a^2b)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+ $$

$$ +\frac{65536a(672430138856900592a^3b - 118482004312390688a^4b + 45551364598847984a^5b)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+ $$

$$ +\frac{65536a(-4435348958817568a^6b + 824274801730864a^7b - 47099407828704a^8b + 4760242317072a^9b)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+ $$

$$ +\frac{65536a(-158593548256a^{10}b + 9023659216a^{11}b - 161683808a^{12}b + 4998224a^{13}b - 36960a^{14}b)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+ $$

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$$+\frac{65536a(528a^{15}b+9403043470403022000b^{2}-16712986646331360 0ab^{2}+1565019021296531592a^{2}b^{2})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(-2061065359127089992a^{3}b^{2}+2252712556906476 72a^{4}b^{2}-1823922724736851 2a^{5}b^{2})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(7239298092789576a^{6}b^{2}-361604047216896a^{7}b^{2}+69113934169896a^{8}b^{2}-2073715803456a^{9}b^{2})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(213043702872a^{10}b^{2}-3508382592a^{11}b^{2}+199059432a^{12}b^{2}-1374912a^{13}b^{2}+40920a^{14}b^{2})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(1348118704909310 40b^{3}+1145685449186532144ab^{3}-420728547803752 32a^{2}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(427559184167559840a^{3}b^{3}-24609722912109280a^{4}b^{3}+2614105452792116 8a^{5}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(-1085549498794112a^{6}b^{3}+422199872988608a^{7}b^{3}-11232092335200a^{8}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(2100369962064a^{9}b^{3}-31783404224a^{10}b^{3}+3163530656a^{11}b^{3}-20569120a^{12}b^{3}+1107568a^{13}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(227974014975506844b^{4}+74389934082676608ab^{4}+340559682051260760a^{2}b^{4})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(-37354554211349 76a^{3}b^{4}+4435779720483315 6a^{4}b^{4}-1251405780261120a^{5}b^{4})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(126697145565 0384a^{6}b^{4}-27724725718272a^{7}b^{4}+10295243007588a^{8}b^{4}-138281447040a^{9}b^{4})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(24624322008a^{10}b^{4}-148097664a^{11}b^{4}+13884156a^{12}b^{4}+23463156071200736b^{5})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(109033109373003376ab^{5}+11644472098601760a^{2}b^{5}+36927624682661328a^{3}b^{5})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(-133812647496000a^4b^5+2004216495193440a^5b^5-28680451208640a^6b^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(27382474864800a^7b^5-298829757600a^8b^5+104344772400a^9b^5-555366240a^{10}b^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(92561040a^11b^5+1086907218781216 8b^6+5985820063371456ab^6+14802425524183176a^2b^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(708321165615360a^3b^6+1713851879026320a^4b^6-1935009901440a^5b^6+41314983328080a^6b^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(-285600648960a^7b^6+255386589480a^8b^6-1091745600a^9b^6+354817320a^{10}b^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(7172607779944464b^7+2594162140235248ab^7+458691506376576a^2b^7+784325815238592a^3b^7)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(18518957835840a^4b^7+35960462445600a^5b^7-9170663040a^6b^7+372721947840a^7b^7)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(-982571040a^8b^7+818809200a^9b^7+149064655240710b^8+97562518884864ab^8)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(182692939157256a^2b^8+137253269514 24a^3b^8+17944376279940a^4b^8+204374776320a^5b^8)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(328424370120a^6b^8+1166803110a^8b^8+62010293660 48b^9+19239357998608ab^9)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(3911767205280a^2b^9+4975885470864a^3b^9+165209589600a^4b^9+173969661360a^5b^9)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(764221920a^6b^9+1037158320a^7b^9+674268049224b^{10}+456852420288ab^{10})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(717189756888a^2b^{10}+56410020480a^3b^{10}+54189742200a^4b^{10}+655047360a^5b^{10})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536a(573166440a^6b^{10}+16994547232b^{11}+46979218832ab^{11}+9049664832a^2b^{11}+9488348064a^3b^{11})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536a(252439200a^4b^{11}+193536720a^5b^{11}+1010111388b^{12}+628138368ab^{12}+860817672a^2b^{12})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536a(49365888a^3b^{12}+38567100a^4b^{12}+13933216b^{13}+34705616ab^{13}+4746720a^2b^{13})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536a(4272048a^3b^{13}+434808b^{14}+196416ab^{14}+237336a^2b^{14}+2464b^{15}+5456ab^{15}+33b^{16})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536b(191898783962510625-158998505377975200a+940304347040302200a^2)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(134811870490931040a^3+227974014975506844a^4+23463156071200736a^5)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(10869072187812168a^6+717260777994464a^7+149064655240710a^8+6201029366048a^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(674268049224a^{10}+16994547232a^{11}+1010111388a^{12}+13933216a^{13}+434808a^{14}+2464a^{15})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(33a^{16}-454441401368236800b+1601586452087647920ab-167129866463313600a^2b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(1145685449186532144a^3b+74389934082676608a^4b+109033109373003376a^5b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(5985820063371456a^6b+2594162140235248a^7b+97562518884864a^8b+19239357998608a^9b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(456852420288a^{10}b+46979218832a^{11}b+628138368a^{12}b+34705616a^{13}b+196416a^{14}b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(5456a^{15}b+421214220916438680b^2-624388051013229216ab^2+1565019021296531592a^2b^2)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-420728547803752322a^{3}b^{2}+3405596820512607 60a^{4}b^{2}+116444720986017 60a^{5}b^{2})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(14802425524183176a^{6}b^{2}+458691506376576a^{7}b^{2}+182692939157256a^{8}b^{2})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(3911767205280a^{9}b^{2}+717189756888a^{10}b^{2}+9049664832a^{11}b^{2}+860817672a^{12}b^{2})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(4746720a^{13}b^{2}+237336a^{14}b^{2}-215245451727154944b^{3}+672430138856900592ab^{3})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-206106535912708992a^{2}b^{3}+427559184167559840a^{3}b^{3}-3735455421134976a^{4}b^{3})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(36927624682661328a^{5}b^{3}+708321165615360a^{6}b^{3}+784325815238592a^{7}b^{3})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(13725326951424a^{8}b^{3}+4975885470864a^{9}b^{3}+56410020480a^{10}b^{3}+9488348064a^{11}b^{3})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(49365888a^{12}b^{3}+4272048a^{13}b^{3}+699531258931396644b^{4}-118482004312390688ab^{4})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(225271255690647672a^{2}b^{4}-24609722912109280a^{3}b^{4}+44357797204833156a^{4}b^{4})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-133812647496000a^{5}b^{4}+1713851879026320a^{6}b^{4}+18518957835840a^{7}b^{4})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(17944376279940a^{8}b^{4}+165209589600a^{9}b^{4}+54189742200a^{10}b^{4}+252439200a^{11}b^{4})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(38567100a^{1}2b^{4}-15611917903312640b^{5}+45551364598847984ab^{5}-18239227247368512a^{2}b^{5})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(26141054527921168a^{3}b^{5}-1251405780261120a^{4}b^{5}+2004216495193440a^{5}b^{5})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-1935009901440a^6b^5+35960462445600a^7b^5+204374776320a^8b^5+173969661360a^9b^5)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(655047360a^{10}b^5+193536720a^{11}b^5+25051858273878880b^6-4435348958817568ab^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(7239298092789576a^2b^6-1085549498794112a^3b^6+1266971455650384a^4b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-28680451208640a^5b^6+41314983328080a^6b^6-9170663040a^7b^6+328424370120a^8b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(764221920a^9b^6+573166440a^10b^6-2974030779939968b^7+824274801730864ab^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-361604047216896a^2b^7+422199872988608a^3b^7-27724725718272a^4b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(27382474864800a^5b^7-285600648960a^6b^7+372721947840a^7b^7+1037158320a^9b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(26569595376038b^8-47099407828704ab^8+69113934169896a^2b^8-11232092335200a^3b^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(10295243007588a^4b^8-298829757600a^5b^8+255386589480a^6b^8-982571040a^7b^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(1166803110a^8b^8-1801370405120b^9+4760242317072ab^9-2073715803456a^2b^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(2100369962064a^3b^9-138281447040a^4b^9+104344772400a^5b^9-1091745600a^6b^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(818809200a^7b^9+92751487400b^{10}-158593548256ab^{10}+213043702872a^2b^{10})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-31783404224a^3b^{10}+24624322008a^4b^{10}-555366240a^5b^{10}+354817320a^6b^{10})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-3600792832b^{11}+9023659216ab^{11}-3508382592a^{2}b^{11}+3163530656a^{3}b^{11}-148097664a^{4}b^{11})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(92561040a^{5}b^{11}+1036136920b^{12}-161683808ab^{12}+199059432a^{2}b^{12}-20569120a^{3}b^{12})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(13884156a^{4}b^{12}-2141440b^{13}+4998224ab^{13}-1374912a^{2}b^{13}+1107568a^{3}b^{13}+30040b^{14})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536b(-36960ab^{14}+40920a^{2}b^{14}-256b^{15}+528ab^{15}+b^{16})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}\Bigg\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{131072(191898783962510625+158998505377975200a+940304347040302200a^{2})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(-1348118704909310400a^{3}+2279740149755068444a^{4}-234631560712007360a^{5})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(10869072187812168a^{6}-717260777994464a^{7}+14906465524071 0a^{8}-6201029366048a^{9})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(674268049224a^{10}-16994547232a^{11}+1010111388a^{12}-13933216a^{13}+434808a^{14}-2464a^{15})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(33a^{16}+4544414013682368000b+1601586452087647920ab+167129866463313600a^{2}b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(1145685449186532144a^{3}b-743899340826766080a^{4}b+109033109373003376a^{5}b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(-5985820063371456a^{6}b+2594162140235248a^{7}b-97562518884864a^{8}b+19239357998608a^{9}b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(-456852420288a^{10}b+46979218832a^{11}b-628138368a^{12}b+34705616a^{13}b-196416a^{14}b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(5456a^{15}b+4212142209164386800b^{2}+624388051013229216ab^{2}+1565019021296531592a^{2}b^{2})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{131072(420728547803752\,32a^3b^2+3405596820512607\,60a^4b^2-1164447209860176\,0a^5b^2)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(148024255241831\,76a^6b^2-458691506376576a^7b^2+182692939157256a^8b^2)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(-3911767205280a^9b^2+717189756888a^{10}b^2-9049664832a^{11}b^2+860817672a^{12}b^2)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(-4746720a^{13}b^2+237336a^{14}b^2+2152454517271549\,44b^3+6724301388569005\,92ab^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(2061065359127089\,92a^2b^3+4275591841675598\,40a^3b^3+3735455421134976a^4b^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(369276246826613\,28a^5b^3-708321165615360a^6b^3+784325815238592a^7b^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(-13725326951424a^8b^3+4975885470864a^9b^3-56410020480a^{10}b^3+9488348064a^{11}b^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(-49365888a^{12}b^3+4272048a^{13}b^3+699531258931396\,44b^4+1184820043123906\,88ab^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(2252712556906476\,72a^2b^4+246097229121092\,80a^3b^4+4435779720483315\,6a^4b^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(133812647496000a^5b^4+1713851879026320a^6b^4-18518957835840a^7b^4+17944376279940a^8b^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(-165209589600a^9b^4+54189742200a^{10}b^4-252439200a^{11}b^4+38567100a^{12}b^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(1561191790331264\,0b^5+455513645988479\,84ab^5+1823922724736851\,2a^2b^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(261410545279211\,68a^3b^5+1251405780261120a^4b^5+2004216495193440a^5b^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

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$$+\frac{131072(1935009901440a^6b^5+35960462445600a^7b^5-204374776320a^8b^5+173969661360a^9b^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(-655047360a^{10}b^5+193536720a^{11}b^5+2505185827387880b^6+4435348958817568ab^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(7239298092789576a^2b^6+1085549498794112a^3b^6+1266971455650384a^4b^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(28680451208640a^5b^6+41314983328080a^6b^6+9170663040a^7b^6+328424370120a^8b^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(-764221920a^9b^6+573166440a^10b^6+2974030779399688b^7+8242748017308 64ab^7)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(361604047216896a^2b^7+422199872988608a^3b^7+27724725718272a^4b^7+27382474864800a^5b^7)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(285600648960a^6b^7+372721947840a^7b^7+1037158320a^9b^7+265695953760 38b^8)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(47099407828704ab^8+69113934169896a^2b^8+11232092335200a^3b^8+10295243007588a^4b^8)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(298829757600a^5b^8+255386589480a^6b^8+982571040a^7b^8+1166803110a^8b^8)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(18013704051 20b^9+4760242317072ab^9+2073715803456a^2b^9+2100369962064a^3b^9)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(138281447040a^4b^9+104344772400a^5b^9+1091745600a^6b^9+818809200a^7b^9+92751487400b^{10})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(158593548256ab^{10}+213043702872a^2b^{10}+31783404224a^3b^{10}+24624322008a^4b^{10})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(555366240a^5b^{10}+354817320a^6b^{10}+3600792832b^{11}+9023659216ab^{11}+3508382592a^2b^{11})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(3163530656a^3b^{11}+148097664a^4b^{11}+92561040a^5b^{11}+103613692b^{12}+161683808ab^{12})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(199059432a^2b^{12}+20569120a^3b^{12}+13884156a^4b^{12}+2141440b^{13}+4998224ab^{13})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(1374912a^2b^{13}+1107568a^3b^{13}+30040b^{14}+36960ab^{14}+40920a^2b^{14}+256b^{15}+528ab^{15}+b^{16})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{131072(191898783962510625+454441401368236800a+421214220916438680a^2)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(215245451727154944a^3+69953125893139644a^4+15611917903312640a^5)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(2505185827387880a^6+297403077939968a^7+26569595376038a^8+1801370405120a^9)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(92751487400a^{10}+3600792832a^{11}+103613692a^{12}+2141440a^{13}+30040a^{14}+256a^{15}+a^{16})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(1589985053779752200b+1601586452087647920ab+624388051013229216a^2b)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(672430138856900592a^3b+118482004312390688a^4b+45551364598847984a^5b)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(4435348958817568a^6b+824274801730864a^7b+47099407828704a^8b+4760242317072a^9b)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(158593548256a^{10}b+9023659216a^{11}b+161683808a^{12}b+4998224a^{13}b+36960a^{14}b+528a^{15}b)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(9403043470403022200b^2+1671298664633136000ab^2+1565019021296531592a^2b^2)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(2061065359127089992a^3b^2+225271255690647672a^4b^2+18239227247368512a^5b^2)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

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$$+\frac{131072(7239298092789576a^{6}b^{2}+361604047216896a^{7}b^{2}+69113934169896a^{8}b^{2}+2073715803456a^{9}b^{2})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(213043702872a^{10}b^{2}+3508382592a^{11}b^{2}+199059432a^{12}b^{2}+1374912a^{13}b^{2}+40920a^{14}b^{2})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(-1348118704909310040b^{3}+1145685449186532144ab^{3}+420728547803752 32a^{2}b^{3})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(427559184167559840a^{3}b^{3}+246097229121092 80a^{4}b^{3}+26141054527921168a^{5}b^{3})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(1085549498794112a^{6}b^{3}+422199872988608a^{7}b^{3}+11232092335200a^{8}b^{3}+2100369962064a^{9}b^{3})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(31783404224a^{10}b^{3}+3163530656a^{11}b^{3}+20569120a^{12}b^{3}+1107568a^{13}b^{3})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(227974014975506844b^{4}-743899340826766 08ab^{4}+340559682051260760a^{2}b^{4})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(3735455421134976a^{3}b^{4}+4435779720483315 6a^{4}b^{4}+1251405780261120a^{5}b^{4})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(1266971455650384a^{6}b^{4}+27724725718272a^{7}b^{4}+10295243007588a^{8}b^{4}+138281447040a^{9}b^{4})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(24624322008a^{1}0b^{4}+148097664a^{1}1b^{4}+13884156a^{1}2b^{4}-234631560712007 36b^{5})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(109033109373003376ab^{5}-116444720986017 60a^{2}b^{5}+36927624682661328a^{3}b^{5})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(133812647496000a^{4}b^{5}+2004216495193440a^{5}b^{5}+28680451208640a^{6}b^{5}+27382474864800a^{7}b^{5})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(298829757600a^{8}b^{5}+104344772400a^{9}b^{5}+555366240a^{10}b^{5}+92561040a^{11}b^{5})}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{131072(10869072187812168b^6-5985820063371456ab^6+1480242552418317 6a^2b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(-708321165615360a^3b^6+1713851879026320a^4b^6+1935009901440a^5b^6+41314983328080a^6b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(285600648960a^7b^6+255386589480a^8b^6+1091745600a^9b^6+354817320a^{10}b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(-7172607779944 64b^7+2594162140235248ab^7-458691506376576a^2b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(784325815238592a^3b^7-18518957835840a^4b^7+35960462445600a^5b^7+9170663040a^6b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(372721947840a^7b^7+982571040a^8b^7+818809200a^9b^7+149064655240710b^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(-97562518884864ab^8+182692939157256a^2b^8-13725326951424a^3b^8+17944376279940a^4b^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(-204374776320a^5b^8+328424370120a^6b^8+1166803110a^8b^8-6201029366048b^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(19239357998608ab^9-3911767205280a^2b^9+4975885470864a^3b^9-165209589600a^4b^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(173969661360a^5b^9-764221920a^6b^9+1037158320a^7b^9+674268049224b^{10})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(-456852420288ab^{10}+717189756888a^2b^{10}-56410020480a^3b^{10}+54189742200a^4b^{10})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(-655047360a^5b^{10}+573166440a^6b^{10}-16994547232b^{11}+46979218832ab^{11}-9049664832a^2b^{11})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(9488348064a^3b^{11}-252439200a^4b^{11}+193536720a^5b^{11}+1010111388b^{12}-628138368ab^{12})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(860817672a^2b^{12}-49365888a^3b^{12}+38567100a^4b^{12}-13933216b^{13}+34705616ab^{13})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{131072(-4746720a^2b^{13}+4272048a^3b^{13}+434808b^{14}-196416ab^{14}+237336a^2b^{14}-2464b^{15})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$\left.+\frac{131072(5456ab^{15}+33b^{16})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}\right] \tag{14}$$

III. Evaluation of Main Result

Substituting $c=\frac{a+b+35}{2}$ and $z=\frac{1}{2}$ in equation (8), we get

$$(a-b)\ {}_2F_1\left[\begin{matrix}a,\ b & ; & \frac{1}{2}\\ \frac{a+b+35}{2} & ; & \end{matrix}\right]=a\ {}_2F_1\left[\begin{matrix}a+1,\ b & ; & \frac{1}{2}\\ \frac{a+b+35}{2} & ; & \end{matrix}\right]-b\ {}_2F_1\left[\begin{matrix}a,\ b+1 & ; & \frac{1}{2}\\ \frac{a+b+35}{2} & ; & \end{matrix}\right]$$

Now involving derived result from Gauss second summation theorem, we get

$$L.H.S=a\,\frac{2^b\ \Gamma(\frac{a+b+35}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{65536(191898783962510625-454441401368236800a)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+\right.\right.$$

$$+\frac{65536(421214220916438680a^2-215245451727154944a^3+69953125893139644a^4)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-15611917903312640a^5+2505185827387880a^6-297403077939968a^7+26569595376038a^8)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-1801370405120a^9+92751487400a^{10}-3600792832a^{11}+103613692a^{12}-2141440a^{13})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(30040a^{14}-256a^{15}+a^{16}-158998505377975200b+160158645208764792 0ab)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-624388051013229216a^2b+672430138856900592a^3b-118482004312390688a^4b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(45551364598847984a^5b-4435348958817568a^6b+824274801730864a^7b-47099407828704a^8b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(4760242317072a^9b-158593548256a^{10}b+9023659216a^{11}b-161683808a^{12}b+4998224a^{13}b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-36960a^{14}b+528a^{15}b+9403043470403022000b^2-16712986646331360ab^2)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(1565019021296531592a^2b^2-206106535912708992a^3b^2+22527125569064767 2a^4b^2)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-18239227247368512a^5b^2+7239298092789576a^6b^2-361604047216896a^7b^2)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(69113934169896a^8b^2-2073715803456a^9b^2+213043702872a^{10}b^2-3508382592a^{11}b^2)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(199059432a^{12}b^2-1374912a^{13}b^2+40920a^{14}b^2+1348118704909310 40b^3)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(1145685449186532144ab^3-420728547803752 32a^2b^3+427559184167559840a^3b^3)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-24609722912109280a^4b^3+26141054527921168a^5b^3-1085549498794112a^6b^3)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(422199872988608a^7b^3-11232092335200a^8b^3+2100369962064a^9b^3-31783404224a^{10}b^3)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(30040a^{14}-256a^{15}+a^{16}-1589985053779752 00b+1601586452087647920ab)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-624388051013229216a^2b+672430138856900592a^3b-118482004312390688a^4b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(45551364598847984a^5b-4435348958817568a^6b+824274801730864a^7b-47099407828704a^8b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(4760242317072a^9b-158593548256a^{10}b+9023659216a^{11}b-161683808a^{12}b+4998224a^{13}b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-36960a^{14}b+528a^{15}b+9403043470403022000b^2-16712986646331360ab^2)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(1565019021296531592a^2b^2-2061065359127089992a^3b^2+2252712556906476672a^4b^2)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(-18239227247368512a^5b^2+7239298092789576a^6b^2-361604047216896a^7b^2)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(69113934169896a^8b^2-2073715803456a^9b^2+213043702872a^{10}b^2-3508382592a^{11}b^2)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(199059432a^{12}b^2-1374912a^{13}b^2+40920a^{14}b^2+1348118704909310400b^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(1145685449186532144ab^3-420728547803752232a^2b^3+427559184167559840a^3b^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(-246097229121092800a^4b^3+26141054527921168a^5b^3-1085549498794112a^6b^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(422199872988608a^7b^3-11232092335200a^8b^3+2100369962064a^9b^3-31783404224a^{10}b^3)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(3163530656a^{11}b^3-20569120a^{12}b^3+1107568a^{13}b^3+227974014975506844b^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(743899340826766608ab^4+340559682051260760a^2b^4-3735455421134976a^3b^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(44357797204833156a^4b^4-1251405780261120a^5b^4+1266971455650384a^6b^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(-27724725718272a^7b^4+10295243007588a^8b^4-138281447040a^9b^4+24624322008a^{10}b^4)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(-148097664a^{11}b^4+13884156a^{12}b^4+234631560712007366b^5+109033109373003376ab^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(116444720986017660a^2b^5+36927624682661328a^3b^5-133812647496000a^4b^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

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$$+\frac{65536(2004216495193440a^5b^5-28680451208640a^6b^5+27382474864800a^7b^5-298829757600a^8b^5)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(104344772400a^9b^5-555366240a^{10}b^5+92561040a^{11}b^5+1086907218781216 8b^6)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(5985820063371456ab^6+14802425524183176a^2b^6+708321165615360a^3b^6)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(1713851879026320a^4b^6-1935009901440a^5b^6+41314983328080a^6b^6-285600648960a^7b^6)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(255386589480a^8b^6-1091745600a^9b^6+354817320a^{10}b^6+7172607779944464b^7)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(2594162140235248ab^7+4586915063765 76a^2b^7+784325815238592a^3b^7+18518957835840a^4b^7)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(35960462445600a^5b^7-9170663040a^6b^7+372721947840a^7b^7-982571040a^8b^7+818809200a^9b^7)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(1490646552407106b^8+97562518884864ab^8+182692939157256a^2b^8+13725326951424a^3b^8)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(17944376279940a^4b^8+204374776320a^5b^8+328424370120a^6b^8+1166803110a^8b^8)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(62010293660048b^9+19239357998608ab^9+3911767205280a^2b^9+4975885470864a^3b^9)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(165209589600a^4b^9+173969661360a^5b^9+764221920a^6b^9+1037158320a^7b^9+674268049224b^{10})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(456852420288ab^{10}+717189756888a^2b^{10}+56410020480a^3b^{10}+54189742200a^4b^{10})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(655047360a^5b^{10}+573166440a^6b^{10}+16994547232b^{11}+46979218832ab^{11}+9049664832a^2b^{11})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(9488348064a^3b^{11}+252439200a^4b^{11}+193536720a^5b^{11}+1010111388b^{12}+628138368ab^{12})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(860817672a^2b^{12}+49365888a^3b^{12}+38567100a^4b^{12}+13933216b^{13}+34705616ab^{13})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(4746720a^2b^{13}+4272048a^3b^{13}+434808b^{14}+196416ab^{14}+237336a^2b^{14}+2464b^{15})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\left.\frac{65536(5456ab^{15}+33b^{16})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}\right\}-$$

$$-\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a+2}{2})}\left\{\frac{65536(191898783962510625+158998505377975200a+940304347040302200a^2)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}\right.$$

$$+\frac{65536(-1348118704909310400a^3+2279740149755068440a^4-234631560712007360a^5)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(108690721878121680a^6-7172607779944640a^7+149064655240710a^8-6201029366048a^9)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(674268049224a^{10}-16994547232a^{11}+1010111388a^{12}-13933216a^{13}+434808a^{14}-2464a^{15})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(33a^{16}+454441401368236800b+1601586452087647920ab+1671298664633136000a^2b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(1145685449186532144a^3b-743899340826766080a^4b+1090331093730033760a^5b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-59858200633714560a^6b+2594162140235248a^7b-97562518884864a^8b+19239357998608a^9b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(-456852420288a^{10}b+46979218832a^{11}b-628138368a^{12}b+34705616a^{13}b-196416a^{14}b)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(5456a^{15}b+4212142209164386800b^2+6243880510132292160ab^2+1565019021296531592a^2b^2)}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(42072854780375232a^{3}b^{2}+340559682051260760a^{4}b^{2}-116444720986017760a^{5}b^{2})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(14802425524183176a^{6}b^{2}-458691506376576a^{7}b^{2}+182692939157256a^{8}b^{2}-3911767205280a^{9}b^{2})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(717189756888a^{10}b^{2}-9049664832a^{11}b^{2}+860817672a^{12}b^{2}-4746720a^{13}b^{2}+237336a^{14}b^{2})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(215245451727154944b^{3}+672430138856900592ab^{3}+206106535912708992a^{2}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(427559184167559840a^{3}b^{3}+3735455421134976a^{4}b^{3}+36927624682661328a^{5}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(-708321165615360a^{6}b^{3}+784325815238592a^{7}b^{3}-13725326951424a^{8}b^{3}+4975885470864a^{9}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(-56410020480a^{10}b^{3}+9488348064a^{11}b^{3}-49365888a^{12}b^{3}+4272048a^{13}b^{3})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(699531258931139644b^{4}+118482004312390688ab^{4}+225271255690647672a^{2}b^{4})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(24609722912109280a^{3}b^{4}+44357797204833156a^{4}b^{4}+133812647496000a^{5}b^{4})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(1713851879026320a^{6}b^{4}-185189578355840a^{7}b^{4}+17944376279940a^{8}b^{4}-165209589600a^{9}b^{4})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(54189742200a^{10}b^{4}-252439200a^{11}b^{4}+38567100a^{12}b^{4}+15611917903312640b^{5})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(45551364598847984ab^{5}+18239227247368512a^{2}b^{5}+26141054527921168a^{3}b^{5})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(1251405780261120a^{4}b^{5}+2004216495193440a^{5}b^{5}+1935009901440a^{6}b^{5}+35960462445600a^{7}b^{5})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

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$$+\frac{65536(-204374776320a^8b^5+173969661360a^9b^5-655047360a^{10}b^5+193536720a^{11}b^5)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(2505185827387880b^6+4435348958817568ab^6+7239298092789576a^2b^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(1085549498794112a^3b^6+1266971455650384a^4b^6+28680451208640a^5b^6+41314983328080a^6b^6)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(9170663040a^7b^6+328424370120a^8b^6-764221920a^9b^6+573166440a^{10}b^6+2974030779399968b^7)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(824274801730864ab^7+361604047216896a^2b^7+422199872988608a^3b^7+27724725718272a^4b^7)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(27382474864800a^5b^7+285600648960a^6b^7+372721947840a^7b^7+1037158320a^9b^7)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(265695953760038b^8+470994078287 04ab^8+69113934169896a^2b^8+11232092335200a^3b^8)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(10295243007588a^4b^8+298829757600a^5b^8+255386589480a^6b^8+982571040a^7b^8)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(1166803110a^8b^8+1801370405120b^9+4760242317072ab^9+2073715803456a^2b^9)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(2100369962064a^3b^9+138281447040a^4b^9+104344772400a^5b^9+1091745600a^6b^9)}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(818809200a^7b^9+92751487400b^{10}+158593548256ab^{10}+213043702872a^2b^{10})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(31783404224a^3b^{10}+24624322008a^4b^{10}+555366240a^5b^{10}+354817320a^6b^{10}+3600792832b^{11})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(9023659216ab^{11}+3508382592a^2b^{11}+3163530656a^3b^{11}+148097664a^4b^{11}+92561040a^5b^{11})}{\left[\prod_{\varsigma=1}^{16}\left\{a-b-(2\varsigma-1)\right\}\right]\left[\prod_{\upsilon=1}^{17}\left\{a-b+(2\upsilon-1)\right\}\right]}+$$

$$+\frac{65536(103613692b^{12}+161683808ab^{12}+199059432a^{2}b^{12}+20569120a^{3}b^{12}+13884156a^{4}b^{12})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(2141440b^{13}+4998224ab^{13}+1374912a^{2}b^{13}+1107568a^{3}b^{13}+30040b^{14}+36960ab^{14})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}+$$

$$+\frac{65536(40920a^{2}b^{14}+256b^{15}+528ab^{15}+b^{16})}{\left[\prod_{\varsigma=1}^{16}\{a-b-(2\varsigma-1)\}\right]\left[\prod_{\upsilon=1}^{17}\{a-b+(2\upsilon-1)\}\right]}\Bigg\}\Bigg]-$$

$$-\frac{2^{b+1}\,\Gamma(\frac{a+b+35}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\Bigg\{\frac{65536(191898783962510625+454441401368236800a)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+\right.$$

$$+\frac{65536(421214220916438680a^{2}+215245451727154944a^{3}+69953125893139644a^{4})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(15611917903312640a^{5}+2505185827387880a^{6}+297403077939968a^{7}+26569595376038a^{8})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(1801370405120a^{9}+92751487400a^{10}+3600792832a^{11}+103613692a^{12}+2141440a^{13})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(30040a^{14}+256a^{15}+a^{16}+158998505377975200b+1601586452087647920ab)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(624388051013229216a^{2}b+672430138856900592a^{3}b+118482004312390688a^{4}b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(45551364598847984a^{5}b+4435348958817568a^{6}b+824274801730864a^{7}b+47099407828704a^{8}b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(4760242317072a^{9}b+158593548256a^{10}b+9023659216a^{11}b+161683808a^{12}b+4998224a^{13}b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(36960a^{14}b+528a^{15}b+940304347040302200b^{2}+1671298664633136600ab^{2})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(1565019021296531592a^{2}b^{2}+2061065359127089920a^{3}b^{2}+2252712556906476720a^{4}b^{2})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(2004216495193440a^5b^5+28680451208640a^6b^5+27382474864800a^7b^5+298829757600a^8b^5)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(104344772400a^9b^5+555366240a^10b^5+92561040a^11b^5+1086907218781216 8b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-5985820063371456ab^6+14802425524183176a^2b^6-708321165615360a^3b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(1713851879026320a^4b^6+1935009901440a^5b^6+41314983328080a^6b^6+285600648960a^7b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(255386589480a^8b^6+1091745600a^9b^6+354817320a^10b^6-7172607779944464b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(2594162140235248ab^7-458691506376576a^2b^7+784325815238592a^3b^7-18518957835840a^4b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(35960462445600a^5b^7+9170663040a^6b^7+372721947840a^7b^7+982571040a^8b^7+818809200a^9b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(1490646552407106^8-975625188848 64ab^8+182692939157256a^2b^8-13725326951424a^3b^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(17944376279940a^4b^8-204374776320a^5b^8+328424370120a^6b^8+1166803110a^8b^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-62010293660486^9+19239357998608ab^9-3911767205280a^2b^9+4975885470864a^3b^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-165209589600a^4b^9+173969661360a^5b^9-764221920a^6b^9+1037158320a^7b^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(674268049224b^{10}-456852420288ab^{10}+717189756888a^2b^{10}-56410020480a^3b^{10})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(54189742200a^4b^{10}-655047360a^5b^{10}+573166440a^6b^{10}-16994547232b^{11}+46979218832ab^{11})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-9049664832a^2b^{11}+9488348064a^3b^{11}-252439200a^4b^{11}+193536720a^5b^{11}+1010111388b^{12})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-628138368ab^{12}+860817672a^2b^{12}-49365888a^3b^{12}+38567100a^4b^{12}-13933216b^{13})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(34705616ab^{13}-4746720a^2b^{13}+4272048a^3b^{13}+434808b^{14}-196416ab^{14}+237336a^2b^{14})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$\left.+\frac{65536(-2464b^{15}+5456ab^{15}+33b^{16})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}\right\}-$$

$$-\frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\left\{\frac{65536(1918987839625 10625-1589985053779752 00a+9403043470403022 00a^2)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}\right.$$

$$+\frac{65536(134811870490931040a^3+227974014975506844a^4+23463156071200736a^5)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(1086907218781216 8a^6+717260777994464a^7+149064655240710a^8+6201029366048a^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(674268049224a^{10}+16994547232a^{11}+1010111388a^{12}+13933216a^{13}+434808a^{14}+2464a^{15})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(33a^{16}-4544414013682368 00b+1601586452087647920ab-1671298664633136 00a^2b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(1145685449186532144a^3b+743899340826766 08a^4b+109033109373003376a^5b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(5985820063371456a^6b+2594162140235248a^7b+97562518884864a^8b+1923935799860 8a^9b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(456852420288a^{10}b+46979218832a^{11}b+628138368a^{12}b+34705616a^{13}b+196416a^{14}b)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(5456a^{15}b+4212142209164386 80b^2-624388051013229216ab^2+1565019021296531592a^2b^2)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-420728547803752 32a^3b^2+340559682051260760a^4b^2+116444720986017 60a^5b^2)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(14802425524183176a^6b^2+458691506376576a^7b^2+182692939157256a^8b^2+3911767205280a^9b^2)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(717189756888a^{10}b^2+9049664832a^{11}b^2+860817672a^{12}b^2+4746720a^{13}b^2+237336a^{14}b^2)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(-2152454517271549 44b^3+672430138856900592ab^3-206106535912708992a^2b^3)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(42755918416755984 0a^3b^3-3735455421134976a^4b^3+3692762468266132 8a^5b^3)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(708321165615360a^6b^3+784325815238592a^7b^3+13725326951424a^8b^3+4975885470864a^9b^3)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(56410020480a^{10}b^3+9488348064a^{11}b^3+49365888a^{12}b^3+4272048a^{13}b^3)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(69953125893139644b^4-118482004312390688ab^4+22527125569064767 2a^2b^4)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(-24609722912109280a^3b^4+4435779720483315 6a^4b^4-133812647496000a^5b^4)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(1713851879026320a^6b^4+18518957835840a^7b^4+17944376279940a^8b^4+165209589600a^9b^4)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(54189742200a^{10}b^4+252439200a^{11}b^4+38567100a^{12}b^4-1561191790331264 0b^5)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(45551364598847984ab^5-18239227247368512a^2b^5+26141054527921168a^3b^5)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(-1251405780261120a^4b^5+2004216495193440a^5b^5-1935009901440a^6b^5)}{\left[\prod_{\varphi=1}^{17}\left\{a-b-(2\varphi-1)\right\}\right]\left[\prod_{\omega=1}^{16}\left\{a-b+(2\omega-1)\right\}\right]}+$$

$$+\frac{65536(35960462445600a^7b^5+204374776320a^8b^5+173969661360a^9b^5+655047360a^{10}b^5)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(193536720a^{11}b^5+2505185827387880b^6-4435348958817568ab^6+7239298092789576a^2b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-1085549498794112a^3b^6+1266971455650384a^4b^6-28680451208640a^5b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(41314983328080a^6b^6-9170663040a^7b^6+328424370120a^8b^6+764221920a^9b^6)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(573166440a^{10}b^6-297403077939968b^7+824274801730864ab^7-361604047216896a^2b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(422199872988608a^3b^7-27724725718272a^4b^7+27382474864800a^5b^7-285600648960a^6b^7)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(372721947840a^7b^7+1037158320a^9b^7+26569595376038b^8-47099407828704ab^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(69113934169896a^2b^8-11232092335200a^3b^8+10295243007588a^4b^8-298829757600a^5b^8)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(255386589480a^6b^8-982571040a^7b^8+1166803110a^8b^8-1801370405120b^9+4760242317072ab^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-2073715803456a^2b^9+2100369962064a^3b^9-138281447040a^4b^9+104344772400a^5b^9)}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-1091745600a^6b^9+818809200a^7b^9+92751487400b^{10}-158593548256ab^{10})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(213043702872a^2b^{10}-31783404224a^3b^{10}+24624322008a^4b^{10}-555366240a^5b^{10})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(354817320a^6b^{10}-3600792832b^{11}+9023659216ab^{11}-3508382592a^2b^{11}+3163530656a^3b^{11})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-148097664a^4b^{11}+92561040a^5b^{11}+103613692b^{12}-161683808ab^{12}+199059432a^2b^{12})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$+\frac{65536(-20569120a^3b^{12}+13884156a^4b^{12}-2141440b^{13}+4998224ab^{13}-1374912a^2b^{13}+1107568a^3b^{13})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}+$$

$$\left.\left.+\frac{65536(30040b^{14}-36960ab^{14}+40920a^2b^{14}-256b^{15}+528ab^{15}+b^{16})}{\left[\prod_{\varphi=1}^{17}\{a-b-(2\varphi-1)\}\right]\left[\prod_{\omega=1}^{16}\{a-b+(2\omega-1)\}\right]}\right\}\right]$$

After simplifing the result (14) is proved.

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Biostatistics : A Probable Integration of Mathematics in Biological Systems. *An Aided Philosophical Account*

By Rishan Singh
University of Kwa Zulu-Natal, South Africa

Abstract - The information supplied below is a philosophical account of the importance of statistics in biology. It is aimed at learners, who aspire to further their careers in biology through research. This paper brings recognition to UKZN and the Republic of South Africa.

Keywords : Probability, Statistics, Bioprobability, Biomathematics.

GJSFR-F Classification MSC 2010: 92B15, 62P10.

BIOSTATISTICS A PROBABLE INTEGRATION OF MATHEMATICS IN BIOLOGICAL SYSTEMS. AN AIDED PHILOSOPHICAL ACCOUNT

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I. Introduction

On a daily basis, a life scientist would encounter times when the only way of understanding important scientific data that is used to write scientific papers, would be through the use of statistics. Therefore, statistics is, as stated by Neuhauser (2004), an indispensible tool for life scientists. However, although one may think that statistics is merely mathematics, in actual fact it is not. By philosophical definition, statistics is a branch of mathematics and it is not confined to its own usage. By this, I mean that statistics is often associated with probability. The implications of this is that we would, therefore, be faced with the possibility of 'guess-work', in order to solve simple biological problems.

Statistics and therefore probability are both reliant on calculus, even though they are not a part of calculus. Therefore, biological systems would have to incorporate such methods in scientific tests so as to test the validity of the results. And because, biostatistics would be testing validity, we can say that it is an objective tool for problem solvers.

II. Statistics Verses Probability

It is possible to separate statistics and probability when defining its usage. The reason for this has to do with the fact that they are both reliant on calculus, as mentioned, and therefore the principle behind the exploitation of these techniques are essentially the same. They can however, be separated on the basis of the type of data that is used and the results that are obtained.

It is important to understand that in biological systems, the sampling of data has to occur randomly for zero bias to be available in the results. The idea of randomness would ensure that no single variable would be favoured, knowingly. Moreover, this would ensure that the results take into account all the possible outcomes, all of which are uncertain but can be repeated. If this rule were violated in any way, the results that would be obtained, without statistical considerations, would be subjective, making the

Author : University of Kwa Zulu-Natal, South Africa. E-Mail : rshnsingh1@webmail.co.za

point of the investigation – pointless. Therefore, during statistical calculations, the probability theory, which forms the foundation of statistics, provides the essential tools for randomness to be modelled. Furthermore, it is for this reason that biologists turn to statistics prior to the design of experiments, and subsequently, the setting up of hypotheses to be tested.

However, statistics can be a very 'heavy' tool to employ to determine the objectivity of results obtained from non-deterministic phenomena. One of the main reasons for this is because statistics needs a huge sample set of data in order to be feasible. Some examples of such phenomena, as stated by Neuhauser (2004) are 'the number of eggs laid by a bird, the lifespan of an organism, the inheritance of genes' and even 'the number of people infected during an outbreak of a disease.'

At this point of the article, I would explain the importance of statistics by using bioprobability as an example.

Suppose a black dog (B) mates with a white female dog (W). After one mating (of adult dogs), the possible outcomes of the F_1 offspring are B and W and therefore the sample space can be defined as:

$$\Omega = \{B, W\}$$

However, suppose the F1 generation, which are all fertile grow to reach reproductive age, mate to produce the F2 generation of offspring, the outcome of the first mating followed by the outcome of the second mating, could be **BW**, which essentially brown male(s) and/or female(s). Now since,

Parents mate : $B \times W$
F1 generation : $BW \times BW$ (fertile)
F2 generation : $BB\ BW\ WB\ WW$

Therefore, the sample size is now given by: $\Omega = \{BB, BW, WB, WW\}$

The case of Fatality – the Rb genes – tumour suppressor genes!

The first case of these genes were identified in a type of eye cancer, called hereditary retinoblastoma, which occurs in young children due to family inheritance of defective gene/disease - children who inherit a deletion in a specific region of chromosome 13. However, the disease only becomes distinct when cell division of defective chromosome 13 occurs to form a duplicate of itself.

Assuming the defective gene is indicated by Rb, and the genetic disorder is due to a X-linked recessive inheritance with the mother being the carrier on one of her X-chromosomes (recessive allele), then:

Carrier mother	Normal father
RbX	**X**Y

(Meiosis, gamete formation)

Rb eggs X	**X** sperm Y

Possible outcome :

XX (Normal Daughter)	XY (Normal Son)
RbX (Carrier Daughter)	RbY (Affected Son)

Notes

Now, assume that **X**, i.e. both mother and father are carriers, taking into account the maternal contributor first and then the paternal consideration. The following probable outcomes would be applicable in the situation:

$$\Omega = \{(Rb, Rb), (Rb, Y), (Rb, X), (X, Y)\}$$

Notes

According to Mendel's Law of Inheritance, since gametes form at random, all possible outcomes of **Ω** are likely to occur equally. Now since, there are four possible outcomes, the absolute sample space i.e. $|\Omega| = 4$ and the probability of each outcome would be 25 % or ¼.

It is important to note the difference between this example and the pea situation with which Mendel experimented. In Mendel's experiments with peas, there were 4 possible outcomes, but essentially only 3 genotypes. In this case there are, however, four genotypes with four possible outcomes.

Therefore,

P(RbRb) = ¼
P(RbY) = ¼
P(RbX) = ¼
P(XY) = ¼

a) *What is the probability that a child not having the genetic disorder be born?*

Since only one genotype would result in a perfectly normal child (son), it follows that,

$$P(\text{normal}) = P(X,Y) = 1/4$$

Hence, the probability of a child born, developing a deletion and duplication of it later in age would be:

$$P(\text{disorder}) = \{(Rb,Rb), (Rb,Y), (Rb,X)\}$$
$$= 3/4$$

The Mark Recapture Method and Probability

A good way of illustrating a probability example of greater complexity is to explain in the lights of the mark recapture technique. The mark recapture technique is often used by zoologists and environmental conservationists so as to estimate the size of a population and in certain cases; this technique is also used when the aim of the conservationist is to conserve genetic diversity.

I would now illustrate this method by using an example of introducing a new population of peppered moths (*Biston betulana*) into an existing population. Suppose M moths, where M is unknown, are present in the area of interest. In order to evaluate the number of M or how large the M population is, the moths are captured and marked/painted (O), by using paint, for example, that would not cause any harm to the moths, and then released back into the population, subsequently. Once released, the moths are allowed to interact and mix with those moths that had already pre-existed before their introduction. This essentially means that the sample size is larger than expected. *m* moths are thereafter captured. Suppose that *m* of the *n* moths are marked, while assuming that *m ¿ 0*, and then released to mix with the population again, the ratio of marked to unmarked moths in the sample size, *n* should be approximately equal to the ratio of marked is to unmarked moths in the area i.e.,

$$\frac{m}{n} = \frac{O}{M}$$

Therefore the size of M is given by,

$$M = O_{n/m}$$

moths in the area.

a) What is the probability of finding m marked moths in a sample of size n?

There are M moths in the area, O of which are marked. Choosing n as the sample size, each outcome becomes a subset of n, which are equally likely.

$$|\Omega| = \binom{M}{n}$$

Let R denote the event that the sample of size n contains exactly m marked moths. Select m moths from M marked moths and, n-m moths from M-O unmarked one. Selecting the m marked moths can be done in $\binom{O}{m}$ ways.

Similarly, n-m unmarked moths may be selected in $\binom{O-M}{n-m}$ ways.

The multiplication principle can be used to find the total number of ways of obtaining a sample size of n with exactly m marked moths. This is because each choice of m marked moths can be combined with any choice of the n-m unmarked moths. Therefore,

$$|R| = \binom{O}{m}\binom{M-O}{n-m}$$

Thus, $P(R) = \frac{|R|}{|\Omega|}$

$$= \frac{\binom{O}{m}\binom{M-O}{n-m}}{\binom{M}{n}}$$

b) Why can the total number of moths in the area be estimated using the formula $M = O\ ^{n}/_{m}$?

Obtaining a sample of size n and observing m marked moths in the area, it is possible to show that the value of M that maximises the probability of finding m marked moths in a sample size n is the largest integer less than or equal to $M = O\ ^{n}/_{m}$. This is used as the estimate for the population size M. This is referred to as the maximum likelihood estimate, because its objective is to maximise the probability of what is observed.

From, $P(R) = \frac{|R|}{|\Omega|}$

$$= \frac{\binom{O}{m}\binom{M-O}{n-m}}{\binom{M}{n}}$$

Notes

Now, considering that P(R) is a function of M, it is possible to express this relationship as p_M. In order to evaluate the value of M that maximises P_M, the ratio of p_M / $p_{M-1.}$ Hence, p_M cannot be differentiated to find its maximum since it is not continuous and can only be defined by integer values of M, the ratio is stated as follows:

Notes

$$\frac{p_M}{p_{M-1}} = \frac{\dfrac{\binom{O}{m}\binom{M-O}{n-m}}{\binom{M}{n}}}{\dfrac{\binom{O}{m}\binom{M-1-O}{n-m}}{\binom{M-1}{n}}}$$

$$\frac{p_M}{p_{M-1}} = \frac{\binom{M-O}{n-m}\binom{M-1}{n}}{\binom{M-1-O}{n-m}\binom{M}{n}}$$

$$= \frac{(M\text{-}O)!(n\text{-}m)!(M\text{-}1\text{-}O\text{-}n+m)!n!(M\text{-}n)!}{(n\text{-}m)!(M\text{-}O\text{-}n+m)!(M\text{-}1\text{-}O)!n!(M\text{-}1\text{-}n)!M}$$

Cancellation gives,

$$\frac{p_M}{p_{M-1}} = \frac{M-O}{M-O-n+m} \quad \frac{M-n}{M}$$

In order to find the values of M at which p_M exceeds p_{M_1}, it is important to obtain the ratio of p_M/p_{M-1} is greater than or equal to. This is because the local maxima are the values of M at which p_M exceeds both p_{M-1} and p_M.

When,

$(M-O)(M-n) \geq M(M-O-n+m)$, the ratio of p_M / p_{M-1} is greater than or equal to . Factorising this equation, we find that

$$M^2 - Mn - OM + Mn \geq M^2 - MO - Mn + Mm$$

Simplifying yields,

$$On \geq m\,M \text{ or/ } M \leq O\ {}^{n}/_{m}$$

As long as $M \leq O\ {}^{n}/_{m}$, $p_M \geq p_{M-1}$. If $M \leq O\ {}^{n}/_{m}$ is an integer, then $p_M = p_{M-1}$ for $M = O\ {}^{n}/_{m}$ and both $O\ {}^{n}/_{m}$ and $O\ {}^{n}/_{m-1}$ maximises the probability of observing m moths in the sample size n and so both values can be chosen as estimates for the number of moths in the area. If $O\ {}^{n}/_{m}$ is not an integer less than $O\ {}^{n}/_{m}$ maximises the probability, p_M . To arrive at just one value, we will always use the largest integer less than or equal to $O\ {}^{n}/_{m}$ to estimate the total number of moths in the area.

c) Assume that there are 32 marked moths in the area. We take a sample of size 20 and observe 10 marked moths. From this data and the information gathered from (b), it is possible to find an estimate of the number of moths in the area.

The estimate of the number of moths in the area is denoted by M, which is the largest integer less than or equal to $O\ ^n/_m$, where $O = 32$, $n = 20$, $m = 10$,

Since $O\ ^n/_m = 32 \times 20/10 = 64$,

It can be estimated that there are 64 moths in the area.

Notes

III. Potential Impacts of Statistics for Future Biologists

The merger of mathematics and life sciences is possible through employing techniques (such as statistics) that can be used to explain phenomena that cannot be seen, generally, through the naked eye. The marriage of these two sciences has enabled the evaluation of a number of facets. Some of these facets have been explained in the examples that are illustrated in this article, for example; it is possible to make an estimation of the number of organisms occupying a particular area, even when an ecological niche has been filled to capacity on introduction of new/different species of the same kind of organisms. Also it is able to estimate the outcome of offspring percentages of normal and defective origin that is unknown. Therefore, the idea of statistics can be exploited as an invaluable tool to estimate lineages that constitute ancestral and descendant traits in evolutionary context, as well. This bridges the gap between statistics and evolutionary biology.

One of the major advantages for understanding the workings of statistics, is that it would provide life scientists with the depth and knowledge to make suitable deductions by knowing the mathematics involved and the terminology associated with it. It would, more importantly, enable scientists to point out important concepts of biology nature through a mathematical medium.

Most scientists often associate biological concepts in isolation. As in the example of the male and female dog, probability deductions of biological nature can be deduced by simply looking at the answers of the F_2 generation. However, when one looks at mathematical probability, this would also be possible, but in this case the sample size would be expressed in the answer. A biologist, on the basis of looking at the total number of offspring and then distinguishing between traits to get ratios, would in most cases miss out 'sample size' in the final answer. Although mathematics and biology are at two different ends of the scientific spectrum, statistics could provide the language to bridge the link between the two sciences. By doing this, biologists, would be able to critically evaluate their findings and express them biomathematically.

Statistics is therefore an objective tool to both mathematicians and other scientists, its aims are to ultimately eliminate subjectivity and to focus on reliable results. Furthermore, this means that our daily operations are confined to mathematics because we are surrounded by biology, which depends on mathematical deductions.

The major concluding remark is that statistics should not be viewed as an effort in identifying unidentifiable problems. Instead, students should be objective in selecting career opportunities that explore a variety of avenues; and by understanding statistics, it would make this exploration an impressive endeavour!

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A Comparison of Adomian's Decomposition Method and Picard Iterations Method in Solving Nonlinear Differential Equations

By Ibijola E.A & Adegboyegun B.J

Ekiti State University, Nigeria

Abstract - In this paper,nonlinear differential equations are solved through Adomian decomposition method(ADM) and the results are compared with those of Picard iterations method. It is noted that ADM takes the form of a convergent series with easily computable components. The ADM method is able to solve large class of nonlinear equations effectively, more easily and accurately; and thus the method has been widely applicable to solve any class of equations in sciences and engineering.

Keywords : Picard iteration, Adomian decomposition, Nonlinear differential equation, Volteral integral equation of first kind.

GJSFR-F Classification MSC 2010: 34L30.

A COMPARISON OF ADOMIANSDECOMPOSITION METHOD AND PICARD ITERATIONS METHOD IN SOLVING NONLINEAR DIFFERENTIAL EQUATIONS

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Ibijola E.A[α] & Adegboyegun B.J[σ]

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Keywords : Picard iteration, Adomian decomposition, Nonlinear differential equation, Volteral integral equation of first kind.

I. Introduction

Large classes of linear differential equations can be solved by the Adomian decomposition method $[1,2,3,5]$ and Picard iteration method $[9]$. The ADM was first compared with the Picard method by Rach $[4]$ and Bellomo and Sarafyan $[6]$ on a number of examples.Golberg$[7]$ showed that the Adomian approach to llinear differential equations was equivalent to the classical method of successive approximations (Picard method). However,we shall show that the equivalence does not hold for nonlinear differential equations. We consider the generalized first order nonlinear differential equations in the form:

$$y' = f(t, y), \quad y \in R^d, \quad f : R \text{x} R^d \to R^d, \qquad 1.1$$

With initial condition $y(0) = y_0 \in R^d$. We shall proceed to discuss the basic concepts of the two methods.

II. Derivation of The Methods

In reviewing the basic methodology, we consider an abstract system of differential equation (1.1) and assume that $f(t, y)$ is nonlinear and analytic near $y = y_0, t = 0$. It is equivalent to solve the initial value problem (1.1) and the Voltera integral equation of first kind,

$$y(t) = y_0 = \int_0^t f(s, y(s))\, ds \qquad 1.2$$

To set up the Adomian method, Rach [4] write (1.1) in an operator form:

$$Ly = f(t, y), \qquad 1.3$$

Author α σ: Department of Mathematical Sciences, Ekiti State University, Nigeria. E-mail : bolucrown2000@yahoo.com

Ref.

1. Ibijola E.A and Adegboyegun B.J on AdomianDecompositoon Method (ADM) for Numerical Solution of Ordinary Differential Equation. Advances in Natural Applied Sciences, 2(3): 165 – 169, 2008 U.S

Where $L = \frac{d}{dt}$.He applied L^{-1} (a one fold integral) on (1.3) and imposed the initial condition to obtain:

$$y(t) = y_0 + L^{-1} f(t,y)$$

Adomian considered the solution $y(t)$ in the series form:

$$y(t) = y_0 + \sum_{n=1}^{\infty} y_n \qquad 1.4$$

And write the nonlinear $f(t,y)$ as the series of function

$$f(t,y) = \sum_{n=0}^{\infty} A_n(t, y_0, y_1, \cdots y_n) \qquad 1.5$$

The dependence of A_n on t and y_0 may be non-polynomial. Formally, A_n is obtained by

$$A_n = \frac{1}{n!} \frac{d^n}{d\ell^n} f(t, \sum_{i=0}^{\infty} \ell^i y_i)|_{\ell=0}, \quad n = 0,1,2\cdots \qquad 1.6$$

Where ℓ is a formal parameter. Functions A_n are polynomials in $(y_1, y_2, \cdots y_n)$, which are referred to as the Adomian polynomials. The first few Adomian for $d=1$ are listed by Zhu and Chang [8] as follows:

$$A_0 = f(t, y_0)$$

$$A_1 = y_1 f'(t, y_0)$$

$$A_2 = y_2 f'(t, y_0) + \frac{1}{2} y_1^2 f''(t, y_0)$$

$$A_3 = y_3 f'(t, y_0) + y_1 y_2 f''(t, y_0) + \frac{1}{6} y_1^3 f'''(t, y_0)$$

where primes denote the partial derivatives with respect to y.

It was shown by Cherruault and Abbaoui [3] that the Adomian polynomials A_n are defined by the explicit formulae:

$$A_n = \sum_{k=1}^{n} \frac{1}{k!} f^k(t, y_0) \left(\sum_{p_1 + \cdots + p_k = n} y_{p_1} \cdots y_{p_k} \right), \quad n \geq 1 \qquad 1.7$$

Khelifa and Cherruault [8] proved a bound for Adomian polynomials by,

$$|A_n| \leq \frac{(n+1)^n}{(n+1)!} M^{n+1} \qquad 1.8$$

Where $\sup_{t \in J} | f^{(k)}(t, y_0)| \leq M$ for a given time interval $J \subset R$.

By substituting (1.4) and (1.5) into (1.2) gives a recursive equation for y_{n+1} in terms of $(y_0, y_1, y_2, \cdots y_n)$:

$$y_{n+1} = \int_0^t A_n(s, y_0(s), y_1(s), \cdots y_n(s)) ds, \quad n = 0,1,2,\cdots \qquad 1.9$$

The Picard's method is used for the proof of existence and uniqueness of solutions of a system of differential equations. The method starts with analysis of Volterra's integral equation (1.2). Assume that $f(t,y)$ satisfies a local Lipschitz condition in a ball around

Ref.

8. Y. Zhu, Q. Chang, and S. Wu, "A new algorithm for calculating Adomian Polynomials", Appl. Math. Comput., 169, 402-416 (2005).
3. N. Himoun, K. Abbaoui, and Y. Cherruault, "Short new results on Adomian method",Kybernetes, 32, 523-539 (2003).

$t=0$ and $y=y_0$:

$$\forall |t| \le t_0,\ \forall |\tilde{y}-y_0| \le \delta_0 : \quad |f(t,y)-f(t,\tilde{y})| \le K|y-\tilde{y}|, \qquad 1.10$$

Where K is lipshitz constant and $|y|$ is any norm in R^d

Let $y^{(0)}=y_0$ and define a recurrence relation

$$y^{(n+1)}(t) = y_0 + \int_0^t f(s, y^n(s))\, ds, \qquad n=0,1,2,\cdots \qquad 1.11$$

If t_0 is small enough, the new approximation $y^{(n+1)}(t)$ belongs to the same ball $|y-y_0| \le \delta_0$ for all $|t| \le t_0$ and (1.11) is a contraction in the sense that

$$\left| \int_0^t [f(s,y(s)) - f(s,\tilde{y}(s))]\, ds \right| \le Q \sup_{|t|\le t_0} |y(t)-\tilde{y}(t)|, \qquad 1.12$$

Where $Q=Kt_0<1$, so that $t_0 < \frac{1}{k}$.

By the Banach fixed point theorem, there exists a unique solution $y(t)$ in $C([-t_0,t_0], B_{\delta_0}(y_0))$ where $B_{\delta_0}(y_0)$ is an open ball in R^d centered at y_0 with radius δ_0. The computations of Picard iterative algorithm were reported recently by Youssef [9].

III. Applications and Results

Since the problems given in the article at reference [9] on the computations and analysis of Picard's iterative scheme are relevant for our aim, we reuse some of the problems.

Problem 1

Consider a scalar first order ODE,

$$\frac{dy}{dt} = y^p, \quad y(0)=1 \qquad 1.13$$

where $p \ge 1$. This differential equation has exact solution;

$$y(t) = \frac{1}{(1-(p-1)t)^{\frac{1}{p-1}}} \qquad 1.14$$

Following the Adomian method, we write (1.13) as $y(t) = 1 + \int_0^t y^p(s)\, ds$ and compute the Adomian polynomials from $f(t,y)=y^p$ in the form:

$$A_0 = y_0^p,$$

$$A_1 = p y_0^{p-1} y_1,$$

$$A_2 = \frac{p(p-1)}{2} y_0^{p-2} y_1^2 + p y_0^{p-1} y_2,$$

$$A_3 = \frac{p(p-1)(p-2)}{6} y_0^{p-3} y_1^3 + p(p-1) y_0^{p-2} y_1 y_2 + p y_0^{p-1} y_3,$$

Ref.

9. I. K. Youssef, "Picard iteration algorithm combined with Gauss-Seidel technique for initial value problems", Applied mathematics and computation, 190, 345-355 (2007).

Using (1.9), we determine few terms of the Adomian series:

$y_0(t) = 1$

$y_1(t) = t$

$y_2(t) = \frac{pt^2}{2}$

$y_3(t) = \frac{p(2p-1)t^3}{3!}$

$y_4(t) = \frac{p(6p^2-7p+2)t^4}{4!}$

From (1.4)

$$y(t) = y_0 + y_1 + y_2 + y_3 + y_4 + \cdots = 1 + t + \frac{pt^2}{2} + \frac{p(2p-1)t^3}{3!} + \frac{p(6p^2-7p+2)t^4}{4!} + \cdots$$

Expanding (1.13) in a power series of t, we can see that theAdomian method recovers the power series solution $y(t) = \frac{1}{(p-(p-1)t)^{\frac{1}{p-1}}}$

$$= 1 + t + \frac{pt^2}{2} + \frac{p(2p-1)t^3}{3!} + \frac{p(6p^2-7p+2)t^4}{4!} + 0(t^5)$$

Using the Picard iterations method,

$$y^{(n+1)} = 1 + \int_0^t (y^{(n)}(s))^p \, ds,$$

We obtain successive approximations in the form:

$y^{(0)} = 1$

$y^{(1)} = 1 + t$

$y^{(2)} = \frac{p}{1+p} + \frac{(1+t)^{1+p}}{1+p}$

$y^{(3)} = 1 + \int_0^t (y^{(2)})^p ds$

It is noted from $y^{(2)}$ that Picard iteration mix up powers of t which make $y^{(n)}$ being different from the nth partial sum of the power series.If $p = 2$, then

$y^{(0)} = 1 = y_0$,

$y^{(1)} = 1 + t = y_0 + y_1$,

$y^{(2)} = 1 + t + t^2 + \frac{t^3}{3} = y_0 + y_1 + y_2 + \frac{t^3}{3}$,

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Since $\sum_{i=0}^{n} y_i(t)$ is a partial sum of the power series (1.13), we conclude that the Adomian method better approximates the exact power series solution compared to the Picard method.

Problem 2

Consider the nonlinear differential equation,

$$\frac{dy}{dt} = 2y - y^2, \quad y(0) = 1 \qquad 1.15$$

with the exact solution $y(t) = 1 + \tanh(t)$

By the ADM, we write (1.15) in the integral form $y(t) = 1 + \int_0^t \left(2y(s) - y^2(s)\right) ds$ and compute the Adomian polynomials for $f(y) = 2y - y^2$ in the form;

$A_0 = 2y_0 - y_0^2$,

$A_1 = 2y_1 - 2y_0 y_1$,

$A_2 = 2y_2 - 2y_0 y_2 - y_1^2$,

$A_3 = 2y_3 - 2(y_0 y_3 + y_1 y_2)$,

$A_4 = 2y_4 - 2(y_0 y_4 + y_1 y_3) - y_2^2$.

Using (1.9) we determine few terms of the series;

$$y_0 = 1; \quad y_1 = t; \quad y_2 = 0; \quad y_3 = \frac{-t^3}{3}; \quad y_4 = 0; \quad y_5 = \frac{2t^5}{15}.$$

Thus, $y(t) = 1 + t - \frac{t^3}{3} + \frac{2t^5}{15} - \cdots = 1 + \tanh(t)$

By using Picard method, we write (1.15) in the integral form, $y^{n+1} = 1 + \int_0^t \left(2y^{(n)}(s) - (y^{(n)}(s))^2\right) d$ and we obtain the successive approximations in the form;

$y^{(0)} = 1 = y_0$,

$y^{(1)} = 1 + t = y_0 + y_1$,

$y^{(2)} = 1 + t - \frac{t^3}{3} = y_0 + y_1 + y_2 - \frac{t^3}{3}$,

$y^{(3)} = 1 + t - \frac{t^3}{3} + \frac{2t^5}{15} - \frac{t^7}{63} = y_0 + y_1 + y_2 + y_3 + \frac{2t^5}{15} - \frac{t^7}{63}$

We can equally see from $y^{(2)}$ and $y^{(3)}$ that Picard method mix up powers of t which also make $y^{(n)}$ being different from the nth partial sum of the series.

IV. Conclusion and Discussion

In this paper, ADM has been successfully applied to finding the solutions of nonlinear ODE. The obtained results are compared with those of Picard iterations method. It is noted that the Picard method mixes up powers of the partial sum for the exact solutions, while the Adomian series is equivalent to the power series in time and Adomian method requires analyticity of $f(t, y)$,which is more restrictive than the Lipschitz condition required for the Picard method.The results show that ADM is a powerful mathematical tool for solving nonlinear differential equations, and therefore, can be widely applied in the field of science and engineering.

Notes

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Dielectric Properties of Order-Disorder Type Crystals

By V. S. Bist & N. S. Panwar
HNB Garhwal University Srinagar, Uttrakhand, India

Abstract - The soft mode dynamical model has been used to study the dielectric properties of order-disorder-type crystals. Using the model Hamiltonian proposed by Blinc [Advances in Phys, 29 (1972) 701] and has been modified by Bist et al [GJSFR,10,18(2010)], the expressions for the dielectric constant and tangent loss have been derived and discussed for order-disorder, KH2PO4 type-crystals with the help of double time temperature dependent Green's function techniques and Dyson's equation treatment. Using appropriate parameters given by Ganguli et al [Phys Rev. B21, 2937 (1980)] the transverse dielectric constant and observed dielectric constant have been calculated and compared with experimental results of Raman Intensity[Ferroelectrics 52, 91 (1983)], Busch [Helv. Phys. Acta. 11, 265 (1938)], Kaminow et al [Phys. Rev. 138A, 1539 (1963)], and Deguchi et al [J. Phys. Soc. Jpn. 49, 1887 (1980)]. The observed dielectric constant explains the Curie-Weiss behaviour of dielectric constant along the c-axis of KH2PO4 crystal in the paraelectric phase. Also the temperature dependence of tangent loss in paraelectric phase for KH2PO4 at 9.2 GHz for field along the a-axis, and the c-axis have been calculated and compared with experimental results of Kaminow et al [Phys. Rev. 138A, 1539 (1963)]. It is observed that these results are in good agreement with each other and with the results obtained by other methods. At higher temperature the loss deviates from the Curie-Weiss type behaviour and increases linearly with temperature. This behaviour suggests that at higher temperatures the phonon anharmonicity contributes significantly in the observed loss.

Keywords : Green's function; Dyson's equation; phonon anharmonicity; transverse and observed dielectric constant; and tangent loss.

GJSFR-F Classification MSC 2010: *82D30.*

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Keywords : Green's function; Dyson's equation; phonon anharmonicity; transverse and observed dielectric constant; and tangent loss.

I. Introduction

The phase transition from ferro to non-ferroelectric (paraelectric - hereafter referred to as PE) phase or vice versa, at the transition temperature, is associated with the change in crystal structure as well as anomaly in certain physical properties. In view of the nature of phase transition occurring at transition temperature, the ferroelectrics are classified in two types (i) order-disorder type, and (ii) displacive type. In order-disorder type, as KH_2PO_4, the transition is associated with the tunneling of proton through a barrier between two positions of minimum potential energy in the double well potential in the hydrogen bond at the transition temperature. In the other types of transitions, which are called displacive type, the transition are associated with the displacement of a whole sublattice of ions of one type relative to other sublattice, e.g., in $BaTiO_3$ and most of the double oxide ferroelectrics.

The tunneling proton model has been long believed to be an established model of the phase transition in KH_2PO_4 and other hydrogen-bonded crystals. Experimental results of dielectric dispersion [1], Brillouin spectroscopy[2] and low frequency Raman spectroscopy[3] have shown, however, that the dynamical spectra of polarization fluctuations in pure and mixed crystals $KDP_{1-x}\,DKDP(KD_2PO_4)_x$ can be well analyzed in terms of the Debye type

Author α : STO (Electronics),USIC, School of Engineering and Technology, HNB Garhwal University Srinagar (Garhwal) Uttrakhand-24 6174,India. E-mail : vsbist_usic68@yahoo.com
Author σ : Professor and Dean, School of Engineering and Technology, Head USIC, HNB Garhwal University Srinagar (Garhwal) Uttrakhand-24 6174, India. E-mail : nspusic@gmail.com

susceptibility[4], which is characterized by a relaxation time. On the other hand, the origin of spontaneous polarization in KDP was established to be shift of P and O ions relative to K ions, so that the site symmetry of a PO_4 tetrahedron was determined to be C_2 below the transition temperature Tc. Consequently, the Debye susceptibility suggests that the order-disorder of PO_4 dipoles with C_2 site symmetry may be the transition mechanism of KDP. Raman spectroscopic studies confirm that the ferroelectric phase transition in KH_2PO_4, KD_2PO_4 and their mixed crystals is due to the "order-disorder dynamics" of PO_4 dipoles[5].

The frequency and temperature dependence of dielectric constant near the Curie points of several ferroelectric crystals have investigated by Kaminow[6]. The theoretical studies of temperature dependence of microwave loss in Rochelle salt crystal have been discussed[7] by considering PLCM model Hamiltonian with phonon anharmonocity up to fourth order and using double time thermal Green's function. Levitsky *et al* [8],on the basis of the model of relaxational dynamics, have shown that the two particle cluster theory treating the ferroelectric phase transition as a result of an instability in a deuteron (proton) subsystem gives a fair quantitative description of the relaxation soft mode dynamics of quasi-one-dimensional hydrogen bonded crystals.

The dielectric loss is associated with the slow establishment of polarization accompanied by absorption currents. In the high fields, the dielectric loss in these ferroelectrics is due to hysteresis. Microwave loss in the KDP has studied by Kaminow[6] and Upadhyaya[9]. The temperature dependence of microwave tangent loss in KDP is empirically represented by $(T-T_C)\tan\delta = \alpha + \beta T + \gamma T^2$, this dependence can be explained in terms of slowing down of a relaxational mode. Recently [10,11] have experimentally studied the temperature dependence of dielectric constant and loss of KDP measured at 23 kHz before and after the proton irradiation. Formulae were developed to explain ferroelectric transitions in order-disorder [7,9,12,13] type crystals.

In our previous studies (hereafter referred to as I[12], II[13]), we have designed the four protons Hamiltonian considers the third-and fourth-order phonon anharmonic interaction terms. Applying the double time thermal Green's function techniques and Dyson's equation the higher order correlations have been evaluated using the renormalized Hamiltonian. The collective mode frequencies and corresponding widths and shifts have been evaluated for PE phase for KDP-type ferroelectric in I. The relaxation processes and ultrasonic attenuation in KDP –type ferroelectric have been studied in II.

In the present study, we use the same Hamiltonian as in I and II, we have evaluated expressions for dielectric constant and tangent loss. Using model parameters given by Ganguli *et al* [14] the transverse dielectric constant and observed dielectric constants have been calculated for KH_2PO_4. Their temperature dependence for KH_2PO_4 have been calculated and compared with experimental results of others[6,15-17] in PE phase. The observed dielectric constant explains the Curie-Weiss behaviour of dielectric constant along the c-axis of KH_2PO_4 crystal in the PE phase. The temperature dependence of tangent loss of KH_2PO_4 at 9.2 GHz for field along the a-axis $(\tan\delta_a)$ along the c-axis $(\tan\delta_c)$ have been calculated and compared with experimental results of Kaminow *et al* [6] in PE phase.

II. Dielectric Susceptibility and Tangent Loss

Using double-time thermal Green's function (see Appendix A) technique and Dyson's equation, the general expressions for collective phonon mode frequencies $(\tilde{\tilde{\omega}}_\pm^2)$ and corresponding widths $\Delta(q,\omega)$ and shifts $\Gamma(q,\omega)$ for KDP-type ferroelectric {represented by

Notes

equations (25), (24) and (28) respectively in I}. We start with acoustic phonon Green's function equation (5) in II) write its equation of motion, Fourier Transform and write it in the Dyson's equation form using model Hamiltonian {equation (3) in I} . In this process we find $\Delta(q,\omega)$ and $\Gamma(q,\omega)$ to be the real and imaginary parts respectively of the response function for soft phonon mode. The response function consists of higher order Green's function, which are solved my renormalized Hamiltonian {equation (27) in I}. Thus dielectric susceptibility, dielectric constant, and dielectric loss (see Appendix B) have been calculated for KDP-type ferroelectrics as.

Notes

a) Dielectric Susceptibility

Following Kuo[18] and Zubarev[19], the dielectric susceptibility is given by

$$\chi(\omega)=\frac{-2N\mu^2\tilde{\omega}}{[\omega^2-\tilde{\tilde{\omega}}^2+2j\omega\,\Gamma_P(\omega)]}, \tag{1}$$

The range of frequencies used in ultrasound[20], Brillouin[2] and susceptibility[20] measurement experiments are such that $\omega << \tilde{\tilde{\omega}}$. Thus equation (1) can be written as

$$\chi(\omega)=\frac{2N\mu^2\tilde{\omega}}{\tilde{\tilde{\omega}}^2[1-j\omega\tau_P]}, \tag{2}$$

Where the polarization relaxation time (τ_P) is given by equation (9) in II. This approximation of equation (1) is equivalent to a Debye relaxation susceptibility. Furthermore, if $\omega\tau_P << 1$, which is true for KDP-system[20], equation (2) can be written as

$$\chi(\omega)=\frac{2N\mu^2\tilde{\omega}}{\tilde{\tilde{\omega}}^2}(1+j\omega\tau_P), \tag{3}$$

Using equation (B.5) in appendix B, the expression for dielectric constant, from equation (1), can be written as

$$\varepsilon(\omega)-1 =\frac{-8\pi N\mu^2\tilde{\omega}[\{(\omega^2-\tilde{\tilde{\omega}}^2)-2j\omega\Gamma_P(\omega)\}]}{[(\omega^2-\tilde{\tilde{\omega}}^2)^2+4\omega^2\Gamma_P^2(\omega)]}, \tag{4}$$

The imaginary part of which can be written as

$$\varepsilon''(\omega)=-\frac{8\pi N\mu^2\tilde{\omega}^2\omega\Gamma_P(\omega)}{(\omega^2-\tilde{\tilde{\omega}}^2)^2+4\omega^2\Gamma_P^2(\omega)}, \tag{5}$$

and the real part as

$$\varepsilon'(\omega)-1=-\frac{8\pi N\mu^2\tilde{\omega}(\omega^2-\tilde{\tilde{\omega}}^2)}{(\omega^2-\tilde{\tilde{\omega}}^2)^2+4\omega^2\Gamma_P^2(\omega)}, \tag{6}$$

for the experimental range of frequencies, $\omega << \tilde{\tilde{\omega}}$ and ($\omega\tau_p << 1$ for KDP), equation (6) reduced to ($\varepsilon' >> 1$)

$$\varepsilon'(\omega) = \frac{8\pi N\mu^2 \tilde{\omega}}{\tilde{\tilde{\omega}}^2 + \omega^2\tau_P^2} = \frac{8\pi N\mu^2 \tilde{\omega}}{\tilde{\tilde{\omega}}^2} \quad (7)$$

where $\tilde{\tilde{\omega}}_{\pm}^2$ and $\tilde{\omega}$ (represented by equation (6) and (7) respectively in II). The $\tilde{\tilde{\omega}}_+$ mode and $\tilde{\tilde{\omega}}_-$ mode are described in II.

Notes

b) *Tangent Loss*

The dielectric tangent loss ($\tan\delta$) for the dissipation of power in a dielectric sample is given by equation (17) in II. For experimental values of the applied field frequency ω, one has $\omega\tau_p << 1$ for KDP system, can be approximated {given by equation (18) in II}.

III. Numerical Calculations

By using Blinc-de Gennes model parameter values for KH_2PO_4 crystal as given by Ganguli *et al*[14](see table 1), we have calculated transverse dielectric constant[$\varepsilon_a(0)$], observed dielectric constant [$\varepsilon_c(0)$] in PE phase for KH_2PO_4, and tangent loss ($\tan\delta$) of KH_2PO_4 at 9.2 GHz for fields along the a-axis ($\tan\delta_a$), and c-axis ($\tan\delta_c$) are calculated using respective equations (see table 2).

Table 1 : Blinc de Genns model parameters for KDP as given by Ganguli *et al* [14]

Ω	J (cm^{-1})	J' (cm^{-1})	T_C(K)	V/kT_C
82	334	440	123	0.299

Table 2 : Calculated values for KDP crystal in PE phase.

Temperature (K)	$\varepsilon_a(0)$	$\varepsilon_c(0)$	$\tan\delta_a$	$\tan\delta_c$
125	63	35714	0.00297	0.068
130	62	6144	0.0048	0.033
135	61	2286	0.00393	0.0279
140	60	874	0.0039	0.0253
145	59	486	0.00393	0.0247
150	58	359	0.00297	0.0241

IV. Temperature Dependence of $\varepsilon_a(0)$, and $\varepsilon_c(0)$ for KH_2PO_4

The results for transverse dielectric constant {$\varepsilon_a(0)$} obtained from integrated intensity of Raman spectroscopy[20] and those measured by Busch[16] and Kaminow *et al* [6], together with the theoretical result of Havlin, Litov and Uehling[21]. Their temperature dependence is shown in figure 1. This indicates that the low frequency $\tilde{\tilde{\omega}}_+$ {E(x, y) mode} is closely related to the macroscopic dielectric constant ε_a'. This also suggests that the E-mode Raman spectrum originates neither from the second order Raman scattering nor from the density of states due to the local disorder above T_c but from one of the collective modes at the centre of the Brillouin zone. It should be mentioned here that the low frequency E - mode continuous appears also in a deuterated KDP (DKDP), although the intensity is about one-third of that of KDP[22], which indicates the possibility that the spectrum is due to the hydrogen collective motion. Using equation (6) given in I, for $\tilde{\tilde{\omega}}_+$ mode, it can be seen that the E-mode collective hydrogen motion has a characteristics damping factor which slowly increases as the temperature approaches T_c, while that of $\tilde{\tilde{\omega}}_-$ $\{B_2(z)\}$ soft mode the damping factor slowly decreases down to a finite value, which agrees with the observations of Kaminow *et al* [6]. The present results agree with the behaviour of the observed E-mode Raman spectrum in the following aspects:

(i) $\tilde{\tilde{\omega}}_+$ does not change appreciably as $T \to T_C$ in PE phase,

(ii) $\Gamma_P(\omega_+)$ is weakly dependent on temperature,

(iii) Because of the factor $(\omega^2 - \tilde{\tilde{\omega}}^2)$ in the numerator of equation (6), the susceptibility derived changes the corresponding spectrum from a simple overdamped form to a more flat one, like the E-mode Raman spectrum of KH_2PO_4 [23].

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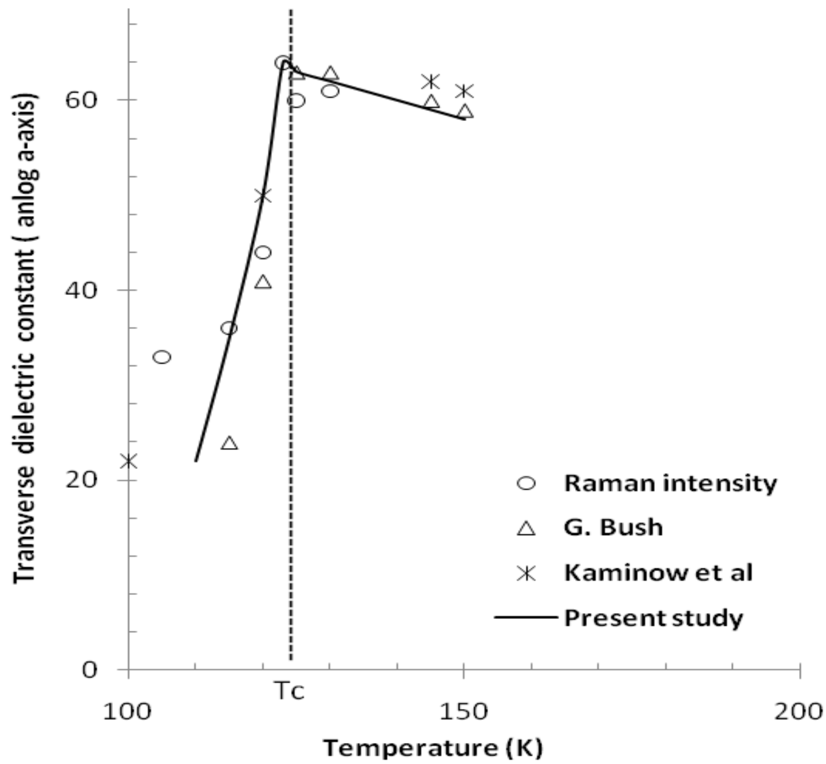

Fig.1 : Temperature dependence of Transverse dielectric constant $\varepsilon_a(0)$ for KH_2PO_4, obtained from Raman intensity [15] (shown by o o o), Busch [16](shown by Δ Δ Δ), Kaminow *et al* [6](shown by***) and solid line represents the present theoretical results.

The observed dielectric constant $\{(\varepsilon_c)\}$ of KH_2PO_4 along c-axis are shown in figure 2. The $\tilde{\tilde{\omega}}_-$ $\{B_2(z)\}$ mode may be assigned for the observed temperature dependence of ε_c. As from equation (23b) in I, $\tilde{\tilde{\omega}}_-^2 \propto (T - T_C)$, the real part of the dielectric constant associated with this mode, from equation (7), can be expressed as given by equation (15) in II , which explains the Curie-Weiss behaviour of dielectric constant along the c-axis of KH_2PO_4 crystal in the PE phase observed by Deguchi *et al* [17], Busch[16] and Kaminow et al[6] shown in figure 2. For temperature $T \to T_c$, ε_c' tends to maximum value, which is consistent with the theory of Hill and Ichiki[24] for TGS and KDP crystals. While Mason monodisperse theory gives $\varepsilon_c' \to 0$ as $T \to T_c$ .The origin of this difference in the temperature dependence of ε_c' is easily traced back in monodisperse theory, the critical slowing down of the relaxation time has a dominant effect over the Curie-Weiss law of static dielectric constant, while the Hill-Ichiki[24] theory of distribution function of relaxation time makes contribution to finite $\tau \neq 0$ to ε_C' more dominant. There are actually, however, many cases in which ε_C' takes a minimum of finite value at $T = T_C$ being neither zero as in Mason's theory nor maximum as in Hill and Ichiki theory.

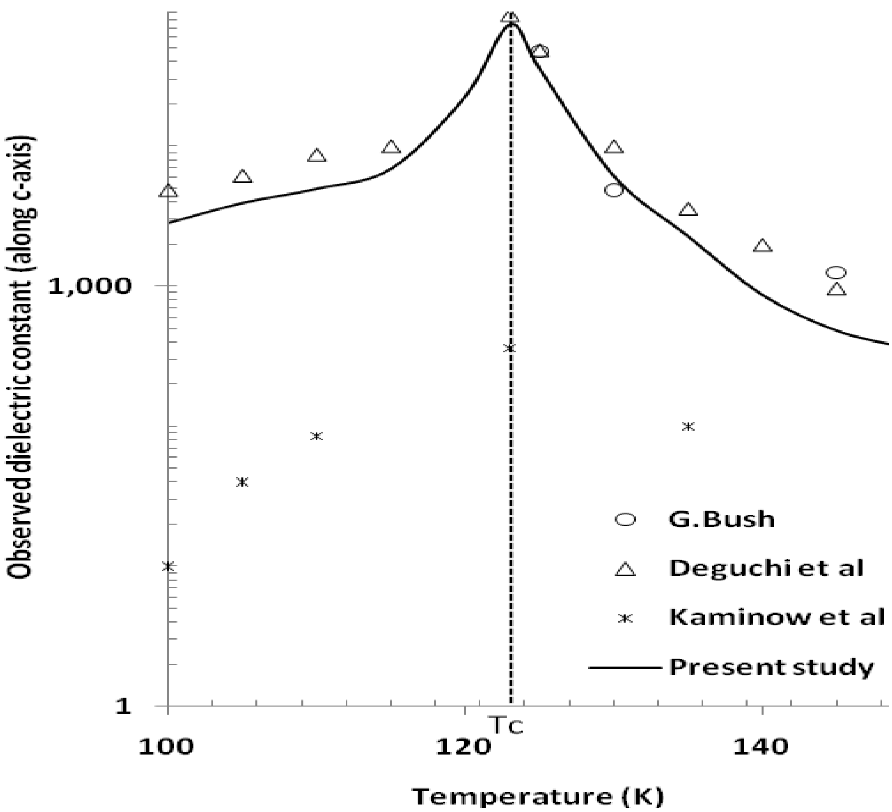

Fig.2 : Temperature dependence of observed dielectric constant $\varepsilon_C(0)$ for KH_2PO_4, obtained by Busch [16](shown by o o o), Δ Deguchi *et al* [17](shown by Δ Δ Δ), Kaminow *et al* [6](shown by***) and solid line represents the present theoretical results.

V. Temperature Dependence of Tangent Loss of KH_2PO_4.

From equations (18) and (9) in II. The $\tilde{\tilde{\omega}}_+$ mode gives the contribution for weakly temperature dependent transverse relaxational behaviour of the observed transverse tangent loss ($\tan\delta_a$) and $\tilde{\tilde{\omega}}_-$ mode contributes to the longitudinal relaxational behaviour of the observed longitudinal tangent loss ($\tan\delta_C$) of KH_2PO_4 are shown in figure 3. Under the assumption that the proton moves in a double well potential the dielectric property of KDP - type ferroelectrics in the transition region may be due to the relaxation processes or due to tunneling mode. An interpretation of equation (19) given in II, for experimental data, is remarkable for temperature region $T > T_C$ and suggests that the relaxation mode is possible in $PbHPO_4$.

The tangent loss is associated with damping parameter Γ, given by equation (17) in II and equation (28) in I. Damping can be understood as the creation of a virtual polarization mode excited by the transverse electromagnetic radiation and its subsequent decay into the real phonons by scattering from crystal defects, higher order phonon anharmonicities, etc. At higher temperature the loss deviates from the Curie-Weiss type behaviour and increases linearly with temperature. This behaviour suggests that at higher temperatures the phonon anharmonicity contributes significantly in the observed loss.

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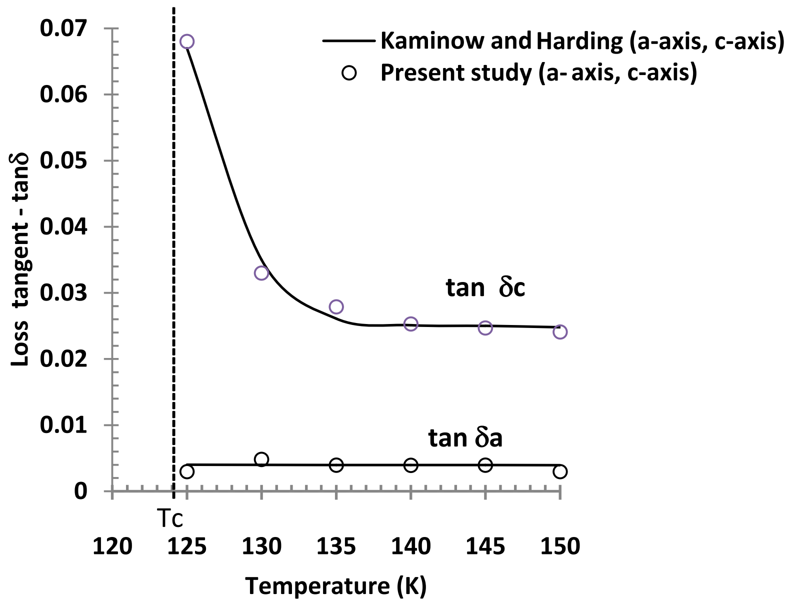

Fig.3 : Temperature dependence of tangent loss of KH_2PO_4 at 9.2 GHz solid line for fields along the a-axis $(\tan\delta_a)^6$, along the c-axis, $(\tan\delta_c)^6$, (o) represent the present theoretical results.

VI. Conclusion

Present study reveal that four cluster Hamiltonian alongwith third and fourth-order anharmonities for the KDP-type ferroelectrics, explains well the temperature dependence of transverse dielectric constant, observed dielectric constant and tangent loss. The present results reduce to the results of others [25,26] if the width and shift are neglected. Many workers[27,28] used four proton cluster model but could not explain most of the features of KDP-system except the difference between the Curie and Curie-Weiss temperature. Vaks and Zinenko [29] and Havlin and Sompolinsky [30] performed extensive calculations for the static thermodynamics behaviour in the four-particle cluster approximation and found satisfactory agreement with the experimental data, but they could not explain the observed relaxational behaviour of dielectric properties and ultrasonic attenuation explicitly. Ganguli *et al* [14] modified Ramakrishnan and Tanaka[31] theory by considering anharmonic interaction. Their treatment explains many features of order-disorder ferroelectrics. However, due to insufficient treatment of anharmonic interactions, they could not obtain quantitatively good results and could not describe some interesting properties, like dielectric, ultrasonic attenuation, etc. Our Theoretical results fairly agree with the experimental results of others[10,11].

The method of double time temperature dependent Green's function and Dyson's equation formalism have been found convenient and systematic to give the static and dynamical properties on a single framework of KDP-type system. The dielectric properties and ultrasonic attenuation strongly depend on the relaxational mode behaviour of stochastic motion of $H_2PO_4^-$ group in KDP-type ferroelectrics.

VII. Acknowledgement

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Appendix A: Green's Function

Green's functions of statistical mechanics are the appropriate generalization of the concept of correlation functions. As in quantum field theory three different types of Green's functions can be defined in terms of the average value of the T-product of operators, namely, causal retarded and advanced Green's functions. Here we shall use only the retarded double-time Green's functions.

A.1. The double-time thermal Green's function

For any pair of operators, the retarded Green's function is defined as

$$G(t,t') = \langle\langle A(t);\ B(t') \rangle\rangle$$
$$= -i\theta(t-t') < [A(t), B(t')] > \quad (A.1)$$

Where $A(t)$ and $B(t')$ are Heisenberg operators, i.e,.

$$A(t) = e^{\frac{iHt}{\hbar}} A e^{-\frac{iHt}{\hbar}} \quad (A.2)$$

H being Hamiltonian of the system. The square bracket denotes the commutator or anticommutator of the operators

$$[A, B] = AB - \eta BA; \qquad \eta = \pm 1 \quad (A.3)$$

η is +1,if A and B are Bose operators, and -1 if they are Fermi operators. Since in the present work we shall be dealing with phonons, these operators are always Bose operators and the square brackets will be commutator. The angular bracket $\langle \ldots\ldots \rangle$ denotes an ensemble average described by the Hamiltonian, namely for any operator 0 it is given by

$$\langle 0 \rangle = \frac{Tr\,(e^{-\beta H}\,0)}{Tr(e^{-\beta H})} \quad (A.4)$$

Where Tr stands for the trace of the expression and $\beta = (K_\beta T)^{-1}$, K_β being the Boltzmann constant and T the absolute temperature. $\theta(t)$ is the Heaviside step function having the property

$$\theta(t) = 1 \text{ for } t > 0$$
$$= 0 \text{ for } t < 0 \quad (A.5)$$

The retarded Green's functions give much detailed dynamical information, because they are expectation values of the ensemble and contain all the statistical mechanical information. For example, the one-particle Green's function has a direct physical interpretation as a particle propagator. It describes the motion of one particle added to the many-particle system.

A.2. Equation for Green's function

The equation of motion for any operator $A(t)$ in Heisenberg representation is

$$i\hbar \frac{d}{dt} A(t) = [A(t), H] = AH - HA \quad (A.6)$$

The right hand side of this equation can be evaluated using the explicit form of the Hamiltonian and the commutation relation for the operators. The equation of motion for the Green's function defined by (A.1), can be obtained by differentiating it with respect to time t

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$$i\hbar \frac{d}{dt} G(t,t^{'}) = \hbar \frac{d}{dt}\theta(t-t^{'}) < \left[A(t), B(t^{'})\right] > + << \left[A(t), H\right]; B(t^{'}) >> \qquad (A.7)$$

In Heisenberg representation $G(t,t')$ depends upon the difference of the arguments t and t'. The step function $\theta(t)$ can be expressed in terms of the δ -function of t as

$$\theta(t) = \lim_{\varepsilon \to 0} \int_{-\infty}^{\infty} \delta(t^{'}) e^{\varepsilon t^{'}} dt^{'} \qquad (A.8)$$

Notes

With the help of equation (A.8), the equation of motion for the Green's function G becomes

$$i\hbar \frac{d}{dt} G(t-t^{'}) = \hbar \delta(t-t^{'}) < \left[A(t), B(t^{'})\right] > + << \left[A(t), H\right]; B(t^{'}) >> \qquad (A.9)$$

The second term in the right hand side of the above equation, in general, involves Green's Functions of higher order than the original Green's function $<< A(t); B(t^{'}) >>$, except for the trivial case of noninteracting systems when exact solution can be obtained. In general the exact solution is very difficult. One is immediately forced to write equation of motion of the kind (A.9) for the Green's function appearing on the right hand side which in turn leads to a hierarchy of coupled equations for Green's functions. One sees that at every time the order of the Green's function increases. In order to get some physical result one must cut off the endless chain at certain stage by decoupling the higher order Green's functions into simpler double-time Green's functions using some suitable decoupling schemes. The type of decoupling of hierarchy of coupled equations generally depends upon the nature of the problem.

A.3. Time Correlations

The experimental study of various dynamical properties is usually carried out by measuring the response of the phonon system to different applied external disturbances. Different types of experiments (e.g., ultrasonic attenuation, inelastic neutron scattering and optical measurements) may be described in a unified way by a single phonon correlation function. In general, physical quantities of interest in many problems are time correlation functions or related to them. We define the correlation function as the thermal expectation value of the product of two Heisenberg operators $A(t)$ and $B(t')$ at difference times, namely

$$F_{BA}(t,t^{'}) = < B(t^{'}) A(t) \qquad (A.10a)$$

$$F_{AB}(t,t^{'}) = < A(t), B(t^{'}) > \qquad (A.10b)$$

On the attainment of the thermal equilibrium, these correlation functions depend only on the difference of their time arguments as do the Green's functions. Equal time $t=t'$ correlation functions are called the auto-correlation functions and they give the average value of the product of the operators,

$$F_{BA}(0) = < B(t) A(t) > = < B(0) A(0) > \qquad (A.11a)$$

$$F_{AB}(0) = < A(t) B(t) > = < A(0) B(0) > \qquad (A.11b)$$

The time correlation functions also satisfy the same equation of motion as the Green's function in Eq. (A.9) except the omission of discontinuous factor $\delta(t-t^{'})$, i.e.,

$$i\hbar\frac{d}{dt}F_{AB}(t-t^{'}) =< [A(t),H]> B(t^{'}) > \quad (A.12a)$$

$$i\hbar\frac{d}{dt}F_{BA}(t-t^{'}) =< B(t^{'})[A(t),H]> \quad (A.12b)$$

The values of the correlation functions can be evaluated directly by integration of Eq. (A.12) using the boundary conditions. However, it is more convenient to evaluate them indirectly by first calculating the Green's function from (A.9). This method is considerably simpler, sine it makes it easier to satisfy the boundary conditions using the spectral theorems.

Notes

A.4. Spectral Representations

In order to evaluate the correlation functions (A.10) with the help of the Green's functions, it is convenient to introduce the spectral representations for them. These spectral representations supplement the necessary boundary conditions. We will now obtain the spectral representations for the time correlation functions F_{AB} and F_{BA} .

Let $|n\rangle$ and E_n be the eigenstates and eigenvalues of the Hamiltonian H, i.e.,

$$H|n\rangle = E_n|n\rangle \quad (A.13)$$

Using the definition of thermal average, the time correlation function F_{BA} can be written

$$F_{BA}(t-t^{'}) = z^{-1}\sum_n \langle n| B(t^{'})A(t)| n\rangle e^{-\beta E_n} \quad (A.14)$$

Where z is the partition function given by $z = Tr(e^{-\beta H})$. By virtue of the completeness of the eigenstates, the expression Eq. (A.14) can be written as

$$F_{BA}(t-t^{'}) = z^{-1}\sum_{n,m} e^{\frac{-i(E_n-E_m)(t-t^{'})}{\hbar}} \times\langle n| B| m \rangle\langle m | A| n\rangle e^{-\beta E_n} \quad (A.15)$$

Similarly

$$F_{AB}(t-t^{'}) = z^{-1}\sum_{n,m} e^{\frac{-i(E_n-E_m)(t^{'}-t)}{\hbar}} \times\langle n | A| m \rangle\langle m | B|n \rangle e^{-\beta E_n} \quad (A.16)$$

Interchanging the summation indices n and m in (A.16) and doing slight manipulation, we can write (A.15) and (A.16) in the form

$$F_{BA}(t-t^{'}) = \int_{-\infty}^{\infty} J(\omega) e^{-i\omega(t-t^{'})} d\omega \quad (A.17a)$$

$$F_{AB}(t-t^{'}) = \int_{-\infty}^{\infty} e^{\beta\hbar\omega} J(\omega) e^{-i\omega(t-t^{'})} d\omega \quad (17b)$$

$J(\omega)$ is called the spectral density of the function $F_{BA}(t-t^{'})$ and is given by

$$J(\omega) = z^{-1}\sum_{n,m} e^{-\beta E_n} \langle n| B| m\rangle\langle m |A|n \rangle \times \delta(\omega - \frac{E_n - E_m}{\hbar}) \quad (A.18)$$

Equation (A.17) are the required spectral representations for the time correlation functions.

Notes

A.5. Spectral representation for Green's functions

The spectral representation for the retarded Green's Function Eq. (A.1) can easily be obtained by means of the spectral representations, (A.17a) and (A.17b), for the time correlation functions. Let $G(\omega)$ be the Fourier transform of the Green's function $G(t-t')$ then

$$G(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}G(t-t')e^{i\omega(t-t')}d(t-t') \tag{A.19}$$

$$G(t-t')=\int_{-\infty}^{\infty}G(\omega)e^{-i\omega(t-t')}d\omega \tag{A.20}$$

Substituting Eq. (A.1) into Eq. (A.19) we get

$$G(\omega)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}d(t-t')e^{i\omega(t-t')}\theta(t-t') \times\left[< A(t)B(t') > -\eta < B(t')A(t) >\right] \tag{A.21}$$

Using the spectral representations (A.17a) and (A.17b) for the time correlation function, we obtain

$$G(\omega)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}(e^{\beta\hbar\omega'}-\eta)J(\omega')d\omega'\int_{-\infty}^{\infty}dt\theta(t)e^{i(\omega-\omega')t} \tag{A.22}$$

Time integration in (A.22) can be carried out by using the integral representations for $\delta(t)$ and $\theta(t)$, namely

$$\delta(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{ixt}\,dx \tag{A.23}$$

and

$$\theta(t)=\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{e^{-ixt}}{x+i\varepsilon}dx \tag{A.24}$$

The result is

$$G(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega' J(\omega')\frac{(e^{\beta\hbar\omega'}-\eta)}{\omega-\omega'+i\varepsilon} \tag{A.25}$$

So far we have considered ω to be a real quantity. Assuming ω to be complex, we have

$$G(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega'(e^{\beta\hbar\omega'}-\eta)J(\omega')\frac{1}{(\omega-\omega')} \tag{A.26}$$

Where ω is having a small imaginary part, is understood. This shows that the function $G(\omega)$ can be considered to be analytic in the upper half of the complex ω - plane with a singularity on the real axis. In the similar manner, expression for the advanced Green's function is obtained except that the small positive quantity $(\varepsilon\to+0)$ is changed by negative sign in the expression (A.25). We can say that the Fourier transform of the advanced Green's function is analytic in the lower half plane.

The spectral density function $J(\omega)$ can be immediately calculated if the function $G(\omega)$ is known. For this we have

$$G(\omega+i\varepsilon)-G(\omega-i\varepsilon)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega' J(\omega')(e^{\beta\hbar\omega'}-\eta)$$

$$\times\left[\frac{1}{\omega-\omega^{'}+i\varepsilon}-\frac{1}{\omega-\omega^{'}+i\varepsilon}\right] \quad (A.27)$$

Using the relation

$$\delta(x)=\frac{1}{2\pi i}\left(\frac{1}{x-i\varepsilon}-\frac{1}{x+i\varepsilon}\right) \quad (A.28)$$

We arrive at

$$G(\omega+i\varepsilon)-G(\omega-i\varepsilon)=-i(e^{\beta\hbar\omega^{'}}-\eta)J(\omega) \quad (A.29)$$

or

$$\lim_{\varepsilon\to 0}J(\omega)=-\frac{2}{(e^{\beta\hbar\omega}-\eta)}\operatorname{Im}G(\omega+i\varepsilon)$$

Where $\varepsilon\to 0$ is implied and Im stands for the imaginary part.

This relation is very important. Once the retarded Green's function is known, the spectral density function can be obtained the help of (A.29) and taking the requisite Fourier transforms (A.17), one can get the correlation functions at all times and at all temperatures. This enables us to calculate the physical properties of a crystal in terms of one set of functions.

Notes

Notes

Appendix B: Dielectric Constant and Tangent Loss

General formulation

The response of a dielectric field is conveniently described by the dielectric susceptibility. Following Kuo[28] and Zubarev[29], the general expression for complex dielectric susceptibility tensor $\chi_{mn}(\omega)$ can be expressed as

$$\chi_{mn}(\omega) = \lim_{\varepsilon \to 0} -2\pi G_{mn}(\omega + j\varepsilon), \qquad (B.1)$$

Where $G_{mn}(\omega)$ the Fourier transform of the retarded double-time thermal Green's function between the m[th] and n[th] components of the crystal dipole moment operators $\vec{M}(t)$ in the Heisenberg representation and is defined as

$$G_{mn}(t-t') = \langle\langle M_m(t); M_n(t') ¿¿$$

$$= -j\theta(t-t') < [M_m(t); M_n(t')] > \qquad (B.2)$$

Where $\theta(t-t')$ is the Heaviside step function and the angular brackets $<-->$ denote the thermal ensemble average. The crystal dipole moment $\vec{M}(t)$ depends on the ionic co-ordinates, like potential energy, i.e., on the lattice configurations and can be expanded in a Taylor's series in terms of ionic displacements. Because of the periodic boundary conditions, i.e., symmetry considerations, imposed on the ionic motions, only the low lying relaxational modes have non-zero polarization associated with them. Thus only the expansion coefficients which correspond to lowest frequency mode, i.e., $\vec{M}(q, j)$ –where q = 0 for ferroelectrics, and j relates the modes of spectrum) contribute to the dielectric susceptibility, significantly.

Thus we can write the dielectric susceptibility as

$$\chi_{mn}(\omega) = \lim_{\varepsilon \to 0} -2\pi N\mu^2 G_{mn}(\omega + j\varepsilon), \qquad (B.3)$$

Where N is the number of unit cells in the sample and μ is the effective dipole moment per unit cell, and

$$G_{mn}(\omega + j\varepsilon) = \langle\langle A_q(t); A_{q'}(t') \rangle\rangle$$

$$= G'(\omega) - jG''(\omega) \qquad (B.4)$$

Where $G'(\omega)$ and $G''(\omega)$ are real and imaginary parts of the Green's function. The dielectric constant can be evaluated using the relation

$$\varepsilon(\omega) = 1 + 4\pi\chi = \varepsilon'(\omega) - j\varepsilon''(\omega) \qquad (B.5)$$

Where $\varepsilon'(\omega)$ and $\varepsilon''(\omega)$ are real and imaginary parts of the dielectric constant. The real part of the dielectric constant can be expressed as

$$\varepsilon'(\omega) - 1 = -8\pi N\mu^2 G'(\omega), \qquad (B.6)$$

and the imaginary part

$$\varepsilon''(\omega) = -8\pi N\mu^2 G''(\omega), \qquad (B.7)$$

The dielectric loss ($\tan\delta$), for the dissipation of power in the dielectric crystal is defined as the ratio of imaginary and real parts of the dielectric constant, i.e.,

$$\tan\delta = \frac{\varepsilon''(\omega)}{\varepsilon'(\omega)} = \frac{G''(\omega)}{G'(\omega)}, \qquad (B.8)$$

The dielectric susceptibility, equation (B.3); dielectric constant equation (B.6); and dielectric loss equation (B.8); can thus be calculated by using the Green's function and the model Hamiltonian.

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Mathematical Modeling and Thin Layer Drying Kinetics of Carrot Slices

By Hosain Darvishi, Ahmad Banakar & Mohammad Zarein

Islamic Azad University, Tehran, Iran

Abstract - The thin-layer drying characteristics of carrot slices were investigated under four microwave powers; 200, 300, 400 and 500 W and slice thickness of 2.5 mm. Data were analyzed to obtain diffusivity values from the period of falling drying rate. Four mathematical models for describing the thin-layer drying behavior of carrot were investigated. The results show that the Midilli et al. is the most appropriate model for drying behaviour of thin layer carrot slices. An analysis of variance (ANOVA) revealed that microwave power significantly affected the drying time, drying rate, effective diffusivity and specific energy consumption. The effective diffusivity varied from 1.90×10^{-8} to 3.99×10^{-8} m^2/s, and the activation energy was determined to be 36.40. Specific energy consumption values ranged 8.58 to 12.46 MJ/kg and the optimized specific energy consumption was obtained 540 W microwave levels.

Keywords : Carrot slices, effective diffusivity, modeling, microwave drying, energy consumption.

GJSFR-F Classification MSC 2010: 92C45.

MATHEMATICAL MODELING AND THIN LAYER DRYING KINETICS OF CARROT SLICES

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Abstract - The thin-layer drying characteristics of carrot slices were investigated under four microwave powers; 200, 300, 400 and 500 W and slice thickness of 2.5 mm. Data were analyzed to obtain diffusivity values from the period of falling drying rate. Four mathematical models for describing the thin-layer drying behavior of carrot were investigated. The results show that the Midilli et al. is the most appropriate model for drying behaviour of thin layer carrot slices. An analysis of variance (ANOVA) revealed that microwave power significantly affected the drying time, drying rate, effective diffusivity and specific energy consumption. The effective diffusivity varied from 1.90×10^{-8} to 3.99×10^{-8} m^2/s, and the activation energy was determined to be 36.40. Specific energy consumption values ranged 8.58 to 12.46 MJ/kg and the optimized specific energy consumption was obtained 540 W microwave levels.

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I. Introduction

Drying is the process of removing the moisture in the product up to certain threshold value by evaporation. In this way, the product can be stored for a long period, since the activities of the microorganisms, enzymes or ferments in the material are suppressed via drying [1]. In particular, convective hot-air drying is extensively employed as a preservation technique. The major draw-back of convective hot-air drying method, from an energy point of view, is the longer drying period, higher drying temperature and therefore high energy consumption, which may be as high as 6000 kJ/kg of water evaporated [2]. In general, energy efficiency in drying is closely related to drying times.

Microwave is an electromagnetic wave in the frequency range of 300–30000 MHz. The conversion of microwave energy into heat in the food is because of the presence of water. The quick absorption of energy by water molecules causes rapid evaporation of water, resulting in high drying rates of the food.

Compared to hot air drying, microwave or hybrid/combined microwave drying techniques can greatly reduce the drying time (up to 50%) of biological materials without quality degradation, therefore microwave method offers significant energy savings [3, 4]. The most relevant aspects of drying technology are the mathematical modeling of the process and the experimental setup [5]. The modeling is basically based on the design of a set of equations to describe the system as accurately as possible. Drying characteristics of the particular products being dried and simulation models are needed in the design, construction and operation of drying systems [6].

The aim of this research was (i) to determine the influence of microwave power on the energy consumption and drying kinetics during microwave dehydration of carrot slices and (ii) to fit the experimental moisture data to four mathematical models.

Author α : Department of Engineering, Shahre Ray Branch, Islamic Azad University, Tehran, Iran. E-mail : Hosaindarvishi@gmail.com.
Author σ: Department of Agricultural Machinery Mechanics, Faculty of Agriculture, Tarbiat Modares University, Tehran, Iran.

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II. Materials and Methods

Carrot samples were procured from local vegetable market in Tehran, Iran. The samples were stored at 4ffi0.5 flC before they were used in experiments. Carrots were washed under running water to remove the adhering impurities, and thinly sliced in thicknesses of 2.5 using a sharp stainless steel knife. The average initial moisture content of the samples were found to be 83.8% wet basis, as determined by using convective oven at 105 ˚C for 24h.

A domestic microwave oven (M945, Samsung Electronics Ins) with maximum output of 1000 W at 2450MHz was used for the drying experiments. The dimensions of the microwave cavity were 327×370×207 mm. The oven has a fan for air flow in drying chamber and cooling of magnetron. The moisture from drying chamber was removed with this fan by passing it through the openings on the right side of the oven wall to the outer atmosphere. The microwave dryer was operated by a control terminal which could control both microwave power level and emission time. Experiments were performed at four microwave powers of 200, 300, 400 and 500 W. The moisture losses of samples were recorded at 30s intervals during the drying process by a digital balance (GF-600, A & D, Japan) and an accuracy of ffi0.01 g. For measuring the weight of the sample during experimentation, the tray with sample was taken out of the drying chamber, weighed on the digital top pan balance and placed back into the chamber. Drying was carried out until the final moisture content reaches to a level less than 5% (w.b.).

The experimental drying data were used to calculate the moisture ratio (MR) and drying rate (DR) using the following equations:

$$MR = \frac{M_t - M_e}{M_0 - M_e} \qquad (1)$$

$$DR = \frac{M_{t+dt} - M_t}{dt} \qquad (2)$$

where M_t, M_0 and M_e are the moisture content at any time of drying; initial moisture content and equilibrium moisture content (kg water/kg dm), respectively, DR is the drying rate (kg waret/kg dm.min), Mt+dt is the moisture content at t+dt (kg water/kg dm), and t is drying time (min). Since the values of me are very small compared to mt or m_0, Eq. (1) can be simplified to M_t/M_0 [7].

Four well-known thin-layer drying models in Table 1 were tested to select the best model for describing the drying curve of the carrot slices. The terms used to evaluate goodness of fit of the tested models to the experimental data were the coefficient of determination (R^2); root mean square error (RMSE) and the reduced chi-square (χ^2) between the experimental and predicted moisture ratio values. Statistical values are defined as follows:

$$\chi^2 = \frac{\sum_{i=1}^{N}\left(MR_{\exp,i} - MR_{pre,i}\right)^2}{N - z} \qquad (3)$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}\left(MR_{\exp,i} - MR_{pre,i}\right)^2}{N}} \qquad (4)$$

In these equations, N is the number of observations, z is the number of constants, MR_{exp} and MR_{pre} are the experimental and predicted moisture ratios, respectively.

Notes

Table 1 : Thin-layer drying models

Model name	Model	References
Page	$MR=\exp(-kt^n)$	[7]
Wang and Singh	$MR = 1 + bt + at^2$	[3]
Logarithmic	$MR=a\exp(-kt) + b$	[5]
Midilli et al.	$MR=a\exp(-kt^n)+ bt$	[10]

**where, k is the drying constant and a, b, n are equation constants*

During the drying process, diffusivity is assumed to be the only physical mechanism for the transfer of water to material surface and can be defined by Fick's second law of diffusion for a slab as follows:

$$\frac{\partial M}{\partial t} = D_{eff}\frac{\partial^2 M}{\partial x^2} \tag{5}$$

By using appropriate initial and boundary conditions, Crank [8] gave the analytical solutions for various geometries and the solution for spherical object with constant diffusivity is given as:

$$MR = \frac{8}{\pi^2}\sum\frac{1}{(2n+1)^2}\exp\left(-\frac{(2n+1)\pi^2 D_{eff}}{4L^2}t\right) \tag{6}$$

where D_{eff} is the effective diffusivity (m^2/s), and L is the half thickness of slab (m). For long drying times, only the first term (n=0) in the series expansion of the above equation can give good estimate of the solution, which is expressed in logarithmic forms as follows:

$$\ln(MR) = \ln\left(\frac{8}{\pi^2}\right) - \left(\frac{\pi^2 D_{eff}}{4L^2}\right)t \tag{7}$$

The diffusion coefficients are typically determined by plotting experimental drying data in terms of ln(MR) versus drying time (t), because the plot gives a straight line with a slope as $\pi^2 D_{eff}/4L^2$.

Specific energy consumption (E_s) of the drying process was expressed in MJ/kg water evaporated. Therefore, the Es could be determined as follows [9]:

$$E_s = \frac{Pt}{m_w} \tag{8}$$

where P is the microwave power input (W); and mw is the mass of water evaporated (kg).

Inasmuch as temperature is not precisely measurable inside the microwave drier, the activation energy is found as modified from the revised Arehnious equation. In this method it is assumed as related to effective diffusion coefficient and the ratio of microwave output power to sample weight (m/p) instead of to air temperature. Then Equation (9) can be effectively used [10] as follows:

$$D_{eff} = D_0\exp\left(-\frac{E_a.m}{P}\right) \tag{9}$$

Ref.

8. Crank J; The Mathematics of Diffusion. 2nd ed. Oxford (UK): Clarendon Press; 1975.
10. Ozbek B. and Dadali G; Thin-layer drying characteristics and modelling of mint leaves undergoing microwave treatment. J Food Eng, 83(2007), 541-549.

where E_a is the activation energy (W/g), m is the mass of raw sample (g), and D_0 is the pre-exponential factor (m^2/s).

All measurements were carried out in triplicate. ANOVA test was performed in order to examine the effect of microwave power on drying kinetics and energy consumption. The SPSS version 17.0 was used for statistical investigations. For all statistical analysis, the level of significance is fixed at 95%. Each factor having a P value ≤ 0.05 was considered significant.

III. Results and Discussion

The moisture ratios versus drying time for the carrot slices at the selected powers are shown in Fig. 1. The total drying times to reach the final moisture content for the carrot sample were 19.5, 15.5, 10.5 and 7 min at 200, 300, 400, and 500 W, respectively. Obviously, within a certain microwave power range (200-500W in this study), increasing output power speeds up the drying process, thus shortening the drying time (up to 61%).

As seen in Figs. 2 and 3, all curves have two stages. The drying rate rapidly increases and then slowly decreases as drying progresses. The moisture content of the material was very high during the initial phase of the drying which resulted in a higher absorption of microwave power and higher drying rates due to the higher moisture diffusion. As the drying progressed, the loss of moisture in the product caused a decrease in the absorption of microwave power and resulted in a fall in the drying rate. Similar findings were reported in previous studies [3,11,12].The drying rate by the microwave method can be described by Eq. (10):

$$DR = At^{u} + \frac{ht}{1+\exp(t^{d})} \tag{10}$$

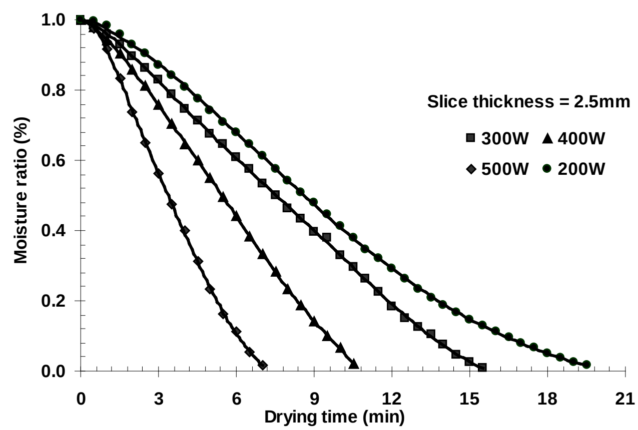

Fig.1 : Variation of moisture ratio with drying time for the carrot slice

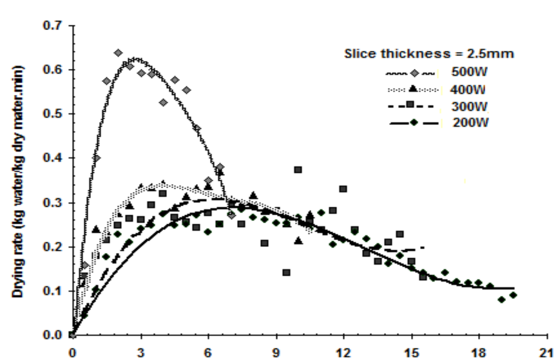

Fig.2 : Variation of drying rate with drying time for the carrot slice

Notes

Notes

Parameters a, b, c and d of that function are given in Table 2. In order to take into account the effect of microwave power on the constant and coefficients of the Eq. (10), namely, A, u, h and d, the regression analysis was used to set up the relations between these parameters and the microwave power. Thus, the regression equations of these parameters against microwave power and the drying rate model are as follows:

$h = 0.0074P - 2.1312 \quad R^2 = 0.971$

$u = 64.431\ P^{-0.84} \quad R^2 = 0.983$

$d = -6\times10^{-7}P^3 + 7\times10^{-4}P^2 - 0.237P + 24.667 \quad R^2 = 0.999$

$A = 6\times10^{-8}P^3 - 6\times10^{-5}P^2 + 0.0192P - 1.549 \quad R^2 = 0.999$

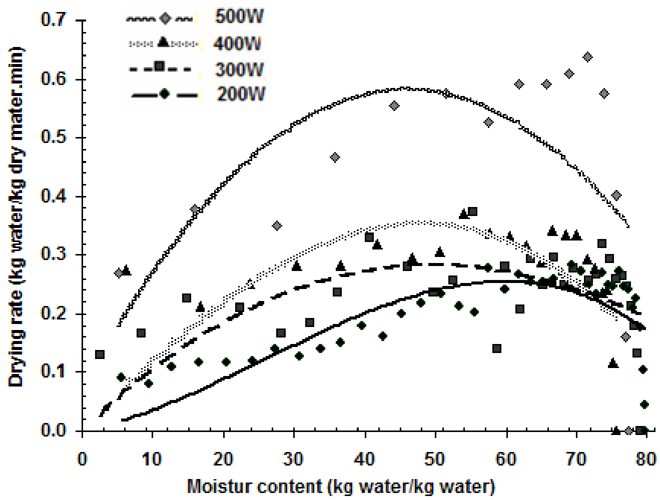

Fig.3 : Variation of drying rate with moisture content for the carrot slice

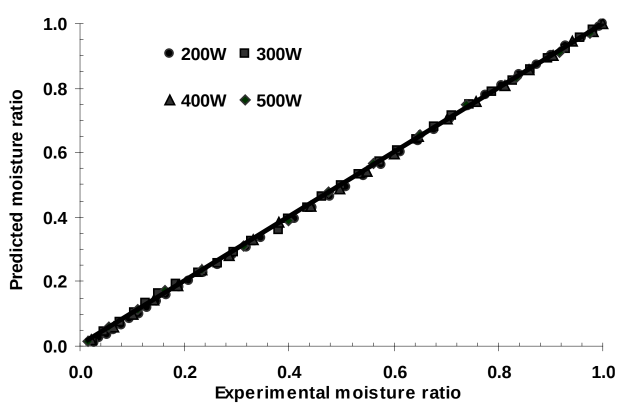

Fig.4 : Comparison of moisture ratios determined by experimentation and prediction using the Midilli et al. model for the carrot slices

The statistical results from models are summarized in Table 3. The best model describing the thin-layer drying characteristics of carrot slices was chosen as the one with the highest R^2 values and the lowest χ^2 and RMSE values. The statistical parameter estimations showed that R^2, χ^2 and RMSE values were ranged from 0.9900 to 0.9999, 0.00001 to 0.10269, and 0.00072 to 0.29833, respectively. Of all the models tested, the Midilli et al. model gives the highest value of R^2 and the lowest values of χ^2 and RMSE. It was determined that the value of the drying rate constant (k) increased with the increase in microwave powers. This implies that with increase in microwave power drying curve becomes steeper indicating increase in drying rate.

Fig. 4 compares experimental data with those predicted with the Midilli et al. model for carrot slice samples at the different microwave powers. The prediction using the model showed MR values banded along the straight line, which showed the suitability of these models in describing drying characteristics of carrot slices.

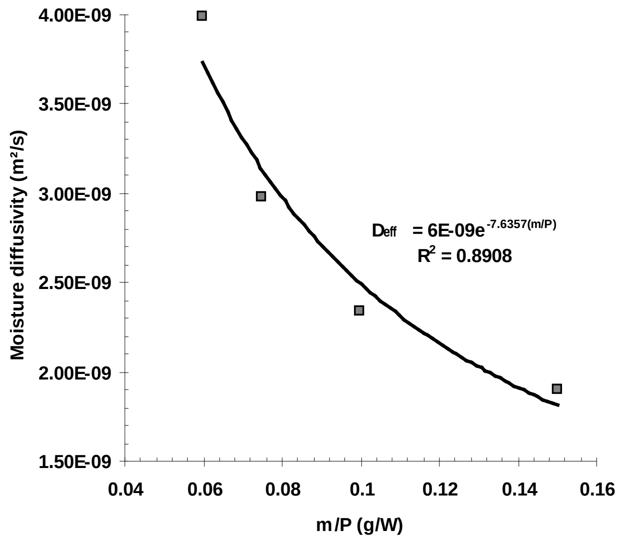

Fig.5 : The relationship between the values of D_{eff} versus sample amount/power

The effective moisture diffusivities of carrot slices for different microwave powers are presented in Table 4. The values ranged from 1.90×10^{-9} to 3.99×10^{-9} m^2/s. In Table 4, it was noted that D_{eff} increased progressively with the increase of drying microwave power. This might be explained by the increased heating energy, which would increase the activity of the water molecules leading to higher moisture diffusivity when samples were dried at higher microwave power. The values for D_{eff} obtained from this study lie within the general range 10^{-11}–10^{-8} m^2/s for drying of food materials [13, 14]. The values of D_{eff} are comparable with the reported values of 1.0465×10^{-8} to 9.1537×10^{-8} m^2/s mentioned for apple pomace microwave drying [11], 1.14×10^{-6} to 6.09×10^{-6} m^2/s for tomato pomace microwave drying at 160-800W [2], 0.55×10^{-7} to 3.5×10^{-7} m^2/s for Gundelia tournefortii microwave drying at 90-800W [15].

The values of effective diffusivity versus m/P shown in Fig. 5 accurately fit to Eq. (9) with coefficient of determination (R^2) of 0.8908. Then, D_0 and E_a values were estimated as 6×10^{-9} m^2/s and 7.636W/g.

Table 4 : Values of effective diffusivity obtained for carrot slice at different microwave powers

P(W)	Effective diffusivity (m^2/s)
200	1.90×10^{-9}
300	2.34×10^{-9}
400	2.98×10^{-9}
500	3.99×10^{-9}

Fig.6 shows the microwave specific energy consumption values at different amounts of microwave powers for the drying of carrot slices. Statistical analyses showed that microwave power was significant on the specific energy consumption values of carrot slices ($\alpha = 0.05$). The values ranged from 8.58 to 12.46 MJ/kg water evaporated.

Ref.

11. Wang Z., Sun J., Chen F., Liao X. and Hu X; Mathematical modelling on thin layer microwave drying of apple pomace with and without hot air pre-drying. J Food Eng, 80 (3007), 536–544.

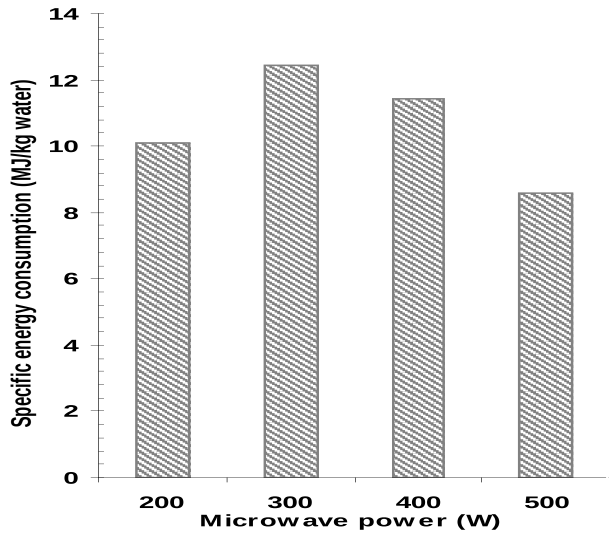

Fig.6 : Specific energy consumption for microwave drying of carrot slices

Table 2 : Parameters A, u, h and d of the functions describing the drying rate of carrot slices

P(W)	Drying kinetic parameters				
	A	u	h	d	R^2
500	0.001	0.336	1.694	0.642	0.904
400	0.001	0.446	0.736	9.521	0.917
300	0.213	0.522	-0.096	-0.958	0.630
200	0.270	0.751	-0.496	0.026	0.931

IV. Conclusion

Thin layer drying experiments were conducted to determine the thin layer drying characteristics and energy consumption of carrot slices in a microwave dryer. Four thin layer drying models were evaluated for their suitability. The Midilli et al. model was found to be the most suitable model for describing the thin layer drying of carrot slices. The effective moisture diffusivity was obtained based on Fick's second law. The values ranged from 1.90×10^{-9} to 3.99×10^{-9} m^2/s. The activation energy required to detach and move the water out from melon slices during the drying process was found to be 7.636 W/g. Increasing the microwave power was caused to increase the drying rate and decrease the energy consumption. The values ranged from 8.58 to 12.46 MJ/kg water evaporated.

Table 3 : Results of statistical analysis on the modeling of moisture content and drying time for the microwave dried carrot slices

Model	P (W)	Model constants	R^2	χ^2	RMSE
Page	200	k=0.017, n=1.749	0.9980	0.00029	0.01673
	300	k=0.023,n=1.722	0.9900	0.00101	0.03084
	400	k=0.034, n=1.824	0.9940	0.00070	0.02532
	500	k=0.075, n=1.859	0.9970	0.00044	0.01947
Wang and Singh	200	a= 0.0004, b= - 0.0610	0.9930	0.00125	0.03450
	300	a= -5E-05, b= - 0.0655	0.9975	0.00026	0.01580
	400	a= -0.0012, b= - 0.0840	0.9960	0.0165	0.12246
	500	a= 0.0142, b= - 0.2489	0.9971	0.10269	0.29833

Logarithmic	200	k= 0.032, a=2.334, b= -1.266	0.9950	0.00064	0.02441
	300	k= 0.015, a=5.019, b= -3.982	0.9990	0.00018	0.01278
	400	k= 0.018, a= 5.867, b= -4.813	0.9970	0.00063	0.02334
	500	k= 0.037, a= 4.656, b= -3.601	0.9950	0.00082	0.02557
Midilli et al.	200	k= 0.020, a= 0.999, b= -0.005, n=1.605	0.9999	0.00014	0.01110
	300	k= 0.027, a= 1.003, b= -0.023, n=1.342	0.9999	0.00006	0.00072
	400	k= 0.034, a= 1.001, b= -0.024, n=1.550	0.9999	0.00001	0.00380
	500	k= 0.080, a=1.007, b= -0.019, n=1.634	0.9990	0.00004	0.00537

Notes

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Analytical Approach to Partial Differential Equations Arising in Mechanics

By B.J Adegboyegun, P.S Ayenimo & E.A Ibijola
Ekiti State University, Nigeria

Abstract - In this article, we implement relatively new analytical approach, Adomian decomposition method (ADM), for solving some selected partial differential equations in mechanics. This method in applied mathematics can be used as alternative method for obtaining exact solution for different types of partial differential equations. The method takes the form of a convergent series with easily computable components. The results obtained with minimum amount of computation indicate that the method is reliable and accurate.

Keywords : *Decomposition method; Partial differential equations.*

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Notes

Analytical Approach to Partial Differential Equations Arising in Mechanics

B.J Adegboyegun[α], P.S Ayenimo[σ] & E.A Ibijola[ρ]

Abstract - In this article, we implement relatively new analytical approach, Adomian decomposition method (ADM), for solving some selected partial differential equations in mechanics. This method in applied mathematics can be used as alternative method for obtaining exact solution for different types of partial differential equations. The method takes the form of a convergent series with easily computable components. The results obtained with minimum amount of computation indicate that the method is reliable and accurate.

Keywords : Decomposition method; Partial differential equations.

I. Introduction

In recent years, much attention has been devoted to the search for reliable and more efficient solution methods for the equations modeling physical phenomena in various fields of sciences and engineering. One of the methods which have received the attention is the Adomian decomposition method. Unlike the other traditional numerical methods, which are usually valid for short period of time, the solution of the presented equation is analytic for $-\infty \leq x \leq \infty$. Also, the method needs no discretization, linearization, transformation or perturbation unlike the prior act.

In general, we shall consider equation of the form;

$$\frac{\partial^\alpha u}{\partial t^\alpha} + a_0(x)u + a_1(x)\frac{\partial u}{\partial x} + a_2(x)\frac{\partial^2 u}{\partial x^2} + \ldots + a_n(x)\frac{\partial^n u}{\partial x^n} = q(x,t), \;\; t>0, x \in \Re, \tag{1}$$

Subject to the initial conditions

$$u(x,0) = f(x), \qquad \alpha = 1, \; u(x,t) \to 0 \;\text{ as }\; |x| \to \infty, \quad t>0 \;\text{ and}$$

$$u(x,0) = f(x), \quad \frac{\partial u(x,0)}{\partial t} = g(x), \;\text{ for }\; \alpha = 2$$

We shall proceed to discuss the basic concepts and theory of the decomposition method.

The basic concepts of Adomian decomposition method

The decomposition approach requires that the partial differential equation (1) be expressed in an operator form as

$$C_t^\alpha u + a_0(x)u + a_1(x)L_{1x}u + a_2(x)L_{2x}u + \,.\;.\;.\; + a_n(x)L_{nx}u = q(x,t) \tag{2}$$

Author α σ ρ : Department of Mathematical Sciences, Ekiti State University, Nigeria. E-mail : bolucrown2000@yahoo.com

Where $L_{1x}=\frac{\partial}{\partial x}$, $L_{2x}=\frac{\partial^2}{\partial x^2}$, . . . $L_{nx}=\frac{\partial^n}{\partial x^n}$

And the differential operator C_t^{α} is defined as $C_t^{\alpha}=\frac{\partial}{\partial t}$ *for* $\alpha=1$ $C_t^{\alpha}=\frac{\partial^2}{\partial t^2}$ *for* $\alpha=2$

The method is based on applying the operator L^{-1}, the inverse of the operator C_t^{α} on both sides of equation (2) to obtain

$$u(x,t)=\sum_{k=0}^{\alpha-1}\frac{\partial^k u(x,t)t^k}{\partial t^k k!}-L^{-1}\left(a_0(x)u+a_1(x)L_{1x}u+a_2(x)L_{2x}u+\ .\ .\ .\ +a_n(x)L_{nx}u-q(x,t)\right) \tag{3}$$

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Where L^{-1} is considered a one fold integral for $\alpha=1$ and two fold integral for $\alpha=2$.

$\sum_{k=0}^{\alpha-1}\frac{\partial^k u(x,t)t^k}{\partial t^k k!}$ is the term(s) that arise from the initial condition(s)

The method assumes a series solution for $u(x,t)$ given by;

$$u(x,t)=\sum_{n=0}^{\infty}u_n(x,t) \tag{4}$$

The components $u_n(x,t)$ will be determined recursively. By substituting (4) into both sides of (3) we obtain

$$\sum_{n=0}^{\infty}u_n(x,t)=\sum_{k=0}^{\alpha=1}\frac{\partial^k u(x,t)t^k}{\partial t^k}-L^{-1}\left(a_0(x)\sum_{n=0}^{\infty}u_n(x,t)\right)+L^{-1}\left(a_1(x)L_{1x}\sum_{n=0}^{\infty}u_n(x,t)\right)$$

$$+L^{-1}\left(a_2(x)L_{2x}\sum_{n=0}^{\infty}u_n(x,t)\right)+\ .\ .\ .\ +L^{-1}\left(a_n(x)L_{nx}\sum_{n=0}^{\infty}u_n(x,t)\right) \tag{5}$$

We next determine the components $u_n(x,t)$ for which $n\geq 0$. We first identify the zeroth component $u_0(x,t)$ by all terms that arise from the initial conditions. The remaining components are determined by using the preceding component. Each term of series (4) is given by the recursive relations

$$u_0(x,t)=\sum_{n=0}^{\alpha-1}\frac{\partial^k u(x,t)t^k}{\partial t^k k!}-L^{-1}\left(q(x,t)\right)$$

$$u_{n+1}(x,t)=-L^{-1}\left(a_0(x)u_n(x,t)+a_1(x)L_{1x}u_n(x,t)+\ .\ .\ .\ +a_n(x)L_n\,u_{xn}(x,t)\right) \tag{6}$$

In practice, all terms of the series (4) cannot be computed. Thus, the solution if equation (1) is approximated by the truncated by the truncated series of the form

$$\phi_N(x,t)=\sum_{n=0}^{N-1}u_n(x,t)$$

And

$$\lim_{N\to\infty}\phi_N(x,t)=u(x,t) \tag{7}$$

The decomposition series solution by (7), in general converges very rapidly in real physical problems (G. Adomian 1993). The theoretical treatment of convergence of the decomposition method has been investigated by (Y. Cherruault and G. Adomian 1993).

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The method provides the solution of (1) in the form of a rapidly convergent series that may lead to the exact solution in the case of linear differential equations. The accuracy of the method can be improved by accommodating more terms in our decomposition series.

II. Numerical Results

Problem 1

Consider the equation;

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},\quad t>0,\ x\in\Re \tag{8}$$

Subject to the initial condition $(x,0)=\sin x$

We apply ADM operator to equation (8)

$$Lu(x,t)=L_{2x}u(x,t) \tag{9}$$

Operating L^{-1} on both sides of (9) and by imposing the initial condition

$$u(x,t)=u(x,0)+L^{-1}L_{2x}u(x,t) \tag{10}$$

L^{-1} is considered as one fold integral

From (10), we obtain the recurrence relations

$$u_0(x,t)=u(x,0)=\sin x$$
$$u_{n+1}(x,t)=L^{-1}(L_{2x}u_n(x,t) \tag{11}$$

From (11), we can proceed to compute the first few terms of the series

$$u_1(x,t)=L^{-1}(L_{2x}u_0(x,t))=\int_0^t\left(\frac{\partial^2 \sin x}{\partial x^2}\right)dt=-t\sin x$$

$$u_2(x,t)=L^{-1}(L_{2x}u_1(x,t))=-\int_0^t\left(\frac{\partial^2 t\sin x}{\partial x^2}\right)dt=\frac{t^2\sin x}{2!}$$

$$u_3(x,t)=L^{-1}(L_{2x}u_2(x,t))=-\int_0^t\left(\frac{\partial^2 t^2\sin x}{2!\partial x^2}\right)dt=-\frac{t^3\sin x}{3!}$$

$$u_4(x,t)=L^{-1}(L_{2x}u_3(x,t))=-\int_0^t\left(\frac{\partial^2 t^3\sin x}{3!\partial x^2}\right)dt=\frac{t^4\sin x}{4!}$$

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$$u_n(x,t)=\frac{(-1)t^n\sin x}{n!}$$

The solution in series form is given by

$$u(x,t)=\sin x-t\sin x+\frac{t^2\sin x}{2!}-\frac{t^3\sin x}{3!}+\frac{t^4\sin x}{4!}+\ldots$$

$$u(x,t)=\left(1-t+\frac{t^2}{2!}-\frac{t^3}{3!}+\frac{t^4}{4!}\ .\ .\ .\right)\sin x=e^{-t}\sin x \tag{12}$$

For application purpose, only the first ten terms of the series will be computed. Table (1) compares the approximate solution of equation (8) with the theoretical solution $u(x,t)=e^{-t}\sin x$. It is obvious that the results are in agreement with the theoretical solution. Higher accuracy of the approach can be obtained by computing more components of the series.

Table 1 : Numerical result when $t=0.5$

x	Exact solution	Adomian result	Error
0.0	0.00000000	0.00000000	0.00000000
0.2	0.12049904	0.12049904	0.00000000
0.4	0.23619416	0.23619416	0.00000000
0.6	0.34247297	0.34247297	0.00000000
0.8	0.43509847	0.43509847	0.00000000
1.0	0.51037794	0.51037788	0.00000006
1.2	0.56531030	0.56531030	0.00000000
1.4	0.59770548	0.59770548	0.00000000
1.6	0.60627204	0.60627210	0.00000006
1.8	0.59066844	0.59066850	0.00000006
2.0	0.55151671	0.55151671	0.00000000

Problem 2

Let us consider the equation

$$\frac{\partial^2 u}{\partial t^2}=\frac{x^2\partial^2 u}{2\partial x^2},\quad t>0 \tag{13}$$

Subject to the initial conditions; $u(x,o)=0,\ \dfrac{\partial u(x,0)}{\partial t}=x^2$

With the theoretical solution $u(x,t)=x+x^2\sinh t$

We apply ADM operator to equation (13)

$$Lu(x,t)=\frac{x^2L_{2x}u(x,t)}{2} \tag{14}$$

Operating L^{-1} on both sides of (14) and impose the initial conditions, we obtain

$$u(x,t)=\sum_{k=0}^{1}\frac{\partial^k u(x,t)t^k}{\partial t^k k!}+\frac{L^{-1}\left(x^2L_{2x}u(x,t)\right)}{2} \tag{15}$$

Where $\sum_{k=0}^{1}\dfrac{\partial^k u(x,t)t^k}{\partial t^k k!}$ is the term that arises from the initial conditions and $L^{-1}=\int_0^t\int_0^t dtdt$

From equation (15),

$$u_0(x,t)=x+x^2t$$

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$$u_{n+1}(x,t)=\frac{L^{-1}\left(x^2 L_{2x} u_n(x,t)\right)}{2} \tag{16}$$

From equation (16), we compute the first few terms of the series as follows;

$$u_1(x,t)=\frac{L^{-1}\left(x^2 L_{2x}(x+x^2)\right)}{2}=\int_0^t\int_0^t x^2 t\,dt\,dt=\frac{x^2t^3}{3!}$$

Notes

$$u_2(x,t)=\frac{L^{-1}\left(x^2 L_{2x}(x^2t^3)\right)}{2(3!)}=\frac{1}{2(3!)}\int_0^t\int_0^t x^2t^3\,dt\,dt=\frac{x^2t^5}{5!}$$

$$u_3(x,t)=\frac{L^{-1}\left(x^2 L_{2x}(x^2t^5)\right)}{2(5!)}=\frac{1}{2(5!)}\int_0^t\int_0^t x^2t^5\,dt\,dt=\frac{x^2t^7}{7!}$$

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$$u_n(x,t)=\frac{x^2t^{2n+1}}{(2n+1)!},\quad n\geq 1 \tag{17}$$

Thus,

$$u(x,t)=x+x^2\left(\sum_{n=0}^{\infty}\frac{t^{2n+1}}{(2n+1)!}\right),\quad n\geq 0 \tag{18}$$

Table (2) compares the result obtained using the ADM with the theoretical solution

Table 2 : Numerical result when $t=0.5$

x	Exact solution	Adomian result	Error
0.0	0.00000000	0.00000000	0.00000000
0.2	0.22084381	0.22084381	0.00000000
0.4	0.48337525	0.48337525	0.00000000
0.6	0.78759451	0.78759443	0.00000008
0.8	1.13350096	1.13350099	0.00000003
1.0	1.52109531	1.52109531	0.00000000
1.2	1.95037724	1.95037725	0.00000001
1.4	2.42134676	2.42134680	0.00000004
1.6	2.93400396	2.93400399	0.00000003
1.8	3.48834859	3.48834880	0.00000021
2.0	4.08438122	4.08438124	0.00000002

III. Conclusions

In this work, we have applied decomposition method to obtain an approximate solution to heat and wave equations. A comparison of the results with the exact solutions suggests that Adomian decomposition method is accurate, reliable and easy to use. It is interesting to note that higher accuracy of the method can be obtained by accommodating more terms of the series.

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Notes

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New Representations in Terms of q-product Identities for Ramanujan's Results III

By M.P. Chaudhary
International Scientific Research and Welfare Organization, New Delhi, India

Abstract - In this paper author has established four q-product identities with the help of Jacobi's triple product identity using elementary method. These identities are new and not available in the literature of special functions.

Keywords : Generating functions, triple product identities.

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New Representations in Terms of q-product Identities for Ramanujan's Results III

M.P. Chaudhary

Abstract - In this paper author has established four q-product identities with the help of Jacobi's triple product identity using elementary method. These identities are new and not available in the literature of special functions.

Keywords : Generating functions, triple product identities.

I. Introduction

For $|q| < 1$,

$$(a;q)_\infty = \prod_{n=0}^{\infty}(1-aq^n) \tag{1.1}$$

$$(a;q)_\infty = \prod_{n=1}^{\infty}(1-aq^{(n-1)}) \tag{1.2}$$

$$(a_1,a_2,a_3,...,a_k;q)_\infty = (a_1;q)_\infty(a_2;q)_\infty(a_3;q)_\infty...(a_k;q)_\infty \tag{1.3}$$

Ramanujan [2, p.1(1.2)]has defined general theta function, as

$$f(a,b) = \sum_{-\infty}^{\infty} a^{\frac{n(n+1)}{2}} b^{\frac{n(n-1)}{2}} \quad ; \quad |ab| < 1, \tag{1.4}$$

Jacobi's triple product identity [3,p.35] is given, as

$$f(a,b) = (-a;ab)_\infty(-b;ab)_\infty(ab;ab)_\infty \tag{1.5}$$

Special cases of Jacobi's triple products identity are given, as

$$\phi(q) = f(q,q) = \sum_{n=-\infty}^{\infty} q^{n^2} = (-q;q^2)_\infty^2(q^2;q^2)_\infty \tag{1.6}$$

$$(q) = f(q,q^3) = \sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}} = \frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.7}$$

$$f(-q) = f(-q,-q^2) = \sum_{n=-\infty}^{\infty}(-1)^n q^{\frac{n(3n-1)}{2}} = (q;q)_\infty \tag{1.8}$$

Equation (1.8) is known as Euler's pentagonal number theorem. Euler's another well known identity is as

$$(q;q^2)_\infty^{-1} = (-q;q)_\infty \tag{1.9}$$

Author : International Scientific Research and Welfare Organization, New Delhi, India. E-mail: mpchaudhary 2000@yahoo.com

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2. B.C. Berndt; What is a q-series?, preprint.
3. B.C. Berndt; Ramanujan's notebook Part III, Springer-Verlag, New York, 1991.

Throughout this paper we use the following representations

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty=(q^a,q^b,q^c\cdots q^t;q^n)_\infty \tag{1.10}$$

$$(q^a;q^n)_\infty(q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty=(q^a,q^b,q^c\cdots q^t;q^n)_\infty \tag{1.11}$$

$$(-q^a;q^n)_\infty(-q^b;q^n)_\infty(q^c;q^n)_\infty\cdots(q^t;q^n)_\infty=(-q^a,-q^b,q^c\cdots q^t;q^n)_\infty \tag{1.12}$$

Now we can have following q-products identities, as

$$(q^2;q^2)_\infty=\prod_{n=0}^{\infty}(1-q^{2n+2})$$

$$=\prod_{n=0}^{\infty}(1-q^{2(4n)+2})\times\prod_{n=0}^{\infty}(1-q^{2(4n+1)+2})\times$$

$$\times\prod_{n=0}^{\infty}(1-q^{2(4n+2)+2})\times\prod_{n=0}^{\infty}(1-q^{2(4n+3)+2})$$

$$=\prod_{n=0}^{\infty}(1-q^{8n+2})\times\prod_{n=0}^{\infty}(1-q^{8n+4})\times\prod_{n=0}^{\infty}(1-q^{8n+6})\times\prod_{n=0}^{\infty}(1-q^{8n+8})$$

or,

$$(q^2;q^2)_\infty=(q^2;q^8)_\infty(q^4;q^8)_\infty(q^6;q^8)_\infty(q^8;q^8)_\infty$$

$$=(q^2,q^4,q^6,q^8;q^8)_\infty \tag{1.13}$$

$$(q^4;q^4)_\infty=\prod_{n=0}^{\infty}(1-q^{4n+4})$$

$$=\prod_{n=0}^{\infty}(1-q^{4(3n)+4})\times\prod_{n=0}^{\infty}(1-q^{4(3n+1)+4})\times\prod_{n=0}^{\infty}(1-q^{4(3n+2)+4})$$

$$=\prod_{n=0}^{\infty}(1-q^{12n+4})\times\prod_{n=0}^{\infty}(1-q^{12n+8})\times\prod_{n=0}^{\infty}(1-q^{12n+12})$$

or,

$$(q^4;q^4)_\infty=(q^4;q^{12})_\infty(q^8;q^{12})_\infty(q^{12};q^{12})_\infty$$

$$=(q^4,q^8,q^{12};q^{12})_\infty \tag{1.14}$$

$$(q^4;q^{12})_\infty=\prod_{n=0}^{\infty}(1-q^{12n+4})$$

$$=\prod_{n=0}^{\infty}(1-q^{12(5n)+4})\times\prod_{n=0}^{\infty}(1-q^{12(5n+1)+4})\times$$

$$\times\prod_{n=0}^{\infty}(1-q^{12(5n+2)+4})\times\prod_{n=0}^{\infty}(1-q^{12(5n+3)+4})\times\prod_{n=0}^{\infty}(1-q^{12(5n+4)+4})$$

$$=\prod_{n=0}^{\infty}(1-q^{60n+4})\times\prod_{n=0}^{\infty}(1-q^{60n+16})\times\prod_{n=0}^{\infty}(1-q^{60n+28})\times$$

$$\times\prod_{n=0}^{\infty}(1-q^{60n+40})\times\prod_{n=0}^{\infty}(1-q^{60n+52})$$

or,

Notes

$$(q^4;q^{12})_\infty = (q^4;q^{60})_\infty(q^{16};q^{60})_\infty(q^{28};q^{60})_\infty(q^{40};q^{60})_\infty(q^{52};q^{60})_\infty$$

$$= (q^4,q^{16},q^{28},q^{40},q^{52};q^{60})_\infty \qquad (1.15)$$

Similarly we can compute following as

$$(q^5;q^5)_\infty = (q^5;q^{15})_\infty(q^{10};q^{15})_\infty(q^{15};q^{15})_\infty$$

$$= (q^5,q^{10},q^{15};q^{15})_\infty \qquad (1.16)$$

$$(q^6;q^6)_\infty = (q^6;q^{24})_\infty(q^{12};q^{24})_\infty(q^{18};q^{24})_\infty(q^{24};q^{24})_\infty$$

$$= (q^6,q^{12},q^{18},q^{24};q^{24})_\infty \qquad (1.17)$$

$$(q^6;q^{12})_\infty = (q^6;q^{60})_\infty(q^{18};q^{60})_\infty(q^{30};q^{60})_\infty(q^{42};q^{60})_\infty(q^{54};q^{60})_\infty$$

$$= (q^6,q^{18},q^{30},q^{42},q^{54};q^{60})_\infty \qquad (1.18)$$

The outline of this paper is as follows. In sections 2, some recent results obtained by the author [1], and also some well known results are recorded in [6;7], those are useful to the rest of the paper. In section 3, we state and prove four q-product identities, which are new and not recorded in the literature of special functions.

II. Preliminaries

In [1], following identities are being established

$$(q;q^2)_\infty = (q,q^3,q^5;q^6)_\infty \qquad (2.1)$$

$$\left[\frac{(-q;q^2)_\infty^8 - (q;q^2)_\infty^8}{q}\right]^{\frac{1}{4}} = \frac{2}{\left[(q^2;q^4)_\infty\right]^2} \qquad (2.2)$$

$$\frac{(q^2;q^2)_\infty}{(q^4;q^4)_\infty} = (q,-q;q^2)_\infty \qquad (2.3)$$

$$(q^2;q^2)_\infty = (q^2;q^4)_\infty(q^4;q^4)_\infty \qquad (2.4)$$

In Ramanujan's notebook [7, p. 209], following entry is recorded as

$$f(-q)f(-q^2) = \phi(-q)\boldsymbol{\psi}(q) \qquad (2.5)$$

In Ramanujan's notebook [6, p. 245], following entry is recorded as

$$\frac{\boldsymbol{\psi}(q)\boldsymbol{\psi}(-q)}{\boldsymbol{\psi}(q^2)\,\boldsymbol{\psi}(-q^2)} = \frac{\boldsymbol{\psi}(-q^2)}{\boldsymbol{\psi}(q^4)} \qquad (2.6)$$

In Ramanujan's notebook [6, p. 254], following entry is recorded as

$$[3\phi(-q^9) - \phi(-q)]^3 = 8\frac{\boldsymbol{\psi}^3(q)}{\boldsymbol{\psi}(q^3)}\phi(-q^3) \qquad (2.7)$$

In Ramanujan's notebook [6, p. 222], following entry is recorded as

$$\frac{\frac{\phi^5(q)}{\phi(q^5)} + 4\frac{^5(q)}{(q^5)}}{\phi(q)\phi^3(q^5) + 4q^2\boldsymbol{\psi}(q)\boldsymbol{\psi}^3(q^5)} = 5\frac{\phi^2(q)}{\phi^2(q^5)} \qquad (2.8)$$

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7. S. Ramanujan; Notebooks (Volume II), Tata Institute of Fundamental Research, Bombay, 1957.

III. Main Results

In this section, we established following new results with the help of $\boldsymbol{\psi}(.)$ and $\phi(.)$ functions or in more general language we can say that by using the properties of Jacobi's triple product identity, as $\boldsymbol{\psi}(.)$ and $\phi(.)$ functions are its special cases. These results are not recorded in the literature of special functions

$$(q;q)_\infty = (q,q;q^2)_\infty \tag{3.1}$$

$$(q^4;q^4)_\infty = (-q^2,q^2;q^4)_\infty(q^8;q^8)_\infty \tag{3.2}$$

$$(q;q^2)^3_\infty + 2(q^3;q^6)_\infty = \frac{3(q;q^2)_\infty(q^9;q^9)_\infty(q^9;q^{18})_\infty}{(q^2;q^2)_\infty} \tag{3.3}$$

$$\frac{5(q^2;q^2)^2_\infty}{(q^5;q^{10})^3_\infty} = \frac{(-q;q^2)^5_\infty(q^2;q^2)^5_\infty(q^{20};q^{20})_\infty + 4(-q^5;q^{10})_\infty(q^{10};q^{10})_\infty(q^4;q^5)^5_\infty}{(-q;q^2)_\infty(q^2;q^2)_\infty(-q^5;q^{10})^3_\infty(q^{10};q^{10})^3_\infty + 4q^2(q^4;q^4)_\infty(q^{20};q^{20})^3_\infty} \tag{3.4}$$

Proof of (3.1): By substituting, $q=q^2$ in (1.8), we have also by substituting, $q=-q$ in (1.6), we get

$$f(-q^2) = (q^2;q^2)_\infty$$

$$\phi(-q) = (q;q^2)^2_\infty(q^2;q^2)_\infty$$

by substituting the values of $f(-q^2), \phi(-q)$, employing (1.7) and (1.8) in (2.5), we get

$$(q;q)_\infty(q^2;q^2)_\infty = (q;q^2)^2_\infty(q^2;q^2)_\infty\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}$$

after simplification, we get (3.1).

Proof of (3.2): By substituting, $q=-q, q=-q^2, q=q^2$ and $q=q^4$ respectively in (1.7), we get

$$\boldsymbol{\psi}(-q) = \frac{(q^2;q^2)_\infty}{(-q;q^2)_\infty}\ ;\ \boldsymbol{\psi}(-q^2) = \frac{(q^4;q^4)_\infty}{(-q^2;q^4)_\infty}$$

and

$$\boldsymbol{\psi}(q^2) = \frac{(q^4;q^4)_\infty}{(q^2;q^4)_\infty}\ ;\ \ \boldsymbol{\psi}(q^4) = \frac{(q^8;q^8)_\infty}{(q^4;q^8)_\infty}$$

by substituting the values of $\boldsymbol{\psi}(-q), \psi(-q^2), \psi(q^2), \psi(q^4)$ and employing (1.7) in (2.6), and further using (2.3), we get

$$\frac{(q^2;q^2)_\infty(q^8;q^8)_\infty}{(q^4;q^8)_\infty} = \frac{(q^4;q^4)_\infty(q^4;q^4)_\infty}{(-q^2;q^4)_\infty(-q^2;q^4)_\infty(q^2;q^4)_\infty}$$

further using (2.4) in above expression, and after simplification, we get (3.2).

Proof of (3.3): By rearranging (2.7) can be written as

$$\frac{[3\phi(-q^9)-\phi(-q)]^3}{[2\boldsymbol{\psi}(q)]^3} = \frac{\phi(-q^3)}{\boldsymbol{\psi}(q^3)} \tag{3.3.1}$$

by substituting $q=-q^3$ in (1.6) and $q=q^3$ in (1.7) respectively, we get

$$\phi(-q^3) = (q^3;q^6)^2_\infty(q^6;q^6)_\infty\ ;\boldsymbol{\psi}(q^3) = \frac{(q^6;q^6)_\infty}{(q^3;q^6)_\infty}$$

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now substituting the values of $\phi(-q^3)$ and $\boldsymbol{\psi}(q^3)$ in (3.3.1), and after simplification, we get

$$\frac{[3\phi(-q^9)-\phi(-q)]}{[2\boldsymbol{\psi}(q)]}=(q^3;q^6)_\infty \tag{3.3.2}$$

by substituting $q=-q^9$ and $q=-q$ respectively in (1.6), we get

$$\phi(-q^9)=(q^9;q^{18})^2_\infty(q^{18};q^{18})_\infty\ ;\ \phi(-q)=(q;q^2)^2_\infty(q^2;q^2)_\infty$$

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now substituting the values of $\phi(-q^9)$, $\phi(-q)$ and employing (1.7) in (3.3.2), after little algebra, we get

$$3(q^9;q^{18})^2_\infty(q^{18};q^{18})_\infty=\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}[2(q^3;q^6)_\infty+(q;q^2)^3_\infty] \tag{3 .3.3}$$

by substituting $q=q^{\frac{9}{2}}$ in (2.4), we get

$$(q^9;q^9)_\infty=(q^9;q^{18})_\infty(q^{18};q^{18})_\infty$$

by using the values of $(q^9;q^9)_\infty$ in (3.3.3), and after simplification, we get (3.3).

Proof of (3.4): By substituting $q=q^5$ in (1.6) and (1.7) respectively, we get

$$\phi(q^5)=(-q^5;q^{10})^2_\infty(q^{10};q^{10})_\infty\ ;\ \boldsymbol{\psi}(q^5)=\frac{(q^{10};q^{10})_\infty}{(q^5;q^{10})_\infty}$$

with values of $\phi(q^5),\boldsymbol{\psi}(q^5)$, and further employing (1.6) and (1.7), we can compute following identities

$$\frac{\phi^5(q)}{\phi(q^5)}+4\frac{\boldsymbol{\psi}^5(q)}{\boldsymbol{\psi}(q^5)}=\frac{(q^2;q^2)^5_\infty}{(q^{10};q^{10})_\infty}\left[\frac{(-q;q^2)^{10}_\infty}{(-q^5;q^{10})^2_\infty}+4\frac{(q^5;q^{10})_\infty}{(q;q^2)^5_\infty}\right] \tag{3.4.1}$$

$$\phi(q)\phi^3(q^5)+4q^2\boldsymbol{\psi}(q)\boldsymbol{\psi}^3(q^5)=(q^2;q^2)_\infty(q^{10};q^{10})^3_\infty\times$$
$$\times\left[(-q;q^2)^2_\infty(-q^5;q^{10})^6_\infty+\frac{4q^2}{(q;q^2)_\infty(q^5;q^{10})^3_\infty}\right] \tag{3.4.2}$$

$$5\frac{\phi^2(q)}{\phi^2(q^5)}=5\frac{(-q;q^2)^4_\infty}{(-q^5;q^{10})^4_\infty} \tag{3.4.3}$$

by substituting the values from (3.4.1), (3.4.2) and (3.4.3) in (2.8), further using the property (2.3), after simplification, we get (3.4).

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Mathematics: Usage in, Multidisciplinary Sciences and, Everyday Life What the World of Work tells us

By Rishan Singh
University of KwaZulu-Natal, South Africa

Abstract - The use of mathematics in everyday life often goes unnoticed by the person who exploits mathematics and also by the person who associates with the person who exploits mathematics. This article looks at mathematics as a discipline and it serves to inform people, in general, about the importance of mathematics in the world we live in and how in many ways, human beings have contributed to its sophistication in the application and working worlds.

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MATHEMATICSUSAGE IN, MULTIDISCIPLINARY SCIENCES AND, EVERYDAY LIFE WHAT THE WORLD OF WORK TELLS US

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Mathematics: Usage in, Multidisciplinary Sciences and, Everyday Life What the World of Work tells us

Rishan Singh

Abstract - The use of mathematics in everyday life often goes unnoticed by the person who exploits mathematics and also by the person who associates with the person who exploits mathematics. This article looks at mathematics as a discipline and it serves to inform people, in general, about the importance of mathematics in the world we live in and how in many ways, human beings have contributed to its sophistication in the application and working worlds.

How many of us ever ask questions pertaining to the importance of mathematics? And how many of us ever commend our friends or neighbours work, or comment on the pivotal nature of their work to the world at large? If you ask me, I would think that very few of us ever do. I believe that the nature of mathematics has become more complicated over the decades by ourselves, by making new discoveries and conveying them to the world. However, although this is true, the dissemination of such innovations and discoveries is mostly only noted in the academic environment. I mean, the ordinary guy on the street, wouldn't even know that mathematics journal articles even exist. A shoe maker may apply mathematics when cutting the sole of a shoe, but he himself is unaware of his importance and is not blinded by reality mathematics. This article takes on a conservative yet mature approach and speaks about mathematics and how we cannot live without it.

From the time we are born, our whole life is programmed in numbers. Our age, is the clock that governs what and when things should be done. Our last breadth on Earth is also numericalised, but we know very little about this. And then there is also adulthood, a time when one decides to get married, have children and leave home. This entails buying a house or building one. Architects utilise mathematics in plan drawing of houses and big industrialised and commercialised buildings. They utilise mathematics from an artistic perspective and use the vision of an artist to create something that suits a person or prospective company or business. The mathematics that is used by architects utilises equipment that is similar to those used by a technical drawer. The t-square, set-square, ruler, protractor and compass, all used in geometrical and analytical mathematics, are used on a larger scale. In addition an architect utilises the knowledge of geometry to understand the landscape of a place and to devise a way of fitting a property, of the clients wants, in that space. However, just like how a graphic designer or fine artist creates 2-dimensional and 3-dimensional pictures, an architect does the same but to create something at a larger scale. While a cartoonist, create frames of accurate precision,

Author : University of KwaZulu-Natal, SOUTH AFRICA. E-mail : rshnsingh1@webmail.co.za

specifically when producing 3-D animation, an architect is precise in a 'concrete' sense. However, just having a plan, as we all know, is not sufficient for one to build a life for him/herself and so the next step would be to build the house.

Someone who has a working knowledge of angles and analytical geometry is required to analyse the plan drawer or architects drawings. Such specialists are generally termed structural engineers. However, for smaller jobs, like the building of smaller houses for example, the term 'builder' comes to mind. By simple definition the term 'structural engineer' is someone who is able to build a particular structure but who is not only confined to building that structure. By this I mean that a structural engineer could by all means be a civil, electronic and electrical engineer. The first and last types of which are pivotal in the building of major commercialised properties and road structures. However, in all types of engineering, a certain degree of mathematical sciences is employed. This could be statistical, physical and even actuary.

In the building of a property (in general), the mathematical knowledge of the structural engineer is highly competitive in the sense that by using his working knowledge of geometry, and basic arithmetic, he has to make decisions as to whether the suggestions given by the land surveyor are correct and feasible. Say, for example, that the plan drawn by the architect shows that the property needs to have a 6 meter yard space in front followed by the building itself, but there is some major underground piping from the point the building needs to be started, then in this case, the structural engineer would have to make mathematical suggestions and recommendations to the plan drawer and second it by the land/building surveyor. The recommendations normally include the angle at which the building would have to be positioned relative to the piping structure. Using this recommendation, the plan drawer would make the relative changes and submit it to the to-be owners of the unbuilt 'structure'. This shows how mathematics has become sophisticated by the sharing and transfer of mathematical knowledge between the architect, structural engineer and surveyor. The purpose of a surveyor is to inspect the landscape or building so as to ensure that all the conditions are optimal to perform a certain action, in a particular area for example. However, it is imperative to note that the job of a structural engineer (builder) is complicated since they use a number of non-mathematical instruments to achieve a 'mathematical' outcome. However, two mathematical instruments that are used are the leveller and in some cases the t-square. In some instances, the land surveyor may request that more intricate instruments are used to flatten the land surface before a trench can be made. Such instrumentation is made using physical mathematics so as to get the desired product design. A good example of the use of physical mathematics is when someone wants to affix a large structure on a huge property. Such structures are usually levitated using cranes and pulleys which requires a working knowledge of physical mathematics, geometry and analytical geometry. This ensures that the weight of the object to be offloaded does not exceed the strength of the suspending rope and that the crane itself is at a specific angle to the rope which is perpendicular to the suspending object. This shows synchronisation between physical mathematicians and structural engineers. The sad situation is that fewer and fewer matriculants, particularly, in South Africa, are failing to pursue careers in mathematics before of its constant and consistent advancement year after year.

However, lets return back to building our house structure. Once the trench for the house is built, the next step is that the structural engineers (mostly builders though) start by laying down the bricks. Although many of us view this to be 'a piece of cake', in actual fact it is not because the bricklayers themselves need to be very accurate to ensure a solid building structure. For example, say a bricklayer is standing within a box structure and is laying bricks for one side of the wall; he needs to have the mathematical knowledge to

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make sure that all four 90o angles are met while bearing in bind which bricks should interlock, where and why. This would ensure that the building structure is solid, safe and secure, creating a home. The principle to building larger properties, aircraft and even ships are the same, as was that of Moses Mabida stadium in South Africa.

Well, and of course once the married couple moves into their new home, they would want to start a family and look for better jobs in hope of securing brighter futures for their children. The couple will then have people plaster the walls of their new homes, paint or renovate the newly wedded structure. And one day, one of the parents would need help to find the reliability of some mathematical answers out at work or perhaps one of their children may need help to check the reliability of a scientific or engineering problem at school of university. All I can say is that this is where the help of a statistician comes in.

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A statistician is a person who is able to perform a range of statistical tests to check whether a research question is valid or invalid. The statistical tests performed usually takes on 3 forms, namely: the difference between samples test, the relationship and trend amongst samples test, and the best of fit amongst samples tests. The type of test performed depends on the data which a person presents the statistician with. The outcome of the result will determine if the results of the experiment performed is reliable, if the experiment has to be performed again, and possibly, if the research question has to be change to suit the type of data ascertained from the study. For example, say you are performing a test on digestive coefficients on cane rats and you are interested to see if there is a difference amongst the eating consumption rates amongst rats of the sample species and weights. From your study you obtain results that support your null hypothesis of there being no difference amongst the consumption rates. This essentially means that the research questions and predictions should be changed so that the null hypothesis is rejected, as in most scientific investigations. Alternatively, also perhaps you should try to check is there is a relationship or trend amongst the consumption rates especially since the rats used in the study all have the same weight and are of the sample species. If the weight of the rats where different and were of a different species in this case, I would opt for a difference test. Therefore it is very important to consider the variables of a scientific investigation, closely, before a statistician is consulted or a statistical test is performed. This highlights the importance of statisticians as number lovers in todays world.

However, we must not forget that the mathematical knowledge gained by architects, structural engineers, physicists, surveyors and statisticians, were first acquired from mathematical textbooks and television content. There are very few self-taught individuals in the fields I have mentioned above today and those individuals possibly have distinguished themselves in these fields. These individuals gained perspective into their careers by understanding the mathematics presented to them in books and tv, and this was only possible because the people behind such texts and programmes were well versed in mathematics. Furthermore, the editors are obviously qualified proficiently to produce such contributions that can be studied by individuals (i.e. students and the general reader). In close association with math textbook editors and math television content editors, are the mathematics opinion researchers who are also versed enough in the field of mathematics are the contributing to. I strongly believe that these 3 mathematic specialists are, probably, the only 'powerhouse' by which the entire sphere of mathematics rests on. Therefore we should have respect for such esteemed individuals.

In addition to the architects, structural engineers, physicists, surveyors and statisticians, in order for a home or company to be safely secured, actuaries, are required. An actuary is a person who has studied accounting mathematics and who is proficient enough to calculate insurance risks and premiums. This means that say for example your

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home or company is robbed, an actuary within a particular insurance company can help you claim the value of the insured goods or content of the home or company. However, in everyday life, there are many different types of actuaries, some work in companies that specialise in law, life insurance, car insurance and many others.

Financial lawyers and advisors also require a basic mathematical knowledge to perform their role in legal matters. However, the amount of mathematical knowledge utilised by them is minimal compared to learners of mathematics in school or university. Accountants, on the other hand, require complicated mathematics of arithmetic nature to perform the basics like bank reconciliation, invoices, financial statements, cash receipts journal, cash payments journal, ledger accounts etc., however, they do not require analytical mathematics, and geometry i.e. no scientific mathematics.

In countries like Japan, China and in various cities of the United States of America, seismologists play a pivotal role. Their roles include communicating with climatologists and reporting important information about earthquake tremors to the communities in those countries and the world at large. A seismologist requires a level of mathematics to make comprehensive deductions about earthquakes and therefore they are important in recording historical information about earthquake disasters. The information supplied by seismologists will have an impact on the information gathered by our children from textbooks and television programmes, possibly in the future, and seismologists, mathematics opinion researchers and editors of textbooks and television programmes are thus very important.

As mentioned previously, surveyors are essential people in building property including ships and aircrafts. A navigator who is steering a ship out at sea for example, will require visual mathematics to make judgements without using any equipment. This becomes extremely important especially in the case of an emergency when the navigator has to make a quick indication of the correct route that the ship should be steered into. In this scenario, the pilot does not require the use of textbook mathematics, but instead his eyes have to be trained to judge timing, distance and speed of the ship from a particular thing (object or obstacle) in order to put the ship on the correct course route.

In the United States, Navy Officers, use hearing mathematics to locate objects like submarines. They achieve this by using an underwater radar that sends and receives sound pulses. In addition, people utilise mathematics in businesses as well, as we know. For example, costing managers, require the familiarity of statistics to determine if the business will benefit if they continue to purchase a particular product from a particular company or not. In many ways, a costing manager is similar to the person who runs a household. The difference is that the person who runs the household does not use statistics but common sense of mathematical judgement. In many ways, if one thinks about it, the costing manager as well the the person who runs a household are the key players in sustaining the economy. However, in order for a business to survive on its own, production managers with an in-depth mathematical knowledge of basic and accounting insight, are key role players in making available resources that are needed and utilised by the general public. For example, to build a fully functional house, the various components of the house require highly specialised equipment to make roof tiles, light bulbs and steel pipes for water for instance. The production manager will ensure that the product supplied to shops for sale to the general public conform to a specific standards and that by supplying the product which the public is in demand of, the business will prosper on its own.

In the world that we live in, we are surrounded by mathematics. In South Africa, from time to time, there are land inspectors who come to check if house properties are built and boundaried according government rules and regulations. These land inspectors have a basic idea of mathematics and are able to tell if something does not conform to standards.

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Conclusion

In South Africa, still a developing country, there are presently many challenges that face society. One of the major challenges is that school finishers pass mathematics and the physical sciences at the lowest percentages compared to their other subjects and fail to gain admission into universities. Very seldomly, but possible, few students with a keen interest in pursuing mathematics at the tertiary level, do pass mathematics with high percentages, but the guarantee to enter university is blur due to space limitations.

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I believe, just like people in many other countries, that society is blinded by the true value of mathematics in the sense that people don't realise that mathematics, if not completely, is probably one of the most important sciences in the world and it integrates itself into our daily lifestyle as well as in other sciences like biology, accounting, engineering, climatology and many other sciences that are aligned or divergent to or of itself, respectively. As can be seen from this article that, from birth to death, mathematics will play an integral parts of our lives. Money management, whether for ourselves or our or other businesses, will ultimately deter how we live our lives and the outcome of our lives. From this article, the usage of mathematics in multidisciplinary sciences and everyday life is told from a working perspective of mathematics (its application) within a variety of independently related scientific fields. However, the examples in this article have been written using the notion that the reader will grasp an understanding of mathematical importance to mankind and civilisation. Image having electronic engineers without mathematics. Can you image being a human being without mathematics being at the center of your existence? Can you picture farmers sowing seeds in fruit and vegetable plantations? So now I am sure that you agree that we would not exist without mathematics having the power to explain the things which other sciences fail to explain. Hence drawing a picture of houses cannot bring itself to reality without mathematics. Physicists and structural engineers will not communicate without plan drawers and architects. The land we live on, will not be surfaced for our feet to walk on. And everything we know about mathematics will be forgotten if mathematics textbooks and television programmes, we unreliable.

Mathematics as a science, is not a core subject, but it can be considered important when integrated into other subjects. I recommend mathematics to the general reader and public and I strongly encourage the children of tomorrow to have a working knowledge of mathematics. I believe that applying mathematics and understanding when we are applying it is just as important as getting ready to start work or going to school. I am not sure if everyone agrees, but mathematics can be used in the academic environment to teach and educate others, students and the less fortunate. Also, we need to start making people aware about the importance of mathematics in the outside world, since we use mathematics in everything that we do. For example, restaurant use mathematics when cooking and baking for customers so that the finances within the business is regulated each month. People are blinded by such, because they are often unaware of how mathematics is used in the real world. I speak to many people daily and they always say that they don't see the need to study mathematics and physical sciences. We need to get out of this negative notion that the world will still exist without mathematics, because we can see that from this article – it won't. We need to educate ourselves and others about the things we do and their relevance when we do them. I hope this article helps to understand how important mathematics is and just how important the people who work in this field are. Nevertheless, we should remember that there is a wider range of careers that that utilise mathematics and that are important to this world.

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Certain Indefinite Integrals Involving Gegenbauer Polynomials

By Salahuddin

P.D.M College of Engineering, Bahadurgarh, Haryana, India

Abstract - In this paper we have established certain indefinite integrals involving Gegenbauer polynomials. The results represent here are assume to be new.

Keywords : Pochhammer Symbol; Gaussian Hypergeometric Function.

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CERTAIN INDEFINITE INTEGRALS INVOLVING GEGENBAUER POLYNOMIALS

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I. Introduction and Preliminaries

Gegenbauer polynomials

Gegenbauer polynomials or ultraspherical polynomials $C_n^{\alpha}(x)$ are orthogonal polynomials on the interval [-1,1] with respect to the weight function $(1-x^2)^{\alpha-\frac{1}{2}}$. They generalize Legendre polynomials and Chebyshev polynomials, and are special cases of Jacobi polynomials. They are named for Leopold Gegenbauer.

Gegenbauer polynomials are particular solutions of the Gegenbauer differential equation

$$(1-x^2)y'' - (2\alpha+1)xy' + n(n+2\alpha)y = 0 \tag{1.1}$$

When $\alpha = \frac{1}{2}$, the equation reduces to the Legendre equation, and the Gegenbauer polynomials reduce to the Legendre polynomials.

They are given as Gaussian hypergeometric series in certain cases where the series is in fact finite:

$$C_n^{\alpha}(z) = \frac{(2\alpha)_n}{n!}\, {}_2F_1\left(-n, (2\alpha+n); \alpha+\frac{1}{2}; \frac{1-z}{2}\right) \tag{1.2}$$

They are special cases of the Jacobi polynomials

$$C_n^{\alpha}(z) = \frac{(2\alpha)_n}{(\alpha+\frac{1}{2})_n} P_n^{(\alpha-\frac{1}{2},\alpha-\frac{1}{2})}(x) \tag{1.3}$$

One therefore also has the Rodrigues formula

$$C_n^{\alpha}(z) = \frac{(-2)^n}{n!}\frac{\Gamma(n+\alpha)\Gamma(n+2\alpha)}{\Gamma(\alpha)\Gamma(2n+2\alpha)}(1-x^2)^{-\alpha+\frac{1}{2}}\frac{d^n}{dx^n}\left[(1-x^2)^{n+\alpha-\frac{1}{2}}\right] \tag{1.4}$$

The Pochhammer's symbol is defined by

$$(b,\ k) = (b)_k = \frac{\Gamma(b+k)}{\Gamma(b)} = \begin{cases} b(b+1)(b+2)\cdots(b+k-1); & \text{if } k=1,2,3,\cdots \\ 1 \quad ; & \text{if } k=0 \\ k! \quad ; & \text{if } b=1,\ k=1,2,3,\cdots \end{cases} \tag{1.5}$$

Author : P.D.M College of Engineering, Bahadurgarh, Haryana, India. E-Mails: Vsludn@Gmail.Com , Sludn@Yahoo.Com

where b is neither zero nor negative integer and the notation Γ stands for Gamma function.

Polylogarithm function

The polylogarithm (also known as Jonquire's function) is a special function$Li_s(z)$ that is defined by the infinite sum, or power series:

$$Li_s(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^s} \tag{1.6}$$

It is in general not an elementary function, unlike the related logarithm function. The above definition is valid for all complex values of the order s and the argument z where$|\ z\ |< 1$.

Generalized Gaussian Hypergeometric Function

Generalized ordinary hypergeometric function of one variable is defined by

$$ {}_AF_B \left[\begin{array}{lll} a_1, a_2, \cdots, a_A & ; & \\ & & z \\ b_1, b_2, \cdots, b_B & ; & \end{array} \right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_A)_k z^k}{(b_1)_k (b_2)_k \cdots (b_B)_k k!}$$

or

$$ {}_AF_B \left[\begin{array}{lll} (a_A) & ; & \\ & & z \\ (b_B) & ; & \end{array} \right] \equiv {}_AF_B \left[\begin{array}{lll} (a_j)_{j=1}^A & ; & \\ & & z \\ (b_j)_{j=1}^B & ; & \end{array} \right] = \sum_{k=0}^{\infty} \frac{((a_A))_k z^k}{((b_B))_k k!} \tag{1.7}$$

where denominator parameters $b_1, b_2, \cdots, b_B$ are neither zero nor negative integers and A, B are non-negative integers.

II. Main Indefinite Integrals

$$\int \frac{\cosh x\ C_2(x)}{\sqrt{(1-\cosh x)}}\,\mathrm{dx} = -\frac{1}{\sqrt{1-\cosh x}}\ 2\sinh\frac{x}{2}\Big[-8x\ \mathrm{Li}_2(-e^{\frac{-x}{2}}) + 8x\ \mathrm{Li}_2(e^{\frac{-x}{2}}) -$$

$$-16\ \mathrm{Li}_3(-e^{\frac{-x}{2}}) + 16\ \mathrm{Li}_3(e^{\frac{-x}{2}}) - 2x^2\log(1-e^{-\frac{x}{2}}) + 2x^2\log(1+e^{-\frac{x}{2}}) - 4x^2\cosh\frac{x}{2} +$$

$$+16x\sinh\frac{x}{2} - 30\cosh\frac{x}{2} + \log\left(\tanh\frac{x}{4}\right)\Big] + Constant \tag{2.1}$$

$$\int \frac{\cosh x\ C_2(x)}{\sqrt{(1-\cos x)}}\,\mathrm{dx} = -\frac{1}{\sqrt{1-\cos x}}\left(\frac{22}{125} - \frac{4\iota}{125}\right) e^{(-1-\frac{\iota}{2})x}\sin\frac{x}{2}\times$$

$$\times\Big[(16-8\iota)e^{2x}x\ {}_3F_2\left(-\frac{1}{2}-\iota, -\frac{1}{2}-\iota, 1; \frac{1}{2}-\iota, \frac{1}{2}-\iota; e^{\iota x}\right) +$$

$$+(16-8\iota)e^{\iota x}x\ {}_3F_2\left(\frac{1}{2}+\iota, \frac{1}{2}+\iota, 1; \frac{3}{2}+\iota, \frac{3}{2}+\iota; e^{\iota x}\right) -$$

$$-16e^{2x}{}_4F_3\left(-\frac{1}{2}-\iota, -\frac{1}{2}-\iota, -\frac{1}{2}-\iota, 1; \frac{1}{2}-\iota, \frac{1}{2}-\iota, \frac{1}{2}-\iota; e^{\iota x}\right) +$$

$$+16e^{\iota x}{}_4F_3\left(\frac{1}{2}+\iota, \frac{1}{2}+\iota, \frac{1}{2}+\iota, 1; \frac{3}{2}+\iota, \frac{3}{2}+\iota, \frac{3}{2}+\iota; e^{\iota x}\right) -$$

$$-(6-8\iota)e^{2x}x^2\ {}_2F_1\left(-\frac{1}{2}-\iota, 1; \frac{1}{2}-\iota; e^{\iota x}\right) + (6-8\iota)e^{\iota x}x^2\ {}_2F_1\left(\frac{1}{2}+\iota, 1; \frac{3}{2}+\iota; e^{\iota x}\right) +$$

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$$+(3-4\iota)e^{2x}\ {}_2F_1\Big(-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota;e^{\iota x}\Big)-(3-4\iota)e^{\iota x}\ {}_2F_1\Big(\frac{1}{2}+\iota,1;\frac{3}{2}+\iota;e^{\iota x}\Big)+$$

$$+(6-8\iota)e^{2x}x^2-(16-8\iota)e^{2x}x+(13+4\iota)e^{2x}\Big]+Constant \tag{2.2}$$

$$\int\frac{\sinh x\ C_2(x)}{\sqrt{(1-\cos x)}}\,\mathrm{dx}=-\frac{1}{\sqrt{1-\cos x}}\Big(\frac{22}{125}-\frac{4\iota}{125}\Big)e^{(-1-\frac{\iota}{2})x}\sin\frac{x}{2}\times$$

$$\times\Big[(16-8\iota)e^{2x}x\ {}_3F_2\Big(-\frac{1}{2}-\iota,-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota,\frac{1}{2}-\iota;e^{\iota x}\Big)-$$

$$-(16-8\iota)e^{\iota x}x\ {}_3F_2\Big(\frac{1}{2}+\iota,\frac{1}{2}+\iota,1;\frac{3}{2}+\iota,\frac{3}{2}+\iota;e^{\iota x}\Big)-$$

$$-16e^{2x}{}_4F_3\Big(-\frac{1}{2}-\iota,-\frac{1}{2}-\iota,-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota,\frac{1}{2}-\iota,\frac{1}{2}-\iota;e^{\iota x}\Big)-$$

$$-16e^{\iota x}{}_4F_3\Big(\frac{1}{2}+\iota,\frac{1}{2}+\iota,\frac{1}{2}+\iota,1;\frac{3}{2}+\iota,\frac{3}{2}+\iota,\frac{3}{2}+\iota;e^{\iota x}\Big)-$$

$$-(6-8\iota)e^{2x}x^2\ {}_2F_1\Big(-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota;e^{\iota x}\Big)-(6-8\iota)e^{\iota x}x^2\ {}_2F_1\Big(\frac{1}{2}+\iota,1;\frac{3}{2}+\iota;e^{\iota x}\Big)+$$

$$+(3-4\iota)e^{2x}\ {}_2F_1\Big(-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota;e^{\iota x}\Big)+(3-4\iota)e^{\iota x}\ {}_2F_1\Big(\frac{1}{2}+\iota,1;\frac{3}{2}+\iota;e^{\iota x}\Big)+$$

$$+(6-8\iota)e^{2x}x^2-(16-8\iota)e^{2x}x+(13+4\iota)e^{2x}\Big]+Constant \tag{2.3}$$

$$\int\frac{\cosh x\ C_1(x)}{\sqrt{(1-\cos x)}}\,\mathrm{dx}=-\frac{1}{\sqrt{1-\cos x}}\Big(\frac{16}{25}-\frac{12\iota}{25}\Big)e^{(-1-\frac{\iota}{2})x}\sin\frac{x}{2}\times$$

$$\times\Big[2e^{2x}\ {}_3F_2\Big(-\frac{1}{2}-\iota,-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota,\frac{1}{2}-\iota;e^{\iota x}\Big)+2e^{\iota x}{}_3F_2\Big(\frac{1}{2}+\iota,\frac{1}{2}+\iota,1;\frac{3}{2}+\iota,\frac{3}{2}+\iota;e^{\iota x}\Big)-$$

$$-(2-\iota)e^{2x}\ x{}_2F_1\Big(-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota;e^{\iota x}\Big)+(2-\iota)e^{\iota x}{}_2F_1\Big(\frac{1}{2}+\iota,1;\frac{3}{2}+\iota;e^{\iota x}\Big)+$$

$$+(2-\iota)e^{2x}x-2e^{2x}\Big]+Constant \tag{2.4}$$

$$\int\frac{\sinh x\ C_1(x)}{\sqrt{(1-\cos x)}}\,\mathrm{dx}=-\frac{1}{\sqrt{1-\cos x}}\Big(\frac{16}{25}-\frac{12\iota}{25}\Big)e^{(-1-\frac{\iota}{2})x}\sin\frac{x}{2}\times$$

$$\times\Big[2e^{2x}\ {}_3F_2\Big(-\frac{1}{2}-\iota,-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota,\frac{1}{2}-\iota;e^{\iota x}\Big)-2e^{\iota x}{}_3F_2\Big(\frac{1}{2}+\iota,\frac{1}{2}+\iota,1;\frac{3}{2}+\iota,\frac{3}{2}+\iota;e^{\iota x}\Big)-$$

$$-(2-\iota)e^{2x}\ x{}_2F_1\Big(-\frac{1}{2}-\iota,1;\frac{1}{2}-\iota;e^{\iota x}\Big)-(2-\iota)e^{\iota x}{}_2F_1\Big(\frac{1}{2}+\iota,1;\frac{3}{2}+\iota;e^{\iota x}\Big)+$$

$$+(2-\iota)e^{2x}x-2e^{2x}\Big]+Constant \tag{2.5}$$

Notes

III. Derivation of The Integrals

Involving the same parallel method of ref[3] , one can derive the integrals.

IV. Applications

The integrals which are presented here are very special integrals.These are applied in the field of engineering and other allied sciences.

V. Conclusion

Notes

In our work we have established certain indefinite integrals involving Gegenbauer polynomials and Hypergeometric function . However, one can establish such type of integrals which are very useful for different field of engineering and sciences by involving these integrals.Thus we can only hope that the development presented in this work will stimulate further interest and research in this important area of classical special functions.

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14. Produce good diagrams of your own: Always try to include good charts or diagrams in your paper to improve quality. Using several and unnecessary diagrams will degrade the quality of your paper by creating "hotchpotch." So always, try to make and include those diagrams, which are made by your own to improve readability and understandability of your paper.

15. Use of direct quotes: When you do research relevant to literature, history or current affairs then use of quotes become essential but if study is relevant to science then use of quotes is not preferable.

16. Use proper verb tense: Use proper verb tenses in your paper. Use past tense, to present those events that happened. Use present tense to indicate events that are going on. Use future tense to indicate future happening events. Use of improper and wrong tenses will confuse the evaluator. Avoid the sentences that are incomplete.

17. Never use online paper: If you are getting any paper on Internet, then never use it as your research paper because it might be possible that evaluator has already seen it or maybe it is outdated version.

18. **Pick a good study spot:** To do your research studies always try to pick a spot, which is quiet. Every spot is not for studies. Spot that suits you choose it and proceed further.

19. Know what you know: Always try to know, what you know by making objectives. Else, you will be confused and cannot achieve your target.

20. Use good quality grammar: Always use a good quality grammar and use words that will throw positive impact on evaluator. Use of good quality grammar does not mean to use tough words, that for each word the evaluator has to go through dictionary. Do not start sentence with a conjunction. Do not fragment sentences. Eliminate one-word sentences. Ignore passive voice. Do not ever use a big word when a diminutive one would suffice. Verbs have to be in agreement with their subjects. Prepositions are not expressions to finish sentences with. It is incorrect to ever divide an infinitive. Avoid clichés like the disease. Also, always shun irritating alliteration. Use language that is simple and straight forward. put together a neat summary.

21. Arrangement of information: Each section of the main body should start with an opening sentence and there should be a changeover at the end of the section. Give only valid and powerful arguments to your topic. You may also maintain your arguments with records.

22. Never start in last minute: Always start at right time and give enough time to research work. Leaving everything to the last minute will degrade your paper and spoil your work.

23. Multitasking in research is not good: Doing several things at the same time proves bad habit in case of research activity. Research is an area, where everything has a particular time slot. Divide your research work in parts and do particular part in particular time slot.

24. Never copy others' work: Never copy others' work and give it your name because if evaluator has seen it anywhere you will be in trouble.

25. Take proper rest and food: No matter how many hours you spend for your research activity, if you are not taking care of your health then all your efforts will be in vain. For a quality research, study is must, and this can be done by taking proper rest and food.

26. Go for seminars: Attend seminars if the topic is relevant to your research area. Utilize all your resources.

27. Refresh your mind after intervals: Try to give rest to your mind by listening to soft music or by sleeping in intervals. This will also improve your memory.

28. Make colleagues: Always try to make colleagues. No matter how sharper or intelligent you are, if you make colleagues you can have several ideas, which will be helpful for your research.

29. **Think technically:** Always think technically. If anything happens, then search its reasons, its benefits, and demerits.

30. Think and then print: When you will go to print your paper, notice that tables are not be split, headings are not detached from their descriptions, and page sequence is maintained.

31. Adding unnecessary information: Do not add unnecessary information, like, I have used MS Excel to draw graph. Do not add irrelevant and inappropriate material. These all will create superfluous. Foreign terminology and phrases are not apropos. One should NEVER take a broad view. Analogy in script is like feathers on a snake. Not at all use a large word when a very small one would be sufficient. Use words properly, regardless of how others use them. Remove quotations. Puns are for kids, not grunt readers. Amplification is a billion times of inferior quality than sarcasm.

32. Never oversimplify everything: To add material in your research paper, never go for oversimplification. This will definitely irritate the evaluator. Be more or less specific. Also too, by no means, ever use rhythmic redundancies. Contractions aren't essential and shouldn't be there used. Comparisons are as terrible as clichés. Give up ampersands and abbreviations, and so on. Remove commas, that are, not necessary. Parenthetical words however should be together with this in commas. Understatement is all the time the complete best way to put onward earth-shaking thoughts. Give a detailed literary review.

33. Report concluded results: Use concluded results. From raw data, filter the results and then conclude your studies based on measurements and observations taken. Significant figures and appropriate number of decimal places should be used. Parenthetical remarks are prohibitive. Proofread carefully at final stage. In the end give outline to your arguments. Spot out perspectives of further study of this subject. Justify your conclusion by at the bottom of them with sufficient justifications and examples.

34. After conclusion: Once you have concluded your research, the next most important step is to present your findings. Presentation is extremely important as it is the definite medium though which your research is going to be in print to the rest of the crowd. Care should be taken to categorize your thoughts well and present them in a logical and neat manner. A good quality research paper format is essential because it serves to highlight your research paper and bring to light all necessary aspects in your research.

Informal Guidelines of Research Paper Writing

Key points to remember:

- Submit all work in its final form.
- Write your paper in the form, which is presented in the guidelines using the template.
- Please note the criterion for grading the final paper by peer-reviewers.

Final Points:

A purpose of organizing a research paper is to let people to interpret your effort selectively. The journal requires the following sections, submitted in the order listed, each section to start on a new page.

The introduction will be compiled from reference matter and will reflect the design processes or outline of basis that direct you to make study. As you will carry out the process of study, the method and process section will be constructed as like that. The result segment will show related statistics in nearly sequential order and will direct the reviewers next to the similar intellectual paths throughout the data that you took to carry out your study. The discussion section will provide understanding of the data and projections as to the implication of the results. The use of good quality references all through the paper will give the effort trustworthiness by representing an alertness of prior workings.

Writing a research paper is not an easy job no matter how trouble-free the actual research or concept. Practice, excellent preparation, and controlled record keeping are the only means to make straightforward the progression.

General style:

Specific editorial column necessities for compliance of a manuscript will always take over from directions in these general guidelines.

To make a paper clear

· Adhere to recommended page limits

Mistakes to evade

- Insertion a title at the foot of a page with the subsequent text on the next page

- Separating a table/chart or figure - impound each figure/table to a single page
- Submitting a manuscript with pages out of sequence

In every sections of your document

· Use standard writing style including articles ("a", "the," etc.)

· Keep on paying attention on the research topic of the paper

· Use paragraphs to split each significant point (excluding for the abstract)

· Align the primary line of each section

· Present your points in sound order

· Use present tense to report well accepted

· Use past tense to describe specific results

· Shun familiar wording, don't address the reviewer directly, and don't use slang, slang language, or superlatives

· Shun use of extra pictures - include only those figures essential to presenting results

Title Page:

Choose a revealing title. It should be short. It should not have non-standard acronyms or abbreviations. It should not exceed two printed lines. It should include the name(s) and address (es) of all authors.

Abstract:

The summary should be two hundred words or less. It should briefly and clearly explain the key findings reported in the manuscript--must have precise statistics. It should not have abnormal acronyms or abbreviations. It should be logical in itself. Shun citing references at this point.

An abstract is a brief distinct paragraph summary of finished work or work in development. In a minute or less a reviewer can be taught the foundation behind the study, common approach to the problem, relevant results, and significant conclusions or new questions.

Write your summary when your paper is completed because how can you write the summary of anything which is not yet written? Wealth of terminology is very essential in abstract. Yet, use comprehensive sentences and do not let go readability for briefness. You can maintain it succinct by phrasing sentences so that they provide more than lone rationale. The author can at this moment go straight to shortening the outcome. Sum up the study, with the subsequent elements in any summary. Try to maintain the initial two items to no more than one ruling each.

- Reason of the study - theory, overall issue, purpose
- Fundamental goal
- To the point depiction of the research
- Consequences, including definite statistics - if the consequences are quantitative in nature, account quantitative data; results of any numerical analysis should be reported
- Significant conclusions or questions that track from the research(es)

Approach:

- Single section, and succinct
- As a outline of job done, it is always written in past tense
- A conceptual should situate on its own, and not submit to any other part of the paper such as a form or table
- Center on shortening results - bound background information to a verdict or two, if completely necessary
- What you account in an conceptual must be regular with what you reported in the manuscript
- Exact spelling, clearness of sentences and phrases, and appropriate reporting of quantities (proper units, important statistics) are just as significant in an abstract as they are anywhere else

Introduction:

The **Introduction** should "introduce" the manuscript. The reviewer should be presented with sufficient background information to be capable to comprehend and calculate the purpose of your study without having to submit to other works. The basis for the study should be offered. Give most important references but shun difficult to make a comprehensive appraisal of the topic. In the introduction, describe the problem visibly. If the problem is not acknowledged in a logical, reasonable way, the reviewer will have no attention in your result. Speak in common terms about techniques used to explain the problem, if needed, but do not present any particulars about the protocols here. Following approach can create a valuable beginning:

- Explain the value (significance) of the study
- Shield the model - why did you employ this particular system or method? What is its compensation? You strength remark on its appropriateness from a abstract point of vision as well as point out sensible reasons for using it.
- Present a justification. Status your particular theory (es) or aim(s), and describe the logic that led you to choose them.
- Very for a short time explain the tentative propose and how it skilled the declared objectives.

Approach:

- Use past tense except for when referring to recognized facts. After all, the manuscript will be submitted after the entire job is done.
- Sort out your thoughts; manufacture one key point with every section. If you make the four points listed above, you will need a least of four paragraphs.
- Present surroundings information only as desirable in order hold up a situation. The reviewer does not desire to read the whole thing you know about a topic.
- Shape the theory/purpose specifically - do not take a broad view.
- As always, give awareness to spelling, simplicity and correctness of sentences and phrases.

Procedures (Methods and Materials):

This part is supposed to be the easiest to carve if you have good skills. A sound written Procedures segment allows a capable scientist to replacement your results. Present precise information about your supplies. The suppliers and clarity of reagents can be helpful bits of information. Present methods in sequential order but linked methodologies can be grouped as a segment. Be concise when relating the protocols. Attempt for the least amount of information that would permit another capable scientist to spare your outcome but be cautious that vital information is integrated. The use of subheadings is suggested and ought to be synchronized with the results section. When a technique is used that has been well described in another object, mention the specific item describing a way but draw the basic principle while stating the situation. The purpose is to text all particular resources and broad procedures, so that another person may use some or all of the methods in one more study or referee the scientific value of your work. It is not to be a step by step report of the whole thing you did, nor is a methods section a set of orders.

Materials:

- Explain materials individually only if the study is so complex that it saves liberty this way.
- Embrace particular materials, and any tools or provisions that are not frequently found in laboratories.
- Do not take in frequently found.
- If use of a definite type of tools.
- Materials may be reported in a part section or else they may be recognized along with your measures.

Methods:

- Report the method (not particulars of each process that engaged the same methodology)
- Describe the method entirely
- To be succinct, present methods under headings dedicated to specific dealings or groups of measures
- Simplify - details how procedures were completed not how they were exclusively performed on a particular day.
- If well known procedures were used, account the procedure by name, possibly with reference, and that's all.

Approach:

- It is embarrassed or not possible to use vigorous voice when documenting methods with no using first person, which would focus the reviewer's interest on the researcher rather than the job. As a result when script up the methods most authors use third person passive voice.
- Use standard style in this and in every other part of the paper - avoid familiar lists, and use full sentences.

What to keep away from

- Resources and methods are not a set of information.
- Skip all descriptive information and surroundings - save it for the argument.
- Leave out information that is immaterial to a third party.

Results:

The principle of a results segment is to present and demonstrate your conclusion. Create this part a entirely objective details of the outcome, and save all understanding for the discussion.

The page length of this segment is set by the sum and types of data to be reported. Carry on to be to the point, by means of statistics and tables, if suitable, to present consequences most efficiently.You must obviously differentiate material that would usually be incorporated in a study editorial from any unprocessed data or additional appendix matter that would not be available. In fact, such matter should not be submitted at all except requested by the instructor.

Content

- Sum up your conclusion in text and demonstrate them, if suitable, with figures and tables.
- In manuscript, explain each of your consequences, point the reader to remarks that are most appropriate.
- Present a background, such as by describing the question that was addressed by creation an exacting study.
- Explain results of control experiments and comprise remarks that are not accessible in a prescribed figure or table, if appropriate.
- Examine your data, then prepare the analyzed (transformed) data in the form of a figure (graph), table, or in manuscript form.

What to stay away from

- Do not discuss or infer your outcome, report surroundings information, or try to explain anything.
- Not at all, take in raw data or intermediate calculations in a research manuscript.

- Do not present the similar data more than once.
- Manuscript should complement any figures or tables, not duplicate the identical information.
- Never confuse figures with tables - there is a difference.

Approach

- As forever, use past tense when you submit to your results, and put the whole thing in a reasonable order.
- Put figures and tables, appropriately numbered, in order at the end of the report
- If you desire, you may place your figures and tables properly within the text of your results part.

Figures and tables

- If you put figures and tables at the end of the details, make certain that they are visibly distinguished from any attach appendix materials, such as raw facts
- Despite of position, each figure must be numbered one after the other and complete with subtitle
- In spite of position, each table must be titled, numbered one after the other and complete with heading
- All figure and table must be adequately complete that it could situate on its own, divide from text

Discussion:

The Discussion is expected the trickiest segment to write and describe. A lot of papers submitted for journal are discarded based on problems with the Discussion. There is no head of state for how long a argument should be. Position your understanding of the outcome visibly to lead the reviewer through your conclusions, and then finish the paper with a summing up of the implication of the study. The purpose here is to offer an understanding of your results and hold up for all of your conclusions, using facts from your research and generally accepted information, if suitable. The implication of result should be visibly described. Infer your data in the conversation in suitable depth. This means that when you clarify an observable fact you must explain mechanisms that may account for the observation. If your results vary from your prospect, make clear why that may have happened. If your results agree, then explain the theory that the proof supported. It is never suitable to just state that the data approved with prospect, and let it drop at that.

- Make a decision if each premise is supported, discarded, or if you cannot make a conclusion with assurance. Do not just dismiss a study or part of a study as "uncertain."
- Research papers are not acknowledged if the work is imperfect. Draw what conclusions you can based upon the results that you have, and take care of the study as a finished work
- You may propose future guidelines, such as how the experiment might be personalized to accomplish a new idea.
- Give details all of your remarks as much as possible, focus on mechanisms.
- Make a decision if the tentative design sufficiently addressed the theory, and whether or not it was correctly restricted.
- Try to present substitute explanations if sensible alternatives be present.
- One research will not counter an overall question, so maintain the large picture in mind, where do you go next? The best studies unlock new avenues of study. What questions remain?
- Recommendations for detailed papers will offer supplementary suggestions.

Approach:

- When you refer to information, differentiate data generated by your own studies from available information
- Submit to work done by specific persons (including you) in past tense.
- Submit to generally acknowledged facts and main beliefs in present tense.

Administration Rules Listed Before
Submitting Your Research Paper to Global Journals Inc. (US)

Please carefully note down following rules and regulation before submitting your Research Paper to Global Journals Inc. (US):

Segment Draft and Final Research Paper: You have to strictly follow the template of research paper. If it is not done your paper may get rejected.

- The **major constraint** is that you must independently make all content, tables, graphs, and facts that are offered in the paper. You must write each part of the paper wholly on your own. The Peer-reviewers need to identify your own perceptive of the concepts in your own terms. NEVER extract straight from any foundation, and never rephrase someone else's analysis.

- Do not give permission to anyone else to "PROOFREAD" your manuscript.

- Methods to avoid Plagiarism is applied by us on every paper, if found guilty, you will be blacklisted by all of our collaborated research groups, your institution will be informed for this and strict legal actions will be taken immediately.)
- To guard yourself and others from possible illegal use please do not permit anyone right to use to your paper and files.

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Topics	Grades		
	A-B	**C-D**	**E-F**
Abstract	Clear and concise with appropriate content, Correct format. 200 words or below	Unclear summary and no specific data, Incorrect form Above 200 words	No specific data with ambiguous information Above 250 words
Introduction	Containing all background details with clear goal and appropriate details, flow specification, no grammar and spelling mistake, well organized sentence and paragraph, reference cited	Unclear and confusing data, appropriate format, grammar and spelling errors with unorganized matter	Out of place depth and content, hazy format
Methods and Procedures	Clear and to the point with well arranged paragraph, precision and accuracy of facts and figures, well organized subheads	Difficult to comprehend with embarrassed text, too much explanation but completed	Incorrect and unorganized structure with hazy meaning
Result	Well organized, Clear and specific, Correct units with precision, correct data, well structuring of paragraph, no grammar and spelling mistake	Complete and embarrassed text, difficult to comprehend	Irregular format with wrong facts and figures
Discussion	Well organized, meaningful specification, sound conclusion, logical and concise explanation, highly structured paragraph reference cited	Wordy, unclear conclusion, spurious	Conclusion is not cited, unorganized, difficult to comprehend
References	Complete and correct format, well organized	Beside the point, Incomplete	Wrong format and structuring

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