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Creation of a Summation Formula Attached with Recurrence Relation and Hypergeometric Function

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Jai Bhagwan [α] & Salahuddin [σ]

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I. Introduction

a) Generalized Hypergeometric Functions

A generalized hypergeometric function ${}_pF_q(a_1, ..., a_p; b_1, ..., b_q; z)$ is a function which can be defined in the form of a hypergeometric series, i.e., a series for which the ratio of successive terms can be written

$$\frac{c_{k+1}}{c_k} = \frac{P(k)}{Q(k)} = \frac{(k+a_1)(k+a_2)...(k+a_p)}{(k+b_1)(K+b_2)...(k+b_q)(k+1)}\, z. \tag{1}$$

Where $k+1$ in the denominator is present for historical reasons of notation[Koepf p.12(2.9)], and the resulting generalized hypergeometric function is written

$$ {}_pF_q\left[\begin{matrix} a_1, a_2, \cdots, a_p & ; \\ & & z \\ b_1, b_2, \cdots, b_q & ; \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k \cdots (a_p)_k z^k}{(b_1)_k (b_2)_k \cdots (b_q)_k k!} \tag{2}$$

or

$$ {}_pF_q\left[\begin{matrix} (a_p) & ; \\ & & z \\ (b_q) & ; \end{matrix}\right] = \sum_{k=0}^{\infty} \frac{((a_p))_k z^k}{((b_q))_k k!} \tag{3}$$

where the parameters $b_1, b_2, \cdots, b_q$ are positive integers.

The ${}_pF_q$ series converges for all finite z if $p \le q$, converges for $|\ z\ |< 1$ if $p = q+1$, diverges for all z , $z \neq 0$ if $p > q+1$[Luke p.156(3)].

The function ${}_2F_1(a, b; c; z)$ corresponding to $p = 2, q = 1$, is the first hypergeometric function to be studied (and, in general, arises the most frequently in physical problems), and so is frequently known as "the" hypergeometric equation or, more explicitly, Gauss's hypergeometric function [Gauss p.123-162]. To confuse matters even more, the term "hypergeometric function" is less commonly used to mean closed form, and "hypergeometric series" is sometimes used to mean hypergeometric function.

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The hypergeometric functions are solutions of Gaussian hypergeometric linear differential equation of second order

$$z(1-z)y'' + [c-(a+b+1)z]y' - aby = 0 \tag{4}$$

The solution of this equation is

$$y = A_0\left[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\right] \tag{5}$$

This is the so-called regular solution, denoted

$${}_2F_1(a,b;c;z) = \left[1 + \frac{ab}{1!\ c}z + \frac{a(a+1)b(b+1)}{2!\ c(c+1)}z^2 + \cdots\cdots\right] = \sum_{k=0}^{\infty} \frac{(a)_k\ (b)_k z^k}{(c)_k k!} \tag{6}$$

which converges if c is not a negative integer for all $\mid z \mid < 1$ and on the unit circle $\mid z \mid = 1$ if $R(c-a-b) > 0$.

It is known as Gauss hypergeometric function in terms of Pochhammer symbol $(a)_k$ or generalized factorial function.

Many of the common mathematical functions can be expressed in terms of the hypergeometric function. Some typical examples are

$$(1-z)^{-a} = z\ {}_2F_1(1,1;2;-z) \tag{7}$$

$$\sin^{-1} z = z\ {}_2F_1(\frac{1}{2},\frac{1}{2};\frac{3}{2};z^2) \tag{8}$$

b) *Gauss' Relations For Contiguous Functions*

The six functions $F(a\pm1,b;c;z), F(a,b\pm1;c;z), F(a,b;c\pm1;z)$ are called contiguous to $F(a,b;c;z)$. Relation between $F(a,b;c;z)$ and any two contiguous functions have been given by Gauss.

[Abramowitz p.558(15.2.19)]

$$(a-b)\ (1-z)\ {}_2F_1\left[\begin{matrix} a,\ b\ ; \\ c\ \ ; \end{matrix}\ z\right] = (c-b)\ {}_2F_1\left[\begin{matrix} a,\ b-1\ \ ; \\ c\ \ ; \end{matrix}\ z\right] + (a-c)\ {}_2F_1\left[\begin{matrix} a-1,\ b\ \ ; \\ c\ ; \end{matrix}\ z\right] \tag{9}$$

c) *Recurrence Relation*

In mathematics, a recurrence relation is an equation that recursively defines a sequence, once one or more initial terms are given: each further term of the sequence is defined as a function of the preceding terms.The recurrence relation of Gamma function is defined by [Temme p.42(3.1.1)]

$$\Gamma(z+1) = z\ \Gamma(z) \tag{10}$$

d) *Legendre's Duplication Formula*

Following Bells and Wong [p.26(2.3.1)], we have

$$\sqrt{\pi}\ \Gamma(2z) = 2^{(2z-1)}\ \Gamma(z)\ \Gamma\left(z+\frac{1}{2}\right) \tag{11}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} = \frac{2^{(b-1)}\ \Gamma(\frac{b}{2})\ \Gamma(\frac{b+1}{2})}{\Gamma(b)} \tag{12}$$

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$$= \frac{2^{(a-1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\Gamma(a)} \tag{13}$$

Following Prudnikov, Brychkov & Marichev [p.491.(7.3.7.3)], we have

$$ {}_2F_1\left[\begin{array}{cc} a, \;\; b \;; & \frac{1}{2} \\ \frac{a+b-1}{2} \;; & \end{array}\right] = \sqrt{\pi}\left[\frac{\Gamma(\frac{a+b+1}{2})}{\Gamma(\frac{a+1}{2})\Gamma(\frac{b+1}{2})} + \frac{2\,\Gamma(\frac{a+b-1}{2})}{\Gamma(a)\Gamma(b)}\right] \tag{14}$$

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Now using Legendre's duplication formula and Recurrence relation for Gamma function, the above formula can be written in the form

$$ {}_2F_1\left[\begin{array}{cc} a, \;\; b \;; & \frac{1}{2} \\ \frac{a+b-1}{2} \;; & \end{array}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})} + \frac{2^{(a-b+1)}\,\Gamma(\frac{a}{2})\,\Gamma(\frac{a+1}{2})}{\{\Gamma(a)\}^2} + \frac{\Gamma(\frac{b+2}{2})}{\Gamma(\frac{a+1}{2})}\right] \tag{15}$$

It is noted that the above formula [Prudnikov,491.(7.3.7.3)], i.e. equation(14) or (15) is not correct.The correct form of equation(14) or (15) is obtained by [Asish et. al(2008), p.337(10)]

$$ {}_2F_1\left[\begin{array}{cc} a, \;\; b \;; & \frac{1}{2} \\ \frac{a+b-1}{2} \;; & \end{array}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-1}{2})}{\Gamma(b)}\left[\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-1}{2})}\left\{\frac{(b+a-1)}{(a-1)}\right\} + \frac{2\,\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a}{2})}\right] \tag{16}$$

Involving the derived formula obtained by [Salahuddin,p .45(9)], we establish the main formula.

II. Main Summation Formula

For the main formula $a \neq b$

For $a < 1$ and $a > 30$

$$ {}_2F_1\left[\begin{array}{cc} a, \;\; b \;; & \frac{1}{2} \\ \frac{a+b-30}{2} \;; & \end{array}\right] = \frac{2^{(b-1)}\,\Gamma(\frac{a+b-30}{2})}{(a-b)\Gamma(b)}\left[\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-29}{2})}\left\{\frac{(-42849873690624000a)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}} + \right.\right.$$

$$+\frac{(102174938785382400a^2 - 90168185255362560a^3 + 48238440075952128a^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-14222757092524032a^5 + 3434748504629248a^6 - 476013869035520a^7 + 63240546512640a^8)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-4603989262336a^9 + 362770106624a^{10} - 14085527040a^{11} + 665833376a^{12} - 12792832a^{13})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(343728a^{14} - 2480a^{15} + 31a^{16} + 42849873690624000b - 31082092167168000ab)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-73098173806018560a^2b + 108868047825272832a^3b - 57797146289504256a^4b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(22066910445879296a^5b-42782189664296696a^6b+807376117319680a^7b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-74955432767488a^8b+8169315368448a^9b-385635936768a^{10}b+25593453184a^{11}b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-582589696a^{12}b+23404256a^{13}b-198896a^{14}b+4464a^{15}b-71092846618214400b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(112755981161594880ab^2-36482063700197376a^2b^2-293431385315082224a^3b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(30395328111255552a^4b^2-10197276680204288a^5b^2+2946095411127296a^6b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-374548696047616a^7b^2+56962219478272a^8b^2-3393695599104a^9b^2+308716484928a^{10}b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-8514867712a^{11}b^2+480008464a^{12}b^2-4847408a^{13}b^2+165416a^{14}b^2+50510377899786240b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-99910969798164480ab^3+54967053635026944a^2b^3-10666441061793792a^3b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-4166767625613312a^4b^3+3206675295074304a^5b^3-686051126108160a^6b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(156167659677696a^7b^3-12540657489408a^8b^3+1572782347392a^9b^3-54323189760a^{10}b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(4180393152a^{11}b^3-51048816a^{12}b^3+2459664a^{13}b^3-20713454402863104b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(40852434546720768ab^4-32041240778039296a^2b^4+8698430393331712a^3b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-1154676393076224a^4b^4-247159742668800a^5b^4+147864940085760a^6b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-19581576698880a^7b^4+3617566362720a^8b^4-166428633600a^9b^4+17522948400a^{10}b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

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$$+\frac{(-266463600a^{11}b^4+17530500a^{12}b^4+5543553731788800b^5-1215540098428108 8ab^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(8180404586819584a^2b^5-3671679306500096a^3b^5+552177907875840a^4b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-54103342402560a^5b^5-6502833792000a^6b^5+3126362400000a^7b^5-233673258240a^8b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(35839030560a^9b^5-719559600a^{10}b^5+64512240a^{11}b^5-1033904237649920b^6)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(2099766917263360ab^6-1867829847333888a^2b^6+595797693046784a^3b^6)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-178743343074816a^4b^6+15207496657920a^5b^6-1156337239680a^6b^6-73365304320a^7b^6)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(29075170320a^8b^6-937908720a^9b^6+121580760a^{10}b^6+139675243868160b^7)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-315216253958144ab^7+210439757914112a^2b^7-108338835331072a^3b^7)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(17944242066432a^4b^7-3924412035840a^5b^7+177220085760a^6b^7-10838056320a^7b^7)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-282861360a^8b^7+94287120a^9b^7-13985569165568b^8+27853051633664ab^8)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-25906772757248a^2b^8+7697887351296a^3b^8-2669359676832a^4b^8+222457190400a^5b^8)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-37501461360a^6b^8+699709680a^7b^8-35357670a^8b^8+1050576384000b^9)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-2375213023744ab^9+1454416768512a^2b^9-790386876288a^3b^9+108636023040a^4b^9)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-27632383200a^5b^9+918058800a^6b^9-124062000a^7b^9-59398127360b^{10})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(111928327680ab^{10}-105528303296a^2b^{10}+25592473088a^3b^{10}-9433933392a^4b^{10})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(490854000a^5b^{10}-96768360a^6b^{10}+2513871360b^{11}-5584355712ab^{11}+2860642304a^2b^{11})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-1587936064a^3b^{11}+134073264a^4b^{11}-36463440a^5b^{11}-78393952b^{12}+132474368ab^{12})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-123782256a^2b^{12}+18886192a^3b^{12}-7152444a^4b^{12}+1747200b^{13}-3719968ab^{13})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\left.\frac{-704816a^3b^{13}-26320b^{14}+34480ab^{14}-31000a^2b^{14}+240b^{15}-464ab^{15}-b^{16})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}\right\}+$$

$$+\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-30}{2})}\left\{\frac{(-42849873690624000a+71092846618214400a^2-50510377899786240a^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+\right.$$

$$+\frac{(20713454402863104a^4-5543553731788800a^5+1033904237649920a^6-139675243868160a^7)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(13985569165568a^8-1050576384000a^9+59398127360a^{10}-2513871360a^{11}+78393952a^{12})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-1747200a^{13}+26320a^{14}-240a^{15}+a^{16}+42849873690624000b+31082092167168000ab)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-112755981161594880a^2b+99910969798164480a^3b-40852434546720768a^4b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(12155400984281088a^5b-2099766917263360a^6b+315216253958144a^7b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-27853051633664a^8b+2375213023744a^9b-111928327680a^{10}b+5584355712a^{11}b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-132474368a^{12}b+3719968a^{13}b-34480a^{14}b+464a^{15}b-102174938785382400b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(73098173806018560ab^2+36482063700197376a^2b^2-54967053635026944a^3b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

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$$+\frac{(32041240778039296a^4b^2-8180404586819584a^5b^2+1867829847333888a^6b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-210439757914112a^7b^2+25906772757248a^8b^2-1454416768512a^9b^2+105528303296a^{10}b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-2860642304a^{11}b^2+123782256a^{12}b^2-1274224a^{13}b^2+31000a^{14}b^2+90168185255362560b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-108868047825272832ab^3+29343138531508224a^2b^3+10666441061793792a^3b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-8698430393331712a^4b^3+3671679306500096a^5b^3-595797693046784a^6b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(108338835331072a^7b^3-7697887351296a^8b^3+790386876288a^9b^3-25592473088a^{10}b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(1587936064a^{11}b^3-18886192a^{12}b^3+704816a^{13}b^3-48238440075952128b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(57797146289504256ab^4-30395328111255552a^2b^4+4166767625613312a^3b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(1154676393076224a^4b^4-552177907875840a^5b^4+178743343074816a^6b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-17944242066432a^7b^4+2669359676832a^8b^4-108636023040a^9b^4+9433933392a^{10}b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-134073264a^{11}b^4+7152444a^{12}b^4+14222757092524032b^5-22066910445879296ab^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(10197276680204288a^2b^5-3206675295074304a^3b^5+247159742668800a^4b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(54103342402560a^5b^5-15207496657920a^6b^5+3924412035840a^7b^5-222457190400a^8b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(27632383200a^9b^5-490854000a^{10}b^5+36463440a^{11}b^5-34347485046292486b^6)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(4278218966429696ab^6-2946095411127296a^2b^6+686051126108160a^3b^6)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-147864940085760a^4b^6+6502833792000a^5b^6+1156337239680a^6b^6-177220085760a^7b^6)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(37501461360a^8b^6-918058800a^9b^6+96768360a^{10}b^6+4760138690355520b^7)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-807376117319680ab^7+374548696047616a^2b^7-156167659677696a^3b^7)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(19581576698880a^4b^7-3126362400000a^5b^7+73365304320a^6b^7+10838056320a^7b^7)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-699709680a^8b^7+124062000a^9b^7-632405465126400b^8+74955432767488ab^8)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-56962219478272a^2b^8+12540657489408a^3b^8-3617566362720a^4b^8+233673258240a^5b^8)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-29075170320a^6b^8+282861360a^7b^8+35357670a^8b^8+4603989262336b^9)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-8169315368448ab^9+3393695599104a^2b^9-1572782347392a^3b^9+166428633600a^4b^9)}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-35839030560a^5b^9+937908720a^6b^9-94287120a^7b^9-362770106624b^{10})}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(385635936768ab^{10}-308716484928a^2b^{10}+54323189760a^3b^{10}-17522948400a^4b^{10})}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(719559600a^5b^{10}-121580760a^6b^{10}+14085527040b^{11}-25593453184ab^{11}+8514867712a^2b^{11})}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-4180393152a^3b^{11}+266463600a^4b^{11}-64512240a^5b^{11}-665833376b^{12}+582589696ab^{12})}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-480008464a^2b^{12}+51048816a^3b^{12}-17530500a^4b^{12}+12792832b^{13}-23404256ab^{13})}{\prod\limits_{\eta=1}^{15}\{a-2\eta\}}+$$

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$$+\frac{(4847408a^2b^{13}-2459664a^3b^{13}-343728b^{14}+198896ab^{14}-165416a^2b^{14}+2480b^{15})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-4464ab^{15}-31b^{16})}{\prod_{\eta=1}^{15}\{a-2\eta\}}\Bigg\}\Bigg] \quad (17)$$

Notes

III. Evaluation Of Main Summation Formula

Substituting $c=\frac{a+b-30}{2}$ and $z=\frac{1}{2}$ in equation (24), we get

$$(a-b)\,{}_2F_1\left[\begin{matrix}a,\ b & ; \\ \frac{a+b-30}{2} & ;\end{matrix}\ \frac{1}{2}\right]=(a-b-30)\,{}_2F_1\left[\begin{matrix}a,\ b-1 & ; \\ \frac{a+b-30}{2} & ;\end{matrix}\ \frac{1}{2}\right]+(a-b+30)\,{}_2F_1\left[\begin{matrix}a-1,\ b & ; \\ \frac{a+b-30}{2} & ;\end{matrix}\ \frac{1}{2}\right]$$

Now involving the the formula obtained by salahuddin[Salahuddin,p .45(9)], we get

$$L.H.S=\frac{2^{(b-1)}\,\Gamma(\frac{a+b-30}{2})}{\Gamma(b)}\Bigg[\frac{(a-b-30)}{(a-b+1)}\,\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-29}{2})}\Bigg\{\frac{(-42849873690624000)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(56468406848716800a+100909430813491200a^2-45646821227298816a^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(30119994532429824a^4-10659242374635520a^5+2384177957457920a^6)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-372420039991552a^7+41100528816128a^8-3372406380160a^9+199271883200a^{10})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-8841030848a^{11}+271930672a^{12}-6056120a^{13}+80260a^{14}-659a^{15}+a^{16})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(113942720308838400b-192432640779878400ab+71179471376547840a^2b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(31565349178638336a^3b-35990351353896960a^4b+14307315699220480a^5b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-3391367994865920a^6b+532460361363712a^7b-59560797031680a^8b+4745684818240a^9b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-279195240480a^{10}b+11619237968a^{11}b-349449360a^{12}b+6800780a^{13}b-84165a^{14}b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(434a^{15}b-121603224518000640b^2+2296633293147340 80ab^2-132562310349127680a^2b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(17716361173647360a^3b^2+12817606114037760a^4b^2-7268501921936640a^5b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1903248210151680a^6b^2-309340014067200a^7b^2+34669345543680a^8b^2-2718132397440a^9b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(154142180400a^{10}b^2-6044516400a^{11}b^2+166890360a^{12}b^2-2737455a^{13}b^2+26970a^{14}b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(7122383230264934 4b^3-142411269625675776ab^3+96502735620096000a^2b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-27158505721528320a^3b^3+10460526963187 20a^4b^3+1645411005646080a^5b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-540144838041600a^6b^3+97422347546880a^7b^3-10689025278240a^8b^3+855046307760a^9b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-43587835200a^{10}b^3+1695549960a^{11}b^3-35236305a^{12}b^3+566370a^{13}b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-26257008134651904b^4+54170505981911040ab^4-39455725614796800a^2b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(13679204053597440a^3b^4-2120890334688000a^4b^4-190255269312 00a^5b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(84920244816000a^6b^4-16677013852800a^7b^4+2116502035200a^8b^4-145414149000a^9b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(8158694700a^10b^4-212995575a^{11}b^4+5259150a^{12}b^4+6577457969438720b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-13820286467276800ab^5+10393670565008640a^2b^5-3918081302580480a^3b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(791845793625600a^4b^5-69492783427200a^5b^5-2483999138880a^6b^5+1910624274720a^7b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

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$$+\frac{(-212369331600a^{8}b^{5}+19049720100a^{9}b^{5}-641090385a^{10}b^{5}+24192090a^{11}b^{5})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-11735794815180800b^{6}+2469322022918400ab^{6}-1911816899339520a^{2}b^{6})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

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$$+\frac{(732192289313280a^{3}b^{6}-163543093881600a^{4}b^{6}+20082084664320a^{5}b^{6}-941035082400a^{6}b^{6})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-48979677600a^{7}b^{6}+18237114000a^{8}b^{6}-901000275a^{9}b^{6}+56448210a^{10}b^{6})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1536608130337728b^{7}-326583853541632ab^{7}+245643692083200a^{2}b^{7}-99106124778240a^{3}b^{7})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(21459053764800a^{4}b^{7}-3012890332320a^{5}b^{7}+216761126400a^{6}b^{7}-4242920400a^{7}b^{7})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-265182525a^{8}b^{7}+58929450a^{9}b^{7}-150361455549568b^{8}+31156475560320ab^{8})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-24451722638400a^{2}b^{8}+8940052710720a^{3}b^{8}-2152280437200a^{4}b^{8}+260046358200a^{5}b^{8})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-23317452900a^{6}b^{8}+795547575a^{7}b^{8}+1109974511360b^{9}-2340242526400ab^{9})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1645789231200a^{2}b^{9}-673510158960a^{3}b^{9}+125836626000a^{4}b^{9}-19235813100a^{5}b^{9})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1052976225a^{6}b^{9}-58929450a^{7}b^{9}-61911998720b^{10}+122169740160ab^{10})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-95144334480a^{2}b^{10}+29274532560a^{3}b^{10}-7159456200a^{4}b^{10}+568514115a^{5}b^{10})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-56448210a^{6}b^{10}+2592265312b^{11}-5337467408ab^{11}+3197563200a^{2}b^{11}-1300061880a^{3}b^{11})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(155144925a^{4}b^{11}-24192090a^{5}b^{11}-80141152b^{12}+142613640ab^{12})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-108689100a^2b^{12}+21562515a^3b^{12}-5259150a^4b^{12}+1773520b^{13}-3493700ab^{13})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(1415925a^2b^{13}-566370a^3b^{13}-26560b^{14}+36735ab^{14}-26970a^2b^{14}+241b^{15})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-434ab^{15}-b^{16})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}\Bigg\}+\frac{(a-b-30)(b-1)}{(a-b+1)}\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-30}{2})}\Bigg\{\frac{(42849873690624000)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-26814643804569600a-21476290178580480a^2+29081047157637120a^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-14200532432486400a^4+4036298413056000a^5-759685713346560a^6+100366817591040a^7)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-9589432195200a^8+671530516800a^9-34499905440a^{10}+1285480560a^{11}-33797400a^{12})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(594300a^{13}-6270a^{14}+30a^{15}-100746609662361600b+105369623089643520ab)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-18497359471509504a^2b-19697219949428736a^3b+13886382286397440a^4b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-4339068030189568a^5b+857574469125888a^6b-113217893178368a^7b+10859483768000a^8b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-731396205504a^9b+36817157936a^{10}b-1257372896a^{11}b+31361460a^{12}b-443548a^{13}b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(4030a^{14}b+92490335838535680b^2-113987664871096320ab^2+46383618741829632a^2b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-19263051474370 56a^3b^2-4380147255427072a^4b^2+1878105598159616a^5b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-386934853266944a^6b^2+54537090886912a^7b^2-4938029127456a^8b^2+342285504528a^9b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-15032099888a^{10}b^2+523484104a^{11}b^2-9419722a^{12}b^2+138446a^{13}b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

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$$+\frac{(-4663216583206502\,4b^3+62163571923615744ab^3-30822099684507648a^2b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(6525530932051968a^3b^3+131032468218624a^4b^3-348720370467840a^5b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(98495967926016a^6b^3-13207757720832a^7b^3+1340626529232a^8b^3-76951667040a^9b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(3775087992a^{10}b^3-84987864a^{11}b^3+1893294a^{12}b^3+14859663097823232b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-20711447922302976ab^4+11169873454153728a^2b^4-2983163756323584a^3b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(363675061996800a^4b^4+20359470898560a^5b^4-11353733939520a^6b^4+2209706471520a^7b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-178725335400a^8b^4+13011676500a^9b^4-369893550a^{10}b^4+12271350a^{11}b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-3271193580412928b^5+4569102979137536ab^5-2610314589022976a^2b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(753958524042240a^3b^5-120686089584000a^4b^5+8674613539200a^5b^5+705515781600a^6b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-150958641600a^7b^5+20749369500a^8b^5-787793700a^9b^5+40320150a^{10}b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(505025512816640b^6-733037562791680ab^6+405911801172480a^2b^6-127441219311360a^3b^6)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(21411848928960a^4b^6-2132804429280a^5b^6+86079178080a^6b^6+8858026800a^7b^6)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-664352010a^8b^6+65132550a^9b^6-59551759843072b^7+81170061386752ab^7)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-48474589768960a^2b^7+13733274514176a^3b^7-2650892519520a^4b^7+253026930240a^5b^7)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-15527599920a^6b^7+282861360a^7b^7+35357670a^8b^7+4947215345536b^8)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-7286981375168ab^8+3740645877984a^2b^8-1232794859472a^3b^8+183751464600a^4b^8)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-22893160860a^5b^8+1003041270a^6b^8-35357670a^7b^8-339191763264b^9+424211354688ab^9)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-257569498128a^2b^9+60858560160a^3b^9-12407548500a^4b^9+800199900a^5b^9)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-65132550a^6b^9+15279519840b^10-22632383280ab^{10}+9573055440a^2b^{10})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-3218599800a^3b^{10}+303277650a^4b^{10}-40320150a^5b^{10}-621837424b^{11}+649171744ab^{11})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-397825480a^2b^{11}+58621992a^3b^{11}-12271350a^4b^{11}+139394326^{12}-20735156ab^{12})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(5539638a^2b^{12}-1893294a^3b^{12}-323708b^{13}+223076ab^{13}-138446a^2b^{13}+2690b^{14})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-4030ab^{14}-30b^{15})}{\prod_{\eta=1}^{15}\{a-2\eta\}}\Bigg\}\Bigg]+\frac{2^{(b-1)}\;\Gamma(\frac{a+b-30}{2})}{\Gamma(b)}\Bigg[\frac{(a-b+30)}{(a-b-1)}\frac{\Gamma(\frac{b}{2})}{\Gamma(\frac{a-30}{2})}\times$$

$$+\frac{(42849873690624000-113942720308838400a+121603224518000640a^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-71223832302649344a^3+26257008134651904a^4-6577457969438720a^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(1173579481518080a^6-153660813033728a^7+15036145549568a^8-1109974511360a^9)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(61911998720a^{10}-2592265312a^{11}+80141152a^{12}-1773520a^{13}+26560a^{14}-241a^{15}+a^{16})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-56468406848716800b+192432640779878400ab-229663329314734080a^2b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

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$$+\frac{(142411269625675776a^3b-541705059811911040a^4b+13820286467276800a^5b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-2469322022918400a^6b+326583853541632a^7b-31156475560320a^8b+2340242526400a^9b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-122169740160a^{10}b+5337467408a^{11}b-142613640a^{12}b+3493700a^{13}b-36735a^{14}b)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(434a^{15}b-100909430813349120b^2-71179471376547840ab^2+132562310349127680a^2b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-96502735620096000a^3b^2+39455725614796800a^4b^2-10393670565008640a^5b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(1911816899339520a^6b^2-245643692083200a^7b^2+24451722638400a^8b^2-1645789231200a^9b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(95144334480a^{10}b^2-3197563200a^{11}b^2+108689100a^{12}b^2-1415925a^{13}b^2+26970a^{14}b^2)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(45646821227298816b^3-31565349178638336ab^3-17716361173647360a^2b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(27158505721528320a^3b^3-13679204053597440a^4b^3+3918081302580480a^5b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-732192289313280a^6b^3+99106124778240a^7b^3-8940052710720a^8b^3+673510158960a^9b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-29274532560a^{10}b^3+1300061880a^{11}b^3-21562515a^{12}b^3+566370a^{13}b^3)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-30119994532429824b^4+35990351353896960ab^4-12817606114037760a^2b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-1046052696318720a^3b^4+2120890334688000a^4b^4-791845793625600a^5b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(163543093881600a^6b^4-21459053764800a^7b^4+2152280437200a^8b^4-125836626000a^9b^4)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(7159456200a^10b^4-155144925a^{11}b^4+5259150a^{12}b^4+106592423746355520b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-14307315699220480ab^5+7268501921936640a^2b^5-1645411005646080a^3b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(19025526931200a^4b^5+69492783427200a^5b^5-20082084664320a^6b^5+3012890332320a^7b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-260046358200a^8b^5+19235813100a^9b^5-568514115a^{10}b^5+24192090a^{11}b^5)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-2384177957457920b^6+3391367994865920ab^6-1903248210151680a^2b^6)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(540144838041600a^3b^6-84920244816000a^4b^6+2483999138880a^5b^6+941035082400a^6b^6)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-216761126400a^7b^6+23317452900a^8b^6-1052976225a^9b^6+56448210a^{10}b^6)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(372420039991552b^7-532460361363712ab^7+309340014067200a^2b^7-97422347546880a^3b^7)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(16677013852800a^4b^7-1910624274720a^5b^7+48979677600a^6b^7+4242920400a^7b^7)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-795547575a^8b^7+58929450a^9b^7-411005288161280b^8+59560797031680ab^8)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-34669345543680a^2b^8+10689025278240a^3b^8-2116502035200a^4b^8+212369331600a^5b^8)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-18237114000a^6b^8+265182525a^7b^8+33724063801600b^9-4745684818240ab^9)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(2718132397440a^2b^9-855046307760a^3b^9+145414149000a^4b^9-19049720100a^5b^9)}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(901000275a^6b^9-58929450a^7b^9-199271883200b^{10}+279195240480ab^{10})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

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$$+\frac{(-154142180400a^2b^{10}+43587835200a^3b^{10}-8158694700a^4b^{10}+641090385a^5b^{10})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-56448210a^6b^{10}+88410308848b^{11}-11619237968ab^{11}+6044516400a^2b^{11}-1695549960a^3b^{11})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(212995575a^4b^{11}-24192090a^5b^{11}-271930672b^{12}+349449360ab^{12}-166890360a^2b^{12})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(35236305a^3b^{12}-5259150a^4b^{12}+6056120b^{13}-6800780ab^{13}+2737455a^2b^{13}-566370a^3b^{13})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+$$

$$+\frac{(-80260b^{14}+84165ab^{14}-26970a^2b^{14}+659b^{15}-434ab^{15}-b^{16})}{\prod_{\eta=1}^{15}\{a-2\eta\}}+\frac{(a-b+30)}{(a-b-1)}\frac{\Gamma(\frac{b+1}{2})}{\Gamma(\frac{a-29}{2})}\times$$

$$\times\left\{\frac{(-42849873690624000+100746609662361600a-92490335838535680a^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+\right.$$

$$+\frac{(46632165832065024a^3-14859663097823232a^4+3271193580412928a^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-505025512816640a^6+59551759843072a^7-4947215345536a^8+339191763264a^9)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-15279519840a^{10}+621837424a^{11}-13939432a^{12}+323708a^{13}-2690a^{14}+30a^{15})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(268146438045696000b-1053696230896435200ab+1139876648710963200a^2b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-621635719236157440a^3b+207114479223029760a^4b-45691029791375360a^5b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(7330375627916800a^6b-811700613867520a^7b+72869813751680a^8b-4242113546880a^9b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(226323832800a^{10}b-6491717440a^{11}b+207351560a^{12}b-2230760a^{13}b+40300a^{14}b)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(214762901785804800b^2+184973594715095040ab^2-463836187418296320a^2b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(30822099684507648a^3b^2 - 11169873454153728a^4b^2 + 2610314589022976a^5b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-405911801172480a^6b^2 + 48474589768960a^7b^2 - 3740645877984a^8b^2 + 257569498128a^9b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-9573055440a^{10}b^2 + 397825480a^{11}b^2 - 5539638a^{12}b^2 + 138446a^{13}b^2)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-2908104715763712 0b^3 + 19697219949428736ab^3 + 1926305147437056a^2b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-6525530932051968a^3b^3 + 2983163756323584a^4b^3 - 753958524042240a^5b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(127441219311360a^6b^3 - 13733274514176a^7b^3 + 1232794859472a^8b^3 - 60858560160a^9b^3)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(3218599800a^10b^3 - 58621992a^{11}b^3 + 1893294a^{12}b^3 + 14200532432486400b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-13886382286397440ab^4 + 4380147255427072a^2b^4 - 131032468218624a^3b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-363675061996800a^4b^4 + 120686089584000a^5b^4 - 21411848928960a^6b^4 + 2650892519520a^7b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-183751464600a^8b^4 + 12407548500a^9b^4 - 303277650a^{10}b^4 + 12271350a^{11}b^4)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-4036298413056000b^5 + 4339068030189568ab^5 - 1878105598159616a^2b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(348720370467840a^3b^5 - 20359470898560a^4b^5 - 8674613539200a^5b^5 + 2132804429280a^6b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-253026930240a^7b^5 + 22893160860a^8b^5 - 800199900a^9b^5 + 40320150a^{10}b^5)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(759685713346560b^6 - 857574469125888ab^6 + 386934853266944a^2b^6 - 98495967926016a^3b^6)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

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$$+\frac{(+113537339395 20a^4b^6-705515781600a^5b^6-86079178080a^6b^6+15527599920a^7b^6)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-1003041270a^8b^6+65132550a^9b^6-100366817591040b^7+113217893178368ab^7)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-54537090886912a^2b^7+13207757720832a^3b^7-2209706471520a^4b^7+150958641600a^5b^7)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-8858026800a^6b^7-282861360a^7b^7+35357670a^8b^7+9589432195200b^8-10859483768000ab^8)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(4938029127456a^2b^8-1340626529232a^3b^8+178725335400a^4b^8-20749369500a^5b^8)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(664352010a^6b^8-35357670a^7b^8-671530516800b^9+731396205504ab^9-342285504528a^2b^9)}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(76951667040a^3b^9-13011676500a^4b^9+787793700a^5b^9-65132550a^6b^9+34499905440b^{10})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-36817157936ab^{10}+15032099888a^2b^{10}-3775087992a^3b^{10}+369893550a^4b^{10})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-403201 50a^5b^{10}-1285480560b^11+1257372896ab^{11}-523484104a^2b^{11}+84987864a^3b^{11})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\frac{(-12271350a^4b^{11}+33797400b^{12}-31361460ab^{12}+9419722a^2b^{12}-1893294a^3b^{12}-594300b^{13})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}+$$

$$+\left.\left.\frac{(443548ab^{13}-138446a^2b^{13}+6270b^{14}-4030ab^{14}-30b^{15})}{\prod_{\zeta=1}^{15}\{a-(2\zeta-1)\}}\right\}\right]$$

In this way, we get the main formula.

IV. Conclusion

In this paper we have derived a summation formula with the help of contiguous relation . However, the formula presented herein may be further developed to extend this result .Thus we can only hope that the development presented in this work will stimulate further interest and research in this important area of classical special functions. Just as the mathematical properties of the Gauss hypergeometric function are already of immense and significant utility in mathematical sciences and numerous other areas of pure and applied mathematics, the elucidation and discovery of the formula of hyperge-

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ometric functions considered herein should certainly eventually prove useful to further developments in the broad areas alluded to above.

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