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Evaluation of Condition Symptoms in Diagnosing Complex Objects

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Abstract- A method is proposed for selecting the most informative diagnostic symptoms in lifetime consumption monitoring of complex objects. Typically in such cases many symptoms are available and their suitability cannot be evaluated even with a detailed knowledge of object layout and operation. The proposed procedure involves two stages. Preliminary symptom selection is based on the Singular Value Decomposition (SVD) method. Second stage is based on the information content assessment and employs the continuous analogue of Shannon entropy. An example is presented for a steam turbine fluid-flow system.

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Abstract - A method is proposed for selecting the most informative diagnostic symptoms in lifetime consumption monitoring of complex objects. Typically in such cases many symptoms are available and their suitability cannot be evaluated even with a detailed knowledge of object layout and operation. The proposed procedure involves two stages. Preliminary symptom selection is based on the Singular Value Decomposition (SVD) method. Second stage is based on the information content assessment and employs the continuous analogue of Shannon entropy. An example is presented for a steam turbine fluid-flow system.

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I. Introduction

Technical condition of any object is described by the *condition parameters vector* $\mathbf{X}(\theta)$, where θ denotes 'operational' time, often–but not necessarily–starting at object commissioning. Condition parameters $X_i(\theta)$ are usually non-measurable, so technical condition is typically determined indirectly, on the basis of the *diagnostic symptoms vector* $\mathbf{S}(\theta)$. In the most general case, relation between these two vectors is given by (see e.g. [1] and references therein):

$$\mathbf{S}(\theta) = \mathbf{S}[\mathbf{X}(\theta), \mathbf{R}(\theta), \mathbf{Z}(\theta)], \qquad (1)$$

where $\mathbf{R}$ and $\mathbf{Z}$ denote control parameters and interferences vectors, respectively. In some specific cases the influences of the $\mathbf{R}$ and $\mathbf{Z}$ vectors can be neglected, but usually in practical applications they have to be accounted for; a brief study can be found in [2].

In diagnosing complex objects, a situation is frequently encountered wherein the number of available diagnostic symptoms $S_i \in \mathbf{S}$ is comparatively large. With some exaggeration it may even be said that this number has no upper limit. Even if we focus our attention on vibration-based symptoms, it has to be kept in mind that vibration signal can be recorded in principle at any available point of the object. Number of these points shall be then multiplied by that of measurement directions (usually three mutually perpendicular ones) and that of frequency bands that contain components generated by elementary sources (determined from the vibrodiagnostic model of the object under consideration). For a large rotating machine, a few hundred is typical. A question therefore arises which of them are the 'best' ones from the point of view of condition assessment and which might be qualified as redundant. Unfortunately, this problem usually cannot be solved on the basis of even detailed knowledge of object layout and operation.

Random damages, or hard faults [3], which are equivalent to stepwise changes of condition parameters, often have their specific representations in diagnostic symptom time histories, although reasoning in such cases is by no means simple (see e.g. [4-7]). Natural damage (soft fault), which may be identified with a continuous lifetime consumption process, is even more difficult to trace, especially when this process is slow and masked by fluctuations caused by control and interference. In such cases, the choice of the most representative symptoms is of prime importance for lifetime consumption assessment and prognosis for further operation.

In this paper a new approach to diagnostic symptoms evaluation is proposed. It consists in a two-stage procedure which involves two distinct methods. The first stage employs the Singular Value Decomposition method, known from linear algebra. The second stage is based on the information contents assessment. For clarity, each stage shall be illustrated by a relevant example. These considerations shall be preceded by a brief description of the object.

II. The Object

a) Brief Presentation

A steam turbine, operated by a utility power plant, is a typical example of a critical machine. It is costly and complex. Potential results of a damage are very serious, in terms of both hazard and generation loss. Maintenance has to be planned very carefully, as spare parts are usually not available off the shelf and have to be manufactured. Moreover, turbines are often operated well beyond the timescale stipulated in the original design. Lifetime consumption assessment and prognosis are thus of paramount importance.

K-200 steam turbines and their derivatives, of which over seventy were built, formed the mainstay of the generating capacity in Poland in the 80s and 90s (Fig.1). A few of them still remain in use. The unit under consideration was commissioned in 1969 and modernized in 1991 (entirely new low-pressure turbine with substantially higher thermal efficiency, new control system and numerous minor improvements).

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Measurements, started in 1997, included recording constant-percentage bandwidth (23% CPB) spectra of vibration velocity in points located at bearing caps and low-pressure turbine casing, using portable equipment (accelerometer and data collector). The unit was finally decommissioned in late 2010 and available database covers 4862 days, with measurements performed at time intervals of approximately two months.

Fig.1 : Machine hall interior with ten K-200 units (source: www.elturow.pgegiek.pl). Letters F and R indicate front and rear HP turbine bearing of the foremost unit

In the following attention shall be focused on the high-pressure (HP) turbine. Vibration velocity was recorded on its front and rear bearings, in vertical, horizontal (i.e. radial) and axial directions. 23% CPB (constant-percentage bandwidth) analysis was employed. HP turbine has twelve stages and, according to the turbine vibrodiagnostic model [8], individual components generated by the fluid-flow system are contained in ten bands with mid-frequencies of 500 Hz, 800 Hz, 1600 Hz, 2000 Hz, 2500 Hz, 3150 Hz, 4000 Hz, 5000 Hz, 6300 Hz and 8000 Hz. This gives sixty individual symptoms in all.

It has to be stressed that time histories of vibration components from the blade frequency range are typically very irregular and exhibit strong fluctuations. This refers in particular to the HP turbine, due to the proximity of the control stage. With the nozzle-type control (partial-arc admission), steam thrust is unevenly distributed over the fluid-flow system cross-section, which strongly influences vibration patterns (see e.g. [9]). This distribution changes as control valves open and close, according to the demanded load profile. This effect is particularly evident at low loads [2] and decreases as we move along the steam expansion path. Examples of symptom time histories are shown in Fig.2.

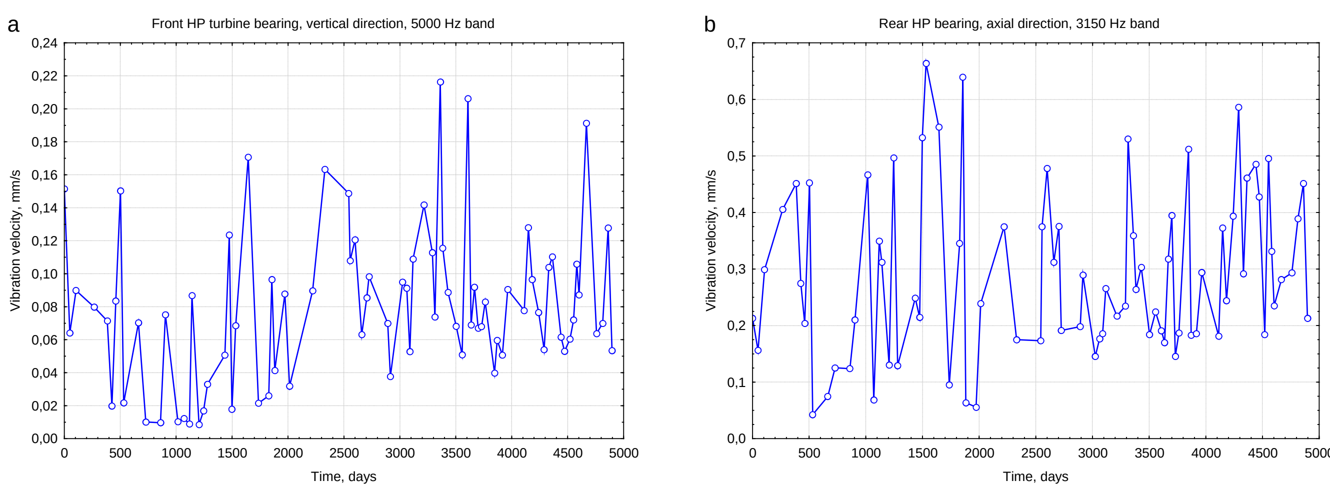

Fig. 2 : Examples of vibration velocity time histories; (a) front HP bearing, vertical direction, 5 kHz band; (b) rear HP turbine bearing, axial direction, 3.15 kHz band

b) *Data Smoothing*

Examples shown in Fig.2 clearly illustrate that some form of data smoothing should be considered. First attempts were based on the observation that operation at low loads usually causes vibration amplitudes in the blade frequency range to rise dramatically, even by one order of magnitude. Normalization of the load influence therefore seemed a reasonable option [10]; in view of Eq. (1) this implies that interference is not accounted for. Experience has shown that some 'peaks' in vibration time histories could be eliminated in that way, but some – probably caused by temporary steam flow instabilities – remained.

A method known as three-point averaging has been proposed [11], wherein kth symptom value reading $S(\theta_k)$ is replaced by the average

$$S'(\theta_k) = \frac{1}{3}[S(\theta_{k-1}) + S(\theta_k) + S(\theta_{k+1})] \qquad (2)$$

In this manner *all* peaks are just 'flattened'. From the statistical point of view, outstandingly high measured symptom values are isolated outliers [12,13]. A procedure of their elimination may consist in excluding peaks supposedly not related to the condition changes, which might be referred to as 'peak trimming' [1]. This approach is based on the assumption that if

$$S(\theta_k)/S(\theta_{k-1}) > c \text{ and } S(\theta_k)/S(\theta_{k+1}) > c, \qquad (3)$$

then the $S(\theta_k)$ value has been strongly influenced by control and/or interference vectors and is therefore suspicious; in such cases, $S(\theta_k)$ is replaced by $S'(\theta_k) = [S(\theta_{k-1}) + S(\theta_{k+1})]/2$. The value of the 'threshold' c should be estimated individually; judging from the author's own experience, $c = 1.5$ is reasonable for steam turbines. It may be noted here that three-point averaging may be considered a limit case of peak trimming, corresponding to $c \to 1$.

III. Stage 1: Svd Method

The idea to employ SVD method in condition monitoring has been first conceived and later developed by Cempel (see e.g. [14,15]). In the following we shall follow the argumentation presented in these references, retaining the original notions. A detailed description of the method itself can be found in specialized reviews and monographs (see e.g. [16]).

In principle, any $m \times n$ matrix $\mathbf{A}$ can be expressed as a product of three matrices $\mathbf{U}$, $\mathbf{\Sigma}$ and $\mathbf{V}^T$:

$$\mathbf{A} = \mathbf{U} * \mathbf{\Sigma} * \mathbf{V}^T, \qquad (4)$$

where $\mathbf{U}$ is a $m \times n$ unitary orthogonal matrix, $\mathbf{\Sigma}$ is a $n \times n$ diagonal matrix and $\mathbf{V}$ is a $n \times n$ unitary orthogonal square matrix (superscript T denotes transpose). The factorization given by the above equation is called a singular value decomposition of the matrix $\mathbf{A}$. The singular values matrix $\mathbf{\Sigma}$ can be written as

$$\mathbf{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_q)\ ,\ q = \max(m,n), \qquad (5)$$

σ_i being non-negative real numbers. If non-zero elements σ_i of the matrix $\mathbf{\Sigma}$ are arranged in such manner that

$$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_p\ ,\ \ p = \min(m,n), \qquad (6)$$

then $\mathbf{\Sigma}$ is uniquely determined by $\mathbf{A}$. In general, columns of $\mathbf{V}$ are orthonormal vectors that are sometimes referred to as 'input' basis vector directions of $\mathbf{A}$; similarly, columns of $\mathbf{U}$ can referred to as 'output' basis vector directions. Singular values σ_i within such approach can be thought of as 'gain' scalars that indicate factors by which 'inputs' are multiplied to give corresponding 'outputs'. In other words, matrices $\mathbf{U}$ and $\mathbf{V}$ form the sets of left-singular vectors $\mathbf{u}_i$ and right-singular vectors $\mathbf{v}_i$, respectively, which obey the relation

$$\mathbf{A} * \mathbf{v}_i = \sigma_i \cdot \mathbf{u}_i\ ,\ \mathbf{A}^{\mathrm{T}} * \mathbf{u}_i = \sigma_i \cdot \mathbf{v}_i. \qquad (7)$$

Let us consider m distinct symptoms $S_i(\theta)$, $i = 1, 2, \ldots, m$, and n symptom readings:

$$S_i(\theta_k) = S_i(\theta = \theta_0 + k\Delta\theta)\ ,\ \Delta\theta << \theta_b\ ,\ k = 1, 2, ..., n. \qquad (8)$$

$\Delta\theta$ denotes here the time interval between consecutive readings and θ_b is time to breakdown. It is assumed that condition monitoring was introduced at $\theta = \theta_0$. The method can handle various symptoms of different physical origin, so it is suggested, in order to make all of them comparable, to normalize them with respect to their initial values and then subtract 1, so that all become dimensionless and start from zero. In this manner, the measurement database is transformed into a $m \times n$ matrix $\mathbf{O}$, known as *symptom observation matrix*. In principle, the above approach means that we have p independent sources of information on object condition and therefore we can trace p independently developing generalized faults.

In view of Eq.(7), we may rewrite Eq.(4) in another form:

$$\mathbf{O} = \sum_{t=1}^{p} \sigma_t \cdot (\mathbf{u}_t * \mathbf{v}_t^T)\,, \qquad (9)$$

so that the tth generalized fault is characterized by the scalar σ_t and singular vectors $\mathbf{u}_t$ and $\mathbf{v}_t$. From Eqs. (7) and (8) we may conclude that this fault can be described by two independent measures or discriminants:

$$\mathbf{SD}_t = \mathbf{O} * \mathbf{v}_t = \sigma_t \cdot \mathbf{u}_t\,, \qquad (10)$$

$$\|\mathbf{SD}_t\| = \sigma_t\,. \qquad (11)$$

$\mathrm{SD}_t(\theta)$ is a time-dependent vector which represents the tth fault profile at a given moment. On the other hand, $\sigma_t(\theta)$ is a time-dependent scalar energy norm of this vector and hence represents fault advancement. Thus, the sum given by

$$F(\theta) = \sum_{i=1}^{p} \sigma_i(\theta) \qquad (12)$$

can be interpreted as a measure of overall lifetime consumption degree and consequently of the overall machine condition. Similarly, the vector given by

$$\mathbf{P}(\theta) = \sum_{i=1}^{p} \mathbf{SD}_i(\theta)\,, \qquad (13)$$

describes the evolution of the total generalized fault profile. In the same manner another discriminant may be defined:

$$\mathbf{AL}_t = \mathbf{u}_t^T \cdot \mathbf{O} = \sigma_t \cdot \mathbf{v}_t^T\,, \qquad (14)$$

which also represents the tth fault profile; obviously,

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$$\|\mathbf{AL}_t\| = \|\mathbf{SD}_t\| . \qquad (15)$$

Elements of both $\mathbf{SD}_t$ and $\mathbf{AL}_t$ vectors represent contributions into the σ_t singular value and hence 'components' of the tth fault profile, which can be expressed in terms of either condition parameters ($\mathbf{SD}_t$) or measurable symptoms ($\mathbf{AL}_t$). Both representations are formally equivalent, but the latter is obviously more useful, as condition parameters are usually 'inaccessible'.

Application of this approach to steam turbines has been described in several earlier publications by the author [1,17]. In general for a unit with short service life no dominant singular value σ_i can be distinguished. As θ_b is approached, however, a dominant failure mechanism appears and with it also a σ_i singular value with the highest contribution into the generalized fault. Symptoms with the highest contributions into this value can then be identified, and these may be viewed most informative from the point of view of lifetime consumption determination.

Let us no return to our example, introduced in Section 2.1. In order to select the most informative symptom for all measuring points and directions, SVD analysis was performed for six sets of symptoms. Following the suggestions given in [14], operational time was included in all sets as the eleventh symptom. Results for one set (front HP turbine bearing, horizontal direction) are presented in Fig.3. It is immediately seen that a dominant fault has already developed, as the contribution of the first singular value is about 54%, while those of the remaining ones do not exceed 15%. The most informative symptom can also be readily identified, namely the 9th one. In this manner the following six symptoms have been specified:

- No.1: front HP turbine bearing, vertical direction, 6300 Hz band;
- No.2: front HP turbine bearing, horizontal direction, 5000 Hz band;
- No.3: front HP turbine bearing, axial direction, 5000 Hz band;
- No.4: rear HP turbine bearing, vertical direction, 8000 Hz band;
- No.5: rear HP turbine bearing, horizontal direction, 5000 Hz band;
- No.6: rear HP turbine bearing, axial direction, 6300 Hz band.

It may be noted here that all frequency bands listed above contain components generated by rotor stages rather than by bladed diaphragms. This indicates that rotor condition deterioration is more pronounced.

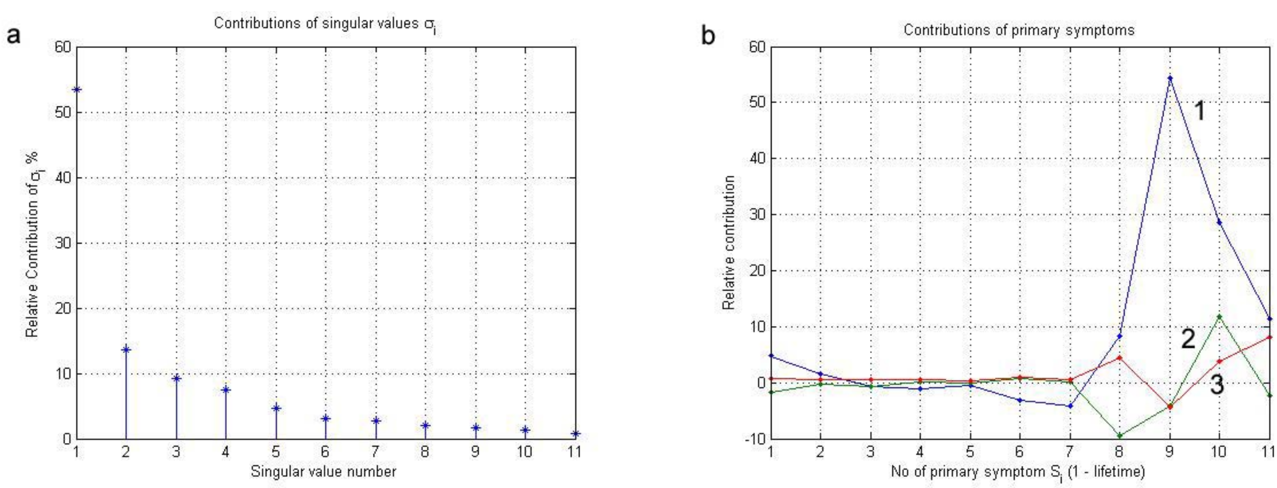

Fig. 3 : Front HP turbine bearing, horizontal direction. (a) Contributions of individual singular values into generalized fault (in descending order). (b) Contri-butions of individual symptoms into first, second and third singular values

Formally the entire SVD procedure may be repeated for six selected symptoms (plus time). Results are shown in Fig.4 and it is easily noticed that they are qualitatively different from those presented in Fig.3. Again, there is a dominant fault (contribution of about 47%), but no dominant symptom can be pointed out. In order to proceed, we shall now evaluate six selected symptoms by applying an information content measure.

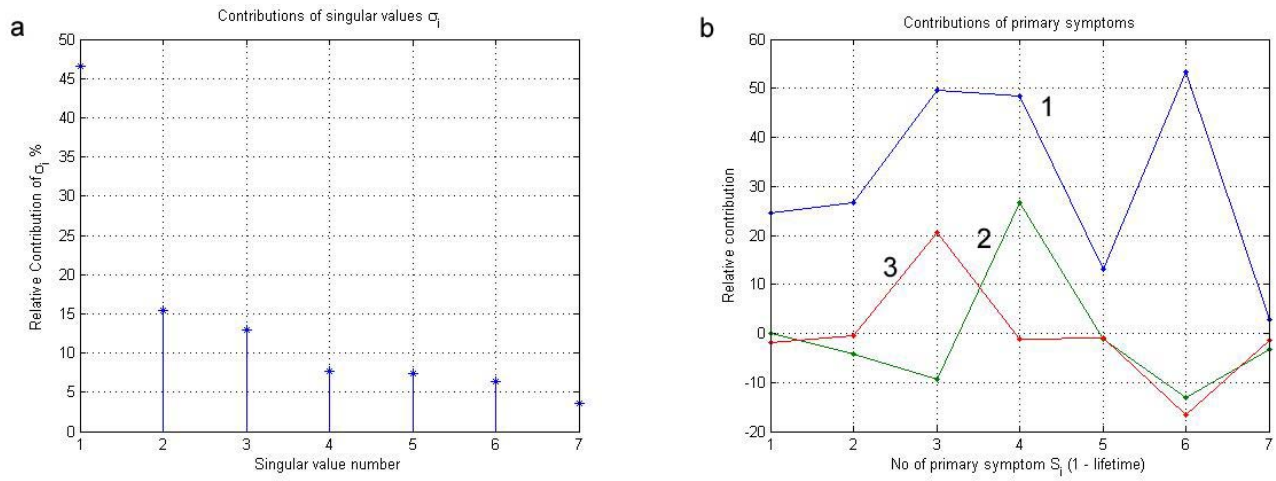

Fig. 4 : SVD evaluation of six selected symptoms (plus time). (a) Contributions of individual singular values into generalized fault (in descending order). (b) Contribution of individual symptoms into first, second and third singular values

IV. Stage 2: Shannon Entropy

Components of the R(θ) and Z(θ) vectors should be considered random variables. The entire symptom time history should thus be treated as a stochastic process rather than a deterministic function of θ. On the other hand, lifetime consumption processes *are* deterministic. It is thus proper to speak in terms of a random variable with time-dependent parameters. With any random variable, a measure of uncertainty can be associated. The most commonly used one is the *Shannon entropy* [18]. For a discrete random variable *Y*, characterized by the probability density function *p*(*y*), Shannon entropy *H*(*Y*) is given by

$$H(Y) = \sum_{i=1}^{n} p(y_i) \log_b \frac{1}{p(y_i)} \qquad (16)$$

with the following obvious conditions:

$$p_i \geq 0 \ (i = 1, 2, \ldots, n) \ , \qquad (17)$$

$$\sum_{i=1}^{n} p_i = 1 \qquad (18)$$

and, by convention [19],

$$0 \log_b 0 = \lim_{t \to 0} t \log_b t = 0 \qquad (19)$$

Typical values for *b* are 2, Euler's number *e* and 10; obviously this is a question of multiplication by a constant only. *H* is expressed in bits, nats or bans, respectively. Shannon entropy may be interpreted as the amount of information that is missing when the exact value of a random variable is not known [18]. Equivalently it may be considered a measure of unpredictability of the outcome of an experiment [20]. Zero entropy means that the outcome is entirely predictable. The concept of the Shannon entropy, originally introduced for discrete random variables, can be extended to include continuous random variables (differential entropy, see e.g. [21]).

For a given symptom time history, Shannon entropy as a function of time may be determined in the following way:

- experimental data histogram is determined within a 'time window' of constant length $\delta\theta$ (in practice, for a meaningful estimation, window containing no less than 25 individual data points is necessary);
- statistical parameters are determined by fitting a distribution to this histogram;
- data window is moved to the next point and the procedure is repeated;
- after the entire period under consideration has been covered, statistical parameters are plotted against time and some function (usually exponential) is fitted to them;
- from the data acquired in the previous step, Shannon entropy as a function of time may be easily calculated.

Second step needs some clarification. Basically right-hand skewed distributions are applicable, as there is no upper limit for *S*(θ) while, at the same time, *S*(θ) > 0. In general, symptom value distribution should satisfy the following requirements:

- $S \in (0, \infty)$;
- low probability for values close to zero;
- probability density function maximum at some value (expected or mean);
- $S \to \infty \Rightarrow p(S) \to 0$.

These conditions are met by the gamma distribution, with the probability density function given by

$$p(S) = \frac{1}{\Gamma(k)\lambda^k} S^{k-1} e^{-S/\lambda} \ , \quad (20)$$

where Γ denotes gamma function and k and λ are shape and scale parameters, respectively. This distribution is commonly used in probabilistic modeling of lifetimes. Alternatively, Weibull distribution may be used:

$$p(S) = \frac{\lambda}{k} S^{-1} e^{-S^k/\lambda} \quad (21)$$

(k and λ as above).

For these two distribution types, Shannon entropy is given by [22]

$$\frac{k-1}{k}\gamma_E + \ln\frac{\lambda}{k} + 1 \quad (22)$$

for the Weibull distribution and

$$\ln(\lambda\Gamma(k)) + (1-k)\psi(k) + k \quad (23)$$

for the gamma distribution, respectively; γ_E is the Euler-Mascheroni constant (≈ 0.5772) and $\psi(k)$ denotes the digamma function.

Fig.5 shows results obtained with the assumption of the Weibull distribution; data pre-processing included peak trimming at $c = 1.5$ followed by three-point averaging. It is easily seen that five from six symptoms exhibit an increase of Shannon entropy with time. This implies that the uncertainty is increasing, so that the random 'component' (resulting from the influence of control and interference) gradually becomes dominant over the deterministic one (which represents lifetime consumption). Only for the sixth symptom (rear HP turbine bearing, axial direction, 6300 Hz band) there is a slight decrease. This system may thus be pointed out as the most informative one from the point of view of lifetime consumption representation and hence the most suitable for prognosis. Results for the gamma distribution (shown in Fig.6) are qualitatively identical, in that they lead to the same conclusions.

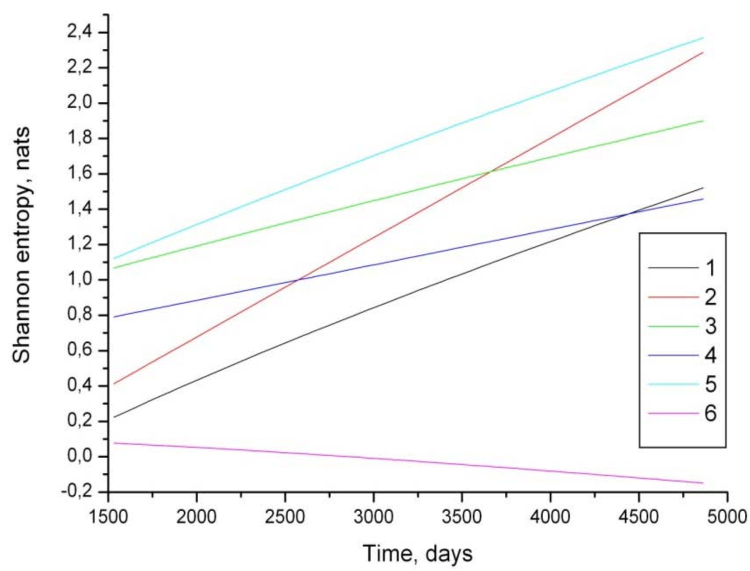

Fig. 5 : Shannon entropy for six symptoms listed in Section 3, plotted against time (Weibull distribution assumption)

V. Discussion And Further Development

The method proposed in this paper has proven capable of selecting the most informative symptom of the turbine fluid-flow system lifetime consumption advancement. Starting from sixty available vibration-based symptoms, one has been selected in a rather unequivocal manner; moreover, this selection is not affected by the assumed distribution type. The procedure may thus be judged suitable for diagnostic symptoms evaluation.

Negative entropy may seem suspicious. It has been pointed out, however, that extension of the Shannon entropy concept onto continuous distributions does not preserve all its properties [21]. In principle $H(Y) \geq 0$ (cf. Eq.(16)), but for certain distribution types differential entropy may be negative.

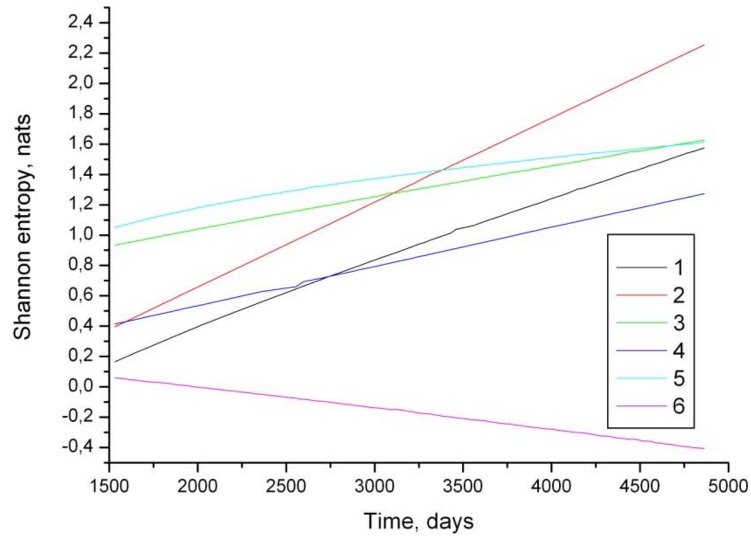

Fig. 6 : Shannon entropy for six symptoms listed in Section 3, plotted against time (gamma distribution assumption)

It has been noted that, even after measurement data pre-processing (smoothing) has been performed, the quality of fitting the shape and scale factors to experimental data was sometimes quite poor, as $\lambda(\theta)$ and $k(\theta)$ tended to be rather irregular. In the author's opinion, time window length is an important and perhaps the key factor. Basically, in order to obtain good fit of Weibull and gamma distributions to experimental histograms, $\delta\theta$ should be as large as possible. On the other hand, when speaking in terms of *fluctuations*, we tacitly assume that they refer to some 'mean' or 'averaged' value, which implies that an increasing trend within the time window is neglected. This may be justified if θ is substantially smaller than θ_b, i.e. for an object with comparatively short operational life. In the case dealt with in this paper, i.e. for large lifetime consumption advancement, this assumption is not valid. It has been suggested (see e.g. [23] and references therein) that, in analyzing time series, it is more appropriate to speak in terms of deviation from a trend than from some 'averaged' value. Such approach has been tested by the author and results have been found promising [1]. Work is currently underway on applying a similar procedure for diagnostic symptoms evaluation and the author hopes to report results in near future.

Finally it has to be noted that the ultimate selection, illustrated graphically in Figs.5 and 6, basically refers to the case when lifetime consumption is assessed directly from the symptom time history. It has been pointed out and proven on the basis of model considerations [1,24]) that information on object condition is also contained in data dispersion measures (standard deviation, median absolute deviation, interquartile range etc.). Moreover, meta-symptoms based on dispersion measures offer certain advantages. Taking this into account, a symptom with the highest increase of Shannon entropy may be considered the most informative one. It is clearly seen in Figs.5 and 6 that also in this case both distribution assumptions yield identical results. Ultimately this choice will thus depend on the very concept of diagnostic information extraction from $S(\theta)$ time histories.

VI. Acknowledgements

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