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Exponential Estimators of Population Mean in Post-Stratified Sampling using Known Value of Some Population Parameters

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GJSFR-F Classification : *MSC 2010: 97K80, 47N30*

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AMS (2013) Classification: 62D05

I. Introduction

Many authors have contributed to the use of known population parameters of an auxiliary character in constructing estimators of the population parameters of a study variate. Notably is the work carried out by Khoshnevisan et al (2007), who discussed a general family of estimators of $\overline{Y}$ under the SRSWOR scheme using known parameters of the auxiliary variable x, such as standard deviation, correlation coefficient, coefficient of skewness, kurtosis and coefficient of variation. Under the stratified random sampling, Koyuncu and Kadilar (2009) discussed a general family of combined estimators of $\overline{Y}$ that makes use of known population parameters. Recently, Onyeka (2012) developed a general family of estimators of $\overline{Y}$ under the post-stratified sampling scheme, using known values of some population parameters of an auxiliary character. The general family of estimators of the population mean $\overline{Y}$ proposed by Onyeka (2012), under the post-stratified sampling scheme is given by

$$t_1 = \bar{y}_{ps}\left(\frac{a\overline{X}+b}{\alpha\left(a\bar{x}_{ps}+b\right)+\left(1-\alpha\right)\left(a\overline{X}+b\right)}\right)^g \tag{1.1}$$

where,

$y_{hi}(x_{hi})$ is the ith observation for the study (auxiliary) variate in stratum h

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$\bar{y}_{ps} = \sum_{h=1}^{L} \omega_h \bar{y}_h$ is the post-stratified estimator of $\bar{Y}$

$\bar{x}_{ps} = \sum_{h=1}^{L} \omega_h \bar{x}_h$ is the post-stratified estimator of $\bar{X}$

$\bar{X} = \sum_{h=1}^{L} \omega_h \bar{X}_h$ is the population mean of auxiliary variate, x.

$a(\neq 0)$, b are either constants or functions of known population parameters of the auxiliary variate, such as coefficient of variation (C_x), correlation coefficient (ρ_{yx}), standard deviation (σ_x), skewness $(\beta_1(x))$, and kurtosis $(\beta_2(x))$

ω_h is stratum weight, L is the number of strata in the population, $\bar{X}_h$ is the population mean of the auxiliary variate in stratum h, $\bar{y}_h(\bar{x}_h)$ is the sample mean of the study (auxiliary) variate in stratum h, N_h is the population size of stratum h, and N is the population size.

More recently, Onyeka (2013), still studying estimation of population mean in post-stratified sampling scheme, proposed a class of dual to ratio estimators of the population mean, $\bar{Y}$ in post-stratified sampling, using known population parameters of an auxiliary character x, as:

$$t_2 = \bar{y}_{ps}\left(\frac{\alpha\left(a\bar{x}^*_{ps} + b\right) + (1-\alpha)\left(a\bar{X} + b\right)}{a\bar{X} + b}\right)^g \qquad (1.2)$$

where $\bar{x}^*_{ps}$ is a transformed sample mean of the auxiliary variable, x, based on the variable transformation, $\bar{x}^*_{hi} = \frac{N\bar{X} - nx_{hi}}{N-n}$ and satisfying the relationship $\bar{X} = f\bar{x}_{ps} + (1-f)\bar{x}^*_{ps}$ in line with the transformation, $x^*_i = \frac{N\bar{X} - nx_i}{N-n}$, $i = 1, 2, \ldots, N$, used by Srivenkataramana (1980) to obtain a dual to ratio estimate of $\bar{Y}$ under the simple random sampling scheme. Other authors who have used the variable transformation introduced by Srivenkataramana (1980) under the simple random sampling scheme include Singh and Tailor (2005), Tailor and Sharma (2009), and Sharma and Tailor (2010). The present study, however, extends the works carried out by Onyeka (2012, 2013) by introducing the use of exponential estimators in post-stratified sampling scheme when using information on some known population parameters.

II. The Proposed Exponential Estimators

The use of exponential estimators was first discussed by Bahl and Tuteja (1991), who, under the simple random sampling scheme, used an exponential ratio-type estimator of the form

$$t = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right) \qquad (2.1)$$

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Singh and Vishwakama (2007) used some modified ratio-type and product-type exponential estimators of population mean in double sampling scheme, while Singh et al. (2009) proposed a class of ratio-type exponential estimators of the population mean using known values of population parameters of an auxiliary character, under the simple random sampling scheme. The class of estimators proposed by Singh et al. (2009) is of the form:

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$$t^* = \bar{y}\exp\left[\frac{(a\bar{X}+b)-(a\bar{x}+b)}{(a\bar{X}+b)+(a\bar{x}+b)}\right] = \bar{y}\exp\left[\frac{a(\bar{X}-\bar{x})}{a(\bar{X}+\bar{x})+2b}\right] \tag{2.2}$$

Following Singh et al. (2009), and extending the works carried out by Onyeka (2012, 2013), we propose a class of ratio-type exponential estimators of the population mean in post-stratified sampling scheme, using known values of population parameters of an auxiliary character, as

$$t_3 = \bar{y}_{ps}\exp\left[\frac{(a\bar{X}+b)-(a\bar{x}_{ps}+b)}{(a\bar{X}+b)+(a\bar{x}_{ps}+b)}\right] = \bar{y}_{ps}\exp\left[\frac{a(\bar{X}-\bar{x}_{ps})}{a(\bar{X}+\bar{x}_{ps})+2b}\right] \tag{2.3}$$

To obtain the properties of the proposed exponential estimator, t_3, we expand (2.3) up to first order approximations in expected values to obtain:

$$\left(t_3-\bar{Y}\right)=\bar{Y}\left(e_0-\tfrac{1}{2}\lambda e_1-\tfrac{1}{2}\lambda e_0e_1+\tfrac{3}{8}\lambda^2e_1^2\right) \tag{2.4}$$

and

$$\left(t_3-\bar{Y}\right)^2=\bar{Y}^2\left(e_0^2+\tfrac{1}{4}\lambda^2e_1^2-\lambda e_0e_1\right) \tag{2.5}$$

where

$$e_0=\frac{\bar{y}_{ps}-\bar{Y}}{\bar{Y}},\quad e_1=\frac{\bar{x}_{ps}-\bar{X}}{\bar{X}}\quad \text{and}\quad \lambda=\frac{a\bar{X}}{a\bar{X}+b} \tag{2.6}$$

Taking the unconditional expectations of (2.4) and (2.5), for repeated samples of fixed size n, gives the approximate bias and mean squared error of the proposed estimator, t_3, respectively as:

$$B(t_3)=E\left(t_3-\bar{Y}\right)=\bar{Y}\left(E(e_0)-\tfrac{1}{2}\lambda E(e_1)-\tfrac{1}{2}\lambda E(e_0e_1)+\tfrac{3}{8}\lambda^2E(e_1^2)\right) \tag{2.7}$$

and

$$MSE(t_3)=E\left(t_3-\bar{Y}\right)^2=\bar{Y}^2\left(E(e_0^2)+\tfrac{1}{4}\lambda^2E(e_1^2)-\lambda E(e_0e_1)\right) \tag{2.8}$$

Following Onyeka (2012), we have

$$E\left(e_0\right)=E\left(e_0\right)=0 \tag{2.9}$$

$$E\left(e_0^2\right)=\frac{V\left(\bar{y}_{ps}\right)}{\bar{Y}^2}=\frac{1}{\bar{Y}^2}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_hS_{yh}^2 \tag{2.10}$$

$$E\left(e_1^2\right) = \frac{V\left(\bar{x}_{ps}\right)}{\bar{X}^2} = \frac{1}{\bar{X}^2}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{xh}^2 \tag{2.11}$$

and

$$E\left(e_0 e_1\right) = \frac{Cov\left(\bar{y}_{ps}, \bar{x}_{ps}\right)}{\bar{Y}\bar{X}} = \frac{1}{\bar{Y}\bar{X}}\left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h S_{yxh} \tag{2.12}$$

where $f = n/N$ is the population sampling fraction, $S_{yh}^2\left(S_{xh}^2\right)$ is the population variance of y(x) in stratum h, and S_{yxh} is the population covariance of y and x in stratum h. Using (2.9) to (2.12) to make the necessary substitutions in (2.7) and (2.8) gives the bias and mean squared error of the proposed estimator, t_3, respectively as:

$$B(t_3) = \frac{1}{\bar{X}}\left(\frac{1-f}{n}\right)\left[\tfrac{3}{8}\lambda^2 R^2 A_{22} - \tfrac{1}{2}\lambda A_{12}\right] \tag{2.13}$$

and

$$MSE\,(t_3) = \left(\frac{1-f}{n}\right)\left[A_{11} + \tfrac{1}{4}\lambda^2 R^2 A_{22} - \lambda R A_{12}\right] \tag{2.14}$$

where

$$R = \frac{\bar{Y}}{\bar{X}},\ A_{11} = \sum_{h=1}^{L}\omega_h S_{yh}^2,\ A_{22} = \sum_{h=1}^{L}\omega_h S_{xh}^2,\ \text{and}\ A_{12} = \sum_{h=1}^{L}\omega_h S_{yxh} \tag{2.15}$$

III. Modified (Composite) Estimators

A linear function or combination of any two of the estimators, t_k, k = 1, 2, 3 respectively given in (1.1), (1.2) and (2.3) would result in a modified (composite-type) estimator of the population mean in post-stratified sampling, using known values of some population parameters of an auxiliary character. Accordingly, we propose the following modified estimators of $\bar{Y}$, as linear functions of pairs of the three estimators, t_k, k = 1, 2, 3.

$$t_4 = \gamma t_1 + (1-\gamma) t_2 \tag{3.1}$$

$$t_5 = \gamma t_1 + (1-\gamma) t_3 \tag{3.2}$$

$$t_6 = \gamma t_2 + (1-\gamma) t_3 \tag{3.3}$$

where γ is a constant or weighting fraction chosen, in practice, to minimize the mean squared error of the respective estimators in (3.1) to (3.3). Notice that the estimator, t_1 proposed by Onyeka (2012), in line with the estimators proposed by Khoshnevisan et.al (2007) under the simple random sampling scheme, is a ratio-type estimator of $\bar{Y}$ in post-stratified sampling scheme for all positive values of g in (1.1). The estimator, t_2, proposed by Onyeka (2013) is a dual to ratio type estimator, while the estimator, t_3, proposed in

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this study is an exponential type estimator. Consequently, the proposed composite estimator, t_4 is a "ratio cum dual to ratio" type estimator, the proposed estimator, t_5, is a "ratio cum exponential" type estimator, while the proposed composite estimator, t_6, is a "dual to ratio cum exponential" type estimator of the population mean in post-stratified sampling, using known values of some population parameters of an auxiliary character. Following a procedure similar to the procedure for obtaining the bias and mean squared error of the proposed exponential estimator, t_3, as described in Section 2.0, the biases and mean squared errors of the proposed composite estimators, t_k, k = 4, 5, 6, are obtained up to first order approximation as:

$$B(t_4) = \frac{1}{\bar{\bar{X}}}\left(\frac{1-f}{n}\right)\left[\left(\gamma + \pi^2 - \gamma\pi^2\right)\tfrac{1}{2}g(g+1)\alpha^2\lambda^2R^2A_{22} - (\gamma + \pi - \gamma\pi)g\alpha\lambda A_{12}\right] \tag{3.4}$$

$$B(t_5) = \frac{1}{\bar{\bar{X}}}\left(\frac{1-f}{n}\right)\left[\left\{\tfrac{1}{2}\alpha^2 g(g+1)\gamma + \tfrac{3}{8} - \tfrac{3}{8}\gamma\right\}\lambda^2R^2A_{22} - \left(\alpha g\gamma + \tfrac{1}{2} - \tfrac{1}{2}\gamma\right)\lambda A_{12}\right] \tag{3.5}$$

$$B(t_6) = \frac{1}{\bar{\bar{X}}}\left(\frac{1-f}{n}\right)\left[\left\{\pi^2\alpha^2 g(g+1)\gamma + \tfrac{3}{8} - \tfrac{3}{8}\gamma\right\}\lambda^2R^2A_{22} - \left(\pi\alpha g\gamma + \tfrac{1}{2} - \tfrac{1}{2}\gamma\right)\lambda A_{12}\right] \tag{3.6}$$

and

$$MSE(t_4) = \left(\frac{1-f}{n}\right)\left[A_{11} + (\gamma + \pi - \gamma\pi)^2 g^2\alpha^2\lambda^2R^2A_{22} - (\gamma + \pi - \gamma\pi)2g\alpha\lambda RA_{12}\right] \tag{3.7}$$

$$MSE(t_5) = \left(\frac{1-f}{n}\right)\left[A_{11} + \left(\alpha g\gamma + \tfrac{1}{2} - \tfrac{1}{2}\gamma\right)^2\lambda^2R^2A_{22} - \left(\alpha g\gamma + \tfrac{1}{2} - \tfrac{1}{2}\gamma\right)2\lambda RA_{12}\right] \tag{3.8}$$

$$MSE(t_6) = \left(\frac{1-f}{n}\right)\left[A_{11} + \left(\pi\alpha g\gamma + \tfrac{1}{2} - \tfrac{1}{2}\gamma\right)^2\lambda^2R^2A_{22} - \left(\pi\alpha g\gamma + \tfrac{1}{2} - \tfrac{1}{2}\gamma\right)\lambda RA_{12}\right] \tag{3.9}$$

where $\pi = f/(1-f) = n/(N-n)$. Notice that Onyeka (2012) obtained the mean squared error of the ratio-type estimator, t_1 as:

$$MSE(t_1) = MSE(\bar{y}_{pss}) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h\left(S_{yh}^2 + \alpha^2\lambda^2g^2R^2S_{xh}^2 - 2\alpha\lambda gRS_{yxh}\right) \tag{3.10}$$

or

$$MSE(t_1) = \left(\frac{1-f}{n}\right)\left[A_{11} + \alpha^2\lambda^2g^2R^2A_{22} - 2\alpha\lambda gRA_{12}\right] \tag{3.11}$$

Similarly, Onyeka (2013) obtained the mean squared error of the dual to ratio type estimator, t_2 as:

$$MSE(t_2) = MSE(\bar{y}_{pss}^*) = \left(\frac{1-f}{n}\right)\sum_{h=1}^{L}\omega_h\left(S_{yh}^2 + \pi^2\alpha^2\lambda^2g^2R^2S_{xh}^2 - 2\pi\alpha\lambda gRS_{yxh}\right) \tag{3.12}$$

or

$$MSE(t_2) = \left(\frac{1-f}{n}\right)\left[A_{11} + \pi^2\alpha^2\lambda^2g^2R^2A_{22} - 2\pi\alpha\lambda gRA_{12}\right] \tag{3.13}$$

It follows, therefore, that the general form of the mean squared errors of the estimators, t_k, $k = 1, 2, \cdots, 6$ could be written as:

$$MSE(t_k) = \left(\frac{1-f}{n}\right)\left[A_{11} + \theta_k^2 R^2 A_{22} - 2\theta_k R A_{12}\right] \tag{3.14}$$

where,

$$\left.\begin{array}{l} \theta_1 = \alpha g\lambda\,, \theta_2 = \pi\alpha g\lambda\,, \theta_3 = \tfrac{1}{2}\lambda\,, \theta_4 = \alpha g\lambda\gamma + \pi\alpha g\lambda - \pi\alpha g\lambda\gamma \\ \theta_5 = \alpha g\lambda\gamma + \tfrac{1}{2}\lambda - \tfrac{1}{2}\lambda\gamma\,, \theta_6 = \pi\alpha g\lambda\gamma + \tfrac{1}{2}\lambda - \tfrac{1}{2}\lambda\gamma \end{array}\right\} \tag{3.15}$$

IV. Efficiency Comparison

Here, we shall compare the efficiency of the proposed exponential estimator (t_3) with those of the customary post-stratified mean estimator $(\bar{y}_{ps})$, the ratio-type estimator (t_1), the dual to ratio estimator (t_2), the ratio cum dual to ratio estimator (t_4), the ratio cum exponential estimator (t_5), and the dual to ratio cum exponential estimator (t_6).

a) *Efficiency of t_3 over $\bar{y}_{ps}$*

Using (2.10), (3.14) and (3.15), the proposed exponential estimator, (t_3) would perform better than the customary post-stratified mean estimator, $(\bar{y}_{ps})$, in terms of having a smaller mean squared error if:

$$\frac{\beta}{\lambda R} > \frac{1}{4} \tag{4.1}$$

where $\beta = \frac{A_{12}}{A_{22}} = \frac{\sum_{h=1}^{L}\omega_h S_{yxh}}{\sum_{h=1}^{L}\omega_h S_{xh}^2}$ is an expression of the population regression coefficient of the study variate (y) on the auxiliary character (x).

b) *Efficiency of t_3 over the estimators t_k, $k \neq 3$*

Here, we compare the efficiency of the proposed exponential estimator (t_3) with those of the ratio-type estimator (t_1) proposed by Onyeka (2012), the dual to ratio estimator (t_2) proposed by Onyeka (2013), and the ratio cum dual to ratio estimator (t_4), the ratio cum exponential estimator (t_5), and the dual to ratio cum exponential estimator (t_6) proposed in the present study. Using (3.14), the proposed exponential estimator (t_3) would perform better than the other five (5) estimators, t_k, k = 1, 2, 4, 5, 6, in terms of having smaller mean squared error if:

$$\left.\begin{array}{lll} & (1) & \theta_k < \theta_3 < \dfrac{\beta}{R} \\ \text{or} & & \\ & (2) & \dfrac{\beta}{R} < \theta_3 < \theta_k \end{array}\right\} \tag{4.2}$$

Notes

where θ_k, k = 1, 2, …, 6 are as given in (3.15).

V. Empirical Illustration

Each of the six estimators, t_k, is a general class of estimators capable of generating an infinite number of combined estimators of $\overline{Y}$ in post-stratified sampling scheme, by making appropriate choices of the values of the constants α, g, a, b and γ in (1.1), (1.2), (2.3), (3.1), (3.2) and (3.3), as the case may be. Authors like Sisodia and Dwivedi (1981), Singh and Kakran (1993), Upadhyaya and Singh (1999), Kadilar and Cingi (2003) and Koyuncu and Kadilar (2009) considered special cases of some of these general classes of estimators using various known population parameters. For illustration and comparison purposes in the present study, we shall consider estimators of the form proposed by Upadhyaya and Singh (1999), by making use of the coefficient of variation (C_x) and coefficient of kurtosis $(\beta_2(x))$, as two known population parameters of the auxiliary variable, x, in estimating the population mean $(\overline{Y})$ in post-stratified random sampling scheme. Consequently, we consider the following special cases of the estimators, t_k, (k = 1, 2, …, 6), of the population mean, $\overline{Y}$, in post-stratified sampling scheme by choosing $a = \beta_2(x)$, $b = C_x$, $\alpha = 1$, $g = 1$ and $0 < \gamma < 1$, following after Upadhyaya and Singh (1999):

Ratio type estimator, $$t_{1US} = \bar{y}_{ps}\left(\frac{\overline{X}\beta_2(x) + C_x}{\bar{x}_{ps}\beta_2(x) + C_x}\right) \tag{5.1}$$

Dual to Ratio type estimator, $$t_{2US} = \bar{y}_{ps}\left(\frac{\bar{x}^*_{ps}\beta_2(x) + C_x}{\overline{X}\beta_2(x) + C_x}\right) \tag{5.2}$$

Exponential type estimator, $$t_{3US} = \bar{y}_{ps}\exp\left(\frac{(\overline{X} - \bar{x}_{ps})\beta_2(x)}{(\overline{X} + \bar{x}_{ps})\beta_2(x) + 2C_x}\right) \tag{5.3}$$

Ratio cum Dual to Ratio type estimator, $$t_{4US} = \gamma t_{1US} + (1-\gamma)t_{2US} \tag{5.4}$$

Ratio cum Exponential type estimator, $$t_{5US} = \gamma t_{1US} + (1-\gamma)t_{3US} \tag{5.5}$$

Dual to Ratio cum Exponential type estimator, $$t_{6US} = \gamma t_{2US} + (1-\gamma)t_{3US} \tag{5.6}$$

where the subscripts (US) indicate that the estimators follow after Upadhyaya and Singh (1999) by utilizing coefficient of variation (C_x) and coefficient of kurtosis $(\beta_2(x))$, as two known population parameters of the auxiliary variable, x. The usual post-stratified mean estimator, is

Usual post-stratified mean estimator, $$\bar{y}_{ps} = \sum_{h=1}^{L}\omega_h\bar{y}_h \tag{5.7}$$

Here, we shall examine the efficiencies of the estimators (5.1) to (5.7) using the data given in Onyeka (2012) on the academic performance of 96 students of Statistics department, Federal University of Technology, Owerri, during the 2008/2009 academic session. From Table 1 of Onyeka (2012), we obtain the following relevant statistics:

Table 1 : Summary Statistics from Onyeka (2012)

$N = 96$	$n = 20$
$\beta_2(x) = 3.83$	$C_x = 0.10$
$\pi = 0.26316$	$\lambda = 0.99962$
$A_{11} = 0.325$	$A_{22} = 49.83$
$A_{12} = 3.26$	$R = 0.03581$

Using the data in Table 1 to make the necessary substitutions in (2.10), (3.14) and (3.15), we obtain the variance of the post-stratified mean estimator, $\bar{y}_{ps}$ and the mean squared errors of the estimators (5.1) to (5.6) as shown in table 2. Using table 2, the percentage relative efficiencies (PRE) of the proposed exponential estimator, t_{3US} over the other estimators are obtained as shown in table 3.

Table 2 : Variance / MSE's of the estimators (5.1) to (5.7)

γ	Estimators						
	$\bar{y}_{ps}$	t_{1US}	t_{2US}	t_{3US}	t_{4US}	t_{5US}	t_{6US}
0.10	0.01286	0.00615	0.01061	0.00888	0.01004	0.00855	0.00904
0.20	0.01286	0.00615	0.01061	0.00888	0.00950	0.00823	0.00920
0.30	0.01286	0.00615	0.01061	0.00888	0.00898	0.00793	0.00937
0.32	0.01286	0.00615	0.01061	0.00888	0.00888	0.00787	0.00940
0.40	0.01286	0.00615	0.01061	0.00888	0.00850	0.00764	0.00954
0.50	0.01286	0.00615	0.01061	0.00888	0.00804	0.00736	0.00971
0.60	0.01286	0.00615	0.01061	0.00888	0.00761	0.00709	0.00988
0.70	0.01286	0.00615	0.01061	0.00888	0.00720	0.00684	0.01006
0.80	0.01286	0.00615	0.01061	0.00888	0.00682	0.00660	0.01024
0.90	0.01286	0.00615	0.01061	0.00888	0.00647	0.00637	0.01042

Table 3 : Percentage Relative Efficiencies (PRE) of t_{3US} over others

γ	Estimators						
	$\bar{y}_{ps}$	t_{1US}	t_{2US}	t_{3US}	t_{4US}	t_{5US}	t_{6US}
0.10	145	69	119	100	113	96	102
0.20	145	69	119	100	107	93	104
0.30	145	69	119	100	101	89	106
0.32	145	69	119	100	100	89	106
0.40	145	69	119	100	96	86	107
0.50	145	69	119	100	91	83	109
0.60	145	69	119	100	86	80	111
0.70	145	69	119	100	81	77	113
0.80	145	69	119	100	77	74	115
0.90	145	69	119	100	73	72	117

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Table 3 shows that for the given set of data, the proposed exponential estimator, t_{3US} has relative positive gain in efficiency over (a) the customary post-stratified mean estimator, $\bar{y}_{ps}$, (b) the dual to ratio type estimator t_{2US} proposed by Onyeka (2013), (c) the proposed dual to ratio cum exponential estimator, t_{6US}, for all values of γ, and (d) the proposed ratio cum dual to ratio estimator, t_{4US} for values of γ less than 0.32. On the other hand, the proposed exponential estimator, t_{3US}, is not more efficient than (i) the ratio type estimator, t_{1US}, proposed by Onyeka (2012), (ii) the proposed ratio cum exponential estimator, t_{5US}, for all values of γ, and (iii) the proposed ratio cum dual to ratio estimator, t_{4US}, for values of γ greater than 0.32. For the given set of data, the best of the seven estimators, (5.1) to (5.7), in terms of having the smallest mean squared error, is the ratio type estimator, t_{1US}, proposed by Onyeka (2012), while the least efficient is the customary post-stratified mean estimator, $\bar{y}_{ps}$.

It is worthy of note that the above empirical inferences are consistent with the theoretical results and efficiency conditions obtained in section 4. Using (3.15) and the data in table 1, the values of θ_3 and β/R are obtained as 0.49981 and 1.82686 respectively, indicating that $\theta_3 < \beta/R$. This means that theoretically, condition (1) of (4.2) needs to be satisfied for the proposed exponential estimator, t_{3US}, to perform better than the other estimators, since $\theta_3 < \beta/R$. Obviously, this reduces the efficiency condition (1) of (4.2) to $\theta_k < \theta_3$. From (3.15) and the data in table 1, the values of θ_1 and θ_2, for instance, are obtained as 0.99962 and 0.26306 respectively. Notice that $\theta_1 = 0.99962$ is not less than $\theta_3 = 0.49981$, hence the estimator t_{3US} is not more efficient than the estimator t_{1US}, as indicated by the empirical results. Again, we observe that $\theta_2 = 0.26306$ is less than $\theta_3 = 0.49981$, hence the estimator t_{3US} is more efficient than the estimator t_{2US}, as already indicated and confirmed from the empirical results. Similar examination of other estimators would confirm that all the empirical results obtained in section 5 are consistent with the theoretical results derived in section 4.

VI. Concluding Remark

The paper first discussed two estimators, the ratio type estimator and the dual to ratio type estimator, earlier proposed by Onyeka (2012, 2013), and then proposed and considered a new exponential type estimator for population mean in post-stratified sampling scheme, using known value of some population parameters of an auxiliary character. The paper further proposed additional three estimators, which are composite or linear functions of the ratio type, the dual to ratio type, and the new exponential type estimators, bringing together, four new estimators of the population mean in post-stratified sampling scheme proposed in the study. Properties of the proposed estimators, including their biases and mean squared errors were obtained up to first order approximations. In particular, theoretical conditions under-which the proposed exponential type estimator would perform better than the other estimators, were obtained. The theoretical results were further verified and confirmed using numerical illustrations.

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