Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 7 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Lorentzian Para Sasakian Manifolds Admitting Special Semi Symmetric Recurrent Metric Connection

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GJSFR-F Classification : *MSC 2000: 53C25*

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Strictly as per the compliance and regulations of :

Global Journal of Science Frontier Research (F) Volume XIII Issue VII Version I Year 2013

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Abstract - Several author as Agashe and Chafle [1], Sengupta, De. U.C, Binh [4] and many other introduced semi symmetric non metric connection in different way. In this paper we have studied LP sasakian manifold with special semi-symmetric recurrent metric connection [2] and discuss it exientance in LP sasakian manifold. In section 3 we establish the relation between the Riemannian connection and special semi-symmetric recurrent metric connection on LP sasakian manifold [4]. The section 4 deals with ξ–conformaly flat and ∅ concircularly flat of n dimensional LP sasakian manifold and we proved that ξ–conformaly flatness with special semi-symmetric recurrent metric connection and Riemannian manifold coincide.

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I. Introduction

An n-dimensional differentiable manifold M^n is a lorentzian para-sasakian manifold if it admits tensor field of type (1,1), a contravariant vector field ξ, a covariant vector field η and a lorentzian metric g satisfying;

$$\emptyset^2 X = X + \eta(x)\xi \tag{1.1}$$

$$\eta(\xi) = -1 \tag{1.2}$$

$$g(\emptyset X, \emptyset Y) = g(X,Y) + \eta(X)\eta(Y) \tag{1.3}$$

$$g(X,\xi) = \eta(X) \tag{1.4}$$

$$(D_X \emptyset)(Y) = g(X,Y)\xi + \eta(Y)X + 2\eta(X)\eta(Y)\xi \tag{1.5}$$

$$D_X \xi = \emptyset X \tag{1.6}$$

For arbitrary vector field *and* Y, whrere D denotes the operator of covariant differentiation with respect to lorentzian metric g [5].

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Ref

1. Agashe, N. S and Chafle, M. R: A semi symmetric non metric connection on a Riema-nnian Manifold. Indian. J. Pure appl. Math. 399-409, 1992.

In a LP sasakian manifold with structure $(\emptyset, \xi, \eta, g)$ the following relation hold.

$$\text{(a)} \quad \emptyset(\xi) = 0 \qquad \text{(b)} \quad \eta(\emptyset X) = 0 \qquad \text{rank}\emptyset = n - 1 \tag{1.7}$$

Let us $put\ F(X,Y) = g(\emptyset X, Y)$ (1.8)

The tensor field F is symmetric $(0,2)$ tensor field ie $F(X,Y) = F(Y,X)$ (1.9)

And
$$(D_X\eta)(Y) = F(X,Y) = g(\emptyset X, Y) \tag{1.10}$$

Also in a LP sasakian manifold the following relation holds;

$$g(R(X,Y)Z,\xi) = \eta(R(X,Y)Z) = g(Y,Z)\eta(X) - g(X,Z)\eta(Y) \tag{1.11}$$

And
$$S(Y,\xi) = (n-1)\eta(X) \tag{1.12}$$

For any vector field X, Y, Z where $R(X,Y)Z$ is the Riemannian curvarture tensor and S is the Ricci tensor.

Let (M^n, g) be an LP sasakian manifold with Levi-Civita connection D. we define a linear connection $\overline{D}$ on M^n by

$$\overline{D}_X Y = D_X Y - \eta(X)Y \tag{1.13}$$

Where η is 1-form associated with vector field ξ on M^n, given by

$$g(X,\xi) = \eta(X) \tag{1.14}$$

Using (1.13) the torsoin tensor $\overline{T}$ of M^n with respect to connection $\overline{D}$ is given by

$$\overline{T}(X,Y) = \overline{D}_X Y - \overline{D}_Y X - [X,Y] = \eta(Y)X - \eta(X)Y \tag{1.15}$$

A linear connection satisfying (1.15) is called semi symmetric connection.
Further from (1.13), we have

$$(\overline{D}_X g)(Y,Z) = 2\eta(X)g(Y,Z) \tag{1.16}$$

A linear connection satisfying (1.16) is called semi symmetric recurrent metric connectionThe word special is used to distinguish it from other connection.

II. Existence of Special Semi-Symmetric Recurrent Metric Connection

Let $\overline{D}$ be a linear connection in M^n, given by

$$\overline{D}_X Y = D_X Y + H(X,Y) \tag{2.1}$$

Where H is a tensor of type (1,2).

Now, we determine the tensor field H such that $\overline{D}$ satifies (1.15) and (1.16).

Notes

Notes

From (2.1), we have

$$\bar{T}(X,Y) = H(X,Y) - H(Y,X) \tag{2.2}$$

$$\text{Let } G(X,Y,Z) = (\bar{D}_X g)(Y,Z) \quad \text{then} \tag{2.3}$$

$$g(H(X,Y),Z) + g(H(X,Z),Y) - -G(X,Y,Z) \tag{2.4}$$

From (2.1),(2.2) and (2.4), we have

$$g(\bar{T}(X,Y),Z) + g(\bar{T}(Z,X),Y) + g(\bar{T}(Z,Y),X)$$

$$= g(H(X,Y),Z) - g(H(Y,X),Z) + g(H(Z,X),Y) - g(H(X,Z),Y) + g(H(Z,Y),X) - g(H(Y,Z),X)$$

$$= 2g(H(X,Y),Z) + 2\eta(X)g(Y,Z) - 2\eta(Z)g(X,Y) + 2\eta(Y)g(X,Z$$

Or

$$H(X,Y) = \frac{1}{2}\{\bar{T}(X,Y) + \bar{T}'(X,Y) + \bar{T}'(Y,X)\} - \eta(X)Y - \eta(Y)X + g(X,Y)\xi \tag{2.5}$$

Where
$$g\left(\bar{T}'(X,Y),Z\right) = g(\bar{T}(Z,X),Y) \tag{2.6}$$

Using (1.5), (2.6) we get

$$\bar{T}'(X,Y) = \eta(X)Y - g(X,Y)\xi \tag{2.7}$$

Then in view of (1.15), (2.5) and (2.7),we get

$$H(X,Y) = -\eta(X)Y$$

This implies

$$\bar{D}_X Y = D_X Y - \eta(X)Y$$

Conversely, a connection $\bar{D}$ given by (1.13), satisfies (1.15) and (1.16) show that $\bar{D}$ is a special semi symmetric recurrent metric connection.
So we state the following theorem.

Theorem 2.1: let (M^n, g) be an LP sasakian manifold with lorentzian para contact metric structure $(\emptyset, \xi, \eta, g)$ admits a special semi-symmetric connection which is given by

$$\bar{D}_X Y = D_X Y - \eta(X)Y$$

III. Curvature Tensor of M^n with Respect to Special Semi –Symmetric Recurrent Metric Metric Connection $\bar{D}$

The Curvature tensor of M^n with respect to special semi - symmetric recurrent metric metric connection $\bar{D}$ is given by

$$\bar{R}(X,Y,Z) = \bar{D}_X \bar{D}_Y Z - \bar{D}_X \bar{D}_Y Z - \bar{D}_{[X,Y]} Z$$

Using (1.13) and (1.10) in above we have

$$\bar{R}(X,Y,Z) = R(X,Y,Z) \tag{3.1}$$

Hence we conculude.

Proposition 3.1: The Curvature tensor of M^n with respect to special semi-symmetric recurrent metric metric connection $\overline{D}$ coincide with the curvature tensor of connection D of Riemannian manifold.
Taking the inner product of (3.1) with W, we have

$$\bar{R}(X,Y,Z,W) = R(X,Y,Z,W) \tag{3.2}$$

Where $\bar{R}(X,Y,Z,W) = g(\bar{R}(X,Y,Z),$

From (3.2), we have

$$\bar{R}(X,Y,Z,W) = -R(Y,X,Z,W) \tag{3.3}$$

$$\bar{R}(X,Y,Z,W) = -R(X,Y,W,Z) \tag{3.4}$$

Combining above two relation, we have

$$\bar{R}(X,Y,Z,W) = R(Y,X,W,Z) \tag{3.5}$$

We also have,

$$\bar{R}(X,Y,Z) + \bar{R}(Y,Z,X) + \bar{R}(Z,X,Y) = 0 \tag{3.6}$$

This is the Bianchi first identity for $\overline{D}$.

Hence we conclude that the curvature tensor of M^n with respect to special semi - symmetric recurrent metric metric connection $\overline{D}$ satifies the first Bianchi identy.
Contracting (3.2) over X and W, we obtain

$$\bar{S}(Y,Z) = S(Y,YZ) \tag{3.7}$$

Where $\bar{S}$ and S denote the Ricci tensor of the connection $\overline{D}$ and D respectively.

From (3.7) we obtain a relation between the scalar curvature of M^n with respect to the Riemannian connection and special semi-symmetric recurrent metric connection which is given by

$$\bar{r} = r \tag{3.8}$$

So we have following

Proposition 3.2: Forn dimensional LP sasakian manifold with special semi symmetric recurrent metric connection $\overline{D}$

(1) The curvature tensor $\bar{R}$ is given by (3.1)

(2) Ricci tensor $\bar{S}$ is given by (3.7)

(3) $\bar{r} = r$

Notes

IV. Concircular Curvature Tensor of Lp Sasakian Manifold with Respect to Special Semi-Symmtric Recurrent Metric Connection

Analogous to the definition of concircularcurvarure tensor in a Riemannian manifold we define concircular curvature tensor with respect to the special semi symmetric recurrent metric connection $\overline{D}$ as

$$\bar{C}(X,Y,Z) = \bar{R}(X,Y,Z) - \frac{r}{n(n-1)}\{g(Y,Z)X - g(X,Z)Y\} \qquad (4.1)$$

Using (3.1) and (3.7) in (4.1), we have

$$\bar{C}(X,Y,Z) = C(X,Y,Z) \qquad (4.2)$$

So we have

Proposition 4.1: $\bar{C}(X,Y,Z) = C(X,Y,Z)$ that is manifold coincide with Riemannian Manifold.

The notion of an ξ-conformaly flat contact manifold was given by Zhen, Cabrezizo and Fermander [3]. In an analogous we define an ξ-conformaly flat n - dimensional LP sasakian manifold.

Defination 5.2: An n-dimensional LP sasakian manifold is called ξ-conformaly flat if the condition $\bar{C}(X,Y)\xi = 0$ holds on M^n.
From (4.2) it is clear that $\bar{C}(X,Y)\xi = C(X,Y)\xi$
So we have the following theorem.

Theorem 4.1: In an n-dimensional LP Sasakian manifold, an ξ–conformaly flatness with respect to special semi-symmetric recurrent metric connection and Riemannian connection coincide.

Defination 4.3: an n dimensional LP Sasakian manifold satisfying the condition

$$\emptyset^2\bar{C}(\emptyset X,\emptyset Y)\emptyset Z = 0 \qquad (4.3)$$

Is called $\emptyset -$ concircularly flat.

Letus suppose that M^n be n dimensional $\emptyset -$ concircularly flat LP sasakian manifold with respect to special semi-symmetric recurrent metric connection. It can easily be seen that $\emptyset^2\bar{C}(\emptyset X,\emptyset Y)\emptyset Z = 0$ if and only if

$$g(\bar{C}(\emptyset X,\emptyset Y)\emptyset Z,\emptyset W) = 0 \qquad (4.4)$$

For all X,Y, Z, Won T(M).

Using (4.1), $\emptyset -$ concircularly flat means

$$g(\bar{R}(\emptyset X,\emptyset Y)\emptyset Z,\emptyset W) = \frac{r}{n(n-1)}\{g(\emptyset Y,\emptyset Z)g(\emptyset X,\emptyset W) - g(\emptyset X,\emptyset Z)g(\emptyset Y,\emptyset W)\} \qquad (4.5)$$

Ref

3. Zhen, G, Cabrezizo J. L, Fernander, L.M and Fernander: on ξ conformally flat contact metric manifold. Indian J. Pure. appl. math 28, 725-734, 1997.

Let $\{e_1, e_2 \ldots., \xi\}$ be a local orthogonal basis of the vector in M^n using the fact that $\emptyset e_1, \emptyset e_2 \ldots. \xi\}$ is also a local orthogonal basis, putting $X = W = e_i$ in (4.5) and summanig with respect to i, we have

$$S(\emptyset Y, \emptyset Z) = \frac{r}{n(n-1)}\{g(\emptyset Y, \emptyset Z)\} \tag{4.6}$$

Notes

Putting $Y = \emptyset Y, Z = \emptyset Z$ in (4.6) and using the fact S is symmetric, we have

$$g(\bar{R}(\emptyset X, \emptyset Y)\emptyset Z, \emptyset W) = 0$$

Hence we have

Theorem 4.2: an n-dimensional LP sasakian manifold is $\emptyset -$ concircularly flat with respect to special semi-symmetric recurrent metric connectionand manifold coincide with Riemannian Manifold.

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