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On the Maximal Ideals in the Banach Space of $^{+}$Quasicontinuous Functions

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ON THE MAXIMAL IDEALS IN THE BANACH SPACE OF +QUASICONTINUOUS FUNCTIONS

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1. **Borsik, J.**, *Points of continuity, Quasicontinuity and Cliquishness*, Pervento in redazoine, pp. 5 – 20, 1993.

On the Maximal Ideals in the Banach Space of $^{+}$Quasicontinuous Functions

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Keywords : $^{+}$quasicontinuity, maximal ideal, space of maximal ideals, weak topology, compact hausdorff space, bounded linear functional, cliquish function.*

I. Introduction

In this paper, it is shown that the set of all bounded real $^{+}$Quasicontinuous functions defined on $[0,1]$ forms a commutative Banach algebra with identity under the supremum norm. The maximal ideals in this Banach algebra are identified to be of the form $M_x = \{f \,/\, f(x) = 0\}$ or $M_x^{+} = \{f \,/\, f(x+) = 0\}$ for $x \in [0,1]$.

In what follows $\mathbb{R}$, I and J stand for the real line, the unit closed interval $[0,1]$ and any closed and bounded interval $[a,b]$ respectively.

a) *Preliminaries*

1.1 Definition: Let $f : J \to \mathbb{R}$. We define $f(a-) = f(a)$ and $f(b+) = f(b)$. We say that $f(p+)$ exists at $p \in [a,b)$ and we write $f(p+) = L$, where $L \in \mathbb{R}$ if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - L| < \varepsilon \;\; \forall \;\; x \in (p, p+\delta) \subset J$

Similarly for $p \in (a,b]$ we write $f(p-) = l \in \mathbb{R}$ if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that $|f(x) - l| < \varepsilon \;\; \forall \; x \in (p-\delta, p) \subset J$

1.2 Definition: A function $f : J \to \mathbb{R}$ is said to be $^{+}$Quasicontinuous on J if

(i) $f(p+)$ exists at every $p \in [a,b)$

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(ii) $f(b-)=f(b)$

1.3 Definition: A function $f:J\to\mathbb{R}$ is said to be cliquish at a point $p\in J$ if for every $\varepsilon>0$ and every neighborhood U of p in J there exists a non-empty open set $W\subset U$ such that $|f(x)-f(y)|<\varepsilon \quad \forall\, x,y\in W$. We say that f is cliquish on J if it is cliquish at every point of J.

1.4 Definition: A mapping T from a linear space $\mathscr{V}$ into a linear space $\mathscr{W}$ is said to be linear if $T(cx+dy)=cT(x)+dT(y)$ for all x and y in $\mathscr{V}$ and constants c and d.

1.5 Definition: Let $\mathscr{V}$ and $\mathscr{W}$ be normed linear spaces. A linear map $T:\mathscr{V}\to\mathscr{W}$ is said to be bounded if there exists a real number $K\geq 0$ such that $\|T(x)\|\leq K\|x\| \quad \forall\; x\in\mathscr{V}$.

1.6 Definition: A linear functional on a vector space $\mathscr{V}$ over a field $\mathscr{K}$ is a linear mapping from $\mathscr{V}$ to $\mathscr{K}$.

b) Properties of +Quasicontinuous functions

2.1 Proposition: Let $c\in\mathbb{R}$. If $f:J\to\mathbb{R}$ and $g:J\to\mathbb{R}$ are +Quasicontinuous on J then $f+g$, cf, fg, $f\vee g$ and $f\wedge g$ are +Quasicontinuous on J, where $(f\vee g)(x)=\max\{f(x),g(x)\}$ and $(f\wedge g)(x)=\min\{f(x),g(x)\}$.

Proof: Let $p\in[a,b)$. (i) Let $\varepsilon>0$ be given. Then there exist $\delta_1>0$ and $\delta_2>0$ such that $|f(x)-f(p+)|<\frac{\varepsilon}{2} \;\forall\, x\in(p,p+\delta_1)\subset J$ and $|g(x)-g(p+)|<\frac{\varepsilon}{2} \;\forall\, x\in(p,p+\delta_2)\subset J$. Put $\delta=\min\{\delta_1,\delta_2\}$.

Then $x\in(p,p+\delta) \;\Rightarrow\; |(f+g)(x)-(f(p+)+g(p+))|\leq|f(x)-f(p+)|+$

$$|g(x)-g(p+)| \;<\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon.$$

Thus for every $\varepsilon>0$ there exists a $\delta>0$ such that

$$|(f+g)(x)-(f(p+)+g(p+))|<\varepsilon \quad \forall \quad x\in(p,p+\delta).$$

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Hence $(f+g)(p+)$ exists and $(f+g)(p+)=f(p+)+g(p+)$. Since f and g are continuous at b, $f+g$ is continuous at b.

Hence $f+g$ is $^+$Quasicontinuous on J.

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(ii) If $c=0$ then $cf=O$, where $O:J\to\mathbb{R}$ is defined by $O(x)=0$.

Then cf is $^+$Quasicontinuous on J. Now suppose that $c\neq 0$.

Let $\varepsilon>0$ be given. Then there exists a $\delta>0$ such that

$$|f(x)-f(p+)|<\frac{\varepsilon}{|c|}\ \ \forall\, x\in(p,p+\delta)\subset J$$

$$\Rightarrow\ |(cf)(x)-(cf)(p+)|<\varepsilon\ \ \forall x\in(p,p+\delta)$$

Hence $(cf)(p+)$ exists and $(cf)(p+)=c\,f(p+)$. Since f is continuous at b, cf is continuous at b. Hence cf is $^+$Quasicontinuous on J.

(iii) Since f and g are $^+$Quasicontinuous at p, for every $\varepsilon>0$ there exists a $\delta>0$ such that $|f(x)-f(p+)|<\varepsilon$ and $|g(x)-g(p+)|<\varepsilon\ \ \forall\, x\in(p,p+\delta)\subset J$

$$\begin{aligned}\Rightarrow|(fg)(x)-f(p+)g(p+)| &= |f(x)g(x)-f(x)g(p+)+f(x)g(p+)-f(p+)g(p+)|\\ &\le |f(x)||g(x)-g(p+)|+|g(p+)||f(x)-f(p+)|\\ &< |f(x)|\varepsilon+|g(p+)|\varepsilon\quad\forall\ x\in(p,p+\delta)\\ &= |f(x)-f(p+)+f(p+)|\varepsilon+|g(p+)|\varepsilon\\ &< \varepsilon\big(\varepsilon+|f(p+)|+|g(p+)|\big)\quad\forall\ x\in(p,p+\delta).\end{aligned}$$

Hence $(fg)(p+)$ exists and $(fg)(p+)=f(p+)g(p+)$. Since f and g are continuous at b, fg is continuous at b.

Hence fg is $^+$Quasicontinuous on J.

It is easy to verify that $f\vee g$ and $f\wedge g$ are $^+$Quasicontinuous on J and we have the following.

$(f\vee g)(p+)=\max\{f(p+),g(p+)\}$ and $(f\wedge g)(p+)=\min\{f(p+),g(p+)\}$.

2.2 Proposition: Let $f_n:J\to\mathbb{R}$, $n=1,2,3,\ldots$, be $^+$Quasicontinuous on J and $f_n\to f$ uniformly on J. Then f is $^+$Quasicontinuous on J.

Proof: Let $p \in [a,b)$. Let $\varepsilon > 0$ be given. Then there exists an integer N such that $n \geq N \Rightarrow |f_n(x) - f(x)| < \frac{\varepsilon}{2} \quad \forall\ x \in J$.

Since f_N is $^+$Quasicontinuous at p, there exists a $\delta > 0$ such that

$$|f_N(x) - f_N(p+)| < \varepsilon \ \forall\ x \in (p, p+\delta) \subset J$$

$$x \in (p, p+\delta) \Rightarrow |f(x) - f_N(p+)| = |f(x) - f_N(x) + f_N(x) - f_N(p+)|$$

$$\leq |f(x) - f_N(x)| + |f_N(x) - f_N(p+)|$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Thus for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$|f(x) - f_N(p+)| < \varepsilon \ \forall\ x \in (p, p+\delta) \subset J.$$

Hence $f(p+)$ exists for every $p \in [a,b)$.

Since each f_n is continuous at b and $f_n \to f$ uniformly on J, f is continuous at b. Hence f is $^+$Quasicontinuous on J.

2.3 Remark: It is not necessary that a $^+$Quasicontinuous function defined on a compact domain is bounded. It can be seen from the following example.

2.4 Example: Define $f : [-1,1] \to \mathbb{R}$ by $f(x) = \begin{cases} 1 & if \quad 0 \leq x \leq 1 \\ \frac{1}{x} & if \quad -1 \leq x < 0 \end{cases}$

This function f is $^+$Quasicontinuous on $[-1,1]$ but it is not bounded.

2.5 Remark: We denote the set of all bounded real valued $^+$Quasicontinuous functions defined on I by the symbol $\mathcal{QC}^+(I)$. By the propositions 2.1 and 2.2 it follows that $\mathcal{QC}^+(I)$ forms a commutative Banach algebra with identity under the supremum norm, where the identity $\boldsymbol{e} : I \to \mathbb{R}$ is defined by $\boldsymbol{e}(x) = 1 \ \forall\ x \in I$.

2.6 Proposition: Let $f : J \to \mathbb{R}$ and $p \in J$. If $f(p+)$ exists then f is cliquish at p.

Proof: Let $\varepsilon > 0$ be given and let U be a neighborhood of p in J. Then there exists a $\delta_1 > 0$ such that $(p - \delta_1, p + \delta_1) \cap J \subset U$.

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Given $f(p+)$ exists. So there exists $\delta_2 > 0$ such that

$$|f(x) - f(p+)| < \frac{\varepsilon}{2} \quad \forall \; x \in (p, p+\delta_2) \subset J.$$

Put $\delta = \min\{\delta_1, \delta_2\}$ and $W = (p, p+\delta)$.

Then for $x, y \in W$, we have $|f(x) - f(y)| = |f(x) - f(p+) + f(p+) - f(y)|$

$$\leq |f(x) - f(p+)| + |f(y) - f(p+)|$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.$$

Thus for every $\varepsilon > 0$ and every neighborhood U of p, there exists a non-empty open set $W \subset U$ such that $|f(x) - f(y)| < \varepsilon \quad \forall \; x, y \in W$

$\Rightarrow$ f is cliquish at p.

2.7 Remark: From the above proposition it is clear that every $^+$Quasicontinuous function is cliquish. The converse is not true as is evident from the following example.

2.8 Example: Define $\mathsf{f}: [-1,1] \to \mathbb{R}$ as follows.

$$\mathsf{f}(x) = \begin{cases} \frac{1}{x} & \text{if} \quad 0 < x \leq 1 \\ 0 & \text{if} \quad -1 \leq x \leq 0 \end{cases}$$

Clearly f is cliquish at $x = 0$ but it is not $^+$Quasicontinuous.

2.9 Theorem [2]: If $f : J \to \mathbb{R}$ is $^+$Quasicontinuous then the set of points of discontinuity of f is atmost countable.

c) Maximal Ideals in $\mathscr{QC}^+(I)$

3.1 Definition: For each $x \in I$, we define the following.

(a) $M_x = \{f \in \mathscr{QC}^+(I) \,/\, f(x) = 0\}$ (b) $M_x^+ = \{f \in \mathscr{QC}^+(I) \,/\, f(x+) = 0\}$.

3.2 Proposition: For each $x \in I$, the sets M_x and M_x^+ are maximal ideals in the commutative Banach algebra $\mathscr{QC}^+(I)$.

Proof: For $x \in I$, define F_x and F_x^+ on $\mathscr{QC}^+(I)$ by $F_x(f) = f(x)$ and $F_x^+(f) = f(x+)$ for $f \in \mathscr{QC}^+(I)$.

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Clearly F_x and F_x^+ are multiplicative linear functionals in the dual space $\mathscr{B}_+$ with kernels M_x and M_x^+ respectively. Hence M_x and M_x^+ are ideals. Moreover M_x and M_x^+ are maximal ideals in $\mathscr{QC}^+(I)$.

3.3 Proposition: If M is a maximal ideal in $\mathscr{QC}^+(I)$ then either $M = M_x$ or $M = M_x^+$ for some $x \in I$.

Proof: For $x \in I$, define F_x and F_x^+ on $\mathscr{QC}^+(I)$ by $F_x(f) = f(x)$ and $F_x^+(f) = f(x+)$ for $f \in \mathscr{QC}^+(I)$.

Clearly F_x and F_x^+ are multiplicative linear functionals in the dual space $\mathscr{B}_+$ with kernels M_x and M_x^+ respectively. Hence M_x and M_x^+ are ideals. Moreover M_x and M_x^+ are maximal ideals in $\mathscr{QC}^+(I)$.

3.3 Proposition: If M is a maximal ideal in $\mathscr{QC}^+(I)$ then either $M = M_x$ or $M = M_x^+$ for some $x \in I$.

Proof: Assume that $M \neq M_x$ and $M \neq M_x^+$ for any $x \in I$.

Then there exist f_x and g_x in M such that $f_x \notin M_x$ and $g_x \notin M_x^+$.

Define $\varphi_x : I \to \mathbb{R}$ by $\varphi_x(t) = f_x^2(t) + g_x^2(t+) \quad \forall \ t \in I$.

Clearly $\varphi_x \in \mathscr{QC}^+(I)$. Since φ_x is $^+$Quasicontinuous at x and $\varphi_x(x) > 0$, there exists $\delta_x > 0$ such that $\varphi_x(t) > 0 \quad \forall \ t \in [0, \delta_x)$ and for $x \neq 1$

We have $\varphi_1(t) = f_1^2(t) + g_1^2(t+) \ \forall \, t \in I$.

Since φ_1 is continuous at 1 and $\varphi_1(1) > 0$ there exists a $\delta > 0$ such that

$\varphi_1(t) > 0 \ \forall \ t \in (1-\delta, 1]$. Then $[0,1] = \left(\bigcup_{x \neq 1} [0, \delta_x) \right) \cup (1-\delta, 1]$. Since I is compact,

there exists $x_1 \neq 1$ in I such that $[0,1] = [0, \delta_{x_1}) \cup (1-\delta, 1]$.

Put $\varphi = \varphi_{x_1}^2 + \varphi_1^2$. Then $\varphi \in M$ and $\varphi(t) > 0 \ \forall \, t \in I \ \Rightarrow \dfrac{1}{\varphi} \in M$

Then $\boldsymbol{e} = \varphi . \dfrac{1}{\varphi} \in M$. This is a contradiction.

Hence it follows that $M = M_x$ or $M = M_x^+$ for some $x \in I$.

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3.4 Remark: Let $\mathscr{M}_+$ be the space of all maximal ideals in $\mathscr{QC}^+(I)$. Then $\mathscr{M}_+$ is a compact Hausdorff space with the weak* topology on $\mathscr{QC}^+(I)$. Hence $\mathscr{M}_+^2 = \mathscr{M}_+ \times \mathscr{M}_+$ is a compact Hausdorff space with the product topology on $\mathscr{QC}^+(I) \times \mathscr{QC}^+(I)$.

3.5 Proposition: Let $\mathscr{A}^+ = \{(M_x, M_x^+) / x \in I\}$. Then there exists a one-to-one correspondence between I and $\mathscr{A}^+$.

Proof: Define $\Psi^+ : I \to \mathscr{A}^+$ by $\Psi^+(x) = (M_x, M_x^+)$.

Clearly Ψ^+ is surjective. If $0 \le s < t \le 1$, the function

$$\Psi_0^+(p) = \begin{cases} 0 & \text{if} \quad t \le p \le 1 \\ \dfrac{1}{x-t} & \text{if} \quad 0 \le p < t \end{cases}$$

satisfies $\Psi_0^+ \in M_t$ and $\Psi_0^+ \notin M_s$.

$\Rightarrow \quad M_s \ne M_t$

$\Rightarrow \quad (M_s, M_s^+) \ne (M_t, M_t^+)$

$\Rightarrow \quad \Psi^+(s) \ne \Psi^+(t)$

Hence Ψ^+ is $1-1$.

Hence Ψ^+ is a one-to-one correspondence between I and $\mathscr{A}^+$.

3.6 Remark: Each maximal ideal in $\mathscr{QC}^+(I)$ is the kernel of some multiplicative linear functional on $\mathscr{QC}^+(I)$, hence can be identified with a multiplicative linear functional on $\mathscr{QC}^+(I)$. Let M_x and M_x^+ be identified with the multiplicative linear functional F_x and F_x^+ respectively. So we can write

$\mathscr{A}^+ = \{(F_x, F_x^+) / x \in I\}$.

3.7 Proposition: $\mathscr{A}^+$ is closed in $\mathscr{B}_+^2 = \mathscr{B}_+ \times \mathscr{B}_+$ and hence compact.

Proof: We prove that $\mathscr{A}^+$ is closed. Compactness is an immediate consequence of the Banach – Alaoglu theorem [5]. If $F = (F_1, F_2) \in \mathscr{B}_+^2$, we define $\|F\| = \max\{\|F_1\|, \|F_2\|\}$. Then $\mathscr{B}_+^2$ is a Banach space under the above norm.

Let $S = \{F / \|F\| \le 1\} \subset \mathscr{B}_+^2$. Put $\mathcal{A} = \mathscr{A}^+ \cup \{O\}$.

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The $\mathscr{A}^{+} \subset \mathscr{M}_{+}^{2} \subset \mathcal{A} \subset S \subset \mathcal{B}_{+}^{2}$.

Define $\mathcal{P}^{+} : \mathcal{A} \to \mathbb{R}$ by

$$\mathcal{P}^{+}(F) = \begin{cases} 1 & \text{if } \quad F \in \mathcal{A} \ \text{ and } \ F \neq O \\ 0 & \text{if } \quad F = O \end{cases}$$

Since $\mathcal{P}^{+}$ is continuous, $\mathscr{A}^{+}$ and $\mathcal{A}$ are closed in $\mathcal{B}_{+}^{2}$.

d) Further Properties

4.1 Proposition: Fix $f \in \mathscr{QC}^{+}(I)$. Define $\psi_f : I \to \mathbb{R}^2$ by $\psi_f(x) = (f(x), f(x+))$, where $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ is considered with the norm $\|(x_1, x_2)\| = \max\{|x_1|, |x_2|\}$. Then ψ_f is continuous on I if and only if f is continuous on I.

Proof: Assume that ψ_f is continuous on I. Let $p \in I$ and let $\varepsilon > 0$ be given.

Since ψ_f is continuous at p, there exists a $\delta > 0$ such that

$\|\psi_f(x) - \psi_f(p)\| < \varepsilon \ \ \forall \ x \in (p-\delta, p+\delta) \cap I$.

$\Rightarrow \|(f(x), f(x+)) - (f(p), f(p+))\| < \varepsilon \quad \forall \ x \in (p-\delta, p+\delta) \cap I$

$\Rightarrow \|(f(x) - f(p), f(x+) - f(p+))\| < \varepsilon \quad \forall \ x \in (p-\delta, p+\delta) \cap I$

$\Rightarrow \max\{|f(x) - f(p)|, |f(x+) - f(p+)|\} < \varepsilon \quad \forall \ x \in (p-\delta, p+\delta) \cap I$

$\Rightarrow |f(x) - f(p)| < \varepsilon \quad \forall \ x \in (p-\delta, p+\delta)$

$\Rightarrow \ f$ is continuous at p.

Thus if ψ_f is continuous at p then f is continuous at p.

Conversely suppose that f is continuous on I.

Then $\psi_f(x) = (f(x), f(x)) \ \forall \ x \in I$.

Hence ψ_f continuous on I.

4.2 Proposition: Let $\mathbf{B} = \{\psi_f \,/\, f \in \mathscr{QC}^{+}(I)\}$. Define $F : \mathscr{QC}^{+}(I) \to \mathbf{B}$ by $F(f) = \psi_f$. Then F is a one-to-one continuous multiplicative linear mapping from $\mathscr{QC}^{+}(I)$ onto **B**.

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Proof: Clearly $F:\mathscr{QC}^+(I)\to\mathbf{B}$ is surjective.

For $f,g\in\mathscr{QC}^+(I)$, $\psi_{f+g}(x)=((f+g)(x),(f+g)(x+))$

$$=(f(x),f(x+))+(g(x),g(x+))$$

$$=\psi_f(x)+\psi_g(x)\quad\forall\ x\in I$$

Hence $\psi_{f+g}=\psi_f+\psi_g\quad\forall\quad f,g\in\mathscr{QC}^+(I)$

$\Rightarrow\quad F(f+g)=F(f)+F(g)\quad\forall\quad f,g\in\mathscr{QC}^+(I)$.

Let $c\in\mathbb{R}$.

Then it is easy to see that $F(cf)=\psi_{cf}=c\psi_f=cF(f)\quad\forall\quad f\in\mathscr{QC}^+(I)$.

Hence F is linear.

Also we have $\psi_{fg}(x)=((fg)(x),(fg)(x+))$

$$=(f(x),f(x+))\ (g(x),g(x+))$$

$$=\psi_f(x)\,\psi_g(x)\ \ \forall\ \ x\in I.$$

Hence $F(fg)=\psi_{fg}=\psi_f\psi_g=F(f)F(g)$.

$\Rightarrow$ F is multiplicative now we prove that F is $1-1$. For this, suppose that $F(f)=F(g)$

$\Rightarrow\quad\psi_f=\psi_g$

$\Rightarrow\quad\psi_f(x)=\psi_g(x)\quad\forall\quad x\in I$

$\Rightarrow\quad(f(x),f(x+))=(g(x),g(x+))\ \ \forall\ x\in I$

$\Rightarrow\quad f(x)=g(x)\quad\forall\quad x\in I$

$\Rightarrow\quad f=g$.

Hence F is $1-1$.

Suppose that $f_n\in\mathscr{QC}^+(I)$, $n=1,2,3,\ldots,$ and $f\in\mathscr{QC}^+(I)$.

Let $f_n\to f$ uniformly on I. Then for a given $\varepsilon>0$ there exists an integer $N>0$ such that $\left|f_n(x)-f(x)\right|<\frac{\varepsilon}{3}$ for all $n\geq N$ and all $x\in I$.

Fix $x\in I$ and $n\geq N$. Since f_n is $^+$Quasicontinuous there exists a $\delta_1>0$ such that $\left|f_n(t)-f_n(x+)\right|<\frac{\varepsilon}{3}\quad\forall\ \ t\in(x,x+\delta_1)$.

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Since f is also ${}^{+}$Quasicontinuous at x, there exists a $\delta_2 > 0$ such that

$$|f(t)-f(x+)| < \frac{\varepsilon}{3} \quad \forall\ t \in (x, x+\delta_2).$$

Put $\delta = \min\{\delta_1, \delta_2\}$. Then for $t \in (x, x+\delta)$ and $n \geq N$,

$$|f_n(x+)-f(x+)| = |f_n(x+)-f_n(t)+f_n(t)-f(t)+f(t)-f(x+)|$$

$$\leq |f_n(x+)-f_n(t)|+|f_n(t)-f(t)|+|f(t)-f(x+)|$$

$$< \frac{\varepsilon}{3}+\frac{\varepsilon}{3}+\frac{\varepsilon}{3} = \varepsilon.$$

Hence $|f_n(x+)-f(x+)| < \varepsilon$ for all $n \geq N$ and all $x \in I$.

$$n \geq N \ \Rightarrow\ \|F(f_n)-F(f)\| = \|\psi_{f_n}-\psi_f\|$$

$$= \sup\left\{\|\psi_{f_n}(x)-\psi_f(x)\| / x \in I\right\} < \varepsilon.$$

$\Rightarrow\ F(f_n) \to F(f)$ uniformly on I.

Hence F is continuous on $\mathcal{QC}^{+}(I)$.

4.3 Proposition: The set $\mathbf{B} = \{\psi_f \,/\, f \in \mathcal{QC}^{+}(I)\}$ is a commutative Banach algebra with identity ψ_e under the norm defined by $\|\psi_f\| = \sup\left\{\|\psi_f(x)\| / x \in I\right\}$, where $\psi_e(x) = (1,1)\ \ \forall\ x \in I$.

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