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Propagation of Stoneley Waves in Couple Stress Generalized Thermoelastic Media

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Abstract - The present paper is concerned with the propagation of Stoneley waves at the interface of two couple stress thermoelastic medium in context with Lord and Shulman (LS) and Green and Lindsay (GL) theories of thermoelasticity. It is observed that shear wave get decoupled from rest of the motion. After developing the formal solution, the secular equation for surface wave propagation is derived. The dispersion curve giving the phase velocity and attenuation coefficient related to wave number are plotted graphically to depict the effect of thermal relaxation times. The amplitude ratios of displacement components and temperature distribution are also computed numerically and shown graphically to depict the effect of thermal relaxation times. Some special cases are also deduced from the present investigation.

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Abstract - The present paper is concerned with the propagation of Stoneley waves at the interface of two couple stress thermoelastic medium in context with Lord and Shulman (LS) and Green and Lindsay (GL) theories of thermoelasticity. It is observed that shear wave get decoupled from rest of the motion. After developing the formal solution, the secular equation for surface wave propagation is derived. The dispersion curve giving the phase velocity and attenuation coefficient related to wave number are plotted graphically to depict the effect of thermal relaxation times. The amplitude ratios of displacement components and temperature distribution are also computed numerically and shown graphically to depict the effect of thermal relaxation times. Some special cases are also deduced from the present investigation.

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I. Introduction

The study of surface wave propagation along the free boundary of an elastic half-space or along the interface between two dissimilar elastic half-spaces has been a subject of continued interest for many years. These waves are well known in the study of seismic waves, geophysics and non- destructive evaluation, and there is a rich literature available in surface waves in terms of classical elasticity (see, e.g., Achenbach (1973); Brekhoviskikh (1960); Bullen and Bolt (1985); Ewing et al. (1957); Love (1911); Udias (1999)). Surface waves propagating along the free boundary of an elastic half-space, non-attenuated in their direction of propagation and damped normal to the boundary are known as Rayleigh waves in the literature, after their discoverer (Rayleigh (1885)). The phase velocity of Rayleigh wave is a single valued function of the parameters of an elastic half-space. These waves are nondispersive and their velocity is somewhat less than the velocity of shear waves in unbounded media.

Stoneley (1924) investigated the possible existence of waves similar to surface waves and propagating along the plane interface between two distinct uniform elastic solid half-spaces in perfect contact, and these are now universally known by his name. Stoneley waves can propagate at interfaces between either two elastic solid media or an elastic solid medium and a liquid medium. These waves are harmonic waves propagating along the interface between two elastic half-spaces, produce continuous traction and displacement across the interface and attenuate exponentially with distance normal to the

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interface in both the half-spaces, provided the range of the elastic constants of the two solids lie within some suitable limits (Scholte, (1947)). Stoneley (1924) obtained the frequency equation for propagation of Stoneley waves and showed that such interfacial waves can exist only if the velocity of distortional waves in the two half-spaces is approximately same.

Nayfeh and Abdelrahman (2000) found an approximate model for wave propagation in rectangular rods and their geometric limits. Tomar (2005) studied the wave propagation in elastic plates with voids. Some problems of wave propagation in micropolar elasticity medium voids were discussed by Kumar and Deswal (2006). Tomar and Singh (2006) discussed the propagation of Stoneley waves at an interface between two microstretch elastic half-spaces. Kumar et al. (2012) discussed the wave propagation in micropolar thermoelastic layer with two temperatures.

The ideas underlying the couple stress linear theory of elasticity were advanced by Voigt (1887) and the Cosserats (1909), but the subject was generalized and reached maturity only with the works of Toupin (1962), Green and Rivlin (1964) in addition, Kröner (1963) gave a physical aspects pertinent to crystal lattices and non local interpretation of the theory. It is noticed that the earlier application of couple stress elasticity theory, manly on quasi-static problems of stress concentration, met with success providing solution more adequate physically than classic solutions (Mindlin and Tiersten (1962), Weitsman (1965, 1967) Bogy and Sternberg (1967), Lakes (1982)).

Aggarwal and Alverson (1969) studied the effects of couple stresses on the diffraction of plane elastic waves by cylindrical discontinuities. Stefeniak (1969) presented the reflection of plane wave from a free surface in Cosserat medium. Sengupta and Ghosh (1974a, 1974b]) studied the effect of couple stress on surface waves in elastic media and propagation of waves in an elastic layer. Sengupta and Benergi (1978) investigated the effects of couple stresses on propagation of waves in an elastic layer immersed in an infinite liquid.

Anthonie (2000) studied the effects of couple stresses on the elastic bending of beams. Lubarda and Markenscoff (2000) investigated the conservation integrals in couple stress elasticity. Chen and Wang (2001) presented strain gradient theory with couple stress for crystalline solids. Circular inclusions in anti-plane strain couple stress elasticity have been investigated by Lubarda (2003). Selim (2006) studied the orthotropic elastic medium under the effect of initial and couple stresses. Shodja and Ghazisaeidi (2007) studied the effects of couple stresses in anti-plane problems of piezoelectric media with inhomogeneties. Radi (2007) find the effects of characteristic material lengths on mode III crack propagation in couple stress elasticity-plasticity materials by adopting an incremental version of the indeterminate theory couple stress plasticity displaying linear and isotropic strain hardening.

Gourgiotis and Gorgiadis (2008) investigated an approach based on distributed dislocations and dislocations for crack problems in couple stress theory of elasticity. Recently, the inplane orthotropic couple stress elasticity constant of elliptical call honey comb have been studied by Chung and Waas (2010). Kumar et al. (2012a, 2012b) studied the wave propagation in couple stress thermoelastic half space underlying an inviscid liquid layer and the propagation of SH-waves in couple stress elastic half space underlying an elastic layer.

The present study is concerned with the propagation of Stoneley waves in couple stress generalized thermoelastic medium in the context of Lord and Shulman (LS), Green and Lindsay (GL) theories of thermoelasticity. The dispersion curves giving the phase

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velocity and amplitude ratios of displacement components and temperature distribution are obtained and depicted graphically. Some special cases of interest are also studied.

II. Basic Equations

Notes

Following Mindlin and Tiersten (1962), Lord and Shulman (1967), Green and Lindsay (1972) the equations governing the couple stress generalized thermoelastic medium in absence of body forces, the constitutive relations are

$$t_{ji,j} = \rho \ddot{u}_i \,, \tag{1}$$

$$m_{ji,j} + e_{ijk} t_{jk} = 0 \,, \tag{2}$$

$$K^* \nabla^2 T - \rho c_e \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2} \right) = T_o \beta \left(\frac{\partial u_{i,i}}{\partial t} + \tau_0 n_o \frac{\partial^2 u_{i,i}}{\partial t^2} \right), \tag{3}$$

$$t_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} - \frac{1}{2} e_{ijk} m_{lk,l} - \beta \left(1 + \tau_1 \frac{\partial}{\partial t} \right) T \delta_{ij} \,, \tag{4}$$

$$m_{ij} = 4\alpha k_{ji} + 4\beta_1 k_{ij} \,, \tag{5}$$

$$k_{ij} = \phi_{j,i} \,, \tag{6}$$

$$\phi_i = \frac{1}{2} e_{ipq} u_{q,p} \quad , \tag{7}$$

where

u_i are the displacement components, ϕ_i is rotational vector, ρ is density, t_{ij} are stress components, m_{ij} are couple stress components, ε_{ij} are strain components, e_{ijk} is alternate tensor, k_{ij} is curvature tensor, δ_{ij} is Kronecker's delta, ∇^2 is Laplacian operator, τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$. Here $n_o = 1, \tau_1 = 0$, for LS theory and $n_o = 0, \tau_1 > 0$, for GL theory.

With the help of equations (2), (4)-(7) and without loss of generality assuming that $\beta' e_{lpq} e_{ijk} u_{q,pkl} = 0$, equation (1) takes the form

$$u_i = (\quad + \quad) u_{j,ij} + \mu \nabla^2 u_i + \alpha \left(e_{ijk} e_{kpq} u_{q,pi} \right)_{,ll} - \beta \left(1 + \tau_1 \frac{\partial}{\partial t} \right) T_{,i} \tag{8}$$

III. Formulation of the Problem

We consider two homogeneous isotropic couple stress generalized thermoelastic half spaces M_1 and M_2 connecting at the interface $x_3 = 0$. We consider a rectangular Cartesian coordinate system, (x_1, x_2, x_3) at any point on the plane horizontal surface and

x_1-axis in the direction of the wave propagation and x_3-axis taking vertically downward into the half-space so that all particles on a line parallel to x_2-axis are equally displaced, therefore, all the field quantities will be independent of x_2-coordinates. Medium M_2 occupies the region $-\infty < x_3 < 0$ and region $x_3 > 0$ is occupied by the half-space (medium M_1). The plane $x_3 = 0$ represents the interface between two media M_1 and M_2.

We take the displacements components as

$$u_i = (u_1, 0, u_3) \tag{9}$$

With the help of following Helmholtz's representation of displacement components u_1 and u_3 in terms of potentials ϕ and ψ

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1} \tag{10}$$

and non dimensional quantities

$$x_1' = \frac{\omega^*}{c_1} x_1, x_3' = \frac{\omega^*}{c_1} x_3, t' = \omega^* t, t_{ij}' = \frac{t_{ij}}{\beta T_0}, m_{ij}' = \frac{\omega^* m_{ij}}{c_1 \beta T_0}, u_1' = \frac{\omega^*}{c_1} u_1,$$
$$u_3' = \frac{\omega^*}{c_1} u_3, T' = \frac{T}{T_0}, \tau_1' = \omega^* \tau_1, \tau_0' = \omega^* \tau_0, t_0' = \omega^* t_0, h' = \frac{c_1 h}{\omega^*}, \omega^{*2} = \frac{\lambda^2}{\rho \alpha}, c_1^2 = \frac{\lambda + 2\mu}{\rho}, \tag{11}$$

where ω^* is the characteristic frequency,
equation (8) with the aid of equations (10) and (11) after suppressing the primes can be written as

$$\nabla^2 \phi - a_5 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T = \ddot{\phi}, \tag{12}$$

$$\nabla^2 \psi - a_6 \nabla^4 \psi = \frac{c_1^2}{c_2^2} \ddot{\psi}, \tag{13}$$

$$\nabla^2 T - a_7 \left(\frac{\partial T}{\partial t} + \tau_0 \frac{\partial^2 T}{\partial t^2}\right) = a_8 \left(\frac{\partial u_{i,i}}{\partial t} + \tau_0 n_o \frac{\partial^2 u_{i,i}}{\partial t^2}\right), \tag{14}$$

where

$$a_1 = \frac{\mu}{\lambda + \mu}, a_2 = \frac{\alpha \omega^{*2}}{c_1^2 (\lambda + \mu)}, a_3 = \frac{\beta T_0}{\lambda + \mu}, a_4 = \frac{\rho c_1^2}{\lambda + \mu}, a_5 = \frac{a_3}{1 + a_1}, a_6 = \frac{a_2}{a_1},$$
$$a_7 = \frac{\rho c_e c_1^2}{K^* \omega^*}, a_8 = \frac{c_1^2 \beta}{K^* \omega^*}, a_9 = \frac{\lambda}{\beta T_0}, a_{10} = \frac{\mu}{\beta T_0}, a_{11} = \frac{\alpha \omega^{*2}}{c_1^2 \beta T_0}, a_{12} = \frac{2 \alpha \omega^{*2}}{c_1^2 \beta T_0}, \tag{15}$$
$$a_{13} = \frac{2 \beta_1 \omega^{*2}}{c_1^2 \beta T_0}$$

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The equation (13) corresponds to purely transverse wave mode that decouples from rest of the motion and is not affected by the thermal effect.

IV. Solution of the Problem

We assume the solution of equations (12)-(14) of the form

$$(\phi,\psi,T)=(\phi_1(x_3),\psi_1(x_3),T_1(x_3))e^{i(kx_1-\omega t)} \tag{16}$$

where $c=\frac{\omega}{k}$ is the non-dimensional phase velocity, ω is the frequency and k is the wave number. Substituting the values ϕ,ψ,T from equation (16) in equations (12)-(14) and satisfying the radiation condition as $\phi,\ \psi,\ T\rightarrow 0$ as $x_3\rightarrow\infty$, we obtain the values of ϕ,ψ,T for medium M_1,

$$\phi=\left(A_1e^{-m_1x_3}+A_2e^{-m_2x_3}\right)e^{i(kx_1-\omega t)}, \tag{17}$$

$$\psi=\left(B_1e^{-m_3x_3}+B_2e^{-m_4x_3}\right)e^{i(kx_1-\omega t)}, \tag{18}$$

$$T=\left(b_1A_1e^{-m_1x_3}+b_2A_2e^{-m_2x_3}\right)e^{i(kx_1-\omega t)} \tag{19}$$

We attach bars for the medium M_2 and write the appropriate values of $\overline{\phi},\overline{\psi},\overline{T}$ for $M_2\left(x_3<0\right)$ satisfying the radiation conditions as:

$$\overline{\phi}=\left(\overline{A}_1e^{\overline{m}_1x_3}+\overline{A}_2e^{\overline{m}_2x_3}\right)e^{i(kx_1-\omega t)}, \tag{20}$$

$$\overline{\psi}=\left(\overline{B}_1e^{\overline{m}_3x_3}+\overline{B}_2{}^{\overline{m}_4x_3}\right)e^{i(kx_1-\omega t)},$$

$$\overline{T}=\left(\overline{b}_1\overline{A}_1e^{\overline{m}_1x_3}+\overline{b}_2\overline{A}_2e^{\overline{m}_2x_3}\right)e^{i(kx_1-\omega t)} \tag{22}$$

where m_1,m_2 are the roots of equation

$$\frac{d^4}{dx_3^4}+S\frac{d^2}{dx_3^2}+P=0 \tag{23}$$

and m_3,m_4 are the roots of equation

$$\frac{d^4}{dx_3^4}+S_1\frac{d^2}{dx_3^2}+P_1=0 \tag{24}$$

where

$$S=\left(A-2k^2\right),P=\left(k^4-k^2A+B\right),l_4=-\left(\frac{c_1^2\omega^2}{a_6c_2^2}\right),$$

$$S_1=-\left(\frac{1}{a_6}\right)-2k^2,P_1=k^4+\frac{k^2}{a_6}-l_4,b_i=-\left(\frac{a_8\omega hl_3}{m_i^2-k^2+a_7\omega^2l_2+ia_8\omega^3l_3a_5l_1}\right),i=1,2,$$

$$A=\left(\omega^2+l_2a_7\omega^2+a_5i\omega l_1a_8\omega^2l_3\right),\ B=\left(\omega^4l_2a_7\right),l_1=\tau_1+i\omega^{-1},l_2=\tau_0+i\omega^{-1},l_3=\tau_0n_0+i\omega^{-1}$$

Substituting the values of $\phi, \psi, T, \bar{\phi}, \bar{\psi}$ and $\bar{T}$ from equations (17)-(22) in equation (10), we obtained the displacement components

For medium M_1

$$u_1 = A_1 ike^{-m_1 x_3} + ikA_2 e^{-m_2 x_3} + m_3 B_1 e^{-m_3 x_3} + m_4 B_2 e^{-m_4 x_3}, \tag{25}$$

$$u_3 = -m_1 A_1 e^{-m_1 x_3} - m_2 A_2 e^{-m_2 x_3} + ikB_1 e^{-m_3 x_3} + ikB_2 e^{-m_4 x_3}, \tag{26}$$

For medium M_2

$$\bar{u}_1 = ik\bar{A}_1 e^{\bar{m}_1 x_3} + ik\bar{A}_2 e^{\bar{m}_2 x_3} - \bar{m}_3 \bar{B}_1 e^{\bar{m}_3 x_3} - \bar{m}_4 \bar{B}_2 e^{\bar{m}_4 x_3}, \tag{27}$$

$$\bar{u}_3 = \bar{m}_1 \bar{A}_1 e^{\bar{m}_1 x_3} \bar{m}_2 \bar{A}_2 e^{\bar{m}_2 x_3} + ik\bar{B}_1 e^{\bar{m}_3 x_3} + ik\bar{B}_2 e^{\bar{m}_4 x_3}. \tag{28}$$

V. Boundary Conditions

a) Mechanical Conditions

The boundary conditions are the continuity of normal stress, tangential stress, tangential couple stress, displacement components, temperature and rotations vector at the interface of elastic half spaces. Mathematically these can be written as

$$\left.\begin{aligned} t_{33} &= \bar{t}_{33} \\ t_{31} &= \bar{t}_{31} \\ m_{32} &= \bar{m}_{32} \\ u_1 &= \bar{u}_1 \\ u_3 &= \bar{u}_3 \\ \phi_2 &= \bar{\phi}_2 \end{aligned}\right\} \text{ at } x_3 = 0 \tag{29}$$

b) Thermal Condition

The thermal condition corresponding to insulated boundary is given by

$$\left.\begin{aligned} T &= \bar{T}, \\ K^* \frac{\partial T}{\partial x_3} &= \bar{K}^* \frac{\partial \bar{T}}{\partial x_3} \end{aligned}\right\} \text{ at } x_3 = 0. \tag{30}$$

Using equations (6) and (7) with aid of (11) in equations (4) and (5), we obtain the values of t_{ij} and m_{ij} as

$$t_{ij} = a_9 \delta_{ij} e + a_{10}\left(u_{i,j} + u_{j,i}\right) - a_{11} e_{ijk} e_{kpq} u_{q,pll} - \beta\left(1 + \tau_1 \frac{\partial}{\partial t}\right) T \delta_{ij}, \tag{31}$$

$$m_{ij} = a_{12} e_{ipq} u_{q,pj} + a_{13} e_{jpq} u_{q,pi}. \tag{32}$$

With the help of equations (31) and (32) with aid of (17)-(22) and (25)-(28) from equations (29) and (30), we obtain a system of eight homogeneous simultaneous linear equations and after some simplification, we obtain the secular equation as

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$$c^2=\frac{-\omega^2 a_9}{QQ}, \tag{33}$$

where

$$QQ=P_{12}Q_1-P_{13}Q_2+P_{13}Q_3-P_{14}Q_4+P_{15}Q_5-P_{16}Q_6+P_{17}Q_7-P_{18}Q_8,$$

$$Q_1=\frac{D2}{D1}, Q_2=\frac{D3}{D1}, Q_3=\frac{D4}{D1}, Q_4=\frac{D5}{D1}, Q_6=\frac{D6}{D1}, Q_7=\frac{D7}{D1}, Q_8=\frac{D8}{D1},$$

$$P_{11}=-a_9k^2+(a_9+a_{10})m_1^{\,2}+b_1l_1, P_{12}=-a_9k^2+(a_9+a_{10})m_2^{\,2}+b_2l_1, P_{13}=-ia_{10}km_3,$$

$$P_{14}=-ia_{10}km_4, P_{15}=-\bar{a}_9k^2+(\bar{a}_9+\bar{a}_{10})\bar{m}_1^{\,2}+(b_1)^2l_1, P_{16}=-\bar{a}_9k^2+(\bar{a}_9+\bar{a}_{10})\bar{m}_2^{\,2}+(b_1)^2l_1,$$

$$P_{17}=-i\bar{a}_{10}km_3, P_{18}=-i\bar{a}_{10}km_4, P_{21}=-2ika_{10}m_1-a_{11}\left[(m_1)^4+m_1ik^3\right],$$

$$P_{22}=-2ika_{10}m_2-a_{11}\left[(m_2)^4+m_2ik^3\right], P_{23}=-a_{10}\left[m_3^{\,2}+k^2\right]-a_{11}\left[ikm_3^{\,3}+2m_3^{\,2}k^2-m_3^{\,4}\right],$$

$$P_{24}=-a_{10}\left[m_4^{\,2}+k^2\right]-a_{11}\left[ikm_4^{\,3}+2m_4^{\,2}k^2-m_4^{\,4}\right], P_{25}=-2i\bar{a}_{10}k\bar{m}_1, P_{25}=-2i\bar{a}_{10}k\bar{m}_2,$$

$$P_{27}=\bar{a}_{10}\left[\bar{m}_3^{\,2}+k^2\right]+\bar{a}_{11}\left[-k^4+2\bar{m}_3^{\,2}k^2-\bar{m}_3^{\,4}\right], P_{28}=\bar{a}_{10}\left[\bar{m}_4^{\,2}+k^2\right]+\bar{a}_{11}\left[-k^4+2\bar{m}_4^{\,2}k^2-\bar{m}_4^{\,4}\right],$$

$$P_{33}=a_{12}\left((m_3)^3+k^2m_3\right), P_{34}=a_{12}\left((m_4)^3-k^2m_4\right), P_{37}=\bar{a}_{12}\left((\bar{m}_3)^3+k^2\bar{m}_3\right),$$

$$P_{38}=\bar{a}_{12}\left((\bar{m}_4)^3+k^2\bar{m}_4\right), P_{41}=-b_1km_1, P_{42}=-b_2km_2, P_{45}=-\bar{b}_1k\bar{m}_1, P_{46}=-\bar{b}_1k\bar{m}_2, P_{51}=ik,$$

$$P_{52}=ik, P_{53}=m_3, P_{54}=m_4, P_{55}=P_{56}=-ik, P_{57}=\bar{m}_3, P_{58}=\bar{m}_4, P_{61}=-m_1, P_{62}=-m_2,$$

$$P_{63}=P_{64}=ik, P_{65}=-\bar{m}_1, P_{66}=-\bar{m}_2, P_{67}=P_{68}=-ik, P_{71}=b_1, P_{72}=b_2, P_{75}=-\bar{b}_1, P_{76}=-\bar{b}_2,$$

$$P_{81}=-2ikm_1, P_{82}=-2ikm_2, P_{83}=-\left\{(m_2)^2+k^2\right\}, P_{84}=-\left\{(m_4)^2+k^2\right\},$$

$$P_{87}=\left\{(\bar{m}_3)^2-k^2\right\}, P_{88}=\left\{(\bar{m}_4)^2-k^2\right\}$$

and

$$D1=\begin{bmatrix} P_{22} & P_{23} & P_{24} & P_{25} & P_{26} & P_{27} & P_{28} \\ P_{32} & P_{33} & P_{34} & P_{35} & P_{36} & P_{37} & P_{38} \\ P_{42} & 0 & 0 & P_{45} & P_{46} & 0 & 0 \\ P_{52} & P_{53} & P_{54} & P_{55} & P_{56} & P_{57} & P_{58} \\ P_{62} & P_{63} & P_{64} & P_{65} & P_{66} & P_{67} & P_{68} \\ P_{72} & P_{73} & P_{74} & P_{75} & P_{76} & 0 & 0 \\ P_{82} & P_{83} & P_{84} & 0 & 0 & P_{87} & P_{88} \end{bmatrix}, \quad D2=\begin{bmatrix} P_{21} & P_{23} & P_{24} & P_{25} & P_{26} & P_{27} & P_{28} \\ P_{31} & P_{33} & P_{34} & P_{35} & P_{36} & P_{37} & P_{38} \\ P_{41} & 0 & 0 & P_{45} & P_{46} & 0 & 0 \\ P_{51} & P_{53} & P_{54} & P_{55} & P_{56} & P_{57} & P_{58} \\ P_{61} & P_{63} & P_{64} & P_{65} & P_{66} & P_{67} & P_{68} \\ P_{71} & P_{73} & P_{74} & P_{75} & P_{76} & 0 & 0 \\ P_{81} & P_{83} & P_{84} & 0 & 0 & P_{87} & P_{88} \end{bmatrix},$$

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$$D3=\begin{bmatrix} P_{21} & P_{22} & P_{24} & P_{25} & P_{26} & P_{27} & P_{28} \\ P_{31} & P_{32} & P_{34} & P_{35} & P_{36} & P_{37} & P_{38} \\ P_{41} & P_{42} & 0 & P_{45} & P_{46} & 0 & 0 \\ P_{51} & P_{52} & P_{54} & P_{55} & P_{56} & P_{57} & P_{58} \\ P_{61} & P_{62} & P_{64} & P_{65} & P_{66} & P_{67} & P_{68} \\ P_{71} & P_{72} & P_{74} & P_{75} & P_{76} & 0 & 0 \\ P_{81} & P_{82} & P_{84} & 0 & 0 & P_{87} & P_{88} \end{bmatrix}, \quad D4=\begin{bmatrix} P_{21} & P_{22} & P_{23} & P_{25} & P_{26} & P_{27} & P_{28} \\ P_{31} & P_{32} & P_{33} & P_{35} & P_{36} & P_{37} & P_{38} \\ P_{41} & P_{42} & 0 & P_{45} & P_{46} & 0 & 0 \\ P_{51} & P_{52} & P_{53} & P_{55} & P_{56} & P_{57} & P_{58} \\ P_{61} & P_{62} & P_{63} & P_{65} & P_{66} & P_{67} & P_{68} \\ P_{71} & P_{72} & P_{73} & P_{75} & P_{76} & 0 & 0 \\ P_{81} & P_{82} & P_{83} & 0 & 0 & P_{87} & P_{88} \end{bmatrix},$$

$$D5=\begin{bmatrix} P_{21} & P_{22} & P_{23} & P_{24} & P_{26} & P_{27} & P_{28} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{36} & P_{37} & P_{38} \\ P_{41} & P_{42} & 0 & 0 & P_{46} & 0 & 0 \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{56} & P_{57} & P_{58} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{66} & P_{67} & P_{68} \\ P_{71} & P_{72} & P_{73} & P_{74} & P_{76} & 0 & 0 \\ P_{81} & P_{82} & P_{83} & P_{84} & 0 & P_{87} & P_{88} \end{bmatrix}, \quad D6=\begin{bmatrix} P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{27} & P_{28} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{37} & P_{38} \\ P_{41} & P_{42} & 0 & 0 & P_{45} & 0 & 0 \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & P_{57} & P_{58} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & P_{67} & P_{68} \\ P_{71} & P_{72} & P_{73} & P_{74} & P_{75} & 0 & 0 \\ P_{81} & P_{82} & P_{83} & P_{84} & 0 & P_{87} & P_{88} \end{bmatrix},$$

$$D7=\begin{bmatrix} P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} & P_{28} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} & P_{38} \\ P_{41} & P_{42} & 0 & 0 & P_{45} & P_{46} & 0 \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & P_{56} & P_{58} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & P_{66} & P_{68} \\ P_{71} & P_{72} & P_{73} & P_{74} & P_{75} & P_{76} & 0 \\ P_{81} & P_{82} & P_{83} & P_{84} & 0 & P_{86} & P_{88} \end{bmatrix} \text{ and } D8=\begin{bmatrix} P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} & P_{27} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} & P_{37} \\ P_{41} & P_{42} & 0 & 0 & P_{45} & P_{46} & 0 \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & P_{56} & P_{57} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & P_{66} & P_{67} \\ P_{71} & P_{72} & P_{73} & P_{74} & P_{75} & P_{76} & 0 \\ P_{81} & P_{82} & P_{83} & P_{84} & 0 & P_{86} & P_{87} \end{bmatrix}.$$

VI. Amplitude Ratios

The amplitude ratios of displacement components and temperature distribution at the interface $x_3 = 0$ for insulated boundary are given by

$$\frac{A_2}{A_1} = -\frac{P_{41}}{P_{42}}, \quad \frac{B_1}{A_1} = L_1 L_2, \quad \frac{B_2}{A_1} = L_3 - \frac{P_{41}}{P_{42}} L_1 L_2,$$

where

$$L_1 = -\frac{P_{21}}{P_{25}} + \frac{P_{22}P_{41}}{P_{42}P_{24}} + \frac{P_{11}P_{23}}{P_{24}P_{13}} - \frac{P_{23}P_{12}P_{24}}{P_{13}P_{42}P_{24}}, L_2 = \frac{P_{24}P_{13}}{(P_{24}P_{13} - P_{23}P_{14})}, L_3 = -\frac{P_{11}}{P_{13}} + \frac{P_{12}P_{14}}{P_{13}P_{24}}.$$

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VII. Special Cases

For LS theory: Taking $\tau_1 = 0, n_0 = 1$ in equation (33), we obtain the secular equation for couple stress thermoelastic media with one relaxation time with the changed values of l_1, l_3 as

$$l_1 = i\omega^{-1}, l_3 = i\omega + \tau_0$$

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For GL theory: In this case, taking $n_0 = 0$ in equation (33), we obtain the secular equation in couple stress thermoelastic medium with two relaxation times with the changed values of l_3 as

$$l_3 = i\omega$$

For C-T theory: Taking $\tau_1 = \tau_0 = 0$, in equation (33), yield the secular equation in couple stress thermoelastic medium with the changed values of l_1, l_2, l_3 as

$$l_1 = l_2 = l_3 = i\omega^{-1}$$

VIII. Numerical Discussion

With view of illustrating theoretical results derived in preceding sections, we now present some numerical results in this section. The physical data for medium M_1 and M_2 is given by

For medium M_1:

$$\lambda = 2.17\, N/m^2, \mu = 3.28\, N/m^2, \rho = 1.74\, kg/m^3, c_e = 1.04 \times 10^3\, Jkg^{-1}\deg^{-1},$$

$$K^* = 1.7 \times 10^2\, Wm^{-1}\deg^{-1}, T_0 = .298^o K, \alpha = 2.05\, N, \beta = 2.68\, Nm^{-1}\deg^{-1}, \beta_1 = 1.5N$$

For Medium M_2:

$$\bar{\lambda} = 2.238\, N/m^2, \bar{\mu} = 2.992\, N/m^2, \rho = 2.65\, kg/m^3, \bar{c}_e = 0.021 \times 10^3\, Jkg^{-1}\deg^{-1},$$

$$\bar{K}^* = 2.4 \times 10^2\, Wm^{-1}\deg^{-1}, \bar{T}_0 = .296^o K, \bar{\alpha} = 1.9\, N, \bar{\beta} = 2.03\, Nm^{-1}\deg^{-1}, \beta_1' = 2.4N$$

Using the above values of parameters, the dispersion curves of non-dimensional phase velocity, attenuation coefficient and amplitude ratios are shown graphically with respect to the non-dimensional wave number for insulated boundary. CGL region corresponds to the coupled theory of Green-Lindsay, region CLS corresponds to coupled theory of Lord-Shulman, region GL corresponds to Green-Lindsay and the region LS corresponds to Lord-Shulman theory.

From fig.1 it is noticed that the values of phase velocity increase for all the theories CLS, CGL, LS and GL for small values of wave number whereas the values of phase velocity decrease for large values of the wave number and the values for CGL remain higher than the other theories.

From fig.2 we observed that the values of attenuation coefficient decrease sharply for all the theories then increase monotonically for all the theories. The values for GL are higher for small values of wave number than CLS, CGL and LS for $0 \le k \le 1.15$ whereas the values for CGL remain higher than the other theories for intermediate values of wave number. But the values for LS are higher than the other as the values of the wave number increase.

Figs. 3, 4 and 5 are showing the behaviors of amplitude ratios of displacements and temperature with respect to wave number. The pattern of amplitude ratios in fig.3 is very similar. It decreases for all the four theories CLS, CGL, LS and GL. From fig.4 it is noticed that the values of amplitude ratio for CLS and CGL increase whereas it decrease for LS and GL theories. From fig. 5 it can be observed that the amplitude ratios increase for CLS and CGL theories.

Notes

IX. Conclusion

The propagation of Stoneley waves at the interface of two couple stress thermoelastic half spaces in the context of LS and GL theories of thermoelasticity have been studied. It has been observed that the transverse wave mode is not affected by thermal effect and get decoupled from rest of motion. The behavior and trends of variation of phase velocity is similar for CGL, CLS, LS and GL theories with difference in their magnitude values whereas the behavior of the attenuation coefficient is similar for all the theories. Appreciable effect of thermal relaxation times is obtained on phase velocity, attenuation coefficient and amplitude ratios of displacement components and thermal distribution.

References Références Referencias

1. Achenbach, J. D., 1973, "Wave Propagation in Elastic Solids", *North Holland Amsterdam, The Netherlands*.
2. Aggarwal, H.R and Alverson, R.C., 1969, "Effects of couple-stresses on the diffraction of plane elastic waves by cylindrical cavity", *International journal of Solids and Structures,* **5**, 491-511.
3. Anthonie, A., 2000, "Effects of couple stresses on the elastic bending of beams", *International journal of Solids and Structures,* **37**, 1003-1018.
4. Bogy, D. B. andSternberg, E., 1967, "The effect of couple-stresses on singularities due to dislocation loadings", *International Journal of Solids and Structures,* **3**, 757-770.
5. Brekhoviskikh, L. M., 1960, "Waves in Layered Media", *Academic Press, NewYork.*
6. Bullen, K. E. and Bolt, B. A., 1985 "An introduction to the theory of Seismology", *Cambridge University Press, Cambridge, UK.*
7. Chen, S. and Wang, T., 2001, "Strain gradient theory with couple stress for crystalline solids", *European Journal Mechanics and Solids*", **20**, 739-756.
8. Chung, J. and Wass, A. M., 2010 "The inplane orthotropic couplet stress elasticity constant of elliptical cell honey combs", *International Journal of Engineering Sciences,* **48(11)**, 1123-1136.

9. Cosserat, E. and Cosserat F., 1909 "Theory des corps deformables", *A. Herman Et. Fils, Paris.*
10. Ewing, W. M., Zardetzky, W. S. and Press, F., 1957, "Elastic Waves in Layered Media", *McGraw-Hill, New York.*
11. Gourdiotis, P. A. and Georgiadis, 2008 "An opporroach based on distributed dislocations and dislocations for crack problems in couple-stress elasticity", *International Journal of Solids and Structures,* **45(21)**, 5521-5539.
12. Green, A.E. and Lindsay, K.A., 1972, *Journal of. Elasticity,* **1**, 2, 1-7.
13. Green, A.E. and Rivlin, R.S., 1964, "Simple force and stress multiples", *Archives of Rational Mechanics and Analysis,* 16, 325-354.
14. Kröner, E., 1963, "On the physical reality of torque stresses in the continuum mechanics", *International Journal of Engineering Sciences,* **1(2),** 261-278.
15. Kumar, R., Deswal, S., 2006, "Some problem of wave propagation in micropolar elasticity medium voids" *Journal of Vibration and Control,* **12(8)**, 647-664.
16. Kumar, R., Kaur, M. and Rajvanshi, S., 2012, "Wave propagation in micropolar thermoelastic layer with two temperatures" *Journal of Vibration and Control,* first published on November 5, 2012.
17. Kumar, R., Kumar, K. and Nautiyal, R. C.,2012a, "Wave Propagation in couple stress thermoelastic half space underlying an inviscid liquid layer" *International of Applied Mathematics and Engineering,* **17(1)**, 161-177.
18. Kumar, R., Kumar, K. and Nautiyal, R. C., 2012b, "Propagation of SH-Waves in Couple Stress Elastic Half Space Underlying an Elastic Layer" *Afrika Matematika,* DOI 10.1007/s13370-012-0073-y.
19. Lakes, R. S., 1982, "Dynamic study of couple stress effects in human compact bone", *ASME Journal of Biomechanical Engineering.,* **104**, 16-21.
20. Lord, H. and Shulman, Y., 1967, "A generalized dynamical theory of elasticity", *Journal of Mechanics and Physics of Solids,* **15**,299-309.
21. Love, A. E. H., 1911"Some problems of Geodynamics", *Cambridge University Press, Cambridge, UK.*
22. Lubarda, V.A. and Markenscoff, X., 2000, "Conservation integrals in couple stress elasticity", *Journal of the Mechanics and Physics of Solids,* **3**, 553-564.
23. Lubarda, V. A., 2003, "Circular inclusions in anti-plane strain couple stress elasticity", *International Journal of Solids and Structures,* **40**, 3827-3851.
24. Mindlin, R. D. and Tiersten, H.F., 1962, "Effects of couple stresses in linear elasticity", *Archive of Rational Mechanics and Analysis,* **11**, 415-448.
25. Nayfeh, A. N. and Abdelrahman, W.G., 2000, "An Approximate Model for Wave Propagation in Rectangular Rods and Their Geometric Limits" *Journal **of** Vibration and Control,* **6(1),** 3-17.
26. Radi, E., 2007, "Effects of characteristic material lengths on Mode III crack propagation in couple stress elastic-plastic materials", *International Journal of Plasticity,* **23**, 1439-1456.

27. Rayleigh, L., 1885, "On waves propagated along the plane surface of an elastic solid," *Proceedings of the London Mathematical Society,* **17**, 4–11.
28. Scholte, J. G., 1947, "The range of existence of Rayleigh and Stoneley waves," *Monthly Notices of Royal Astronomical Society, Geophysical Supplement* **5**, 120–126.
29. Selim, M. M., 2006, "Orthotropic elastic medium under the effect of initial and couple stress", *Applied Mathematics and computations,* **181(1)**, 185-192.
30. Sengupta, P. R. and Ghosh, B., 1974a, "Effects of couple stresses on the surface waves in elastic media", *Gerlands Beitr. Geophysik, Leipzig,* **82**, 309-318.
31. Sengupta, P. R. and Ghosh, B., 1974b, "Effects of couple stresses on propagation of waves in an elastic layer", *Pure Applied Physics,* **112**, 331-338.
32. Sengupta, P. R. and Benerji, D. K., 1978 "Effects of couple-stresses on propagation of waves in an elastic layer immersed in an infinite liquid", *International Journal of Pure and Applied Maths,* **9**,17-28.
33. Shodja, H. M. and Ghagisaeidi, M., 2007, "Effects of couple stresses on anti plane problems of piezoelectric media with inhomogeneties", *European Journal of Mechanics and Solids,* **26**, 26, 647-658.
34. Stefeniak, J., 1969, "Reflection of plane longitudinal wave from a free plane in cosserat medium*", Archives of Mechanics, Polish Academy of Sciences,* **21 (6)**, 745.
35. Stoneley, R., 1924, "Elastic waves at the surface of separation of two solids," *Proceedings of Royal Society of London,* **106**, 416–428.
36. Tomar, S. K., 2005, "Wave propagation in elastic plates with voids" *Journal of Vibration and Control,* **11(6)**, 849-863.
37. Tomar, S. K. and Singh, D., 2006, "Propagation of stoneley waves at an interface between two microstretch elastic half-spaces" *Journal of Vibration and Control*, **12(9),** 995-1009.
38. Toupin, R.A., 1962 "Elastic materials with couple-stresses"*, Archive of Rational Mechanics and Analysis,* **11**, 385-413.
39. Udias, A., 1999, "Principles of Seismology", *Cambridge University Press, Campridge, UK.*
40. Voigt, W., 1887, **"**theoretische studien uber die elastizitatsverhaltnisse der kristalle", *Abh. Ges. Wiss. Gottingen*, **34**.
41. Weitsman, Y., 1965, "Couple stress effects on stress concentration around a cylindrical inclusion in a field of uniaxial tension", *ASME Journal of Applied Mechanics,* **32**, 424-428.
42. Weitsman, Y., 1967, "A note on singularities in a cosserat medium", *Qly. Appl. Mech.,* **25**, 213-217.

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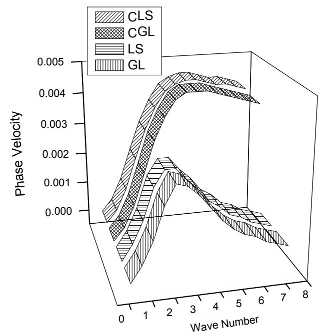

Figure 1 : variation of phase velocity w. r. t. wave number

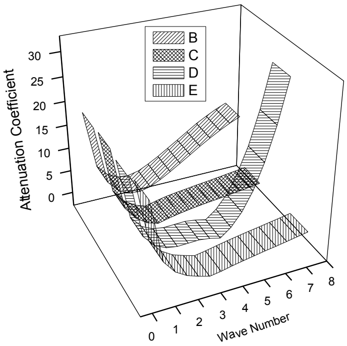

Figure 2 : variation of attenuation coefficient w. r. t. wave number

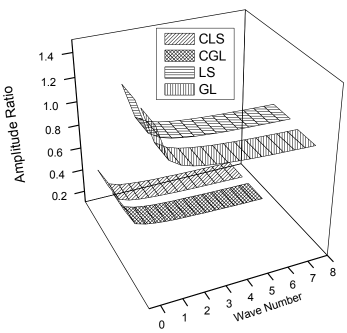

Figure 3 : variation of tangential amplitude verses wave number

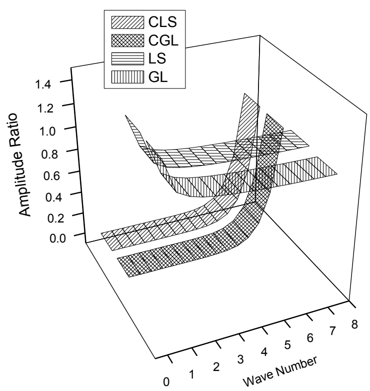

Figure 4 : variation of normal amplitude verses wave number

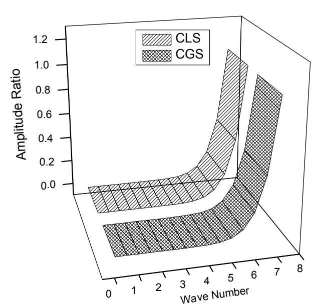

Figure 5 : variation of temperature distribution verses wave number

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