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Some Mathematical and Physical Errors of Wald on General Relativity

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Some Mathematical and Physical Errors of Wald on General Relativity

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Abstract - In spite of including crucial errors, Wald's book has become a standard reference, in part, because the 1993 Nobel Prize Committee for physics made the same mistakes. He also has circumvented some errors of Misner, Thorne and Wheeler, but he still fails to understand Einstein's equivalence principle. Moreover, he maintains the major common errors, the existence of dynamic and wave solutions for the Einstein equation, and thus also the claimed validity of linearization for weak gravity and the perturbation approach. Another problem is that he failed to see the invalidity of Einstein's covariance principle in physics. This is due to that in spite of his being additionally cautious, Wald was often not able to tell the difference between mathematics and physics. Although his main errors have been shown in the literature, some theorists may not have the mathematical background or the time to go through these. In this paper, his errors are illustrated and explained in mathematics at the undergraduate level.

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"Science sets itself apart from other paths to truth by recognizing that even its greatest practitioners sometimes err"

-- S. Weinberg, Physics Today, November 2005.

I. INTRODUCTION

In celebrating a wonderful tradition, the installation of a new MIT president, I had a chance to talk to MIT President, Dr. L. Rafael Reif. Since he asked the community, including MIT Alumni to help him to do a better job, I ventured again to tell that in my 20 years of research on fundamental problems in physics, I discovered that there are many problems in current theories of general relativity. He asked me to give him a report with details on these.

I examined the MIT open course Phy. 8.962 (instructed by Prof. Bertschinger¹⁾) with the textbook [1] by Sean Carroll.²⁾ As expected, there still are major errors that started from the beginning of general relativity. For instance, as Gullstrand [2, 3] pointed out in 1921, there is actually no dynamic solution for the non-linear Einstein equation,

$$\mathbf{G}_{\mu\nu} = \mathbf{R}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa \mathbf{T}_{\mu\nu}, \kappa = 8\pi K/c^2 = 1.86 \times 10^{-27} \quad (1)$$

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Where $g_{\mu\nu}$ is the space-time metric, $R_{\mu\nu}$ is Ricci tensor, and $T_{\mu\nu}$ is the source, energy momentum tensor of matter [4]. Also K is the gravitational constant in Newtonian theory, and c is the velocity of light in vacuum.

Carroll stated that he has often leaned heavily on the book of Wald [5] as a primary source since it has become a standard reference in the field. Thus, I decided to publish another paper on general relativity to illustrate and explain errors of Wald because this spreading of errors should be stopped as soon as possible. Previously, I have reported problems on MIT open course, Phy. 8.033 to former MIT President Susan Hockfield, but that involved mainly the errors of the Wheeler School [6].

The issue of whether the Einstein equation has far reaching consequences leads to answering issues such as whether $E = mc^2$ is only conditionally valid, (Appendix A), whether all the coupling constants have the same sign, and whether general relativity is not applicable to microscopic phenomena as Penrose and Hawking claimed, etc. In fact, the mistakes on the question of dynamic solutions are responsible for many subsequent absurd claims [7].

II. EQUIVALENCE OF THE INERT & THE GRAVITATIONAL MASS, AND EINSTEIN'S EQUIVALENCE PRINCIPLE

Wald [5] circumvented Einstein's equivalence principle, but claimed the equivalence of inert mass and the gravitational mass due to Galileo and Newton as "the equivalence principle", but the 1993 Nobel Prize Committee for Physics adapted this view. In so doing, Wald avoided criticizing the error of Misner, Thorne, and Wheeler [6] because they have misidentified Einstein's equivalence principle of 1916 as the invalid 1911 assumption of equivalence between acceleration and Newtonian gravity. However, this also exposed that Wald does not understand Einstein's equivalence principle-historical errors in mathematics and physics.

Einstein's equivalence principle is based on the then highly accurate experimental fact that the inert mass and the gravitational mass can be considered the same although they are defined very differently. Then, he considered two space coordinate systems: an inertial

system K and a system K' that is uniformly accelerated with respect to K. Then masses that are free from acceleration with respect to K, would have equal and parallel acceleration with respect to K'. They behave just as if a gravitational field were present and K' is unaccelerated. Einstein called the assumption of the complete physical equivalence of the systems of coordinates, K and K' the "principle of equivalence". Einstein considered that this principle is evidently intimately connected with the law of the equality between the inert and the gravitational mass, and signifies an extension of the principle of relativity to coordinate systems which are in non-uniform motion relatively to each other.

Nevertheless, Wald claimed "This fact, known as the equivalence principle, is expressed in Newtonian theory of gravitation by the statement that the gravitational force on a body is proportional to its inertial mass." Thus Wald changed Einstein's equivalence principle completely, and reduced it to the level of Galileo and Newton. Then, he omitted the Einstein-Minkowski condition completely that follows Einstein's equivalence principle. Thus, Wald could avoid the obvious conflict between the Einstein-Minkowski condition and Einstein's covariance principle [7] that Wald subsequently heavily leaned on.

Moreover, he derived the bending of a light without going through the Einstein-Minkowski condition [4]. Then he omitted the justifications for the adaptation of Einstein's theory of measurement and thus avoided approving the invalid applications of Einstein with special relativity to justify his theory of measurement [8]. He also avoided the issue of inconsistency with observed light bending from using Einstein's theory of measurement. However, he still failed to achieve the consistency with Einstein's theory of measurement because the interior solution and the exterior solution must be continuous at the surface of a massive ball. For the Schwarzschild gauge, this means as shown in his equation (6.2.10) [5] that the total mass

$$M = m(R) = 4\pi \int_0^R \rho(r)r^2 dr \quad (2)$$

where $\rho(r)$ is the density of the mass. However, this also means that Einstein's theory of measurement is not valid because as shown in his equation (6.2.11), it would require that.

$$M' = m(R) = 4\pi \int_0^R \rho(r)r^2 \left[1 - \frac{2m(r)}{r} \right]^{1/2} dr \quad (2')$$

Wald's interpretation is that M' is total proper mass. However, he failed to explain why the difference ($M' - M$) does not contribute to gravity.

Apparently, Wald was unaware of that Einstein's equivalence principle plays a crucial role in deriving the Maxwell-Newton Approximation (see Appendix B) which is identical to the linearized equation with massive sources in the harmonic gauge [9]. This derivation is necessary because the non-linear Einstein equation has been found [10, 11] to be invalid for the dynamic case. However, Wald also has mistaken that the Einstein equation had dynamic solutions. He failed to see that a mathematical equation may not necessarily have a valid solution for physics, and thus he often claimed a solution for an equation without a necessary proof.³⁾

III. WEAK GRAVITY, GRAVITATIONAL WAVES, AND THE PRINCIPLE OF CAUSALITY

According to Einstein [4], in general relativity weak sources would produce a weak field, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu}, \text{ where } |\gamma_{\mu\nu}| \ll 1 \quad (3)$$

and $\eta_{\mu\nu}$ is the flat metric when there is no source. For the static case, condition (3) is verified. However, for the dynamic case, Gullstrand [2, 3] suspected that condition (3) may not be valid. According to the principle of causality, condition (3) should be valid; but this is true only if the equation is valid in physics. Many theorists failed to see this because they failed to see the difference between physics and mathematics clearly [7]. In other words, condition (3) and whether the principle of causality is applicable need a rigorous proof.

Unfortunately, many believe that condition (3) is always valid for general relativity because Einstein has produced accurate predictions for the static case. When the Einstein equation has a weak solution, an approximate weak solution can be derived through the approach of the field equation being linearized. The linearized Einstein equation with the linearized harmonic gauge $\partial^\mu \bar{\gamma}_{\mu\nu} = 0$ is

$$\frac{1}{2} \partial^\alpha \partial_\alpha \bar{\gamma}_{\mu\nu} = \kappa T_{\mu\nu} \quad \text{where} \quad \bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \gamma$$

$$\text{and} \quad \gamma = \eta^{\alpha\beta} \gamma_{\alpha\beta} \quad (4)$$

Note that we have

$$G_{\mu\nu} = G_{\mu\nu}^{(1)} + G_{\mu\nu}^{(2)} \quad \text{and}$$

$$G_{\mu\nu}^{(1)} = \frac{1}{2} \partial^\alpha \partial_\alpha \bar{\gamma}_{\mu\nu} - \partial^\alpha \partial_\mu \bar{\gamma}_{\nu\alpha} - \partial^\alpha \partial_\nu \bar{\gamma}_{\mu\alpha} + \frac{1}{2} \eta_{\mu\nu} \partial^\alpha \partial^\beta \bar{\gamma}_{\alpha\beta} \quad (5)$$

The linearized vacuum Einstein equation means

$$G_{\mu\nu}^{(1)}[\gamma_{\alpha\beta}^{(1)}] = 0 \quad (6)$$

Thus, as pointed out by Wald, in order to maintain a solution of the vacuum Einstein equation to second order we must correct $\gamma^{(1)}_{\mu\nu}$ by adding to it the term $\gamma^{(2)}_{\mu\nu}$, where $\gamma^{(2)}_{\mu\nu}$ satisfies

$$G^{(1)}_{\mu\nu}[\gamma^{(2)}_{\alpha\beta}] + G^{(2)}_{\mu\nu}[\gamma^{(1)}_{\alpha\beta}] = 0, \text{ where } \gamma_{\mu\nu} = \gamma^{(1)}_{\mu\nu} + \gamma^{(2)}_{\mu\nu} \quad (7)$$

Which is the correct form of eq. (4.4.52) in Wald's book. (In Wald's book, he did not distinguish $\gamma_{\mu\nu}$ from $\gamma^{(1)}_{\mu\nu}$) This equation does have a solution for the static case. However, detailed calculation shows that this equation does not have a solution for the dynamic case [10, 11]. The fact that there is no solution for eq. (7) means also that the Einstein equation does not have a bounded dynamic solution.

An independent supplementary convincing evidence for the absence of a bounded dynamic solution is, as shown by Hu, Zhang & Ting [12], that gravitational radiation calculated would depend on the approach used. This is also a manifestation that there is no bounded solution. A similar problem in approximation schemes such as post-Newtonian approximation [2, 3] is that their validity is also only assumed.

IV. ILLUSTRATIVE EXAMPLES FOR THE NONEXISTENCE OF WALD'S EQUATION

To illustrate Wald's error, one can consider the example provided by Misner et al. [6]. They claimed that a plane-wave solution is of the form as follows:

$$ds^2 = c^2 dt^2 - dx^2 - L^2(e^{2\beta} dy^2 + e^{-2\beta} dz^2) \quad (8)$$

where $L = L(u)$, $\beta = \beta(u)$, $u = ct - x$, and c is the light speed. Then, the Einstein equation $G_{\mu\nu} = 0$ becomes

$$\frac{d^2 L}{du^2} + L \left(\frac{d\beta}{du} \right)^2 = 0 \quad (9)$$

Misner et al. [6] claimed that Eq. (9) has a bounded solution, compatible with a linearization of Einstein equation (1). It has been shown that Misner et al. are incorrect and Eq. (9) does not have a physical solution that satisfies Einstein's requirement on weak gravity [13, 14]. In fact, $L(u)$ is unbounded even for a very small $\beta(u)$.

On the other hand, from the linearization of the Einstein equation (the Maxwell-Newton approximation) in vacuum, Einstein [15] obtained a solution as follows:

$$ds^2 = c^2 dt^2 - dx^2 - (1 + 2\phi) dy^2 - (1 - 2\phi) dz^2 \quad (10)$$

where ϕ is a bounded function of $u (= ct - x)$. Note that metric (10) is the linearization of metric (8) if $\phi = \beta(u)$. Thus, the problem of waves illustrates that the linearization may not be valid for the dynamic case when gravitational waves are involved since eq. (9) does not have a weak wave solution.

In conclusion, due to confusion between mathematics and physics, Wald [5] made errors in mathematics at the undergraduate level. The principle of causality requires the existence of a dynamic solution, but Wald did not see that the Einstein equation can fail this requirement.

Another well-known counter example is the metric obtained by Bondi, Pirani, & Robinson [16] as follows:

$$ds^2 = e^{2\phi} (d\tau^2 - d\xi^2) - u^2 \begin{bmatrix} \cosh 2\beta (d\eta^2 + d\zeta^2) \\ + \sinh 2\beta \cos 2\theta (d\eta^2 - d\zeta^2) \\ - 2 \sinh 2\beta \sin 2\theta d\eta d\zeta \end{bmatrix} \quad (11a)$$

where ϕ , β and θ are functions of $u (= \tau - \xi)$. It satisfies the differential equation (i.e., their Eq. [2.8]),

$$2\phi' = u (\beta'^2 + \theta'^2 \sinh^2 2\beta) \quad (11b)$$

They claimed this is a wave from a distant source. (11b) implies ϕ cannot be a periodic function. The metric is irreducibly unbounded because of the factor u^2 . Both eq. (9) and eq. (11b) are special cases of $G_{\mu\nu} = 0$. However, linearization of (11b) does not make sense since variable u is not bounded. Thus, they claim Einstein's notion of weak gravity invalid because they do not understand the principle of causality adequately.

Moreover, when gravity is absent, it is necessary to have $\phi = \sinh 2\beta = \sin 2\theta = 0$. These would reduce (11a) to

$$ds^2 = (d\tau^2 - d\xi^2) - u^2 (d\eta^2 - d\zeta^2) \quad (11c)$$

However, this metric is not equivalent to the flat metric. Thus, metric (11c) violates the principle of causality. Also it is impossible to adjust metric (11a) to become equivalent to the flat metric.

This challenges the view that both Einstein's notion of weak gravity and his covariance principle are valid. These conflicting views are supported respectively by the editorials of the "Royal Society Proceedings A" and the "Physical Review D"; thus there is no general consensus. As the Royal Society correctly pointed out [17, 18], Einstein's notion of weak gravity is inconsistent with his covariance principle. However, Einstein's covariance principle has been proven invalid since counter examples have been found [19, 20].

There are other theorists who also ignore the principle of causality. For example, another "plane wave", which is intrinsically non-physical, is the metric accepted by Penrose [21] as follows:

$$ds^2 = du dv + H du^2 - dx_i dx_i, \text{ where } H = h_{ij}(u) x_i x_j \quad (12)$$

Where $u = ct - z$, $v = ct + z$. However, there are arbitrary non-physical parameters (the choice of origin) that are

unrelated to any physical causes. Being a mathematician, Penrose [21] over-looked the principle of causality.

The plane wave solution of Liu & Zhou [22], which satisfies the harmonic gauge, is as follows:

$$ds^2 = dt^2 - dx^2 + 2 F(dt - dx)^2 - \cosh 2\psi(e^{2\phi} dy^2 + e^{-2\phi} dz^2) - 2\sinh 2\psi dy dz. \quad (13)$$

where $\phi = \phi(u)$ and $\psi = \psi(u)$. Moreover, $F = F_p + H$, where

$$F_p = \frac{1}{2} (\dot{\psi}^2 + \dot{\phi}^2 \cosh^2 2\psi) [\cosh 2\psi (e^{2\phi} y^2 + e^{-2\phi} z^2) + 2\sinh 2\phi yz], \quad (14)$$

and H satisfies the equation,

$$\cosh 2\psi (e^{2\phi} H_{,22} + e^{-2\phi} H_{,33}) - 2\sinh 2\psi H_{,23} = 0. \quad (15)$$

For the weak fields one has $1 \gg |\phi|$, $1 \gg |\psi|$, but there is no weak approximation as claimed to be

$$ds^2 = dt^2 - dx^2 - (1 + 2\phi) dy^2 - (1 - 2\phi) dz^2 - 4\psi dy dz \quad (16)$$

because F_p is not bounded unless $\dot{\phi}$ and $\dot{\psi}$ are zero (i.e., no wave).

The linearized equation for a dynamic case has been illustrated as incompatible with the non-linear Einstein equation, which has no bounded dynamic solutions. Thus, Eq. (9), Eq. (11b), and Eq. (13) serve as good simple examples that can be shown through explicit calculation that linearization of the Einstein equation is not valid. Also, metric (12) suggests that the cause of having no physical solution would be due to inadequate source terms [10, 12, 23].

V. THE SO-CALLED SPACE-TIME SINGULARITY THEOREMS, POSITIVE MASS THEOREM, AND $E = MC^2$

A surprising conclusion, from the investigation of the Einstein equation, is that the space-time singularity theorems of Penrose and Hawking are actually irrelevant to physics. This is so because their theorems have a common implicit assumption that all the couplings have the same sign. However, from the investigation of dynamic solutions, such an assumption is necessarily invalid in physics [9, 10]. These theorems were accepted because Penrose won the arguments against a well-known Russian scientist E. M. Lifshitz who claimed, with the same set of assumptions, that there is no space-time singularity [24]. However, the problem is not the mathematics in the theorems, but the earlier historical errors in mathematics and physics.

As Pauli [25] pointed out, in principle general relativity can have different signs for their coupling constants. The fact that nobody questioned the assumption of unique sign for all coupling, is probably

due to the unverified speculation of formula $E = mc^2$ being generally true. This formula comes from special relativity, and the conversion of some mass to various combinations of energy is verified by the fission and fusion in nuclear physics. However, the conversion of a single type of energy to mass actually has never been verified [7].

Einstein and theorists have shown that the photons can be converted into mass through absorption [26]. This conversion is supported by the fact that the π_0 meson can be decayed into two photons. Thus, it was claimed that the electromagnetic energy can be converted into mass because they failed to see that the photons must have non-electromagnetic energy. When Einstein proposed the notion of photons, he had not conceived general relativity yet. Thus, understandably he neglected the gravitational component of light. However, after general relativity, a light ray consists of a gravitational component is natural because the electron has a mass. Besides, the electromagnetic energy-momentum tensor has a zero trace, but the energy-momentum tensor of massive matter has a non-zero trace. In fact, Einstein failed to show the general validity of $E = mc^2$ in spite of several years effort [27]. Experimentally, in contrast of Einstein's claim, $E = mc^2$ is not always valid because a piece of heated up metal has reduced weight [28].

Although the Einstein equation does not have a dynamic solution, physically the dynamic solution must exist for a rectified equation. A problem of the Einstein equation is that it does not include the gravitational energy-stress tensor of its gravitational waves in the source and thus the principle of causality is violated. Since a gravitational wave carried energy-momentum and the source of gravity is the energy-stress tensors, as Hogarth [29] pointed out, the presence of a non-zero energy-momentum in the source is necessary for a gravitational wave. Thus, to fit the Hulse-Taylor data of the binary pulsar, it is necessary to modify the Einstein equation [10] to

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa [T(m)_{\mu\nu} - t(g)_{\mu\nu}] \quad (17)$$

Where $t(g)_{\mu\nu}$ is the energy-stress tensor for gravity. For radiation, the tensor $t(g)_{\mu\nu}$ is equivalent to Einstein's notion of the gravitational energy-stress. However, his notion is a pseudo-tensor and can become zero by choosing a suitable coordinate system, but the energy-momentum of a radiation cannot be zero, and thus must be a tensor [10].

It is crucial to note that the new tensor necessarily has a different sign for its coupling. Thus, the implicit assumption of Penrose and Hawking is proven necessarily invalid. Note that the absence of a dynamic solution and the presence of space-time singularities are related to the same invalid assumption.

It is the long standing bias and errors in mathematics that some theorists accepted one but rejected the other. Now, clearly the space-time singularity theorems dramatically illustrate that what could an applied mathematician do without the proper guidance of physics. Other victims are the positive mass theorem of Yau [30] and the positive energy theorem of Witten [31] because they used the same invalid implicit assumption as Hawking and Penrose.

VI. DISCUSSIONS AND CONCLUSIONS

Einstein was the major architect or foundation builder of three great theories of modern physics, namely: special relativity, quantum mechanics and general relativity. However, he was also the source of oversight in each theory [7]. In special relativity, he failed to see that $E = mc^2$ is only conditionally valid. In quantum theory, he failed to recognize that photons must include non-electromagnetic energy [32]. In general relativity, his principle of covariance and theory of measurement are invalid [8, 33]. However, related criticisms of Whitehead [34] and Zhou [35] were ignored. The lack of examples to illustrate his equivalence principle makes it possible to have popular misinterpretations and confusions in physics [36].

Some theorists such as Carroll [1] claimed "General relativity is the most beautiful physical theory ever invented." This reaffirms that beauty is in the eyes of the beholder. He probably simply regurgitates what he heard in Caltech, where Thorne [23] erroneously claimed his student's opinion, that Einstein neglected the tidal force as Einstein's, although Einstein explicitly pointed it out as wrong [37]⁴. In addition to that analysis shows clearly that there are no bounded wave-solutions; there are many necessarily unbound "wave". Thus, it is a puzzle how the "experts" never reexamine the hand-waving "proofs" for such a long time. A problem of the Wheeler School and her associates is that they seldom read papers that are written by "outsiders" and thus their errors would continue regardless.

However, now it is clearly an incomplete theory that remains to be explored. In terms of physics, a basic problem is that just as in Maxwell's classical electromagnetism [38], there is also no radiation reaction force in general relativity. Although an accelerated massive particle would create radiation [24], the metric elements in the geodesic equation are created by particles other than the test particle [4]. (Thus, Carroll's [1] tendency, to think of general relativity as a field, would be valid.) Another problem is the exact field equation for the dynamic case. Because of the misinterpretation of $E = mc^2$, the study of gravitational effects of electromagnetism is clearly inadequate [7]. The discovery of the charge-mass interaction is a good beginning since the need for Einstein's unification is confirmed.

Unfortunately, these potentially great developments have been blocked because of the inadequacy of the theorists in mathematics and historical inadequacy in physics. Half of the book of Wald [4] dealt with "advanced topics", but they are actually at most unverified speculations. It is obvious that there is no perturbation approach since there is no bounded dynamic solution. The misunderstanding on the notion of gauge invariance [39] persistently presented by C. N. Yang [40, 41] was probably responsible for prolonging the incorrect acceptance of Einstein's covariance principle after it has been found to be invalid through explicit examples [33].

Historically, in the US, the Wheeler School has been responsible for recognizing the importance of general relativity. It is hoped that they are able to continue their efforts after rectifying their mistakes. A lesson to be learned is that nothing can damage sciences more than biased authority worship. Currently, the Einstein equation is served as guidance for the research in string theory. It would be a great disservice to the string theorists if the problem in the Einstein equation is withheld from them. It seems to me that the string theory, if correct, must be able to include the experimentally verified charge-mass interaction, newly discovered from an analysis of general relativity [7, 28]. It is hoped this paper would help theorists to look at unsolved problems squarely.⁵

IV. ACKNOWLEDGMENTS

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a) Appendix A: on Invalidity of $e = mc^2$ and Related Errors

Based on special relativity, it is conjectured that mass can be equivalent to energy with the relation $E = mc^2$ [4]. This conversion is supported by fission and fusion in nuclear physics. In such a conversion the resulting energy is a combination of different types of energy.

However, there is no example that a single type of energy can be converted to mass. Einstein claimed that electromagnetic energy can be equivalent to mass. However, this is actually due to that Einstein failed to see that photons actually consist of gravitational energy [7]. This inclusion is natural after general relativity since a charged particle always has mass. Einstein tried to extend this relation to other types of energy for several years (1905-1909) but failed [27]. Recently, experiments have shown that electromagnetic energy alone is not equivalent to mass [28].

Some argued that the mass of a particle can be defined with the formula $f = ma$. A problem is that such a mass may not be related to the energy of such a force. For instance, although the electromagnetic force acting on a charged particle, this does not enable one to establish a relation between such a mass and the electromagnetic energy. In fact, this is also an error of Nobel Laureate G.'t Hooft made in his 1999 Nobel speech.

Moreover, before direct experimental evidence against the formula $E = mc^2$ are known, there exists a theoretical puzzle because the photons have no mass. For this, some defined an electromagnetic mass with $m_e = E_e/c^2$ in terms of the electromagnetic energy E_e , but few theorists questioned whether such a definition makes sense in physics. Now, it has been shown that this formula may fail, and thus cannot be considered as generally valid. Hence, one should have asked whether such a new definition of mass is equivalent to the inert mass and/or the gravitational mass. Now, clearly this line of thinking actually does not lead to any meaningful physics. In fact, it is probably due to a failure in distinguishing mathematics from physics that must be additionally supported by experiments.³⁾

b) Appendix B: The Question of Dynamic Solutions and the Maxwell-Newton Approximation

A problem in general relativity [10] is that, for a dynamic case, there is no bounded solution,

$$|g_{ab}(x, y, z, t)| < \text{constant}, \tag{A1}$$

for the Einstein equation, where g_{ab} is the space-time metric [4]. In fact, eq. (A1) is also a necessary implicit assumption in calculating Einstein's radiation formula [42] and the light bending [23]. One might argue that requirement (A1) violates the covariance principle. However, the covariant principle is invalid in physics [33].

Moreover, Einstein's notion of weak gravity [4] is also in agreement with the principle of causality. However, although such a requirement can be satisfied for the static case, it fails for a dynamic case [10].

The question of valid dynamic solutions was raised by Gullstrand [2] the chairman of Nobel Prize Committee for Physics (1922-1929). He challenged Einstein and also Hilbert who approved Einstein's calculations [43]. Apparently Hilbert was unaware of the need of a bounded dynamic solution for the perturbation approach to this issue. However, Hilbert, being an excellent mathematician, did not participate in the subsequent efforts for the defense of Einstein's claim. Nevertheless, theorists such as Misner, Thorne and Wheeler [6] and Christodoulou & Klainerman [44], etc. failed to see this, and tried very hard to prove otherwise. Their efforts have been proven as futile [10, 45].

The failure of producing a dynamic solution would cast a strong doubt to the validity of the linearized equation that produces many effects including the

gravitational waves. In fact, for the case that the source is an electromagnetic plane wave, the linearized equation actually does not have a bounded solution [46].

Nevertheless, when the sources are massive, some of such results from the linearized equation have been verified by observation. Thus, there must be a way to justify the linearized equation, independently. Such an investigation has led additionally to a modified Einstein equation that would have dynamic solutions. To this end, Einstein's equivalence principle [9] is needed, and thus this principle, though rejected by the 1993 Nobel Prize Committee for Physics implicitly [47], is crucial in general relativity. As a result, it becomes even clearer that the non-existence of a bounded dynamic solution for massive sources is due to a violation of the principle of causality [13].

i. Gravitational Waves and the Einstein Equation of 1915

Relativity requires the existence of gravitational waves because physical influence must be propagated with a finite speed [48]. To this end, let us consider the Einstein equation of 1915 [4]. Einstein believed that his equation satisfied this requirement since its linearized "approximation" gives a wave solution.

The linearized equation with massive sources [4] is the Maxwell-Newton Approximation [10],

$$\frac{1}{2} \partial_c \partial^c \bar{\gamma}_{ab} = -\kappa T(m)_{ab}, \tag{A2}$$

where $\bar{\gamma}_{ab} = \gamma_{ab} - (1/2)\eta_{ab}$, $\gamma_{ab} = g_{ab} - \eta_{ab}$, $\gamma = \eta^{cd} \gamma_{cd}$, and η_{ab} is the flat metric. Eq. (A2) has a mathematical structure similar to that of Maxwell's equations. A solution of eq. (A2) is

$$\bar{\gamma}_{ab}(x_i, t) = -\frac{\kappa}{2\pi} \int \frac{1}{R} T_{ab} [y^i, (t - R)] d^3y,$$

$$\text{where } R^2 = \sum_{i=1}^3 (x^i - y^i)^2 \tag{A3}$$

Note that the Schwarzschild solution, after a gauge transformation, can also be approximated by (A3). Solution (A3) would represent a wave if T_{ab} has a dynamical dependency on time t' ($= t - R$). Thus, the theoretical existence of gravitational waves seems to be assured as a certainty as believed [25, 42, 49].

However, for non-linear equations, the physical second order terms can be crucial for the mathematical existence of bounded solutions. For Einstein equation (1), the Cauchy initial condition is restricted by four constraints since there is no second order time derivatives in G_{at} ($a = x, y, z, t$) [42]. This suggests that Einstein equation (1) and eq. (A1) may not be compatible for a dynamic problem. Einstein discovered that his equation does not admit a propagating wave

solution [50, 51]. Recently, it has been shown that the linearization procedure is not generally valid in mathematics [10, 52]. Thus, it is necessary to justify wave solution (A3) independently.

ii. *The Weak Gravity of Massive Matter and Einstein Equation of the 1995 Update*

For a massive source, the linear equation (A2), as a first order approximation, is supported by experiments [10, 39]. However, for the dynamic case, the Einstein equation is clearly invalid.

It will be shown that eq. (A2) can be derived from Einstein's equivalence principle. Based on this principle, the equation of motion for a neutral particle is the geodesic equation. In comparison with Newton's second law, one obtains that the Newtonian potential of gravity is approximately $c^2 g_{tt}/2$. Then, in accord with the Poisson equation and special relativity, the most general equation for the first order approximation of g_{ab} is,

$$\frac{1}{2} \partial_c \partial^c \gamma_{ab} = -\frac{\kappa}{2} [\alpha T(m)_{ab} + \beta \hat{T}(m) \eta_{ab}], \quad (A4a)$$

where

$$\hat{T}(m) = \eta^{cd} T(m)_{cd}, \quad \kappa = 8\pi K c^{-2}, \quad \text{and} \quad \alpha + \beta = 1, \quad (A4b)$$

where α and β are constants since Newton's theory is not gauge invariant.

Then, according to Riemannian geometry [42], the exact equation would be

$$R_{ab} + X^{(2)}_{ab} = -\frac{\kappa}{2} [\alpha T(m)_{ab} + \beta T(m) g_{ab}], \quad (A5a)$$

where

$$T(m) = g^{cd} T(m)_{cd}$$

and $X^{(2)}_{ab}$ is an unknown tensor of second order in K , if R_{ab} consists of no net sum of first order other than the term $(1/2) \partial_c \partial^c \gamma_{ab}$. This requires that the sum

$$-\frac{1}{2} \partial^c \partial^c [\partial_b \gamma_{ac} + \partial_a \gamma_{bc}] + \frac{1}{2} \partial_a \partial_b \gamma, \quad (A5b)$$

must be of second order. To this end, let us consider eq. (A4a), and obtain

$$\frac{1}{2} \partial_c \partial^c (\partial^a \gamma_{ab}) = -\frac{\kappa}{2} [\alpha \partial^a T(m)_{ab} + \beta \partial_b \hat{T}(m)] \quad (A6a)$$

From $\nabla^c T(m)_{cb} = 0$, it is clear that $K \partial^c T(m)_{cb}$ is of second order but $K \partial_b \hat{T}(m)$ is not. However, one may obtain a second order term by a suitable linear combination of $\nabla^c \gamma_{cb}$ and $\partial_b \gamma$. From (A6a), one has

$$\frac{1}{2} \partial_c \partial^c (\partial^a \gamma_{ab} + C \partial_b \gamma) = -\frac{\kappa}{2} [\alpha \partial^a T(m)_{ab} + (\beta + 4C\beta + C\alpha) \partial_b \hat{T}(m)]. \quad (A6b)$$

Thus, the harmonic coordinates (i.e., $\partial^a \gamma_{ab} - \partial_b \gamma/2 \approx 0$), can lead to inconsistency. It follows eqs. (A5b) and (A6b) that, for the other terms to be of second order, one must have $C = -1/2$, $\alpha = 2$, and $\beta = -1$. Hence, eq. (A4a) becomes,

$$\frac{1}{2} \partial_c \partial^c \gamma_{ab} = -\kappa [T(m)_{ab} - \frac{1}{2} \hat{T}(m) \eta_{ab}]. \quad (A7)$$

Which is equivalent to eq. (A2a), has been determined to be the field equation of massive matter. This derivation is independent of the exact form of equation (A5a). The implicit gauge condition is that the flat metric η_{ab} is the asymptotic limit. Eq. (A7) is compatible with the equivalence principle as demonstrated by Einstein in his calculation of the bending of light. Thus, the derivation is self-consistent.

Einstein obtained the same values for α and β by considering eq. (A5a) after assuming $X^{(2)}_{ab} = 0$ [43]. His equation (A2) could also be "derived" from a more general linear equation, if one regards the gravitational field as a spin 2 field coupled to the energy-stress tensor [48, 49], and the existence of bounded dynamic solutions be assumed.

An advantage of the approach of considering eq. (A4) and eq. (A5b) is that the over simplification $X^{(2)}_{ab} = 0$ is not needed. Then, it is possible to obtain from eq. (A5a) an equation different from eq. (A2),

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} R = -\kappa [T(m)_{ab} - Y^{(1)}_{ab}]$$

where

$$-\kappa Y^{(1)}_{ab} = X^{(2)}_{ab} - \frac{1}{2} g_{ab} \{ X^{(2)}_{cd} g^{cd} \}. \quad (A8)$$

The conservation law $\nabla^c T(m)_{cb} = 0$ and $\nabla^c G_{cb} = 0$ implies also $\nabla^a Y^{(1)}_{ab} = 0$. If $Y^{(1)}_{ab}$ is identified as the gravitational energy tensor of $t(g)_{ab}$, Einstein equation of the 1995 update [10] is reaffirmed. Note that eq. (A2a) is the first order approximation of eq. (A8) but may not be of eq. (A2). Note, however, that in Einstein's initial consideration, $t(g)_{ab}$ is a pseudo-tensor. It has been shown that it must be a tensor [10].

ENDNOTES

1. Edmond Bertschinger, B.S. (1979) in Phys., Caltech, Ph.D. (1984), Princeton Univ., advisor: Jeremiah P. Ostriker, whose advisor is S. Chandrasekhar. In 1995 Chandrasekhar, as the editor of the Astrophysical Journal, was finally convinced that there is no dynamic solution for the Einstein equation [10]. Another instructor is Scott A. Hughes, B.A. (1993) in Phys. Cornell Univ., Ph.D. (1998) Caltech under Kip Thorne.
2. Sean M. Carroll, Ph.D. (1993) Harvard University, Advisor George Field, a chemist turned Astrophysicist.

3. For a theoretical physicist, it is important to tell the difference between mathematics and physics. A good example is that although mathematically a non-Abelian gauge theory can be totally gauge invariant, a physical theory cannot be totally gauge invariant because a totally gauge invariant theory cannot represent distinct particles.
4. One should not be too surprised if some graduates of Caltech make errors in general relativity since Kip Thorne often made errors in general relativity, including mathematical errors at the undergraduate level [13, 14].
5. A well-known problem is NASA's discovery of Pioneer Space-Probe Anomaly. Recently, it was claimed that this problem has been resolved by a heat-radiation model. However, Erik Anderson (April 1, 2011 at 12:57) a discoverer of the anomaly commented, "I take the opposite viewpoint of Paul and Daniel. Science will have suffered the worst sort of dysfunction if the Pioneer Anomaly gets swept under the convenient rug of "the plausible." Even so, we will still have the Earth flyby anomalies and the so-called "A.U." anomaly left unco-vered. All three anomalies seem to be manifestations of a singular phenomenon — the latter two cannot be dismissed as heat radiation. Heat-radiation models, like string theory, can be customized to fit any set of observational parameters. There is no limit on sophistication. We should not be so easily impressed. Nothing has been resolved.

And it needn't have cost 100's of millions of dollars to do some authentic observational research. The New Horizons mission to Pluto could have been adapted to re-test the Anomaly if it was taken seriously enough in the first place. But Daniel is correct to point out that it wasn't even then. Still, that sum of money would have been a better investment than the 100's of millions of dollars wasted on LIGO — which I expect future historians of science to gossip about as their white elephant of choice."

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