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Con-s-k-EP Generalized Inverses

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2. Rao, C.R. and Mitra, S.K., "Generalized Inverse of Matrices and Its Applications", ***Wiley and Sons, New York***, 1971.

Con-s-k-EP Generalized Inverses

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Abstract- In this paper, equivalent conditions for various generalized inverses of a con-s-k-EPr matrix to be con-s-k-EPr are determined. Generalized inverses belonging to the sets A{1,2}, A{1,2,3} and A{1,2,4} of a con-s-k-EPr matrix *A* are characterized.

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I. Introduction

Let c_{nxn} be the space of nxn complex matrices of order n. let C_n be the space of all complex n tuples. For Aϵc_{nxn} . Let $\bar{A}, A^T, A^*, A^S, \bar{A}^S, A^\dagger$, R(A), N(A) and ρ$(A)$ denote the conjugate, transpose, conjugate transpose, secondary transpose , conjugate secondary transpose, Moore Penrose inverse range space, null space and rank of A respectively. A solution X of the equation AXA = A is called generalized inverse of A and is denoted by A^- . If A ϵ c_{nxn} then the unique solution of the equations A XA =A , XAX = X, $[AX]^*$ = AX , $(\mathrm{XA})^*$ = XA **[2]** is called the Moore-Penrose inverse of A and is denoted by $A^\dagger$. A matrix A is called con-s-k-EP$_r$ if $(\mathrm{A}) = \mathrm{r}$ and N(A) = N(A^TVK) (or) R(A)=R(KVA^T). Throughout this paper let "$\hbar$" be the fixed product of disjoint transposition in S_n= { 1,2,….n} and k be the associated permutation matrix .

Let us define the function $\hbar(\mathrm{x}) = \left(x_{k(1)}, x_{k(2)}, \ldots, x_{k(n)}\right)$. A matrix A = (a_{ij}) ϵ c_{nxn} is s-k-symmetric if a_{ij}= $a_{n-k(j)+1, n-k(i)+1}$ for i, j = 1,2,…..n . A matrix A ϵ c_{nxn}

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is said to be Con-s-k-EP if it satisfies the condition $A_x = 0 <=> A^s \hbar(x) = 0$ or equivalently N(A) =N(A^TVK). In addition to that A is con-s-k-EP <=> KVA is con-EP or AVK is con-EP and A is con-s-k-EP<=> A^T is con-s-k-EP$_r$ moreover A is said to be Con-s-k-EP$_r$ if A is con-s-k-EP and of rank r. For further properties of con-s-k-EP matrices one may refer **[1]**.

In **Theorem (2.11) [1]**, it is shown that $\boldsymbol{A}$ is con-s-k-EP$_r$, if and only if $\boldsymbol{A}^\dagger$ is con-s-k-EP$_r$. Thus the con-s-k-EP$_r$ property of complex matrices is preserved for its Moore-Penrose inverses. However, all other generalized inverses of a con-s-k-EP$_r$ matrix need not be con-s-k-EP$_r$. For instance, let $\boldsymbol{A}=\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$ for k=(1,2)(3), the associated permutation matrix be $\boldsymbol{K}=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$ and $\boldsymbol{V}=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$. Here, $\boldsymbol{A}$ is con-s-k-EP$_1$. But $\boldsymbol{A}^- =\begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}$ is a inverse of $\boldsymbol{A}$ which is not con-s-k-EP$_1$.

A generalized inverse $\boldsymbol{A}^= \in \boldsymbol{A}\{1,2\}$ is shown to be con-s-k-EP$_r$ whenever $\boldsymbol{A}$ is con-s-k-EP$_r$ under certain conditions in the following way.

Theorem 2.1

Let $\boldsymbol{A}\in \boldsymbol{C}_{n\times n}, \boldsymbol{X}\in \boldsymbol{A}\{1,2\}$ and $\boldsymbol{AX}, \boldsymbol{XA}$ are con-s-k-EP$_r$ matrices. Then $\boldsymbol{A}$ is con-s-k-EP$_r$ $\Leftrightarrow$ $\boldsymbol{X}$ is con-s-k-EP$_r$.

Proof

Since $\boldsymbol{AX}$ and $\boldsymbol{XA}$ are con-s-k-EP$_r$, by **Theorem (2.11) [1]**, we have $\boldsymbol{R}(\boldsymbol{AX}) = \boldsymbol{R}(\boldsymbol{KV}(\boldsymbol{AX})^{\boldsymbol{T}})$ and $\boldsymbol{R}(\boldsymbol{KV}(\boldsymbol{XA})^{\boldsymbol{T}}) = \boldsymbol{R}(\boldsymbol{XA})$. Since $\boldsymbol{X}\in \boldsymbol{A}\{1,2\}$ we have $\boldsymbol{AXA}=\boldsymbol{A}, \boldsymbol{XAX}=\boldsymbol{X}$.

Ref

1. Krishnamoorthy, S., Gunasekaran, K. and Muthugobal, B.K.N., "Con-s-k-EP matrices", ***Journal of Mathematical Sciences and Engineering Applications***, Vol. 5, No.1, 2011, 353 – 364.

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Now,

$$\begin{aligned} R(A) &= R(AX) \\ &= R(KV(AX)^T) \\ &= R(KVX^TA^T) \\ &= R(KVX^T) \end{aligned}$$

$$\begin{aligned} R(KVA^T) &= R(KVA^TX^T) \\ &= R(KV(XA)^T) \\ &= R(XA) \\ &= R(X) \end{aligned}$$

Now, A is con-s-k-EP$_r$ $\Leftrightarrow R(A) = R(KVA^T)$ and $(A) = r$ $\Leftrightarrow R(KVX^T) = R(X)$ and $(A) = (X) = r \Leftrightarrow X$ is con-s-k-EP$_r$.

Remark 2.2

In the above Theorem, the conditions that both AX and XA to be con-s-k-EP$_r$ are essential. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ for k=(1,2)(3), the associated permutation matrix $K = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $V = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. A is con-s-k-EP$_1$. $X = A^- = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in A\{1,2\}$ $AX = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; XA = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. AX and XA are not con-s-k-EP$_1$. Also X is not con-s-k-EP$_1$.

Now, we show that generalized inverses belonging to the sets A{1,2,3} and A{1,2,4} of a con-s-k-EP$_r$ matrix A is also con-s-k-EP$_r$, under certain conditions in the following Theorems.

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Theorem 2.3

Let $A \in C_{n\times n}, X \in A\{1,2,3\}, R(X) = R(A^T)$. Then A is con-s-k-EP$_r$ $\Leftrightarrow$ X is con-s-k-EP$_r$.

Proof

Since $X \in A\{1,2,3\}$, we have $AXA = A, XAX = X, (AX)^T = AX$. Therefore, $R(A) = R(AX) = R((AX)^T) = R(X^T)$

$$\begin{aligned} R(X) = R(A^T) &\Rightarrow XX^\dagger = A^T(A^T)^\dagger \\ &\Rightarrow XX^\dagger = A^T(A^\dagger)^T \\ &\Rightarrow XX^\dagger = (A^\dagger A)^T \\ &\Rightarrow XX^\dagger = A^\dagger A \\ &\Rightarrow KVXX^\dagger VK = KVA^\dagger AVK \\ &\Rightarrow (KVX)(KVX)^\dagger = (AVK)^\dagger AVK \\ &\Rightarrow R(KVX) = R((AVK)^T) \\ &\Rightarrow R(KVX) = R(KVA^T) \end{aligned}$$

A is con-s-k-EP$_r$ $\Leftrightarrow R(A) = R(KVA^T)$ and $(A) = r \Leftrightarrow R(X^T) = R(KVX)$ and $(A) = (X) = r \Leftrightarrow X$ is con-s-k-EP$_r$.

Theorem 2.4

Let $A \in C_{n\times n}, X \in A\{1,2,4\}, R(A) = R(X^T)$. Then A is con-s-k-EP$_r$ $\Leftrightarrow$ X is con-s-k-EP$_r$.

Proof

Since $X \in A\{1,2,4\}$, we have $AXA = A, XAX = X, (XA)^T = XA$ and $R(A) = R(X^T)$.

Now, $$\begin{aligned} R(KVA^T) &= R(KVA^T X^T) \\ &= R(KV(XA)^T) \\ &= R(KVXA) \\ &= R(KVX) \end{aligned}$$

Notes

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A is con-s-k-EP$_r$ $\Leftrightarrow R(A) = R(KVA^T)$ and $(A) = r \Leftrightarrow R(X^T) = R(KVX)$ and $(A) = (X) = r \Leftrightarrow X$ is con-s-k-EP$_r$.

Remark 2.5

In particular, if $X = A^\dagger$ then $R(A^\dagger) = R(A^T)$ holds. Hence A is con-s-k-EP$_r$ is equivalent to $A^\dagger$ is con-s-k-EP$_r$.

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