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Con-s-k-EP Generalized Inverses

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Abstract- In this paper, equivalent conditions for various generalized inverses of a con-s-k-EPr matrix to be con-s-k-EPr are determined. Generalized inverses belonging to the sets A{1,2}, A{1,2,3} and A{1,2,4} of a con-s-k-EPr matrix *A* are characterized.

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I. Introduction

Let c_{nxn} be the space of nxn complex matrices of order n. let C_n be the space of all complex n tuples. For Aϵc_{nxn} . Let $\bar{A}, A^T, A^*, A^S, \bar{A}^S, A^\dagger$, R(A), N(A) and $\rho(A)$ denote the conjugate, transpose, conjugate transpose, secondary transpose , conjugate secondary transpose, Moore Penrose inverse range space, null space and rank of A respectively. A solution X of the equation AXA = A is called generalized inverse of A and is denoted by A^- . If A ϵ c_{nxn} then the unique solution of the equations A XA =A , XAX = X, $[AX]^* = AX$, $(XA)^* = XA$ **[2]** is called the Moore-Penrose inverse of A and is denoted by $A^\dagger$. A matrix A is called con-s-k-EP$_r$ if (A) = r and N(A) = N(A^TVK) (or) R(A)=R(KVA^T). Throughout this paper let "$\mathscr{k}$" be the fixed product of disjoint transposition in S_n = { 1,2,….n} and k be the associated permutation matrix .

Let us define the function $\mathscr{k}(x) = \left(x_{k(1)}, x_{k(2)}, \ldots, x_{k(n)}\right)$. A matrix A = (a_{ij}) ϵ c_{nxn} is s-k-symmetric if $a_{ij} = a_{n-k(j)+1, n-k(i)+1}$ for i, j = 1,2,…..n . A matrix A ϵ c_{nxn}

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Ref

2. Rao, C.R. and Mitra, S.K., "Generalized Inverse of Matrices and Its Applications", ***Wiley and Sons, New York***, 1971.

is said to be Con-s-k-EP if it satisfies the condition $A_x = 0 <=> A^s\, k\,(x) = 0$ or equivalently N(A) =N(A^TVK). In addition to that A is con-s-k-EP <=> KVA is con-EP or AVK is con-EP and A is con-s-k-EP<=> A^T is con-s-k-EP$_r$ moreover A is said to be Con-s-k-EP$_r$ if A is con-s-k-EP and of rank r. For further properties of con-s-k-EP matrices one may refer **[1]**.

In **Theorem (2.11) [1]**, it is shown that $\boldsymbol{A}$ is con-s-k-EP$_r$, if and only if $\boldsymbol{A}^\dagger$ is con-s-k-EP$_r$. Thus the con-s-k-EP$_r$ property of complex matrices is preserved for its Moore-Penrose inverses. However, all other generalized inverses of a con-s-k-EP$_r$ matrix need not be con-s-k-EP$_r$. For instance, let $\boldsymbol{A}=\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}$ for k=(1,2)(3), the associated permutation matrix be $\boldsymbol{K}=\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$ and $\boldsymbol{V}=\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}$. Here, $\boldsymbol{A}$ is con-s-k-EP$_1$. But $\boldsymbol{A}^- = \begin{pmatrix}1&0&0\\0&0&0\\0&0&0\end{pmatrix}$ is a inverse of $\boldsymbol{A}$ which is not con-s-k-EP$_1$.

A generalized inverse $\boldsymbol{A}^= \in \boldsymbol{A}\{1,2\}$ is shown to be con-s-k-EP$_r$ whenever $\boldsymbol{A}$ is con-s-k-EP$_r$ under certain conditions in the following way.

Theorem 2.1

Let $\boldsymbol{A}\in \boldsymbol{C}_{n\times n}, \boldsymbol{X}\in \boldsymbol{A}\{1,2\}$ and $\boldsymbol{AX}, \boldsymbol{XA}$ are con-s-k-EP$_r$ matrices. Then $\boldsymbol{A}$ is con-s-k-EP$_r$ $\Leftrightarrow$ $\boldsymbol{X}$ is con-s-k-EP$_r$.

Proof

Since $\boldsymbol{AX}$ and $\boldsymbol{XA}$ are con-s-k-EP$_r$, by **Theorem (2.11) [1]**, we have $\boldsymbol{R}(\boldsymbol{AX}) = \boldsymbol{R}(\boldsymbol{KV}(\boldsymbol{AX})^{\boldsymbol{T}})$ and $\boldsymbol{R}(\boldsymbol{KV}(\boldsymbol{XA})^{\boldsymbol{T}}) = \boldsymbol{R}(\boldsymbol{XA})$. Since $\boldsymbol{X}\in \boldsymbol{A}\{1,2\}$ we have $\boldsymbol{AXA} = \boldsymbol{A}, \boldsymbol{XAX} = \boldsymbol{X}$.

Ref

1. Krishnamoorthy, S., Gunasekaran, K. and Muthugobal, B.K.N., "Con-s-k-EP matrices", ***Journal of Mathematical Sciences and Engineering Applications***, Vol. 5, No.1, 2011, 353 – 364.

Notes

Now,

$$\begin{aligned} \boldsymbol{R}(\boldsymbol{A}) &= \boldsymbol{R}(\boldsymbol{AX}) \\ &= \boldsymbol{R}(\boldsymbol{KV}(\boldsymbol{AX})^{\boldsymbol{T}}) \\ &= \boldsymbol{R}(\boldsymbol{KVX}^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}}) \\ &= \boldsymbol{R}(\boldsymbol{KVX}^{\boldsymbol{T}}) \end{aligned}$$

$$\begin{aligned} \boldsymbol{R}(\boldsymbol{KVA}^{\boldsymbol{T}}) &= \boldsymbol{R}(\boldsymbol{KVA}^{\boldsymbol{T}}\boldsymbol{X}^{\boldsymbol{T}}) \\ &= \boldsymbol{R}(\boldsymbol{KV}(\boldsymbol{XA})^{\boldsymbol{T}}) \\ &= \boldsymbol{R}(\boldsymbol{XA}) \\ &= \boldsymbol{R}(\boldsymbol{X}) \end{aligned}$$

Now, $\boldsymbol{A}$ is con-s-k-EP$_r$ $\Leftrightarrow \boldsymbol{R}(\boldsymbol{A}) = \boldsymbol{R}(\boldsymbol{KVA}^{\boldsymbol{T}})$ and $(\boldsymbol{A}) = \boldsymbol{r}$ $\Leftrightarrow \boldsymbol{R}(\boldsymbol{KVX}^{\boldsymbol{T}}) = \boldsymbol{R}(\boldsymbol{X})$ and $(\boldsymbol{A}) = \ (\boldsymbol{X}) = \boldsymbol{r} \Leftrightarrow \boldsymbol{X}$ is con-s-k-EP$_r$.

Remark 2.2

In the above Theorem, the conditions that both $\boldsymbol{AX}$ and $\boldsymbol{XA}$ to be con-s-k-EP$_r$ are essential. Let $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ for k=(1,2)(3), the associated permutation matrix $\boldsymbol{K} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $\boldsymbol{V} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$. $\boldsymbol{A}$ is con-s-k-EP$_1$. $\boldsymbol{X} = \boldsymbol{A}^{-} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in \boldsymbol{A}\{1,2\}$ $\boldsymbol{AX} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}; \boldsymbol{XA} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. $\boldsymbol{AX}$ and $\boldsymbol{XA}$ are not con-s-k-EP$_1$. Also $\boldsymbol{X}$ is not con-s-k-EP$_1$.

Now, we show that generalized inverses belonging to the sets A{1,2,3} and A{1,2,4} of a con-s-k-EP$_r$ matrix $\boldsymbol{A}$ is also con-s-k-EP$_r$, under certain conditions in the following Theorems.

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Notes

Theorem 2.3

Let $A\in C_{n\times n}, X\in A\{1,2,3\}, R(X)=R(A^T)$. Then A is con-s-k-EP$_r$ $\Leftrightarrow$ X is con-s-k-EP$_r$.

Proof

Since $X\in A\{1,2,3\}$, we have $AXA=A, XAX=X, (AX)^T=AX$. Therefore, $R(A)=R(AX)=R((AX)^T)=R(X^T)$

$$\begin{aligned}
R(X)=R(A^T) &\Rightarrow XX^{\dagger}=A^T(A^T)^{\dagger}\\
&\Rightarrow XX^{\dagger}=A^T(A^{\dagger})^T\\
&\Rightarrow XX^{\dagger}=(A^{\dagger}A)^T\\
&\Rightarrow XX^{\dagger}=A^{\dagger}A\\
&\Rightarrow KVXX^{\dagger}VK=KVA^{\dagger}AVK\\
&\Rightarrow (KVX)(KVX)^{\dagger}=(AVK)^{\dagger}AVK\\
&\Rightarrow R(KVX)=R((AVK)^T)\\
&\Rightarrow R(KVX)=R(KVA^T)
\end{aligned}$$

A is con-s-k-EP$_r$ $\Leftrightarrow R(A)=R(KVA^T)$ and $(A)=r \Leftrightarrow R(X^T)=R(KVX)$ and $(A)=\ (X)=r \Leftrightarrow X$ is con-s-k-EP$_r$.

Theorem 2.4

Let $A\in C_{n\times n}, X\in A\{1,2,4\}, R(A)=R(X^T)$. Then A is con-s-k-EP$_r$ $\Leftrightarrow$ X is con-s-k-EP$_r$.

Proof

Since $X\in A\{1,2,4\}$, we have $AXA=A, XAX=X, (XA)^T=XA$ and $R(A)=R(X^T)$.

Now,
$$\begin{aligned}
R(KVA^T)&=R(KVA^TX^T)\\
&=R(KV(XA)^T)\\
&=R(KVXA)\\
&=R(KVX)
\end{aligned}$$

$\boldsymbol{A}$ is con-s-k-EP$_r$ $\Leftrightarrow \boldsymbol{R}(\boldsymbol{A}) = \boldsymbol{R}(\boldsymbol{KVA}^{\boldsymbol{T}})$ and $(\boldsymbol{A}) = \boldsymbol{r} \Leftrightarrow \boldsymbol{R}(\boldsymbol{X}^{\boldsymbol{T}}) = \boldsymbol{R}(\boldsymbol{KVX})$ and $(\boldsymbol{A}) = (\boldsymbol{X}) = \boldsymbol{r} \Leftrightarrow \boldsymbol{X}$ is con-s-k-EP$_r$.

Remark 2.5

Notes

In particular, if $\boldsymbol{X} = \boldsymbol{A}^{\dagger}$ then $\boldsymbol{R}(\boldsymbol{A}^{\dagger}) = \boldsymbol{R}(\boldsymbol{A}^{\boldsymbol{T}})$ holds. Hence $\boldsymbol{A}$ is con-s-k-EP$_r$ is equivalent to $\boldsymbol{A}^{\dagger}$ is con-s-k-EP$_r$.

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