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I. Introduction

Observational data like Ia Supernovae suggest that the universe is dominated by two dark components containing dark energy (DE) and dark matter (DM). Dark energy with negative pressure is used to explain the present cosmic accelerating expansion while dark matter is used to explain galactic curves and large-scale structure formation.

Origin of the dark energy and dark matter and their natures remains unknown and we hope that Large Hadron Collider (LHC) can gives us these hints. Certainly the nature of dark energy component of the universe (Riess et al. [1], Perlmutter et al. [2]; Sahani [3]) is one of the mysteries of cosmology. Cosmological constant, Quitessence [4, 5, 6], K-essence [7, 8, 9], Phanton energy [10, 11, 12] are some of the candidates of dark energy. To explain the acceleration of the Universe, Cardassion expansion [13, 14, and 15] and what might be derived from brane cosmology [16, 17] have been used. Also there are interacting DE models like Chaplygin gas [18, 19], holographic models [20, 21, 22] etc. have been proposed but none of these models are entirely convincing so far.

In the spirit of the generalized Chaplygin gas (GCG), we use Wet Dark Fluid (WDE) as a model for dark energy. Here the motivation stemmed from empirical equation of state proposed by Tait [23] and Haywords [24] to treat water and aqueous solution.

There has been a lot of interest in scalar-tensor theories of gravitation proposed by Brans and Dicke [25], Nordtvedt [26] and Saez-Ballaster [27] among them Saez-Ballaster scalar-tensor theory is considered to be viable alternative to general relativity, in this theory metric is coupled with a dimensionless scalar-field in a simple manner. Reddy et al. [28] has given a brief review of Saez-Ballaster [27] theory for the combined scalar and tensor field are

$$G_{ij}-\omega\phi^{n}\left(\phi_{,i}\phi_{,j}-\frac{1}{2}g_{ij}\phi_{,a}\phi^{,a}\right)=-kT_{ij}\,, \qquad (1.1)$$

and the scalar-field ϕ satisfies the equation

$$2\phi^{n}\phi^{i}_{;j}+n\phi^{n-1}\phi_{,a}\phi^{,a}=0\,, \qquad (1.2)$$

where $G_{ij}=R_{ij}-\frac{1}{2}g_{ij}R$ is the Einstein tensor, R the scalar curvature, ω and n are constants, T_{ij} is the stress tensor of matter, and comma and semicolon denote partial and covariant differentiation respectively.

Many authors have studied the different cosmological models in the framework of scalar-tensor theories . Particularly Reddy and Naidu [29], Rao et al. [30,31] are some of the authors who have investigated several aspects of the cosmological models in Saez-Ballaster scalar-tensor theory. Recently Rao et al. [32] and Chirde et al. [33] have studied dark-energy cosmological models in this theory.

The equation of WDF is

$$p_{WDF}=\gamma(\rho_{WDF}-\rho_*)\,. \qquad (1.3)$$

Here the parameters γ and ρ_* are taken to be positive and we restrict ourselves $0\le\gamma\le1$. If C_s denotes the adiabatic sound speed in WDF then $\gamma=C_s^2$ (Babichev et al.).

To find the WDF energy density, we use the energy conservation equation

$$\dot{\rho}_{WDF}+3H(p_{WDF}+\rho_{WDF})=0\,. \qquad (1.4)$$

Using the equation of WDF (1.3) and the relation $3H=\frac{\dot V}{V}$ in above equation, we get

$$\rho_{WDF}=\frac{\gamma}{1+\gamma}\rho_*+\frac{C}{V^{(1+\gamma)}}\,, \qquad (1.5)$$

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where C is the constantof integration and V is the volume expansion. WDF includes two components, a piece that behaves a cosmological constant as well as a standard fluid with an equation of state $p = \gamma\rho$.

If $C > 0$, the strong energy condition $p + \rho \geq 0$ will not be violated by this fluid

$$p_{WDF} + \rho_{WDF} = (1+\gamma)\rho_{WDF} - \gamma\rho_*$$

$$= (1+\gamma)\frac{C}{V^{(1+\gamma)}} \geq 0 . \qquad (1.6)$$

The wet dark fluid has been used as dark energy in the homogeneous isotropic FRW case by Holman and Naidu. Singh and Chaubey [35] studied Bianchi-I Universe with wet dark fluid in general relativity. Katore et al. [36] studied plane symmetric Universe with wet dark fluid in general relativity. Recently Adhav et al.[37,38] studied Bianchi type-III Magnetized wet dark universe and Einstein-Rosen wet dark universe in general relativity respectively. R Chaubey [39] studied Bianchi type-V wet dark universe in general relativity. Very recently Samanta et al. [41] studied, five dimensional LRS Bianchi type-I bulk viscous cosmological model with wet dark fluid in general relativity. This motivates the authors to study Bianchi type-I went dark fluid in Scalar-tensor theory of gravitation.

II. The Metric and Field Equations

We consider Bianchi type-I space time given by

$$ds^2 = dt^2 - A^2dx^2 - B^2dy^2 - C^2dz^2, \qquad (2.1)$$

where the metric potentials A, B, C are functions of t only.

The Einstein's field equations for the metric (2.1) are written in the form

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = kT_1^1, \qquad (2.2)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = kT_2^2, \qquad (2.3)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = kT_3^3, \qquad (2.4)$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = kT_4^4 \qquad (2.5)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{V}}{V}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 . \qquad (2.6)$$

Here k is the gravitational constant and overhead dot denotes differentiation with respect to t .

The energy-momentum tensor of the source is given by

$$T_i^j = (\rho_{WDF} + p_{WDF})u_iu^j - p_{WDF}\delta_i^j , \qquad (2.7)$$

where u^i is the flow vector satisfying

$$g_{ij}u^iu^j = 1 . \qquad (2.8)$$

In co-moving co-ordinate system, using equation (2.7), we get

$$T_1^1 = T_2^2 = T_3^3 = -p_{WDF} \text{ and } T_4^4 = \rho_{WDF}. \qquad (2.9)$$

Using equations (2.2)-(2.6) and equation (2.9), we get

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -kp_{WDF}, \qquad (2.10)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -kp_{WDF}, \qquad (2.11)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\omega}{2}\phi^n\dot{\phi}^2 = -kp_{WDF}, \qquad (2.12)$$

$$\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\omega}{2}\phi^n\dot{\phi}^2 = k\rho_{WDF}, \qquad (2.13)$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{V}}{V}\right) + \frac{n}{2}\frac{\dot{\phi}^2}{\phi} = 0 . \qquad (2.14)$$

Using equations (2.10) and (2.11), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = -\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \qquad (2.15)$$

Let V be a function of t defined by

$$V = ABC . \qquad (2.16)$$

From equation (2.15), we get

$$\frac{d}{dt}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = -\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)\frac{\dot{V}}{V} \qquad (2.17)$$

Integrating above equation, we get

$$\frac{A}{B} = d_1 e^{x_1\int\frac{1}{V}dt} . \qquad (2.18)$$

Similarly using equations (2.10), (2.11) and (2.12), we get

$$\frac{B}{C} = d_2 e^{x_2 \int \frac{1}{V} dt}, \tag{2.19}$$

$$\frac{A}{C} = d_3 e^{x_3 \int \frac{1}{V} dt}, \tag{2.20}$$

where d_1, d_2, d_3 and x_1, x_2, x_3 are constants of integration.

In view of $V = ABC$, we get

$$A(t) = D_1 V^{\frac{1}{3}} e^{X_1 \int \frac{1}{V} dt}, \tag{2.21}$$

$$B(t) = D_2 V^{\frac{1}{3}} e^{X_2 \int \frac{1}{V} dt}, \tag{2.22}$$

$$C(t) = D_3 V^{\frac{1}{3}} e^{X_3 \int \frac{1}{V} dt}, \tag{2.23}$$

where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Now using the equations (2.10)-(2.13), we have

$$2\frac{\ddot{A}}{A} + 2\frac{\ddot{B}}{B} + 2\frac{\ddot{C}}{C} + 4\left(\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C}\right) = \frac{3k}{2}\left(\rho_{WDF} - p_{WDF}\right) \tag{2.24}$$

Using $V = ABC$, equation (2.24) becomes

$$\frac{\ddot{V}}{V} = \frac{3k}{2}\left(\rho_{WDF} - p_{WDF}\right). \tag{2.25}$$

The conservational law for the energy-momentum tensor gives

$$\dot{\rho}_{WDF} = -\frac{\dot{V}}{V}\left(\rho_{WDF} + p_{WDF}\right). \tag{2.26}$$

Using equations (2.25) and (2.26), we get

$$\dot{V} = \pm\sqrt{2\left(\frac{3k}{2}\rho_{WDEF} V^2 + C_1\right)}, \tag{2.27}$$

where C_1 is the constant of integration.

From equation (2.26), we obtain

$$\frac{\dot{\rho}}{\rho_{WDF} + p_{WDF}} = -\frac{\dot{V}}{V}. \tag{2.28}$$

By considering that the energy density obeying an equation of state $p_{WDF} = f(\rho_{WDF})$, we conclude ρ_{WDF} and p_{WDF}.

Therefore the right hand side of equation (2.25) is a function of V only. From equation (2.25)

$$\ddot{V} = \frac{3k}{2}\left(\rho_{WDF} - p_{WDF}\right)V \equiv F(V). \tag{2.29}$$

The equation (2.29) can be interpreted as equation of motion of a single particle with unit mass under the force $F(V)$ therefore,

$$\dot{V} = \sqrt{2(\in - U(V))} \tag{2.30}$$

Here $\in$ is taken as energy and $U(V)$ as the potential of the force F.

Comparing equations (2.27) and (2.30), we have

$$\in = C_1 \text{ and } U(V) = \frac{3k}{2}\rho_{WDF} V^2.$$

Therefore the solution of equation (2.27) can be written in quadrature form as

$$\int \frac{dV}{\sqrt{2\left[C_1 + \frac{3k}{2}\rho_{WDF} V^2\right]}} = t + t_0, \tag{2.31}$$

where t_0 is the constant of integration and can be taken to be zero, since it gives only shift in time.

Using equations (1.5) and (2.31), we get

$$\int \frac{dV}{\sqrt{2\left[\frac{3k}{2}\left(\frac{\gamma}{1+\gamma}\rho_* + \frac{C}{V^{1+\gamma}}\right)V^2 + C_1\right]}} = t. \tag{2.32}$$

From equation (2.14) ,we yield

$$\phi = \left[\frac{(n+2)\alpha}{2}\int \frac{1}{V} dt\right]^{\frac{2}{n+2}}. \tag{2.33}$$

III. Some Particular Cases

CaseI. $\gamma = 1$ (Zel'dovich fluid)

Equation (2.32) reduces to

$$V = \sqrt{\frac{6kC + 4C_1}{3k\rho_*}} \sinh\left(\sqrt{\frac{3k\rho_*}{2}}\, t\right). \tag{3.1}$$

Using equations (2.21)-(2.23) and equation (3.1), we get

$$A(t) = D_1\left(\frac{6kC + C_1}{3k\rho_*}\right)^{\frac{1}{6}}\left(\sinh\sqrt{\frac{3k\rho_*}{2}}\ t\right)^{\frac{1}{3}}\exp\left\{\frac{X_1}{\sqrt{3kC+2C_1}}\log\tanh\left(\frac{\sqrt{3k\rho_*}}{2\sqrt{2}}\ t\right)\right\}, \tag{3.2}$$

$$B(t) = D_2\left(\frac{6kC + C_1}{3k\rho_*}\right)^{\frac{1}{6}}\left(\sinh\sqrt{\frac{3k\rho_*}{2}}\ t\right)^{\frac{1}{3}}\exp\left\{\frac{X_2}{\sqrt{3kC+2C_1}}\log\tanh\left(\frac{\sqrt{3k\rho_*}}{2\sqrt{2}}\ t\right)\right\}, \tag{3.3}$$

$$C(t) = D_3\left(\frac{6kC + C_1}{3k\rho_*}\right)^{\frac{1}{6}}\left(\sinh\sqrt{\frac{3k\rho_*}{2}}\ t\right)^{\frac{1}{3}}\exp\left\{\frac{X_3}{\sqrt{3kC+2C_1}}\log\tanh\left(\frac{\sqrt{3k\rho_*}}{2\sqrt{2}}\ t\right)\right\}, \tag{3.4}$$

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Where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.
Using equations (1.5) and (3.1), we obtain

$$\rho_{WDF} = \frac{\rho_*}{2} + \frac{3k\rho_* C}{6kC + 4C_1}\cos ech^2\left(\sqrt{\frac{3k\rho_*}{2}}\ t\right). \tag{3.5}$$

Using equations (1.3)and(3.5), we get

$$p_{WDF} = -\frac{\rho_*}{2} + \frac{3k\rho_* C}{6kC + 4C_1}\cos ech^2\left(\sqrt{\frac{3k\rho_*}{2}}\ t\right). \tag{3.6}$$

Using equation (2.33), we yield

$$\phi = \left[\frac{(n+2)\alpha}{2\sqrt{3kC+2C_1}}\log\tanh\left(\frac{\sqrt{3k\rho_*}}{2\sqrt{2}}t\right)\right]^{\frac{2}{n+2}}. \tag{3.7}$$

The physical quantities in cosmology are the expansion scalar θ, the mean anisotropy parameter A_m, the shear scalar σ^2 and the deceleration parameter q are defined as

$$\theta = 3H, \tag{3.8}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{3.9}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right)$$

$$= \frac{3}{2}A_m H^2, \tag{3.10}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{3.11}$$

Using equations (3.8)-(3.11) the physical quantities can be expressed as

$$\theta = \sqrt{\frac{3k\rho_*}{2}}\ \coth\left(\sqrt{\frac{3k\rho_*}{2}}\ t\right), \tag{3.12}$$

$$A_m = \frac{3X^2}{3kC + 2C_1}\sec h^2\left(\sqrt{\frac{3k\rho_*}{2}}\ t\right), \tag{3.13}$$

$$\sigma^2 = \frac{3X^2 k\rho_*}{4(3kC + 2C_1)}\cos ech^2\left(\sqrt{\frac{3k\rho_*}{2}}\ t\right), \tag{3.14}$$

$$q = 3\sec h^2\left(\sqrt{\frac{3k\rho_*}{2}}\ t\right) - 1, \tag{3.15}$$

where $X^2 = X_1^2 + X_2^2 + X_3^2 =$ Constant .

CaseII. $\gamma = 0$ **(Dust fluid)**

Equation (2.32) reduces to

$$\int\frac{dV}{\sqrt{3kC + 2C_1}} = t, \tag{3.16}$$

which gives

$$V = \frac{3kC}{4}t^2 - \frac{2C_1}{3kC}. \tag{3.17}$$

CaseIIa. When $t > \frac{2\sqrt{2C_1}}{3kC}$

Using equations (2.21)-(2.23) and (3.17), we get

$$A(t) = D_1\left(\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right)^{\frac{1}{3}} \exp\left\{-X_1\sqrt{\frac{2}{C_1}}\coth^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right\} \quad (3.18)$$

$$B(t) = D_2\left(\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right)^{\frac{1}{3}} \exp\left\{-X_2\sqrt{\frac{2}{C_1}}\coth^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right\} \quad (3.19)$$

$$C(t) = D_3\left(\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right)^{\frac{1}{3}} \exp\left\{-X_3\sqrt{\frac{2}{C_1}}\coth^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right\} \quad (3.20)$$

where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Using equations (1.5) and (3.17), we obtain

$$\rho_{WDF} = C\left[\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right]^{-1}. \quad (3.21)$$

Using equations (1.3) and (3.21), we get

$$p_{WDF} = 0, \quad (3.22)$$

Using equation (2.33), we get

$$\phi = \left[\frac{-(n+2)\alpha}{\sqrt{2C_1}}\coth^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right]^{\frac{2}{n+2}}. \quad (3.23)$$

Using equations (3.8)-(3.11), the physical quantities can be written as

$$\theta = \frac{3kCt}{2\left[\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right]}, \quad (3.24)$$

$$A_m = \frac{4X^2}{3k^2C^2t^2}, \quad (3.25)$$

$$\sigma^2 = \frac{72k^2C^2t^2}{\left(9k^2C^2t^2 - 8C_1\right)^2}, \quad (3.26)$$

$$q = \frac{1}{2} + \frac{4C_1}{3k^2C^2t^2}, \quad (3.27)$$

where $X^2 = X_1^2 + X_2^2 + X_3^2 =$ constant

The model becomes isotropic, for large value of t.

CaseIIb. When $t < \frac{2\sqrt{2C_1}}{3kC}$

Using equations (2.21)-(2.23) and (3.17), we get

$$A(t) = D_1\left(\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right)^{\frac{1}{3}} \exp\left\{-X_1\sqrt{\frac{2}{C_1}}\tanh^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right\} \quad (3.28)$$

$$B(t) = D_2\left(\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right)^{\frac{1}{3}} \exp\left\{-X_2\sqrt{\frac{2}{C_1}}\tanh^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right\} \quad (3.29)$$

$$C(t) = D_3\left(\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right)^{\frac{1}{3}} \exp\left\{-X_3\sqrt{\frac{2}{C_1}}\tanh^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right\} \quad (3.30)$$

where $D_i (i = 1,2,3)$ and $X_i (i = 1,2,3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Using equations (1.5) and (3.17), we get

$$\rho_{WDF} = C\left[\frac{3kC}{4}t^2 - \frac{2C_1}{3kC}\right]^{-1}. \quad (3.31)$$

Using equations (1.3) and (3.31), we get

$$p_{WDF}=0, \tag{3.32}$$

Using equations (2.33), we get

$$\phi=\left[\frac{-(n+2)\alpha}{\sqrt{2C_1}}\tanh^{-1}\left(\frac{3kC}{2\sqrt{2C_1}}t\right)\right]^{\frac{2}{n+2}}. \tag{3.33}$$

Using equations (3.8)-(3.11), the physical quantities can be written as

$$\theta=\frac{3kCt}{2\left[\frac{3kC}{4}t^2-\frac{2C_1}{3kC}\right]}, \tag{3.34}$$

$$A_m=\frac{4X^2}{3k^2C^2t^2}, \tag{3.35}$$

$$\sigma^2=\frac{72k^2C^2t^2}{\left(9k^2C^2t^2-8C_1\right)^2} \tag{3.36}$$

$$q=\frac{1}{2}+\frac{4C_1}{3k^2C^2t^2} \tag{3.37}$$

where $X^2=X_1^2+X_2^2+X_3^2=$ Constant.

The model becomes isotropic, for large value of t.

IV. Models with Constant Deceleration Parameter

CaseI. Power law:

Here we take

$$V=at^b, \tag{4.1}$$

where a and b are constants.

Using equations (2.21), (2.22), (2.23) and (4.1), we get

$$A(t)=D_1a^{\frac{1}{3}}t^{\frac{b}{3}}\exp\left\{\frac{X_1}{a(1-b)}t^{(1-b)}\right\}, \tag{4.2}$$

$$B(t)=D_2a^{\frac{1}{3}}t^{\frac{b}{3}}\exp\left\{\frac{X_2}{a(1-b)}t^{(1-b)}\right\}, \tag{4.3}$$

$$C(t)=D_3a^{\frac{1}{3}}t^{\frac{b}{3}}\exp\left\{\frac{X_3}{a(1-b)}t^{(1-b)}\right\}, \tag{4.4}$$

where $D_i(i=1,2,3)$ and $X_i(i=1,2,3)$ satisfy the relation $D_1D_2D_3=1$ and $X_1+X_2+X_3=0$.

Using equations (1.5) and (4.1), we get

$$\rho_{WDF}=\left(\frac{\gamma}{1+\gamma}\right)\rho_*+\frac{C}{a^{1+\gamma}}t^{-(1+\gamma)b}. \tag{4.5}$$

Using equations (1.3) and (4.5), we get

$$p_{WDF}=\gamma\left[\frac{C}{a^{(1+\gamma)}}t^{-(1+\gamma)b}-\frac{1}{1+\gamma}\rho_*\right]. \tag{4.6}$$

Using equations (2.33) and (4.1), we get

$$\phi=\left[\frac{(n+2)\alpha}{2a(1-b)}t^{(1-b)}\right]^{\frac{2}{n+2}}. \tag{4.7}$$

Using equations (3.8)-(3.11), the physical quantities can be written as

$$\theta=\frac{b}{t}, \tag{4.8}$$

$$A_m=\frac{3X^2}{a^2b^2t^{2(b-1)}}, \tag{4.9}$$

$$\sigma^2=\frac{X^2}{2a^2t^{2b}} \tag{4.10}$$

$$q=\frac{3}{b}-1, \tag{4.11}$$

where $X^2=X_1^2+X_2^2+X_3^2=$ constant

The model becomes isotropic, for large value of t and $b>1$.

CaseII. Exponential type:

Here we take

$$V=\alpha_1e^{\beta t}, \tag{4.12}$$

where α_1 and β are constants.

Using equations (2.21), (2.22), (2.23) and (4.12), we get

$$A(t)=D_1\alpha_1^{\frac{1}{3}}e^{\frac{\beta t}{3}}\exp\left\{\frac{-X_1}{\alpha_1\beta}e^{-\beta t}\right\}, \tag{4.13}$$

$$B(t)=D_2\alpha_1^{\frac{1}{3}}e^{\frac{\beta t}{3}}\exp\left\{\frac{-X_2}{\alpha_1\beta}e^{-\beta t}\right\}, \tag{4.14}$$

$$C(t)=D_3\alpha_1^{\frac{1}{3}}e^{\frac{\beta t}{3}}\exp\left\{\frac{-X_3}{\alpha_1\beta}e^{-\beta t}\right\}, \tag{4.15}$$

Where $D_i(i=1,2,3)$ and $X_i(i=1,2,3)$ satisfy the relation $D_1D_2D_3=1$ and $X_1+X_2+X_3=0$.

Using equations (1.5) and (4.12), we get

$$\rho_{WDF} = \left(\frac{\gamma}{1+\gamma}\right)\rho_* + \frac{C}{\alpha_1^{1+\gamma}} e^{-(1+\gamma)\beta t}. \qquad (4.16)$$

Using equations (1.3) and (4.12), we get

$$p_{WDF} = \gamma\left[\frac{C}{\alpha_1^{(1+\gamma)}} e^{-(1+\gamma)\beta t} - \frac{1}{1+\gamma}\rho_*\right]. \qquad (4.17)$$

Using equations (2.33) and (4.12), we get

$$\phi = \left[\frac{-(n+2)\alpha}{2\alpha_1\beta} e^{-\beta t}\right]^{\frac{2}{n+2}}. \qquad (4.18)$$

Using equations (3.8)-(3.11), the physical quantities can be written as

$$\theta = \beta\,, \qquad (4.19)$$

$$A_m = \frac{3X^2}{\alpha_1^2\beta^2} e^{-2\beta t}\,, \qquad (4.20)$$

$$\sigma^2 = \frac{X^2}{2\alpha_1^2} e^{-2\beta t}\,, \qquad (4.21)$$

$$q = -1\,, \qquad (4.22)$$

where $X^2 = X_1^2 + X_2^2 + X_3^2$ = constant

The model becomes isotropic, for large value of t.

V. Conclusion

In this paper we have studied Bianchi Type-I Universe with wet dark fluid in Saez-Ballaster scalar – tensor theory of gravitation. The new equation of state for the dark-energy component (known as wet dark fluid) has been considered. The solution has been obtained in quadrature form. The model with constant deceleration parameter has been discussed in detail. The behaviour of the models for large time have been analyzed. When $\phi \to 0$ our results resemble with the results obtained by Singh and Chaubey[35].

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