Dual to Ratio Estimators of Population Mean in Post-Stratified Sampling using Known Value of Some Population Parameters

By Onyeka, A.C.
Federal University

Abstract - This paper extends the work carried out by Onyeka (2012), by proposing a class of dual to ratio combined estimators of the population mean in post-stratified sampling when using known value of some population parameters. The proposed estimators, under certain conditions, are shown to be more efficient than some existing estimators, including the usual poststratified estimator and the estimators proposed by Onyeka (2012). Properties of the proposed class of estimators, including conditions for optimal efficiency, are obtained up to first order approximation. The results are illustrated using empirical data.

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GJSFR-F Classification: MSC 2010: 62D05
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Abstract - This paper extends the work carried out by Onyeka (2012), by proposing a class of dual to ratio combined estimators of the population mean in post-stratified sampling when using known value of some population parameters. The proposed estimators, under certain conditions, are shown to be more efficient than some existing estimators, including the usual poststratified estimator and the estimators proposed by Onyeka (2012). Properties of the proposed class of estimators, including conditions for optimal efficiency, are obtained up to first order approximation. The results are illustrated using empirical data.

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I. INTRODUCTION

Many authors have considered the use of some known population parameters of an auxiliary character in formulating estimators of population parameters of a variable of interest. A lot of theoretical and empirical studies have been carried out along this line. Some known population parameters of an auxiliary character, which have been considered for the purpose of constructing estimators for some population parameters of the study variate include coefficient of variation, (CV), used by Searls (1964) and Sisodia-Dwivedi (1981); coefficient of kurtosis, used by Singh et al. (1973) and Upadhyaya-Singh (1999); coefficient of skewness, used by G.N. Singh (2003); standard deviation, used by G.N. Singh (2003); and correlation coefficient, used by Singh and Tailor (2003). A general family of estimators of \( \bar{Y} \) under the SRSWOR scheme was discussed by Khoshnevisan et.al. (2007), using known parameters of the auxiliary variable \( x \), such as standard deviation, coefficient of variation, coefficient of skewness, kurtosis and correlation coefficient. Koyuncu and Kadilar (2009) also proposed a general family of combined estimators of \( \bar{Y} \) in stratified random sampling. Onyeka (2012), motivated by the works carried out by Khoshnevisan et.al. (2007) and Koyuncu and Kadilar (2009), developed a general family of estimators of \( \bar{Y} \) under the post-stratified sampling scheme using known values of some population parameters of an auxiliary character. The family of estimators discussed by Onyeka (2012), was found, under some optimum conditions, to be as efficient as the post-stratified regression estimator \( \bar{y}_{psREG} \), but more efficient, in terms of having a smaller mean squared error, than the usual poststratified sampling estimator, \( \bar{y}_{ps} \), and other particular cases of the proposed estimators. The present study is aimed at utilizing some variable transformation of an auxiliary character \( x \), to extend the work carried out by Onyeka (2012) in poststratified sampling scheme. Srivenkataramana
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(1980) used the transformation, \( x^*_i = \frac{N \bar{X} - nx_i}{N-n} \), \( i = 1, 2, \ldots, N \), to obtain a dual to ratio estimate of \( \bar{Y} \) in simple random sampling scheme. Authors, like Singh and Tailor (2005), Tailor and Sharma (2009), and Sharma and Tailor (2010) have used the same transformation to improve estimates under the simple random sampling scheme. Motivated by these studies, we intend, in the present work, to use the same transformation to extend the work carried out by Onyeka (2012) in poststratified sampling scheme.

Let \( y_{hi} (x_{hi}) \) denote the \( i \)th observation in stratum \( h \) for the study (auxiliary) variate in poststratified sampling scheme. Let a random sample of size \( n \) be drawn from a population of \( N \) units using SRSWOR method, and let the sampled units be allocated to their respective strata, where \( n_h \) (a random variable) is the number of units that fall into stratum \( h \) such that \( \sum_{h=1}^L n_h = n \). It is assumed that \( n \) is large enough such that \( P(n_h = 0) = 0, \forall h \). Onyeka (2012) proposed the following general family of combined estimators of the population mean \( \bar{Y} \) in post-stratified sampling scheme:

\[
\bar{Y}_{ps} = \bar{Y}_{ps} \left( \frac{a\bar{X} + b}{\alpha(a\bar{X}_{ps} + b) + (1-\alpha)(a\bar{X} + b)} \right)^g
\]

(1.1)

where,

- \( \bar{Y}_{ps} = \sum_{h=1}^L \omega_h \bar{Y}_h \) is the usual post-stratified estimator of \( \bar{Y} \)
- \( \bar{X}_{ps} = \sum_{h=1}^L \omega_h \bar{X}_h \) is the usual post-stratified estimator of \( \bar{X} \)
- \( \bar{X} = \sum_{h=1}^L \omega_h \bar{X}_h \) is the known population mean of the auxiliary variate \( x \).
- \( a \) and \( b \) are either constants or functions of known population parameters of the auxiliary variate, such as standard deviation \( (\sigma_x) \), coefficient of variation \( (C_v) \), skewness \( (\beta_1(x)) \), kurtosis \( (\beta_2(x)) \), and correlation coefficient \( (\rho_{xy}) \).
- \( \omega_h = N_h / N \) is stratum weight, \( L \) is the number of strata in the population, \( N_h \) is the number of units in stratum \( h \), \( N \) is the number of units in the population, \( \bar{X}_h \) is the population mean of the auxiliary variate in stratum \( h \), and \( \bar{y}_h (\bar{X}_h) \) is the sample mean of the study (auxiliary) variate in stratum \( h \).

Under the unconditional argument, that is, for repeated samples of fixed size \( n \), the variances and covariance of the estimators, \( \bar{Y}_{ps} \) and \( \bar{X}_{ps} \), obtained up to first order approximation are:

\[
V(\bar{Y}_{ps}) = \left( \frac{1-f}{n} \right) \sum_{h=1}^L \omega_h S_{yh}^2,
\]

(1.2)

\[
V(\bar{X}_{ps}) = \left( \frac{1-f}{n} \right) \sum_{h=1}^L \omega_h S_{xh}^2,
\]

(1.3)
\[
\text{Cov}(\bar{y}_{ps}, \bar{x}_{ps}) = \left(1 - \frac{f}{n}\right) \sum_{h=1}^{l} \omega_h S_{yhx}. 
\]

where \(f = n/N\) is the population sampling fraction, \(S_{yhx} (S_{xhx}^2)\) is the population variance of \(y(x)\) in stratum \(h\), and \(S_{yhx}\) is the population covariance of \(y\) and \(x\) in stratum \(h\). Let

\[
e_0 = \frac{\bar{y}_{ps} - \bar{Y}}{\bar{Y}} \text{ and } e_1 = \frac{\bar{x}_{ps} - \bar{X}}{\bar{X}}. 
\]

Under the unconditional argument, it follows that

\[
\text{E}(e_0) = \text{E}(e_1) = 0 
\]

\[
\text{E}(e_0^2) = \frac{\text{V}(ar{y}_{ps})}{\bar{Y}^2} = \frac{1}{\bar{Y}^2} \left(1 - \frac{f}{n}\right) \sum_{h=1}^{l} \omega_h S_{yhx}^2 
\]

\[
\text{E}(e_1^2) = \frac{\text{V}(ar{x}_{ps})}{\bar{X}^2} = \frac{1}{\bar{X}^2} \left(1 - \frac{f}{n}\right) \sum_{h=1}^{l} \omega_h S_{xhx}^2 
\]

and

\[
\text{E}(e_0e_1) = \frac{\text{Cov}(ar{y}_{ps}, \bar{x}_{ps})}{\bar{Y}\bar{X}} = \frac{1}{\bar{Y}\bar{X}} \left(1 - \frac{f}{n}\right) \sum_{h=1}^{l} \omega_h S_{yhx} 
\]

Accordingly, Onyeka (2012) obtained the unconditional bias and mean squared error of \(\bar{y}_{ps}\), up to first order approximation, respectively as

\[
\text{B}(\bar{y}_{ps}) = \frac{\alpha \lambda g}{2X} \left(1 - \frac{f}{n}\right) \sum_{h=1}^{l} \omega_h (\alpha \lambda (g + 1)RS_{xhx}^2 - 2S_{yhx}) 
\]

and

\[
\text{MSE}(\bar{y}_{ps}) = \left(1 - \frac{f}{n}\right) \sum_{h=1}^{l} \omega_h (S_{yhx}^2 + \alpha^2 \lambda^2 g^2 R^2 S_{xhx}^2 - 2\alpha \lambda gRS_{yhx}) 
\]

where \(\lambda = \frac{aX}{\bar{X}} + b\) and \(R = \frac{\bar{Y}}{\bar{X}}\). The (optimum) choice of \(\alpha\) that minimizes (1.11) is \(\alpha_{opt} = \frac{\beta_0}{\lambda gR}\), and the resulting optimum unconditional mean squared error of \(\bar{y}_{ps}\) is obtained as

\[
\text{MSE}_{opt}(\bar{y}_{ps}) = \left(1 - \frac{f}{n}\right) \left(1 - \rho_0^2\right) \sum_{h=1}^{l} \omega_h S_{yhx}^2 
\]
where
\[ \beta_0 = \frac{1}{\sum_{h=1}^{L} \omega_h S_{y|h}}, \quad \text{and} \quad \rho_0 = \frac{1}{\sqrt{\sum_{h=1}^{L} \omega_h S_{y|h}}}, \]

Notice that (1.12) is the same as the unconditional variance of the usual combined post-stratified regression estimator, \( \bar{y}_{ps\text{REG}} = \bar{y}_{ps} - \hat{\beta}_0 (\bar{x}_{ps} - \bar{X}) \). This implies that the efficiency of the general family of estimators, \( \bar{y}_{ps} \), proposed by Onyeka (2012), may not be improved beyond the efficiency of the customary combined regression-type estimator in post-stratified sampling.

II. THE PROPOSED CLASS OF ESTIMATORS

Motivated by Onyeka (2012) and Srivenkataramana (1980), we propose a class of dual to ratio estimators of the population mean, \( \bar{Y} \), in poststratified sampling, using known population parameters of an auxiliary character x, as:

\[ \bar{y}_{ps}^{\ast} = \bar{y}_{ps} \left( \frac{\alpha (a\bar{x}_{ps}^{\ast} + b) + (1-\alpha)(a\bar{X} + b)}{a\bar{X} + b} \right)^{g} \quad (2.1) \]

where \( \bar{x}_{ps}^{\ast} \) is a transformed sample mean of the auxiliary variable, x, based on the variable transformation, \( \bar{x}_{hi}^{\ast} = \frac{N\bar{X} - nx_{hi}}{N - n} \) and satisfying the relationship:

\[ \bar{X} = f\bar{x}_{ps} + (1-f)\bar{x}_{ps}^{\ast} \quad (2.2) \]

The transformed sample mean, \( \bar{x}_{ps}^{\ast} \), in poststratified sampling, is defined along the line of authors like Srivenkataramana and Srinath (1976), Srivenkataramana (1980), and Sharma and Tailor (2010). Using the transformation, \( \bar{x}_{i}^{\ast} = \frac{N\bar{X} - nx_{i}}{N - n}, \quad i = 1, 2, \ldots, N, \) Srivenkataramana (1980) obtained a dual to ratio estimate of \( \bar{Y} \) in simple random sampling scheme as

\[ \bar{y}_{R}^{(d)} = \bar{y}\left( \frac{\bar{x}_{i}^{\ast}}{\bar{X}} \right) \quad (2.3) \]

This means that the proposed estimator in (2.1) is a type of dual to ratio estimator in poststratified sampling when using information on known parameters of an auxiliary character, x, provided the constant \( g \) is positive. The proposed estimator in (2.1) becomes a type of dual to product estimator if the constant \( g \) is negative. Notice that the transformed sample mean, \( \bar{x}_{ps}^{\ast} \), in (2.2) can be written in terms of \( e_i \) as

\[ \bar{x}_{ps}^{\ast} = \bar{X}(1 - \pi e_i) \quad (2.4) \]
where \( \pi = \frac{f}{1-f} = \frac{n}{N-n} \). Consequently, the proposed class of estimators, \( \bar{y}_{pss}^* \) in (2.1), can be rewritten in terms of \( e_0 \) and \( e_1 \) as

\[
\bar{y}_{pss}^* = \bar{Y}(1 + e_0)(1 - \pi\alpha\lambda e_1)^g
\]

(2.5)

Assuming \( |\pi\alpha\lambda e_1| < 1 \), so that the series \( (1 - \pi\alpha\lambda e_1)^g \) converges, and expanding (2.5) up to first order approximation in expected value, we obtain

\[
(\bar{y}_{pss}^* - \bar{Y}) = \bar{Y}(e_0 - \pi\alpha\lambda ge_1 - \pi\alpha\lambda ge_0 e_1 + \frac{1}{2}(g+1)\pi^2\lambda^2 e_1^2)
\]

(2.6)

and

\[
(\bar{y}_{pss}^* - \bar{Y})^2 = \bar{Y}^2(e_0^2 + \pi^2\lambda^2 g^2 e_1^2 - 2\pi\alpha\lambda ge_0 e_1)
\]

(2.7)

To obtain the unconditional bias and mean squared error of the proposed estimators \( \bar{y}_{pss}^* \) we take the unconditional expectations of (2.6) and (2.7), and use (1.6) – (1.9) to make the necessary substitutions. This gives the unconditional bias and mean squared error of the proposed class of estimators, \( \bar{y}_{pss}^* \), up to first order approximation, respectively as

\[
B(\bar{y}_{pss}) = \left(1 - \frac{f}{n}\right)\left(\frac{\pi\alpha\lambda g}{2\bar{X}}\right)\sum_{h=1}^{L} \omega_h (\pi\alpha\lambda (g+1)RS^2_{xh} - 2S_{yxh})
\]

(2.8)

and

\[
MSE(\bar{y}_{pss}) = \left(1 - \frac{f}{n}\right)\sum_{h=1}^{L} \omega_h (S_{yh}^2 + \pi^2\lambda^2 g^2 R^2 S_{xh}^2 - 2\pi\alpha\lambda gRS_{yxh})
\]

(2.9)

Applying the least squares method, the (optimum) choice of \( \alpha \) that minimizes (2.9), is obtained as

\[
\alpha_{opt} = \frac{\hat{\beta}_0}{\pi\lambda gR}
\]

(2.10)

and the resulting optimum unconditional mean squared error of \( \bar{y}_{pss}^* \) is obtained as

\[
MSE_{opt}(\bar{y}_{pss}^*) = \left(1 - \frac{f}{n}\right)(1 - \rho_0^2)\sum_{h=1}^{L} \omega_h S_{yh}^2
\]

(2.11)

We observe that the optimum mean square error of \( \bar{y}_{pss}^* \), given in (2.11), is the same as the unconditional variance of the usual post-stratified regression estimator, \( \bar{y}_{psREG} = \bar{y}_{ps} - \hat{\beta}_0 (\bar{x}_{ps} - \bar{X}) \), indicating that the efficiency of the proposed class of estimators, \( \bar{y}_{pss}^* \), just like the estimators, \( \bar{y}_{ps} \), proposed by Onyeka (2012), may not be improved beyond the efficiency of the customary regression-type estimator in post-stratified sampling.
III. Efficiency Comparisons

Here, we shall compare the efficiency of the proposed class of dual to ratio estimators, \( \bar{y}_{ps}^* \), with those of some existing estimators of \( \bar{y} \), including the usual poststratified sampling estimator, \( \bar{y}_{ps} \), and the estimator, \( \bar{y}_{ps} \), proposed by Onyeka (2012).

a) Efficiency Comparison of \( \bar{y}_{ps}^* \) and \( \bar{y}_{ps} \)

To compare the efficiencies of the proposed dual to ratio estimator, \( \bar{y}_{ps}^* \), and the usual poststratified sampling estimator, \( \bar{y}_{ps} \), we let \( A_0 = \frac{1}{\sum_{h=1}^{L} \omega_h S_y^2} \) and \( A_1 = \frac{1}{\sum_{h=1}^{L} \omega_h S_x^2} \). Then, we can rewrite (1.2) and (2.9), respectively as:

\[
V(\bar{y}_{ps}) = \left(1 - \frac{f}{n}\right) A_0^2 \tag{3.1}
\]

and

\[
\text{MSE}(\bar{y}_{ps}^*) = \left(1 - \frac{f}{n}\right) \left( A_0^2 + \pi^2 \alpha^2 \gamma^2 g^2 R^2 A_1^2 - 2\pi \alpha \lambda g \rho_0 A_0 A_1 \right) \tag{3.2}
\]

so that

\[
V(\bar{y}_{ps}) - \text{MSE}(\bar{y}_{ps}^*) = \left(1 - \frac{f}{n}\right) \left(2\pi \alpha \lambda g \rho_0 A_0 A_1 - \pi^2 \alpha^2 \gamma^2 g^2 R^2 A_1^2 \right) \tag{3.3}
\]

This shows that the proposed class of estimators, \( \bar{y}_{ps}^* \), is more efficient than the estimator, \( \bar{y}_{ps} \), in terms of having a smaller mean squared error, if

\[
\frac{\beta_0}{\pi \alpha \lambda g R} > \frac{1}{2} \tag{3.4}
\]

provided \( a \neq 0, \alpha \neq 0 \) and \( g \neq 0 \). Note that if \( a = 0, \alpha = 0 \) and \( g = 0 \) separately, the proposed estimator, \( \bar{y}_{ps}^* \) in (2.1) reduces to the usual poststratified estimator, \( \bar{y}_{ps} \).

b) Efficiency Comparison of \( \bar{y}_{ps}^* \) and \( \bar{y}_{ps}^{(R)} \)

Here, we compare the efficiencies of the proposed estimator, \( \bar{y}_{ps}^* \) and the ratio-type combined estimator in poststratified sampling, given by

\[
\bar{y}_{ps}^{(R)} = \frac{\Sigma_{ps}}{\bar{x}_{ps}} \bar{X} \tag{3.5}
\]

with mean squared error, approximated up to first order, as

\[
\text{MSE}(\bar{y}_{ps}^{(R)}) = \left(1 - \frac{f}{n}\right) \left( A_0^2 + R^2 A_1^2 - 2R \rho_0 A_0 A_1 \right) \tag{3.6}
\]
Using (3.2) and (3.6), it can be shown that the proposed class of estimators, \( \bar{y}^*_{pss} \), is more efficient than the ratio-type estimator, \( \bar{y}^{(R)}_{ps} \), in terms of having a smaller mean squared error, if
\[
\frac{\beta_0 (1 - \pi \alpha \lambda g)}{R} < \frac{1}{2} \tag{3.7}
\]

**c) Efficiency Comparison of \( \bar{y}^*_{pss} \) and \( \bar{y}^{(P)}_{ps} \)**

Here, we compare the efficiencies of the proposed estimator, \( \bar{y}^*_{pss} \), and the product-type combined estimator in poststratified sampling, given by
\[
\bar{y}^{(P)}_{ps} = \frac{\bar{y}_{ps} X_{ps}}{X} \tag{3.8}
\]
with mean squared error, approximated up to first order, as
\[
\text{MSE}(\bar{y}^{(P)}_{ps}) = \left(1 - \frac{f}{n}\right) \left(A_0^2 + R^2 A_1^2 + 2R \rho_0 A_0 A_1\right) \tag{3.9}
\]
Using (3.2) and (3.9), it can be shown that the proposed class of estimators, \( \bar{y}^*_{pss} \), is more efficient than the product-type estimator, \( \bar{y}^{(P)}_{ps} \), in terms of having a smaller mean squared error, if
\[
\frac{\beta_0 (1 + \pi \alpha \lambda g)}{R} > \frac{1}{2} \tag{3.10}
\]

Note that the ratio-type and product-type estimators, \( \bar{y}^{(R)}_{ps} \) and \( \bar{y}^{(P)}_{ps} \), are both members of the family of combined-type estimators, \( \bar{y}_{pss} \), proposed by Onyeka (2012).

**d) Efficiency Comparison of \( \bar{y}^*_{pss} \) and \( \bar{y}_{pss} \)**

Here, we compare the efficiencies of the proposed estimator, \( \bar{y}^*_{pss} \), and the estimator, \( \bar{y}_{pss} \), proposed by Onyeka (2012), whose mean squared error can be rewritten from (1.11) as:
\[
\text{MSE}(\bar{y}_{pss}) = \left(1 - \frac{f}{n}\right) \left(A_0^2 + \alpha^2 \lambda^2 g^2 R^2 A_1^2 - 2\alpha \lambda g R \rho_0 A_0 A_1\right) \tag{3.11}
\]
Using (3.2) and (3.11), it can be shown that the proposed class of estimators, \( \bar{y}^*_{pss} \), is more efficient than the estimator, \( \bar{y}_{pss} \), in terms of having a smaller mean squared error, if
\[
\frac{\beta_0 (1 - \pi)}{\alpha \lambda g R} < \frac{1}{2} \tag{3.12}
\]
provided \( \alpha \neq 0 \), \( \alpha \neq 0 \) and \( g \neq 0 \), as expected. However, it is worthy of note that the estimators, \( \bar{y}^*_{pss} \) and \( \bar{y}_{pss} \), have equal efficiency under certain optimality conditions, namely,
if we choose $\alpha_{\text{opt}} = \frac{\beta_0}{\lambda g R}$ for $\bar{y}_{\text{ps}}$ and $\alpha_{\text{opt}} = \frac{\beta_0}{\pi \lambda g R}$ for $\bar{y}^*_{\text{ps}}$. Under these conditions, both estimators have the same optimum mean squared error, (1.12) and (2.11), which is easily recognized as the variance of the usual poststratified regression-type estimator, $\bar{y}_{\text{psREG}}$.

IV. Empirical Illustration

Here, we use the data given in Onyeka (2012) to illustrate the properties of the estimators proposed in the present study. The data statistics, consisting mainly of population parameters, are shown in Table 1, while Table 2 shows the percentage relative efficiencies (PRE) of the proposed class of estimators, $\bar{y}_{\text{ps}}$ and the estimator, $\bar{y}^*_{\text{ps}}$, proposed by Onyeka (2012), over the usual poststratified estimator $\bar{y}_{\text{ps}}$ of $\bar{Y}$ in poststratified sampling scheme. We shall consider special cases of the proposed estimator, $\bar{y}^*_{\text{ps}}$, corresponding to the same special cases of $\bar{y}_{\text{ps}}$ discussed in Onyeka (2012).

Table 1: Data Statistics

<table>
<thead>
<tr>
<th>POPULATION</th>
<th>MALES = STRATUM 1</th>
<th>FEMALES = STRATUM 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N = 96$</td>
<td>$N_1 = 72$</td>
<td>$N_2 = 24$</td>
</tr>
<tr>
<td>$n = 20$</td>
<td>$n_1 = 8$</td>
<td>$n_2 = 12$</td>
</tr>
<tr>
<td>$\bar{X} = 68.13$</td>
<td>$\bar{X}_1 = 68.11$</td>
<td>$\bar{X}_2 = 68.17$</td>
</tr>
<tr>
<td>$\bar{Y} = 2.44$</td>
<td>$\bar{Y}_1 = 2.44$</td>
<td>$\bar{Y}_2 = 2.46$</td>
</tr>
<tr>
<td>$S_x = 7.03$</td>
<td>$S_{x1} = 7.28$</td>
<td>$S_{x2} = 6.36$</td>
</tr>
<tr>
<td>$S_y = 49.37$</td>
<td>$S_{y1} = 52.97$</td>
<td>$S_{y2} = 40.41$</td>
</tr>
<tr>
<td>$\rho_{yx} = 0.33$</td>
<td>$\rho_{yx1} = 0.35$</td>
<td>$\rho_{yx2} = 0.25$</td>
</tr>
<tr>
<td>$\rho_{xy} = 3.26$</td>
<td>$\rho_{xy1} = 3.43$</td>
<td>$\rho_{xy2} = 2.75$</td>
</tr>
<tr>
<td>$\beta_1(x) = -1.10$</td>
<td>$\beta_{11}(x) = -1.23$</td>
<td>$\beta_{12}(x) = 0.50$</td>
</tr>
<tr>
<td>$\beta_1(y) = -0.11$</td>
<td>$\beta_{11}(y) = -0.14$</td>
<td>$\beta_{12}(y) = 0.14$</td>
</tr>
<tr>
<td>$\beta_2(x) = 3.83$</td>
<td>$\beta_{21}(x) = 4.33$</td>
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</tr>
<tr>
<td>$\beta_2(y) = 1.27$</td>
<td>$\beta_{21}(y) = 1.40$</td>
<td>$\beta_{22}(y) = 0.31$</td>
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<td>$= 0.04$</td>
<td>$\gamma_1 = 0.05$</td>
<td>$\gamma_2 = 0.16$</td>
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<td>$- -$</td>
<td>$\omega_1 = 0.75$</td>
<td>$\omega_2 = 0.25$</td>
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<tr>
<td>$- -$</td>
<td>$\omega_1^2 = 0.56$</td>
<td>$\omega_2^2 = 0.06$</td>
</tr>
</tbody>
</table>

Notes
Table 2: Pre of $\bar{y}^*$ and $\bar{y}_{pss}$ over $\bar{y}_{ps}$

<table>
<thead>
<tr>
<th>ESTIMATORS</th>
<th>Constants &amp; Parameters</th>
<th>$\bar{y}_{ps}$</th>
<th>$\bar{y}^*_{pss}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$g$</td>
<td>$a$</td>
</tr>
<tr>
<td>1. Usual poststratified estimator, $\bar{y}_{ps}$</td>
<td>–</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2. Ratio-type estimator,</td>
<td>1</td>
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<td>1</td>
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<tr>
<td>3. Sisodia-Dwivedi (1981) estimator,</td>
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<td>1</td>
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<td>4. Singh-Kakran (1993) estimator (1),</td>
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<td>1</td>
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<tr>
<td>5. Upadhyaya-Singh (1999) estimator (1),</td>
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<td>6. Upadhyaya-Singh (1999) estimator (2),</td>
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<td>1</td>
<td>$C_x$</td>
</tr>
<tr>
<td>7. Singh-Tailor (2003) estimator (1),</td>
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<td>1</td>
<td>1</td>
</tr>
<tr>
<td>8. Product-type estimator,</td>
<td>1</td>
<td>–1</td>
<td>1</td>
</tr>
<tr>
<td>9. Pandey-Dubey (1988) estimator,</td>
<td>1</td>
<td>–1</td>
<td>1</td>
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<tr>
<td>10. Upadhyaya-Singh (1999) estimator (3),</td>
<td>1</td>
<td>–1</td>
<td>$\beta_2(x)$</td>
</tr>
<tr>
<td>11. Upadhyaya-Singh (1999) estimator (4),</td>
<td>1</td>
<td>–1</td>
<td>$C_x$</td>
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<tr>
<td>12. G.N. Singh (2003) estimator (1),</td>
<td>1</td>
<td>–1</td>
<td>1</td>
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<td>13. G.N. Singh (2003) estimator (2),</td>
<td>1</td>
<td>–1</td>
<td>$\beta_1(x)$</td>
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<tr>
<td>14. G.N. Singh (2003) estimator (3),</td>
<td>1</td>
<td>–1</td>
<td>$\beta_2(x)$</td>
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<td>15. Singh-Tailor (2003) estimator (2),</td>
<td>1</td>
<td>–1</td>
<td>1</td>
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<tr>
<td>16. Singh-Kakran (1993) estimator (2),</td>
<td>1</td>
<td>–1</td>
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<td>17. Regression-type (Optimum) estimators</td>
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</table>

Table 2 shows that the estimators in the proposed class of estimators, $\bar{y}^*_{pss}$ are not always more efficient than the usual poststratified estimator $\bar{y}_{ps}$. The proposed class of estimators, $\bar{y}^*_{pss}$ is more efficient than the usual poststratified estimator $\bar{y}_{ps}$ only if the efficiency
condition (3.4) is satisfied. The table also shows that the proposed dual to ratio-type estimator, \( \hat{Y}^{(R^*)}_{ps} = \hat{Y}_{ps} \left( \frac{X^*_{ps}}{\bar{X}} \right) \) with PRE of 113.76\%, is more efficient than the usual poststratified estimator \( \hat{Y}_{ps} \), while the proposed dual to product-type estimator, \( \hat{Y}^{(P^*)}_{ps} = \hat{Y}_{ps} \left( \frac{X}{\bar{X}_{ps}} \right) \) with PRE of 79.54\%, is less efficient than the usual poststratified estimator \( \hat{Y}_{ps} \). In fact, table 2 reveals that all the dual to ratio-type estimators (for all \( g > 0 \)) perform better than the usual poststratified estimator \( \hat{Y}_{ps} \), while the dual to product-type estimators (for all \( g < 0 \)) are less efficient than the usual poststratified estimator \( \hat{Y}_{ps} \). Onyeka (2012) noted that this is expected since the given data set shows a strong positive correlation \( (\rho_{yx} = 0.82, \text{Table 1}) \), between the study and auxiliary variables. The dual to product-type estimators are expected to perform better than \( \hat{Y}_{ps} \) and the dual to ratio-type estimators when there is a strong negative correlation between the study and auxiliary variables. Using table 2 to further compare the general performance of the proposed class of estimators, \( \hat{Y}^*_{ps} \) and the estimator, \( \hat{Y}_{ps} \) proposed by Onyeka (2012), we observed that for dual to ratio-type estimators, the estimator \( \hat{Y}_{ps} \) performs better than the estimator \( \hat{Y}^*_{ps} \), while for dual to product-type estimators, the estimator \( \hat{Y}^*_{ps} \) performs better than the estimator \( \hat{Y}_{ps} \), in terms of having a smaller mean squared error. This is equally in line with the efficiency condition in (3.12). With the understanding that product-type estimators perform well when there is a strong negative correlation between the study and auxiliary variates, it therefore follows that the proposed estimator \( \hat{Y}^*_{ps} \) should be preferred to the estimator \( \hat{Y}_{ps} \), proposed by Onyeka (2012), when there is highly negative correlation between the study and auxiliary characters and we are using the dual to product-type estimators (instead of dual to ratio-type estimators) within the proposed class of combined estimators, \( \hat{Y}^*_{ps} \).

V. Concluding Remark

We have extended the work carried out by Onyeka (2012) by considering a general family of dual to ratio-type (and/or dual to product-type) combined estimators of \( \bar{Y} \), in poststratified sampling (PSS) scheme, using information on some known parameters of an auxiliary character. The proposed class of estimators is found, under some optimum conditions, to be as efficient as the poststratified regression estimator \( \hat{Y}_{ps\text{REG}} \). We also obtained conditions under which the proposed estimator performs better (in terms of having a smaller mean squared error) than the usual poststratified estimator and the estimator proposed by Onyeka (2012). Properties of the proposed general family of estimators are obtained up to first order approximation and supported with some empirical illustration.

References Références Referencias


