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Generalizations of 2D-Canonical Sine-Sine Transform

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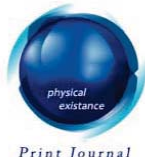
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1. Chavhan S.B. Borkar V.C., "Operator of Two Dimensional Generalized Canonical Sine Transform. IJERD.Vol.4 Issue 2. (2012). 10-14

Generalizations of 2D-Canonical Sine-Sine Transform

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Abstract - Integral transform, fractional integral transform is a flourishing field of active research due to its wide range of application. Fourier transform, fractional Fourier transform is probably the most intensively studied among all fractional transforms, similarly 2D canonical sine-sine transforms, and 2D canonical cosine-cosine is a powerful mathematical tool for processing images. In this paper the canonical 2D sine-sine transform is defined in generalized sense. And various testing functions spaces defined by using Gelfand-shilov technique. Also uniqueness theorem, modulation theorems are proved.

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I. INTRODUCTION

Integral transforms have been successfully used for almost two centuries in solving many problems in mathematical physics, applied mathematics and engineering science. Historically origin of the integral transform is P.S. Laplace and J.Fourier. Laplace transform is useful, for evaluating certain definite integral [2].

The definition of canonical sine-sine transform as follows [1].

$$\{2DCSST f(t, x)\}(s, w) = \langle f(t, x), K_{s_1}(t, x) K_{s_2}(x, w) \rangle$$

In the present paper, 2D sine-sine transform is extended in the distribution sense. The plan of the paper is as follows. The definitions are given in section 2. In section 3, testing function space is defined by Gelfand-shilov technique [3],[4]. Section 4 some results on countable union space are proved. In section 5, inversion and uniqueness theorem are stated. In section 6, modulation theorems are given. The notations and terminology are per zemanian [5],[6].

II. DEFINITION TWO DIMENSIONAL CANONICAL SINE-SINE TRANSFORM

Let $E'(R \times R)$ denote the dual of $E(R \times R)$. Therefore the generalized canonical sine transform of $f(t, x) \in E'(R \times R)$ is defined as

$$\begin{aligned} & \{2DCST f(t, x)\}(s, w) \\ &= (-1) \frac{1}{\sqrt{2\pi i b}} \frac{1}{\sqrt{2\pi i b}} e^{\frac{i(d}{b})s^2} e^{\frac{i(d}{b})w^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right) \sin\left(\frac{w}{b}x\right) e^{\frac{i(a}{2(b)}t)^2} e^{\frac{i(a}{2(b)}x)^2} f(t, x) dx dt \end{aligned}$$

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Where

$$\gamma_{E,k} \left\{ K_{s_1}(t,s) K_{s_2}(x,w) \right\} = \sup_{\substack{-\infty < t < \infty \\ -\infty < x < \infty}} \left| D_t^k D_x^l K_{s_1}(t,s) K_{s_2}(x,w) \right| < \infty$$

III. DIFFERENT S-TYPE TESTING FUNCTION SPACES

In this section we have defined s-type testing function spaces by imposing conditions not only on the decreases of the fundamental functions at infinity, but also on the growth of their derivatives as the order of derivative increases. Clearly, $SS^{a,b}$ space will be extension of testing function space D, so that these spaces have been successfully, used in pseudo differential operator theory.

a) The space $SS_\gamma^{a,b}$:

It is given by

$$SS_\gamma^{a,b} = \left\{ \phi : \phi \in E_+ / \sigma_{l,k,p} \phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \leq C_{kp} A^l l^{\gamma} \right\} \quad (3.1)$$

The constant $C_{k,p}$ and A depend on ϕ .

b) The space $SS^{a,b,\beta}$:

$SS^{a,b,\beta}$ this space is given by

$$SS^{a,b,\beta} = \left\{ \phi : \phi \in E_+ / \rho_{l,k,p} \phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \leq C_{l,p} B^k k^{\beta} \right\} \quad (3.2)$$

The constants $C_{l,p}$ and B depend on ϕ .

c) The space $SS_\gamma^{a,b,\beta}$

This space is formed by combining the condition (3.1) and (3.2)

$$SS_\gamma^{a,b,\beta} = \left\{ \phi : \phi \in E_+ / \xi_{l,k,p} \phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \leq C A^l l^{\gamma} B^k k^{\beta} \right\} \quad (3.3)$$

$l, k, p = 0, 1, 2, \dots$ Where A, B, C depend on ϕ .

d) The space $SS_{\gamma,m}^{a,b,\beta}$

It is defined as,

$$SS_{\gamma,m}^{a,b} = \left\{ \phi : \phi \in E_+ / \sigma_{l,k,p} \phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \leq C_{k,p,\mu} (m + \mu)^l l^{\gamma} \right\} \quad (3.4)$$

For any $\mu > 0$ where m is the constant, depending on the function ϕ .

e) The space $SS^{a,b,\beta,n}$

This space is given by

$$SS^{a,b,\beta,n} = \left\{ \phi : \phi \in E_+ / \rho_{l,k,p} \phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \leq C_{l,p,\delta} (n + \delta)^k k^{\beta} \right\} \quad (3.5)$$

For any $\delta > 0$ where n the constant is depends on the function ϕ .

f) The space $SS_{\gamma,m}^{a,b,\beta,n}$

This space is defined by combining the conditions in (3.4) and (3.5).

$$SS_{\gamma,m}^{a,b,\beta,n} = \left\{ \phi : \phi \in E_+ / \xi_{l,k,p} \phi(t,x) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \leq C_{\mu\delta} (m+\mu)^l (n+\delta)^k l^{\gamma} k^{\beta} \right\} \quad (3.6)$$

IV. RESULTS ON COUNTABLE UNIONS-TYPE SPACE

Proposition 4.1: If $m_1 < m_2$ then $SS_{\gamma,m_1}^{a,b} \subset SS_{\gamma,m_2}^{a,b}$. The topology of $SS_{\gamma,m_1}^{a,b}$ is equivalent to the topology induced on $SS_{\gamma,m_1}^{a,b}$ by $SS_{\gamma,m_2}^{a,b}$

$$\text{i.e. } T_{\gamma,m_1}^{a,b} \sim T_{\gamma,m_2}^{a,b} / SS_{\gamma,m_1}^{a,b}$$

Proof: For $\phi \in SS_{\gamma,m_1}^{a,b}$ and $\delta_{l,k,p}(\phi) \leq C_{k,\mu} (m_1 + \mu)^l l^{\gamma}$

$$\leq C_{k,\mu,p} (m_2 + \mu)^l l^{\gamma} \quad \text{Thus, } SS_{\gamma,m_1}^{a,b} \subset SS_{\gamma,m_2}^{a,b}$$

The space $SS_{\gamma}^{a,b}$ can be expressed as union of countable normed spaces.

Proposition 4.2: $SS_{\gamma}^{a,b} = \bigcup_{i=1}^{\infty} SS_{\gamma,m_i}^{a,b}$ and if the space $SS_{\gamma}^{a,b}$ is equipped with strict inductive limit topology $S_{a,b,m}$ defined by injective map from $SS_{\gamma,m_1}^{a,b}$ to $SS_{\gamma}^{a,b}$ then the sequence $\{\phi_n\}$ in $SS_{\gamma}^{a,b}$ converges to zero.

Proof: we show that $SS_{\gamma}^{a,b} = \bigcup_{i=1}^{\infty} SS_{\gamma,m_i}^{a,b}$

Clearly $\bigcup_{i=1}^{\infty} SS_{\gamma,m_i}^{a,b} \subset SS_{\gamma}^{a,b}$ for proving the other inclusion, let $\phi \in SS_{\gamma}^{a,b}$ then

$$\delta_{l,k,p}(\phi(t,x)) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right| \leq C_{k,p} A^l l^{\gamma}, \quad (4.1)$$

where A is some positive constant, choose an integer $m = m_A$ and $\mu = 0$ such that $C_{k,p} A^l \leq C_{k,p} (m + \mu)^l$.

Then (4.1) we get $\phi \in SS_{\gamma,m_1}^{a,b}$ implying that $SS_{\gamma}^{a,b} = \bigcup_{i=1}^{\infty} SS_{\gamma,m_i}^{a,b}$

Proposition 4.3: If $\gamma_1 < \gamma_2$ and $\beta_1 < \beta_2$ then $SS_{\gamma_1}^{a,b,\beta_1} \subset SS_{\gamma_2}^{a,b,\beta_2}$ and the topology of $SS_{\gamma_1}^{a,b,\beta_1}$ is equivalent to the topology induced on $SS_{\gamma_1}^{a,b,\beta_1}$ by $SS_{\gamma_2}^{a,b,\beta_2}$.

Proof: Let $\phi \in SS_{\gamma_1}^{a,b,\beta_1}$

$$\xi_{l,k,p}(\phi) = \sup_{I_1} \left| t^l D_t^k D_x^p \phi(t,x) \right|$$

$$\leq CA^l l^{\gamma_1} B^k k^{\beta_1}$$

$$\leq CA^l l^{\gamma_2} B^p k^{p, \beta_2} \quad \text{where } l, k, p = 0, 1, 2, 3$$

Hence $\phi \in SS_{\gamma_2}^{a,b,\beta_2}$. Consequently, $SS_{\gamma_1}^{a,b,\beta_1} \subset SS_{\gamma_2}^{a,b,\beta_2}$. The topology of $SS_{\gamma_1}^{a,b,\beta_1}$

Is equivalent to the topology $T_{\gamma_2}^{a,b,\beta_2} / SS_{\gamma_2}^{a,b,\beta_2}$

It is clear from the definition of topologies of these spaces.

Proposition 4.4: $SS^{a,b} = \bigcup_{\gamma_i, \beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i}$ and if the space $SS^{a,b}$ is equipped with the strict $SS^{a,b}$ inductive limit topology defined by the injective maps from $SS_{\gamma_i}^{a,b,\beta_i}$ to $SS^{a,b}$ then the sequence $\{\phi_n\}$ in $SS^{a,b}$ converges to zero iff $\{\phi_n\}$ is contained in some $SS_{\gamma_i}^{a,b,\beta_i}$ and converges to zero.

Proof: $SS^{a,b} = \bigcup_{\gamma_i, \beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i}$

Clearly

$$\bigcup_{\gamma_i, \beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i} \subset SS^a$$

For proving other inclusion, let $\phi(t, x) \in SS^{a,b}$ then

$$\eta_{l,k,p}(\phi) = \sup_{I_1} |t^l D_t^k D_x^p \phi(t, x)|,$$

is bounded by some number. We can choose integers γ_m and β_m such that

$$\eta_{l,k,p}(\phi) \leq CA^l l^{\gamma} B^{k,m} k^{k,\beta,m}$$

$\therefore \phi \in SS_{\gamma_i}^{a,b,\beta_i}$ for some integer γ_i and β_i

Hence $SS^{a,b} \subset \bigcup_{\gamma_i, \beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i}$ Thus $SS^{a,b} = \bigcup_{\gamma_i, \beta_i=1}^{\infty} SS_{\gamma_i}^{a,b,\beta_i}$

V. INVERSION AND UNIQUENESS THEOREMS

Theorem 5.1: (Inversion) If $\{2DCSST f(t, x)\}(s, w)$ is canonical sine-sine transform of $f(t, x)$ then inverse of transform is given by

$$f(t, x) = -\sqrt{\frac{2\pi i}{b}} \sqrt{\frac{2\pi i}{b}} e^{\frac{-i(a)}{2(b)}t^2} e^{\frac{-i(a)}{2(b)}x^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right) \sin\left(\frac{w}{b}x\right) e^{\frac{-i(d)}{2(b)}s^2} e^{\frac{-i(d)}{2(b)}w^2} \{2DCSST f(t, x)\}(s, w) ds dw$$

Theorem 5.2: (Uniqueness) If $\{2DCSST f(t, x)\}(s, w)$ and $\{2DCSST g(t, x)\}(s, w)$ are 2D canonical sine-sine transform and $\sup pf \subset s_a$ and s_b also $\sup pg \subset s_a$ and s_b

Where $s_a = \{t : t \in R^n, |t| \leq a, a > 0\}$ and $s_b = \{x : x \in R^n, |x| \leq b, b > 0\}$

If $\{2DCSST f(t, x)\}(s, w) = \{2DCSST g(t, x)\}(s, w)$

then, $f = g$ in the sense of equality in $D'(I)$

Proof: By inversion theorem

$$\begin{aligned}
 f - g &= -e^{\frac{-i}{2}\left(\frac{a}{b}\right)^2} e^{\frac{-i}{2}\left(\frac{a}{b}\right)^2} \left[\left(\sqrt{\frac{2\pi i}{b}} \sqrt{\frac{2\pi i}{b}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{-i}{2}\left(\frac{d}{b}\right)^2 s^2} e^{\frac{-i}{2}\left(\frac{d}{b}\right)^2 w^2} \sin\left(\frac{s}{b}t\right) \sin\left(\frac{w}{b}x\right) \{2DCSST f(t,x)\}(s,w) dsdw, \right. \\
 &\quad \left. - \left(\sqrt{\frac{2\pi i}{b}} \sqrt{\frac{2\pi i}{b}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{-i}{2}\left(\frac{d}{b}\right)^2 s^2} e^{\frac{-i}{2}\left(\frac{d}{b}\right)^2 w^2} \sin\left(\frac{s}{b}t\right) \sin\left(\frac{w}{b}x\right) \{2DCSST g(t,x)\}(s,w) dsdw, \right) \right] \\
 \therefore f - g &= -\sqrt{\frac{2\pi i}{b}} \sqrt{\frac{2\pi i}{b}} e^{\frac{i}{2}\left(\frac{a}{b}\right)^2} e^{\frac{-i}{2}\left(\frac{a}{b}\right)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{\frac{-i}{2}\left(\frac{d}{b}\right)^2 s^2} e^{\frac{-i}{2}\left(\frac{d}{b}\right)^2 w^2} \sin\left(\frac{s}{b}t\right) \sin\left(\frac{w}{b}x\right) \\
 &\quad \left[\{2DCSST f(t,x)\} - \{2DCSST g(t,x)\} \right] dsdw
 \end{aligned}$$

Thus $f = g$ in $D'(I)$

VI. MODULATION THEOREMS FOR CANONICAL SINE-SINE TRANSFORM

Theorem 6.1: If $\{2DCSST f(t,x)\}(s,w)$ is canonical sine-sine transform of $f(t,x)$ then

$$\begin{aligned}
 &\{2DCSST \cos \mu t f(t,x)\}(s,w) \\
 &= \frac{e^{\frac{-i}{2}(\mu^2 bd)}}{2} \left[e^{-i(s\mu d)} \{2DCSST f(t,x)\}(s+\mu b, w) + e^{i(\mu s d)} \{2DCSST f(t,x)\}(s-\mu b, w) \right]
 \end{aligned}$$

Proof: Definition of two dimensional canonical sine-sine transform $f(t,x)$ is

$$\begin{aligned}
 &\{2DCSST f(t,x)\}(s,w) \\
 &= -\frac{1}{\sqrt{2\pi i b}} \frac{1}{\sqrt{2\pi i b}} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 w^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right) \sin\left(\frac{w}{b}x\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 x^2} f(t,x) dxdt \\
 &\{2DCSST \cos \mu t f(t,x)\}(s,w) \\
 &= -\frac{1}{\sqrt{2\pi i b}} \frac{1}{\sqrt{2\pi i b}} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 w^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin\left(\frac{s}{b}t\right) \sin\left(\frac{w}{b}x\right) \cos \mu t e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 x^2} f(t,x) dxdt \\
 &= -\frac{1}{2} \left[\frac{1}{\sqrt{2\pi i b}} \frac{1}{\sqrt{2\pi i b}} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 w^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin\left(\frac{s+\mu b}{b}t\right) t \sin\left(\frac{w}{b}x\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 x^2} f(t,x) dxdt \right. \\
 &\quad \left. + \frac{1}{\sqrt{2\pi i b}} \frac{1}{\sqrt{2\pi i b}} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 s^2} e^{\frac{i}{2}\left(\frac{d}{b}\right)^2 w^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sin\left(\frac{s-\mu b}{b}t\right) t \sin\left(\frac{w}{b}x\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 t^2} e^{\frac{i}{2}\left(\frac{a}{b}\right)^2 x^2} f(t,x) dxdt \right] \\
 &= \frac{1}{2} \left[e^{-i(s\mu d)} e^{\frac{-i}{2}(\mu^2 bd)} \{2DCSST f(t,x)\}(s+\mu b, w) + e^{i(s\mu d)} e^{\frac{-i}{2}(\mu^2 bd)} \{2DCSST f(t,x)\}(s-\mu b, w) \right] \\
 &\{2DCSST \cos \mu t f(t,x)\}(s,w) \\
 &= \frac{e^{\frac{-i}{2}(\mu^2 bd)}}{2} \left[e^{-i(s\mu d)} \{2DCSST f(t,x)\}(s+\mu b, w) + e^{i(s\mu d)} \{2DCSST f(t,x)\}(s-\mu b, w) \right]
 \end{aligned}$$

Theorem 6.2 If $\{2DCSST f(t, x)\}(s, w)$ is canonical sine-sine transform of $f(t, x)$ then

$$\{2DCSST \sin \mu t f(t, x)\}(s, w) = \frac{ie^{-\frac{i}{2}(\mu^2 bd)}}{2} \left[e^{-i(s\mu d)} \{2DCCST f(t, x)\}(s + \mu b, w) - e^{i(s\mu d)} \{2DCCST f(t, x)\}(s - \mu b, w) \right]$$

Theorem 6.3 If $\{2DCSST f(t, x)\}(s, w)$ is canonical sine-sine transform of $f(t, x)$ then

$$\begin{aligned} & \{2DCSST e^{i\mu t} f(t, x)\}(s, w) \\ &= \frac{e^{-\frac{i}{2}(\mu^2 bd)}}{2} \left[e^{-i(s\mu d)} \left(\{2DCSST f(t, x)\}(s + \mu b, w) - \{2DCCST f(t, x)\}(s + \mu b, w) \right) \right. \\ & \quad \left. + e^{i(s\mu d)} \left(\{2DCSST f(t, x)\}(s - \mu b, w) + \{2DCCST f(t, x)\}(s - \mu b, w) \right) \right] \end{aligned}$$

Proof: Since $\{2DCSST e^{i\mu t} f(t, x)\}(s, w) = \{2DCSST (\cos \mu t + i \sin \mu t) f(t, x)\}(s, w)$

$$\begin{aligned} & \{2DCSST e^{i\mu t} f(t, x)\}(s, w) = \{2DCSST \cos \mu t f(t, x)\}(s, w) + i \{2DCSST \sin \mu t f(t, x)\}(s, w) \\ &= \frac{e^{-\frac{i}{2}(\mu^2 bd)}}{2} \left[e^{-i(s\mu d)} \{2DCCST f(t, x)\}(s + \mu b, w) - e^{i(s\mu d)} \{2DCCST f(t, x)\}(s - \mu b, w) \right] \\ & \quad + \frac{e^{\frac{i}{2}(\mu^2 bd)}}{2} \left[e^{-i(s\mu d)} \{2DCSST f(t, x)\}(s + \mu b, w) + e^{i(s\mu d)} \{2DCSST f(t, x)\}(s - \mu b, w) \right. \\ & \quad \left. - e^{-i(s\mu d)} \{2DCCST f(t, x)\}(s + \mu b, w) + e^{i(s\mu d)} \{2DCSCT f(t, x)\}(s - \mu b, w) \right] \\ & \{2DCSST e^{i\mu t} f(t, x)\}(s, w) = \frac{e^{-\frac{i}{2}(\mu^2 bd)}}{2} \left[e^{-i(s\mu d)} \left(\{2DCSST f(t, x)\}(s + \mu b, w) - \{2DCCST f(t, x)\}(s + \mu b, w) \right) \right. \\ & \quad \left. + e^{i(s\mu d)} \left(\{2DCSST f(t, x)\}(s - \mu b, w) + \{2DCCST f(t, x)\}(s - \mu b, w) \right) \right] \end{aligned}$$

VII. CONCLUSION

In this paper 2D canonical sine-sine transform is generalized in the distributional sense. Uniqueness theorem is proved and various testing functions specs defined by using Gelfand-shilov technique, topology properties are discussed. And lastly modulation theorems are proved.

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