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Traveling Wave Solutions of the (1+1)- Dimensional Compound KdVB Equation by Exp($-\Phi(\eta)$)-Expansion Method

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TRAVELINGWAVESOLUTIONSOFTHEDIMENSIONALCOMPOUNDKDVBEQUATIONBYEPEXPANSIONMETHOD

Strictly as per the compliance and regulations of :

Traveling Wave Solutions of the (1+1)-Dimensional Compound KdVB Equation by Exp(–Φ(η))-Expansion Method

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Abstract- In this article, we apply the exp (–Φ(η))-expansion method for seeking the exact solutions of NLEEs via the (1+1)-Dimensional Compound KdvB equation. Plentiful traveling wave solutions with arbitrary parameters are successfully obtained by this method and the wave solutions are expressed in terms of the hyperbolic, trigonometric, and rational functions. The obtained results show that exp(–Φ(η))-expansion method is very powerful and concise mathematical tool for nonlinear evolution equations in science and engineering.

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I. Introduction

Nowadays NLEEs have been the subject of all-embracing studies in various branches of nonlinear sciences. A special class of analytical solutions named traveling wave solutions for NLEEs has a lot of importance, because most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs are frequently used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity etc. Therefore investigation traveling wave solutions is becoming more and more attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the sine-cosine method [1], the extended tanh-function method [2, 3], the homogeneous balance method [4], the tanh-function method [5], the modified Exp-function method [6], the Exp-function method [7, 8], the generalized Riccati equation [9], the Jacobi elliptic function expansion method [10, 11], the Hirota's bilinear method [12], extended (G'/G) -expansion method [13], the (G'/G) –expansion method [14-18], the novel (G'/G) -expansion method [19, 20], the modified simple equation method [21, 22], the improved (G'/G) -expansion method [23], the inverse scattering transform [24], the Jacobi elliptic function expansion method [25, 26], the new generalized (G'/G) -expansion method [27-31], the exp(–Φ(η)) -expansion method [32, 33] and so on.

The objective of this article is to apply the exp(–Φ(η)) -expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the (1+1)- dimensional compound KdVB equation.

The outline of this paper is organized as follows: In Section 2, we give the description of the exp(–Φ(η)) -expansion method. In Section 3, we apply this method to the (1+1)-dimensional compound KdVB equation, graphical representation of solutions. Conclusions are given in the last section.

II. Description of the Exp (–Φ(η)) -Expansion Method

Let us consider a general nonlinear PDE in the form

$$F(v, v_t, v_x, v_{xx}, v_{tt}, v_{tx}, \ldots), \qquad (1)$$

where $v = v(x,t)$ is an unknown function, F is a polynomial in $v(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. In the following , we give the main steps of this method:

Step 1: We combine the real variables x and t by a complex variable η

$$v(x,t) = v(\eta), \quad \eta = x \pm V t, \qquad (2)$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $v = v(\eta)$:

$$\Re(v, v', v'', v''', \cdots), \qquad (3)$$

where $\Re$ is a polynomial of v and its derivatives and the superscripts indicate the ordinary derivatives with respect to η.

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Step 2. Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$v(\eta)=\sum_{i=0}^{N}A_i(\exp(-\Phi(\eta)))^{i}, \qquad (4)$$

where A_i $(0\le i\le N)$ are constants to be determined, such that $A_N\neq 0$ and $\Phi=\Phi(\eta)$ satisfies the following ordinary differential equation:

$$\Phi'(\eta)=\exp(-\Phi(\eta))+\mu\exp(\Phi(\eta))+\lambda, \qquad (5)$$

Eq. (5) gives the following solutions:

Family 1: When $\mu\neq 0,\ \lambda^2-4\mu>0,$

$$\Phi(\eta)=ln\left(\frac{-\sqrt{(\lambda^2-4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2-4\mu)}}{2}(\eta+E)\right)-\lambda}{2\mu}\right) \qquad (6)$$

Family 2 : When $\mu\neq 0,\ \lambda^2-4\mu<0,$

$$\Phi(\eta)=ln\left(\frac{\sqrt{(4\mu-\lambda^2)}\tanh\left(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\eta+E)\right)-\lambda}{2\mu}\right) \qquad (7)$$

Family 3 : When $\mu=0,\ \lambda\neq 0,$ and $\lambda^2-4\mu>0,$

$$\Phi(\eta)=-ln\left(\frac{\lambda}{\exp(\lambda(\eta+E))-1}\right) \qquad (8)$$

Family 4 : When $\mu\neq 0,\ \lambda\neq 0,$ and $\lambda^2-4\mu=0,$

$$\Phi(\eta)=ln\left(-\frac{2(\lambda(\eta+E)+2)}{\lambda^2(\eta+E)}\right) \qquad (9)$$

Family 5 : When $\mu=0,\ \lambda=0,$ and $\lambda^2-4\mu=0,$

$$\Phi(\eta)=ln(\eta+E) \qquad (10)$$

$A_N,\cdots\cdots,V,\lambda,\mu$ are constants to be determined latter, $A_N\neq 0,$ the positive integer N can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 3: We substitute Eq. (4) into Eq. (3) and then we account the function exp(−Φ(η)) . As a result of this substitution, we get a polynomial of exp(−Φ(η)) . We equate all the coefficients of same power of exp(−Φ(η)) to zero. This procedure yields a system of algebraic equations whichever can be solved to find $A_N,\cdots\cdots,V,\lambda,\mu$ Substituting the values of $A_N,\cdots\cdots,V,\lambda,\mu$ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

III. Application of the Method

In this section, we will present the exp (−Φ(η)) -expansion method to construct the exact solutions and then the solitary wave solutions of the (1+1)-dimensional compound KdVB equation. Let us consider the (1+1)-dimensional compound KdVB equation,

$$v_t+\alpha v v_x+\beta v^2 v_x+\gamma v_{xx}-\delta v_{xxx}=0 \qquad (11)$$

We utilize the traveling wave variable $v(\eta)=v(x,t),\eta=x-\omega t$ Eq. (11) is carried to an ODE

$$-Vv'+\alpha vv'+\beta v^2v'+\gamma v''-\delta v'''=0. \qquad (12)$$

Eq. (12) is integrable, therefore, integrating with respect to η once yields:

$$P-Vv+\frac{1}{2}\alpha v^2+\frac{1}{3}\beta v^3+\gamma v'-\delta v''=0, \qquad (13)$$

where P is an integration constant which is to be determined.

Taking the homogeneous balance between highest order nonlinear term 3 v and linear term of the highest order v'' in Eq. (13), we obtain N=1. Therefore, the solution of Eq. (13) is of the form:

$$v(\eta)=A_0+A_1(\exp(-\Phi(\eta))), \qquad (14)$$

where A_0, A_1 are constants to be determined such that $A_N\neq 0,$ while λ,μ are arbitrary constants.

Substituting Eq. (14) into Eq. (13) and then equating the coefficients of exp(−Φ(η)) to zero, we get

$$\frac{1}{3}\beta A_1^{\ 3} - 2\delta A_1 = 0 \tag{15}$$

$$\frac{1}{2}\alpha A_1^{\ 2} - \gamma A_1 - 3\delta A_1\lambda + \beta A_0 A_1^{\ 2} = 0 \tag{16}$$

$$\alpha A_0 A_1 - \delta A_1\lambda^2 - \gamma A_1\lambda + \beta A_0^{\ 2} A_1 - 2\delta A_1\mu - V = 0 \tag{17}$$

$$P - VA_0 - \delta A_1\mu\lambda + \frac{1}{2}\alpha A_0^{\ 2} + \frac{1}{3}\beta A_0^{\ 3} - \gamma A_1\mu = 0 \tag{18}$$

Solving the Eq. (15)-Eq. (18) yields

$$P = \frac{1}{72}\frac{1}{\beta^2}\left(-18\beta\alpha\delta^2\lambda^2 - 6\beta\alpha\gamma^2 + 72\alpha\delta^2\mu\beta + 3\alpha^3\delta\right) \pm \frac{\gamma(72\delta^2\lambda^2 + 8\gamma^2 + 288\delta^2\mu)}{72\sqrt{6\delta\beta}}$$

$$V = -\frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}$$

$$A_0 = -\frac{1}{2}\frac{\alpha - 2\gamma - 6\delta\lambda}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}}$$

$$A_1 = \pm\sqrt{6\delta/\beta}$$

where λ, μ are arbitrary constants.

Now substituting the values of V, A_o, A_1 into Eq. (14) yields

$$v(\eta) = -\frac{1}{2}\frac{\alpha}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}} \pm \sqrt{6\delta/\beta}(\exp(-\Phi(\eta))), \tag{19}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$.

Now substituting Eq. (6)- Eq. (10) into Eq. (19) respectively, we get the following five traveling wave solutions of the $(1+1)$ dimensional compound KdVB equation.

When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$v_{1,2}(\eta) = -\frac{1}{2}\frac{\alpha}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\left(\sqrt{6\delta\beta}\right)} \mp \frac{2\mu\sqrt{6\delta/\beta}}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + E)\right) + \lambda}.$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$. E is an arbitrary constant.

When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$v_{3,4}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma - 6\delta\lambda}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}} \mp 2\mu \frac{\sqrt{6\delta/\beta}}{\sqrt{(4\mu-\lambda^2)}\tanh(\frac{\sqrt{(4\mu-\lambda^2)}}{2}(\eta+E)) - \lambda}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$. E is an arbitrary constant.

When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$v_{7,8}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma - 6\delta\lambda}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}} \mp \frac{\sqrt{6\delta/\beta}\lambda^2(\eta+E)}{2(\lambda(\eta+E)+2)}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$, E is an arbitrary constant.

When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$

$$v_{9,10}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma}{\beta} \pm \frac{\gamma}{\sqrt{6\delta\beta}} \mp \frac{\sqrt{6\delta/\beta}}{In(\eta+E)}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$, E is an arbitrary constant.

When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$

$$v_{9,10}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma}{\beta} \pm \frac{\gamma}{\sqrt{6\delta\beta}} \mp \frac{\sqrt{6\delta/\beta}}{In(\eta+E)}$$

where $\eta = x + \frac{1}{12}\frac{-2\beta\delta^2 + 3\delta\alpha^2}{\beta\delta}t$, E is an arbitrary constant.

Global Journal of Science Frontier Research (A) Volume XIII Issue VIII Version I Year 2013

IV. Graphical Representation of the Solutions

The graphical illustrations of the solutions are given below in the figures with the aid of Maple.

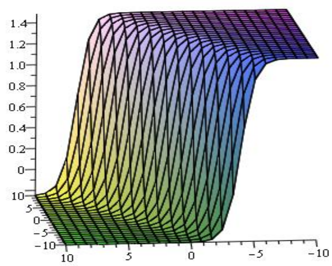

Fig. 1 : Traveling wave solution of $v_{1,2}$ with $E=2, \alpha=1, \beta=2, a=4, \gamma=3, \delta=1, \lambda=3, \mu=2$ and $-10 \le x,t \le 10$.

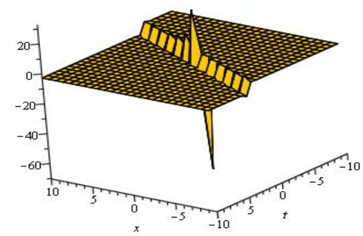

Fig. 2 : Traveling wave solution of $v_{3,4}$ with $E=2, \alpha=1, \beta=2, a=4, \gamma=3, \delta=1, \lambda=1, \mu=2$ and $-10 \le x,t \le 10$

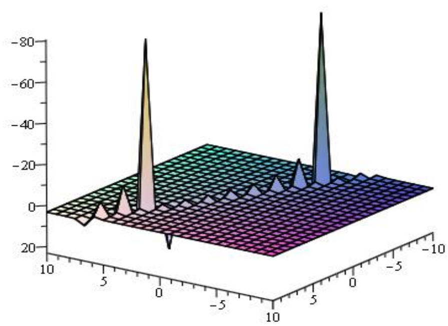

Fig. 3 : Traveling wave solution of $v_{5,6}$ with $E=2, \alpha=1, \beta=2, a=4, \gamma=3, \delta=1, \lambda=1, \mu=0$ and $-10 \le x,t \le 10$

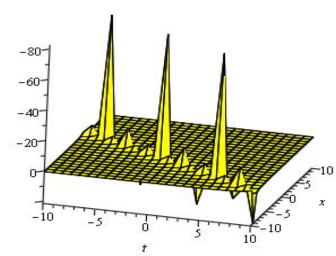

Fig. 4 : Traveling wave solution of $v_{7,8}$ with $E=2, \alpha=1, \beta=2, a=4, \gamma=3, \delta=1, \lambda=2, \mu=2$ and $-10 \le x,t \le 10$

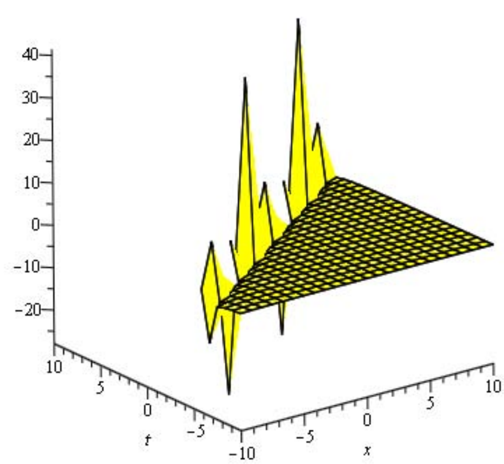

Fig. 5 : Traveling wave solution of $v_{9,10}$ with $E=2, \alpha=1, \beta=2, a=4, \delta=1, \gamma=3, \lambda=0, \mu=0$ and $-10 \le x,t \le 10$.

V. Conclusion

In this article, the exp(–Φ(η))-expansion method has been successfully applied to find new traveling wave solutions for nonlinear wave equation via the (1+1)-dimensional compound KdVB equation. We obtain some new traveling wave solutions including hyperbolic function solutions, trigonometric function solutions and rational solutions. The results show that the method is trustworthy and helpful and gives more solutions. This method can be also applied to other method.

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