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Traveling Wave Solutions of the (1+1)-Dimensional Compound KdVB Equation by Exp(–Φ(η))-Expansion Method

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Abstract- In this article, we apply the exp (–Φ(η))-expansion method for seeking the exact solutions of NLEEs via the (1+1)-Dimensional Compound KdvB equation. Plentiful traveling wave solutions with arbitrary parameters are successfully obtained by this method and the wave solutions are expressed in terms of the hyperbolic, trigonometric, and rational functions. The obtained results show that exp(–Φ(η))-expansion method is very powerful and concise mathematical tool for nonlinear evolution equations in science and engineering.

Keywords: the exp(–Φ(η)) -expansion method; complixiton soliton solutions; the (1+1)- dimensional compound kdvb equations; traveling wave solutions; nonlinear evolution equation.

I. Introduction

Nowadays NLEEs have been the subject of all-embracing studies in various branches of nonlinear sciences. A special class of analytical solutions named traveling wave solutions for NLEEs has a lot of importance, because most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs are frequently used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity etc. Therefore investigation traveling wave solutions is becoming more and more attractive in nonlinear sciences day by day. However, not all equations posed of these models are solvable. As a result, many new techniques have been successfully developed by diverse groups of mathematicians and physicists, such as the sine-cosine method [1], the extended tanh-function method [2, 3], the homogeneous balance method [4], the tanh-function method [5], the modified Exp-function method [6], the Exp-function method [7, 8], the generalized Riccati equation [9], the Jacobi elliptic function expansion method [10, 11], the Hirota's bilinear method [12], extended (G'/G) -expansion method [13], the (G'/G) –expansion method [14-18], the novel (G'/G) -expansion method [19, 20], the modified simple equation method [21, 22], the improved (G'/G) -expansion method [23], the inverse scattering transform [24], the Jacobi elliptic function expansion method [25, 26], the new generalized (G'/G) -expansion method [27-31], the exp(–Φ(η)) -expansion method [32, 33] and so on.

The objective of this article is to apply the exp(–Φ(η)) -expansion method to construct the exact solutions for nonlinear evolution equations in mathematical physics via the (1+1)- dimensional compound KdVB equation.

The outline of this paper is organized as follows: In Section 2, we give the description of the exp(–Φ(η)) -expansion method. In Section 3, we apply this method to the (1+1)-dimensional compound KdVB equation, graphical representation of solutions. Conclusions are given in the last section.

II. Description of the Exp (–Φ(η)) -Expansion Method

Let us consider a general nonlinear PDE in the form

$$F(v, v_t, v_x, v_{xx}, v_{tt}, v_{tx}, \ldots), \qquad (1)$$

where $v = v(x,t)$ is an unknown function, F is a polynomial in $v(x,t)$ and its derivatives in which highest order derivatives and nonlinear terms are involved and the subscripts stand for the partial derivatives. In the following , we give the main steps of this method:

Step 1: We combine the real variables x and t by a complex variable η

$$v(x,t) = v(\eta), \quad \eta = x \pm V t, \qquad (2)$$

where V is the speed of the traveling wave. The traveling wave transformation (2) converts Eq. (1) into an ordinary differential equation (ODE) for $v = v(\eta)$:

$$\Re(v, v', v'', v''', \cdots), \qquad (3)$$

where $\Re$ is a polynomial of v and its derivatives and the superscripts indicate the ordinary derivatives with respect to η.

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Step 2. Suppose the traveling wave solution of Eq. (3) can be expressed as follows:

$$v(\eta) = \sum_{i=0}^{N} A_i (\exp(-\Phi(\eta)))^i, \quad (4)$$

where A_i $(0 \le i \le N)$ are constants to be determined, such that $A_N \neq 0$ and $\Phi = \Phi(\eta)$ satisfies the following ordinary differential equation:

$$\Phi'(\eta) = \exp(-\Phi(\eta)) + \mu \exp(\Phi(\eta)) + \lambda, \quad (5)$$

Eq. (5) gives the following solutions:

Family 1: When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$\Phi(\eta) = ln\left(\frac{-\sqrt{(\lambda^2 - 4\mu)}\tanh\left(\frac{\sqrt{(\lambda^2 - 4\mu)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \quad (6)$$

Family 2 : When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$\Phi(\eta) = ln\left(\frac{\sqrt{(4\mu - \lambda^2)}\tanh\left(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\eta + E)\right) - \lambda}{2\mu}\right) \quad (7)$$

Family 3 : When $\mu = 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu > 0,$

$$\Phi(\eta) = -ln\left(\frac{\lambda}{\exp(\lambda(\eta + E)) - 1}\right) \quad (8)$$

Family 4 : When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$\Phi(\eta) = ln\left(-\frac{2(\lambda(\eta + E) + 2)}{\lambda^2(\eta + E)}\right) \quad (9)$$

Family 5 : When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$

$$\Phi(\eta) = ln(\eta + E) \quad (10)$$

$A_N, \cdots\cdots, V, \lambda, \mu$ are constants to be determined latter, $A_N \neq 0,$ the positive integer *N* can be determined by considering the homogeneous balance between the highest order derivatives and the nonlinear terms appearing in Eq. (3).

Step 3: We substitute Eq. (4) into Eq. (3) and then we account the function exp(–Φ(η)) . As a result of this substitution, we get a polynomial of exp(–Φ(η)) . We equate all the coefficients of same power of exp(–Φ(η)) to zero. This procedure yields a system of algebraic equations whichever can be solved to find $A_N, \cdots\cdots, V, \lambda, \mu$ Substituting the values of $A_N, \cdots\cdots, V, \lambda, \mu$ into Eq. (4) along with general solutions of Eq. (5) completes the determination of the solution of Eq. (1).

III. Application of the Method

In this section, we will present the exp (–Φ(η)) -expansion method to construct the exact solutions and then the solitary wave solutions of the (1+1)-dimensional compound KdVB equation. Let us consider the (1+1)-dimensional compound KdVB equation,

$$v_t + \alpha v v_x + \beta v^2 v_x + \gamma v_{xx} - \delta v_{xxx} = 0 \quad (11)$$

We utilize the traveling wave variable $v(\eta) = v(x,t), \eta = x - \omega t$ Eq. (11) is carried to an ODE

$$-Vv' + \alpha v v' + \beta v^2 v' + \gamma v'' - \delta v''' = 0. \quad (12)$$

Eq. (12) is integrable, therefore, integrating with respect to η once yields:

$$P - Vv + \frac{1}{2}\alpha v^2 + \frac{1}{3}\beta v^3 + \gamma v' - \delta v'' = 0, \quad (13)$$

where *P* is an integration constant which is to be determined.

Taking the homogeneous balance between highest order nonlinear term 3 *v* and linear term of the highest order v'' in Eq. (13), we obtain *N*=1. Therefore, the solution of Eq. (13) is of the form:

$$v(\eta) = A_0 + A_1(\exp(-\Phi(\eta))), \quad (14)$$

where A_0, A_1 are constants to be determined such that $A_N \neq 0,$ while λ, μ are arbitrary constants.

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Substituting Eq. (14) into Eq. (13) and then equating the coefficients of exp(–Φ(η)) to zero, we get

$$\frac{1}{3}\beta A_1^{\,3} - 2\delta A_1 = 0 \tag{15}$$

$$\frac{1}{2}\alpha A_1^{\,2} - \gamma A_1 - 3\delta A_1\lambda + \beta A_0 A_1^{\,2} = 0 \tag{16}$$

$$\alpha A_0 A_1 - \delta A_1\lambda^2 - \gamma A_1\lambda + \beta A_0^{\,2} A_1 - 2\delta A_1\mu - V = 0 \tag{17}$$

$$P - VA_0 - \delta A_1\mu\lambda + \frac{1}{2}\alpha A_0^{\,2} + \frac{1}{3}\beta A_0^{\,3} - \gamma A_1\mu = 0 \tag{18}$$

Solving the Eq. (15)-Eq. (18) yields

$$P = \frac{1}{72}\frac{1}{\beta^2}\left(-18\beta\alpha\delta^2\lambda^2 - 6\beta\alpha\gamma^2 + 72\alpha\delta^2\mu\beta + 3\alpha^3\delta\right) \pm \frac{\gamma(72\delta^2\lambda^2 + 8\gamma^2 + 288\delta^2\mu)}{72\sqrt{6\delta\beta}}$$

$$V = -\frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}$$

$$A_0 = -\frac{1}{2}\frac{\alpha - 2\gamma - 6\delta\lambda}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}}$$

$$A_1 = \pm\sqrt{6\delta/\beta}$$

where λ, μ are arbitrary constants.

Now substituting the values of V, A_o, A_1 into Eq. (14) yields

$$v(\eta) = -\frac{1}{2}\frac{\alpha}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}} \pm \sqrt{6\delta/\beta}(\exp(-\Phi(\eta))), \tag{19}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$.

Now substituting Eq. (6)- Eq. (10) into Eq. (19) respectively, we get the following five traveling wave solutions of the (1+1) dimensional compound KdVB equation.

When $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$v_{1,2}(\eta) = -\frac{1}{2}\frac{\alpha}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\left(\sqrt{6\delta\beta}\right)} \mp \frac{2\mu\sqrt{6\delta/\beta}}{\sqrt{\lambda^2 - 4\mu}\tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + E)\right) + \lambda}.$$

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where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$. E is an arbitrary constant.

When $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$v_{3,4}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma - 6\delta\lambda}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}} \mp 2\mu \frac{\sqrt{6\delta/\beta}}{\sqrt{(4\mu - \lambda^2)}\tanh(\frac{\sqrt{(4\mu - \lambda^2)}}{2}(\eta + E)) - \lambda}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$. E is an arbitrary constant.

When $\mu \neq 0,\ \lambda \neq 0,$ and $\lambda^2 - 4\mu = 0,$

$$v_{7,8}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma - 6\delta\lambda}{\beta} \pm \frac{(\gamma + 3\delta\lambda)}{\sqrt{6\delta\beta}} \mp \frac{\sqrt{6\delta/\beta}\lambda^2(\eta + E)}{2(\lambda(\eta + E) + 2)}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$, E is an arbitrary constant.

When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$

$$v_{9,10}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma}{\beta} \pm \frac{\gamma}{\sqrt{6\delta\beta}} \mp \frac{\sqrt{6\delta/\beta}}{In(\eta + E)}$$

where $\eta = x + \frac{1}{12}\frac{-6\delta^2\lambda^2\beta - 2\beta\delta^2 + 3\delta\alpha^2 + 24\delta^2\mu\beta}{\beta\delta}t$, E is an arbitrary constant.

When $\mu = 0,\ \lambda = 0,$ and $\lambda^2 - 4\mu = 0,$

$$v_{9,10}(\eta) = -\frac{1}{2}\frac{\alpha - 2\gamma}{\beta} \pm \frac{\gamma}{\sqrt{6\delta\beta}} \mp \frac{\sqrt{6\delta/\beta}}{In(\eta + E)}$$

where $\eta = x + \frac{1}{12}\frac{-2\beta\delta^2 + 3\delta\alpha^2}{\beta\delta}t$, E is an arbitrary constant.

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IV. Graphical Representation of the Solutions

The graphical illustrations of the solutions are given below in the figures with the aid of Maple.

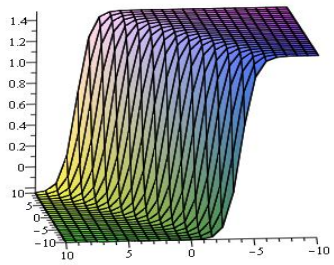

Fig. 1 : Traveling wave solution of $v_{1,2}$ with $E = 2, \alpha = 1,\ \beta = 2, a = 4, \gamma = 3, \delta = 1, \lambda = 3, \mu = 2$ and $-10 \le x, t \le 10$.

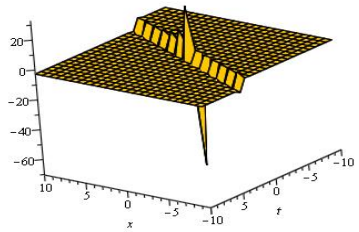

Fig. 2 : Traveling wave solution of $v_{3,4}$ with $E = 2,\ \alpha = 1,\ \beta = 2,\ a = 4,\ \gamma = 3, \delta = 1,\ \lambda = 1 \mu = 2$ and $-10 \le x, t \le 10$

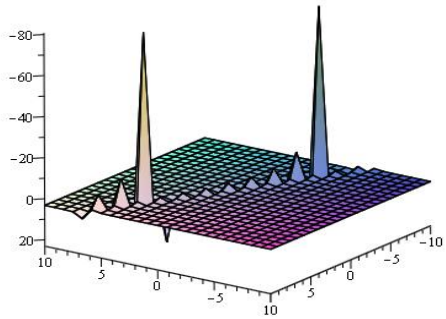

Fig. 3 : Traveling wave solution of $v_{5\,6}$ with $E = 2, \alpha = 1,\ \beta = 2, a = 4, \gamma = 3, \delta = 1, \lambda = 1, \mu = 0$ and $-10 \le x, t \le 10$

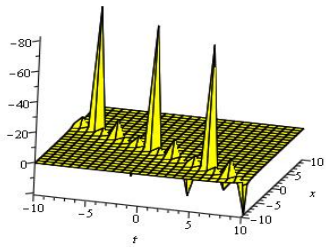

Fig. 4 : Traveling wave solution of $v_{7,8}$ with $E = 2,\ \alpha = 1,\ \beta = 2,\ a = 4,\ \gamma = 3, \delta = 1,\ \lambda = 2, \mu = 2$ and $-10 \le x, t \le 10$

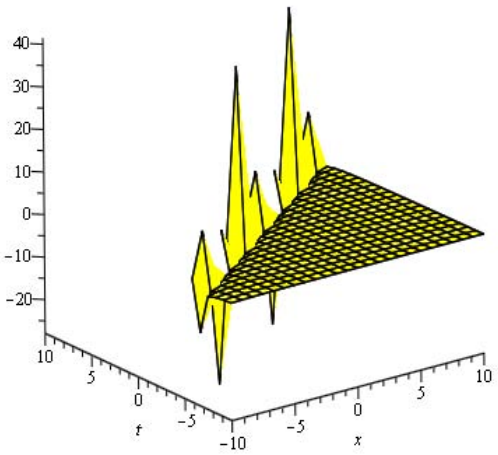

Fig. 5 : Traveling wave solution of $v_{9,10}$ with $E = 2,\ \alpha = 1, \beta = 2,\ a = 4,\ \delta = 1,\ \gamma = 3,\ \lambda = 0, \mu = 0$ and $-10 \le x, t \le 10$.

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V. Conclusion

In this article, the exp(–Φ(η))-expansion method has been successfully applied to find new traveling wave solutions for nonlinear wave equation via the (1+1)-dimensional compound KdVB equation. We obtain some new traveling wave solutions including hyperbolic function solutions, trigonometric function solutions and rational solutions. The results show that the method is trustworthy and helpful and gives more solutions. This method can be also applied to other method.

References Références Referencias

1. A.M. Wazwaz, A sine-cosine method for handle nonlinear wave equations, Applied Mathematics and Computer Modeling, 40 (2004) 499-508.
2. E. Fan, Extended tanh-function method and its applications to nonlinear equations. Phys Lett A 277(4– 5) (2000) 212-218
3. S.A. El-Wakil, M.A. Abdou, New exact travelling wave solutions using modified extended tanh-function method. Chaos Solitons Fractals 31(4) (2007) 840-852.
4. E.G. Fan, Two new applications of the homogeneous balance method. Phys. Lett. A. 265 (2000) 353-357.
5. A.M. Wazwaz, The tanh method for generalized forms of nonlinear heat conduction and Burgers–Fisher equations, Appl. Math.Comput. 169 (2005) 321–338.
6. M. Usman, A. Nazir, T. Zubair, I. Rashid, Z. Naheed S.T. Mohyud-Din, Solitary wave solutions of (3+1)-dimensional Jimbo Miwa and Pochhammer-Chree equations by modified Exp-function method, Int. J. Modern Math. Sci. 5(1) (2013) 27-36.
7. J.H. He and X.H. Wu, Exp-function method for nonlinear wave equations, Chaos, Solitons Fract. 30 (2006) 700-708.
8. M.A. Akbar and N.H.M. Ali, New Solitary and Periodic Solutions of Nonlinear Evolution Equation by Exp-function Method, World Appl. Sci. J., 17(12) (2012) 1603-1610.
9. Z. Yan and H. Zhang, New explicit solitary wave solutions and periodic wave solutions for Whitham Broer-Kaup equation in shallow water, Physics Letters. A, vol. 285 no.5-6 (2001) 355-362.
10. D. Liu, Jacobi elliptic function solutions for two variant Boussinesq equations, Chaos solitons Fractals, 24 (2005) 1373-85.
11. Y. Chen and Q. Wang, Extended Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic functions solutions to (1+1)-dimensional dispersive long wave equation, Chaos solitons Fractals, 24 (2005) 745-57.
12. A.M. Wazwaz, Multiple-soliton solutions for a (3+1)-dimensional generalized KP equation, Commun Nonlinear Sci Numer Simulat 17 (2012) 491–495.
13. H.O. Roshid, N. Rahman and M.A. Akbar, Traveling wave solutions of nonlinear Klein- Gordon equation by extended ($G\square/G$) -expansion method, Annals of pure and applied math., Vol. 3(1), 2013, 10-16.
14. M.L. Wang, X.Z. Li and J. Zhang, The (G'/G) -expansion method and traveling wave solutions of nonlinear evolution equations in mathematical physics, Phys. Lett. A, 372 (2008) 417-423.
15. A. Bekir, Application of the (G'/G) -expansion method for nonlinear evolution equations, Phys. Lett. A 372 (2008) 3400-3406.
16. H.O. Roshid, M.N. Alam, M.F. Hoque and M.A. Akbar, A new extended (G'/G) - expansion method to find exact traveling wave solutions of nonlinear evolution equations, mathematics and Statistics, Vol. 1(3) 2013 162-166. DOI: 10.13189/ms. 2013. 010308
17. S. Zhang, J. Tong and W. Wang, A generalized (G'/G) -expansion method for the mKdV equation with variable coefficients, Phys. Lett. A, 372 (2008) 2254-2257.
18. M.A. Akbar, N.H.M. Ali and E.M.E. Zayed, A generalized and improved (G'/G) - expansion method for nonlinear evolution equations, Math. Prob. Engr., Vol. 2012 (2012) 22 pages. doi: 10.1155/2012/459879.
19. M.N. Alam, M.A. Akbar and S.T. Mohyud-Din, A novel (G'/G)-expansion method and its application to the Boussinesq equation, Chin. Phys. B, vol. 23(2), 2014, 020203-020210, DOI: 10.1088/ 1674-1056/23/2/020203
20. M.N. Alam and M.A. Akbar, Traveling wave solutions of the nonlinear (1+1)- dimensional modified Benjamin-Bona-Mahony equation by using novel (G'/G) - expansion, Physical Review & Research International, 4(1), 2014, 147-165.
21. A.J.M. Jawad, M.D. Petkovic and A. Biswas, Modified simple equation method for nonlinear evolution equations, Appl. Math. Comput., 217 (2010) 869-877.
22. K. Khan, M.A. Akbar and M.N. Alam, Traveling wave solutions of the nonlinear Drinfel'd-Sokolov-Wilson equation and modified Benjamin-Bona-Mahony equations, J. Egyptian Math. Soc., 21, 2013, 233-240, http://dx.doi.org/10.1016/j.joems.2013.04.010.
23. J. Zhang, F. Jiang and X. Zhao, An improved (G'/G) -expansion method for solving nonlinear evolution equations, Int. J. Com. Math., 87(8) (2010) 1716-1725.

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24. M.J. Ablowitz and P.A. Clarkson, Soliton, nonlinear evolution equations and inverse scattering (Cambridge University Press, New York, 1991).
25. D. Liu, Jacobi elliptic function solutions for two variant Boussinesq equations, Chaos solitons Fractals, 24 (2005) 1373-85.
26. Y. Chen and Q. Wang, Extended Jacobi elliptic function rational expansion method and abundant families of Jacobi elliptic functions solutions to (1+1)-dimensional dispersive long wave equation, Chaos solitons Fractals, 24 (2005) 745-57.
27. H. Naher and F.A. Abdullah, New approach of (G'/G) -expansion method and new approach of generalized (G'/G) -expansion method for nonlinear evolution equation, AIP Advances 3, 032116 (2013) DOI: 10. 1063/1.4794947.
28. M.N. Alam, M.A. Akbar and K. Khan, Some new exact traveling wave solutions to the (2+1)-dimensional breaking soliton equation, World Applied Sciences Journal, 25(3), 2013, 500-523.
29. M.N. Alam and M.A. Akbar, Exact traveling wave solutions of the KP-BBM equation by using the new generalized (G'/G) -expansion method, Springer Plus, 2(1), 2013, 617. DOI: 10.1186/2193-1801-2-617.
30. M.N. Alam and M.A. Akbar, Traveling wave solutions of nonlinear evolution equationsvia the new generalized (G'/G) -expansion method, Universal Journal of Computational Mathematics, 1(4), 2013, 129-136 DOI: 10.13189/ujcmj.2013.010403.
31. M.N. Alam, M.A. Akbar and H.O. Roshid, Study of nonlinear evolution equations to construct traveling wave solutions via the new approach of generalized (G'/G) - expansion method, Mathematics and Statistics, 1(3), 2013, 102-112, DOI: 10.13189/ ms. 2013.010302.
32. M.M. Zhao and C. Li, The exp(–Φ(η))-expansion method applied to nonlinear evolution equations, http://www. Paper. Edu. Cn.
33. R. Islam, M.N. Alam, A.K.M.K.S. Hossain, H.O. Roshid and M.A. Akbar, traveling wave solutions of nonlinear evolution equations via exp (–Φ(η)) -expansion method, Golbal J. of sci. frontier research, Vol. 13(11), 2013, XXX-XXX.

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