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Accuracy in Collaborative Robotics: An Intuitionistic Fuzzy Multiset Approach

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Accuracy in Collaborative Robotics: An Intuitionistic Fuzzy Multiset Approach

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Abstract - In this paper, an application of Intuitionistic Fuzzy Multiset in Robotics is discussed. The basic operations on Intuitionistic Fuzzy Multisets such as union, intersection, addition, multiplication etc. are discussed. Accuracy of Collaborative Robots using the concept of Intuitionistic Fuzzy Multiset is discussed.

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I. Introduction

Modern set theory formulated by George Cantor is fundamental for the whole Mathematics. One issue associated with the notion of a set is the concept of vagueness. Mathematics requires that all mathematical notions including set must be exact. This vagueness or the representation of imperfect knowledge has been a problem for a long time for philosophers, logicians and mathematicians. However, recently it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situations like this, many tools were suggested. They include Fuzzy sets, Multi sets, Rough sets, Soft sets and many more.

Considering the uncertainty factor, Lofti Zadeh [1] introduced Fuzzy sets in 1965, in which a membership function assigns to each element of the universe of discourse, a number from the unit interval [0,1] to indicate the degree of belongingness to the set under consideration. In 1983, Krassimir. T. Atanassov [2,3] introduced the concept of Intuitionistic Fuzzy sets (IFS) by introducing a non-membership fuction together with the membership function of the fuzzy set. Among the various notions of higher-order Fuzzy sets, Intuitionistic Fuzzy sets proposed by Atanassov provide a flexible framework to explain uncertainity and vagueness. IFS reflect better the aspects of human behavior.

A human being who expresses the degree of belongingness of a given element to a set, does not often expresses the corresponding degree of non-belongingness as the complement. This psychological fact states that linguistic negation does not always coincides with logical negation. This idea of Intuitionistic fuzzy sets, which is a natural generalization of a standard Fuzzy set, seems to be useful in modelling many real life situations, like negotiation processes, psychological investigations, reasoning etc. The relation between Intuitionistic Fuzzy sets and other theories modeling imprecision can be seen in [4,5].

Many fields of modern mathematics have been emerged by violating a basic principle of a given theory only because useful structures could be defined this way. Set is a well-defined collection of distinct objects, that is, the elements of a set are pair wise

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1. L. A. Zadeh (1965) Fuzzy sets, *Inform. and Control* 8 338-353.

different. If we relax this restriction and allow repeated occurrences of any element, then we can get a mathematical structure that is known as Multisets or Bags. For example, the prime factorization of an integer n>0 is a Multiset whose elements are primes. The number 120 has the prime factorization $120 = 2^3 3^1 5^1$ which gives the Multiset {2, 2, 2, 3, 5}. A complete account of the development of multiset theory can be seen in [6,7]. As a generalization of multiset, Yager [8] introduced the concept of Fuzzy Multiset (FMS). An element of a Fuzzy Multiset can occur more than once with possibly the same or different membership values.

This paper explains how the concept of Intuitionist Fuzzy Multisets can be applied in the field of Robotics. Robots are machines which reduces human effort. Robots can be given intelligence to perform tasks that humans can and cannot do. They can be programmed for doing a task monotonously or they can work intelligently or dynamically according to the situations around them. Some of the applications of a mobile Robot include mine detection, surveillance, bomb detection, remote surgery, welding, cleaning small pipes, window panes and glass doors of buildings using snake-like Robots etc.

A Robot mainly contains: sensors, actuators and a controller. An accelerometer sensor is used for detecting shock/vibration, a temperature sensor can detect the temperature variations, an ultrasonic sensor/Infra-Red sensor/PIR sensor is used to detect obstacles, bump sensor senses a bump (collision), cliff sensor senses the presence of a cliff and so on. With the help of these sensors a Robot moves easily through its programmed path. iRobot Create is one such mobile robot as shown in Figure 1.

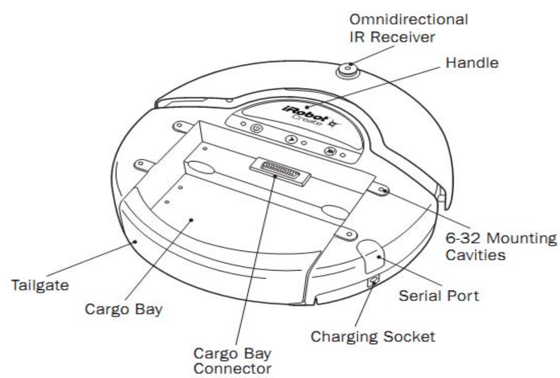

Figure 1 : iRobot Create.

When multiple Robots are used for completing a task, the system is called a multi-Robot system. Task accomplished by multiple Robots saves time and cost. To explain the concept of IFMS, a multi Robot scenario is considered consisting of a central server and a group of mobile Robots patrolling a given area for surveillance application.

Using the distance function, the sensor readings were properly interpreted for proper identification of the problem faced by the Robot.

II. Preliminaries

2.1 Definition [1] Let X be a nonempty set. A *Fuzzy set A* drawn from X is defined as $A = \{< x : \mu_A(x) > : x \in X\}$. Where : $X \to [0,1]$ is the membership function of the Fuzzy Set A.

2.2. Definition [8] Let X be a nonempty set. A *Fuzzy Multiset* (FMS) A drawn from X is characterized by a function, 'count membership' of A denoted by CM_A such that $CM_A : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval

Notes

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[0,1]. Then for any $x \in X$, the value $CM_A(x)$ is a crisp multiset drawn from [0,1]. For each $x \in X$, the membership sequence is defined as the decreasingly ordered sequence of elements in $CM_A(x)$. It is denoted by $(\mu^1_A(x), \mu^2_A(x), \ldots, \mu^P_A(x))$ where $\mu^1_A(x) \geq \mu^2_A(x) \geq, \ldots, \geq \mu^P_A(x)$.

A complete account of the applications of Fuzzy Multisets in various fields can be seen in [9].

2.3 Definition [3] Let X be a nonempty set. An *Intuitionistic Fuzzy Set* (IFS) A is an object having the form $A = \{< x : \mu_A(x), \nu_A(x) > : x \epsilon X\}$, where the functions $\mu_A: X \to [0,1]$ and $\nu_A: X \to [0,1]$ define respectively the degree of membership and the degree of non membership of the element $x \in X$ to the set A with $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \epsilon X$.

2.4 Remark Every Fuzzy set A on a nonempty set X is obviously an IFS having the form

$$A = \{ < x : \mu_A(x), 1 - \mu_A(x) > : x \epsilon X \}$$

Using the definition of FMS and IFS, a new generalized concept called Intuitionistic Fuzzy Multiset (IFMS) is defined in [10].

III. Intuitionistic Fuzzy Multiset

3.1 Definition Let X be a nonempty set. An *Intuitionistic Fuzzy Multiset* A denoted by IFMS drawn from X is characterized by two functions: 'count membership' of A (CM_A) and 'count non membership' of A (CN_A) given respectively by $CM_A : X \to Q$ and $CN_A : X \to Q$ where Q is the set of all crisp multisets drawn from the unit interval [0, 1] such that for each $x \epsilon X$, the membership sequence is defined as a decreasingly ordered sequence of elements in $CMA(x)$ which is denoted by $(\mu^1_A(x), \mu^2_A(x), \ldots, \mu^P_A(x))$ where $(\mu^1_A(x) \geq \mu^2_A(x) \geq, \ldots \geq, \mu^P_A(x)$ and the corresponding non membership sequence will be denoted by $(\nu^1_A(x), \nu^2_A(x), \ldots, \nu^P_A(x))$ such that $0 \leq \mu^i_A(x) + \nu^i_A(x) \leq 1$ for every $x \epsilon X$ and $i = 1,2,\ldots,p$.

An IFMS A is denoted by

$$A = \{ < x: (\mu^1_A(x), \mu^2_A(x), \ldots, \mu^P_A(x)), (\nu^1_A(x), \nu^2_A(x), \ldots, \nu^P_A(x)) > : x \epsilon X \}$$

3.2. Remark We arrange the membership sequence in decreasing order but the corresponding non membership sequence may not be in decreasing or increasing order.

3.3. Definition Length of an element x in an IFMS A is defined as the Cardinality of $CM_A(x)$ or $CN_A(x)$ for which $0 \leq \mu^j_A(x) + \nu^j_A(x) \leq 1$ and it is denoted by $L(x: A)$. That is

$$L(x:A) = |CM_A(x)| = |CN_A(x)|$$

3.4 Definition If A and B are IFMSs drawn from X then $L(x:A,B) = Max\{L(x : A), L(x : B)\}$. Alternatively we use $L(x)$ for $L(x : A, B)$.

33.5.Example Consider the set $X = \{x, y, x, w\}$ with A= {< x : (0.3, 0.2), (0.4, 0.5) >, < y : (1,0.5,0.5), (0.0.5,0.2) >, < z : (0.5, 0.4, 0.3, 0.2), (0.4, 0.6, 0.6, 0.7)>}, B= {< x : (0.4), (0.2) >, < y : (1,0.3, 0.2), (0,0.4, 0.5) >, < w : (0.2, 0.1), (0.7, 0.8) >}.

Now we define basic operations on IFMS. Note that we can make L(x : A) = L(x : B) by appending sufficient number of 0's and 1's with the membership and non membership values respectively.

3.6 Definition Let A and B be two IFMS. The distance function is defined as

$$d(A,B) = \left(\frac{1}{2}\sum_i\left((\mu^i_A(x) - \mu^i_B(x))^2 + (\nu^i_A(x) - \nu^i_B(x))^2 + (\Pi^i_A(x) - \Pi^i_B(x))^2\right)\right)^{\frac{1}{2}}$$

where $\Pi^i_A = 1 - {}^{\backprime i}_{A}(x) - \nu^i_A(x)$ called the IFMS index or hesitation margin.

3.7 Definition For any two IFMSs A and B drawn from a set X, the following operations and relations will hold. Let $A = \{< x : (\mu^1_A(x), \mu^2_A(x),\ldots,\mu^P_A(x)), (\nu^1_A(x), \nu^2_A(x),\ldots,\nu^P_A(x)) > : x \in X\}$ and $B = \{< x : (\mu^1_B(x), \mu^2_B(x),\ldots, \mu^P_B(x)), (\nu^1_B(x), \nu^2_B(x),\ldots,\nu^P_B(x)) > : x \in X\}$ then

1. Inclusion

$A \subset B \Leftrightarrow \mu^j_A(x) \leq \mu^j_A(x)$ and $\nu^j_A(x) \geq \nu^j_B(x)$;

$j = 1,2,\ldots, L(x), x \in X$

$A = B \Leftrightarrow A \subset B$ and $B \subset A$

2. Complement

$\neg A = \{< x : (\nu^1_A(x),\ldots,\nu^P_A(x)), (\mu^1_A(x),\ldots\ldots\ldots, \mu^P_A(x)) > : x \in X\}$

3. Union $(A \cup B)$

In $A \cup B$ the membership and non membership values are obtained as follows.

$\mu^j_{A\cup B}(x) = \mu^j_A(x) \vee \mu^j_B(x)$

$\nu^j_{A\cup B}(x) = \nu^j_A(x) \wedge \nu^j_B(x)$

$j = 1, 2,\ldots,L(x), x \in X.$

4. Intersection $(A \cap B)$

In $A \cap B$ the membership and non membership values are obtained as follows.

$\mu^j_{A\cap B}(x) = \mu^j_A(x) \wedge \mu^j_B(x)$

$\nu^j_{A\cap B}(x) = \nu^j_A(x) \vee \nu^j_B(x)$

$j = 1, 2,\ldots,L(x), x \in X.$

5. Addition $(A \oplus B)$

In $A \oplus B$ the membership and non membership values are obtained as follows.

$\mu^j_{A\oplus B}(x) = \mu^j_A(x) + \mu^j_B(x) - \mu^j_A(x).\ \mu^j_B(x)$

$\nu^j_{A\oplus B}(x) = \nu^j_A(x).\ \nu^j_B(x)$

Notes

$j = 1, 2,...,L(x), x \in X.$

6. Multiplication (A ⊗ B)

In $A \otimes B$ the membership and nonmembership values are obtained as follows.

$$\mu^j_{A\otimes B}(x) = \mu^j_A(x).\ \mu^j_B(x)$$

$$\nu^j_{A\otimes B}(x) = \nu^j_A(x) + \nu^j_B(x) - \nu^j_A(x).\ \nu^j_B(x)$$

$j = 1, 2,...,L(x), x \in X.$

here ∨, ∧ , . , +, - denotes maximum, minimum, multiplication, addition, subtraction of real numbers respectively.

IV. IFMS Theory for Multi Robot System

Most of human reasoning involves the use of variables whose values are fuzzy sets. This is the basis for the concept of a linguistic variable, that is, a variable whose values are words rather than numbers. But in some situations like decision making problems (such as Medical diagnosis, Sales analysis, Marketing etc.) the description by a linguistic variable in terms of membership function only is not adequate. There is chance of existing a non-null complement. IFS can be used in this context as a proper tool for representing both membership and non-membership of an element to a set. Such situations are explained in [11]. But there are situations that each element has different membership values. In such situations IFMS is more adequate. Here we present IFMS as a tool for reasoning such a situation.

An example of a multi Robot system is presented. The multi Robot system [12] considered consists of a central controller and four patrolling Robots in a large area. The total area is divided into four equal parts and assigned to each Robot. The Robot patrols in its assigned area. Each Robot is equipped with ultrasonic sensor, accelerometer sensor, cliff sensor, bump sensor and temperature sensor and is wirelessly controlled by the controller. The controller makes decisions depending upon the sensor readings. For example, if the cliff sensor value in Robot1 indicates the presence of a cliff, the controller can change the commands that are sent to the Robot1; that is, the controller can direct the Robot1 towards the right, left or backward directions. Similar is the case with every other sensor reading.

Let R = {R1, R2, R3, R4} be a set of four Robots, C = {Fire, Obstacle, Bump, Cliff, Vibration} be a set of situations or conditions and S = {Temperature sensor, Ultrasonic sensor, Bump sensor, Cliff sensor, Accelerometer sensor} be a set of sensors deployed on each Robot. A single Robot can be assigned different membership and non membership values for the five different sensor readings. This is where IFMS comes into picture.

Ref

11. De S.K., Biswas R. and Roy A.R. (2001) An Application of Intuitionistic fuzzy sets in medical diagnosis. *Fuzzy Sets and Systems*, Vol. 117, No. 2, pp. 209-213.

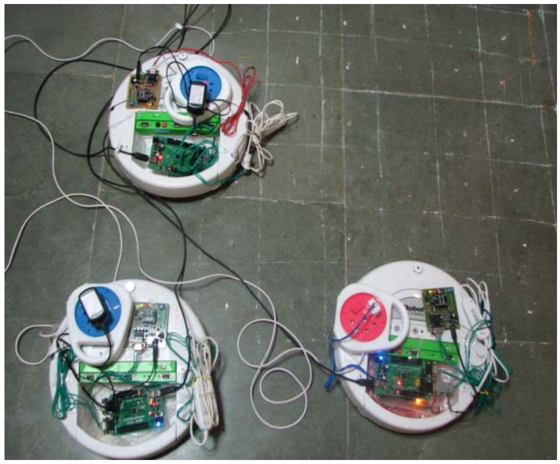

Figure 2 : A multi robot system with three patrolling robots

Whether from a single reading can we conclude what are the situations faced by the Robots? The sensor readings from the Robots have to be monitored for a particular time, say for three minutes. If for example, the ultrasonic sensor in Robot1 indicates an obstacle, it sends a message to the controller so that the corrective measure could be taken. The controller has to make sure whether the Robot1 is really faced with an obstacle or not. For that purpose, the controller monitors the ultrasonic sensor reading for three minutes. Depending upon the consistency of the readings, the controller identifies the situation.

To understand IFMS theory, let us consider the situation where the Robot1 faces an obstacle, Robot2 experiences a shock/vibration, Robot3 faces a bump and Robot4 detects a cliff. Thus whenever the ultrasonic sensor detects an obstacle and the accelerometer sensor detects a vibration, alert is sent to the controller and the controller monitors the situation for 3 minutes.

In Table-I each sensor reading is described by three numbers: Membership μ, non-membership ν and hesitation margin Π.

Table II shows the Robots and the corresponding membership functions to the sensor values.

Table 1

	Fire	Obstacle	Bump	Cliff	Shock/Vibration
Temperature sensor	(0.8,0.,1,0.1)	(0.2,0.7,0.1)	(0.1,0.7,0.2)	(0.2,0.5,0.3)	(0.5,0.2,0.3)
Ultrasonic sensor	(0.2,0.,7,0.1)	(0.8,0.1,0.1)	(0.6,0.3,0.1)	(0.2,0.,7,0.1)	(0.1,0.7,0.2)
Bump sensor	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.9,0.1,0.0)	(0.1,0.7,0.2)	(0.2,0.5,0.3)
Cliff sensor	(0.2,0.5,0.3)	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.7,0.1,0.2)	(0.1,0.7,0.2)
Accelerometer sensor	(0.1,0.7,0.2)	(0.2,0.5,0.3)	(0.1,0.7,0.2)	(0.1,0.7,0.2)	(0.8,0.2,0.0)

The objective is to make a proper decision for each Robot. Hence the readings are monitored for a particular interval time (3 minutes).

Notes

Notes

Table II

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Accelerometer sensor
R_1	(0.8,0.1, 0.1)	(0.8, 0.1, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.8,0.0)	(0.3, 0.6, 0.1)
R_2	(0.4, 0.5, 0.1)	(0.3, 0.7,0.0)	(0.1, 0.7, 0.2)	(0.2, 0.6, 0.2)	(0.8, 0.1, 0.1)
R_3	(0.1, 0.8, 0.1)	(0.6, 0.4,0.0)	(0.8, 0.1, 0.1)	(0.1, 0.9, 0.0)	(0.2, 0.7, 0.1)
R_4	(0.1, 0.7, 0.2)	(0.3, 0.6, 0.1)	(0.2, 0.7, 0.1)	(0.7, 0.2, 0.1)	(0.1, 0.7, 0.2)

Table III shows the sensor readings monitored for 3 minutes, one reading per minute. Table IV shows the distances of each Robot to the situation considered. Thus, using this distance function, IFMS theory is able to make out the correct situation of each Robot.

Table III

	Temperature sensor	Ultrasonic sensor	Bump sensor	Cliff sensor	Accelerometer sensor
R_1	(0.8,0.7, 0.9) (0.1, 0.2, 0.0) (0.1, 0.1, 0.1)	(0.8, 0.8, 0.9) (0.1, 0.1, 0.1) (0.1, 0.1, 0.0)	(0.1, 0.2, 0.0) (0.9, 0.7, 0.8) (0.0, 0.1, 0.2)	(0.2, 0.1, 0.0) (0.8, 0.6, 0.7) (0.0, 0.3, 0.3)	(0.3, 0.3, 0.4) (0.6, 0.4, 0.4) (0.1, 0.3, 0.2)
R_2	(0.4, 0.3, 0.3) (0.5, 0.4, 0.6) (0.1, 0.3, 0.1)	(0.3, 0.2, 0.3) (0.7, 0.6, 0.1) (0, 0.2, 0.7)	(0.1, 0.2, 0.4) (0.7, 0.6, 0.4) (0.2, 0.2, 0.2)	(0.2, 0.5, 0.2) (0.6, 0.4, 0.7) (0.2, 0.1, 0.1)	(0.8, 0.7, 0.6) (0.1, 0.2, 0.3) (0.1, 0.1, 0.1)
R_3	(0.1, 0.2, 0.1) (0.8, 0.6, 0.9) (0.1, 0.2, 0.0)	(0.6, 0.2, 0.1) (0.4, 0.0, 0.7) (0, 0.8, 0.2)	(0.8, 0.7, 0.8) (0.1, 0.1, 0.1) (0.1, 0.2, 0.1)	(0.1, 0.2, 0.2) (0.9, 0.7, 0.6) (0.0, 0.1, 0.2)	(0.2, 0.3, 0.2) (0.7, 0.7, 0.7) (0.1, 0.0, 0.1)
R_4	(0.1, 0.4, 0.5) (0.7, 0.4, 0.3) (0.2, 0.2, 0.2)	(0.3, 0.3, 0.4) (0.6, 0.3, 0.5) (0.1, 0.4, 0.1)	(0.2, 0.1, 0.0) (0.7, 0.6, 0.7) (0.1, 0.3, 0.3)	(0.8, 0.6, 0.9) (0.2, 0.3, 0.0) (0, 0.1, 0.1)	(0.1, 0.5, 0.4) (0.7, 0.4, 0.3) (0.2, 0.1, 0.3)

Table IV

	Fire	Obstacle	Bump	Cliff	Shock/Vibration
R_1	0.72	**0.65**	1.07	1.06	0.90
R_2	0.84	0.79	0.97	0.83	**0.52**
R_3	1.07	0.89	**0.50**	1.03	1.02
R_4	0.79	0.84	1.07	**0.46**	0.87

In the above table the lowest distance point gives the accuracy of the Robot. Robot $\mathbf{R_1}$ is near an obstacle, R2 experiences a vibration, R3 is bumped and R4 is near a cliff.

V. Conclusions

In this paper, we have discussed the various basic operations of Intuitionistic Fuzzy Multiset and its application in Robotics. In the proposed method, we measured the distances of each Robot from each situation by considering the sensor readings. The concept of multiness is incorporated by taking the samples from the same Robot for a particular time.

VI. Acknowledgements

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