Global Journal of Science Frontier Research
Mathematics and Decision Sciences
Volume 13 Issue 1 Version 1.0 Year 2013
Type : Double Blind Peer Reviewed International Research Journal
Publisher: Global Journals Inc. (USA)
Online ISSN: 2249-4626 & Print ISSN: 0975-5896

Certain New Subclasses of Analytic Univalent Functions in the Unit Disk

By B. Srutha Keerthi & M. Revathi
Sri Venkateswara College of Engineering Sriperumbudur

Abstract - Introduction of new subclasses $K(\omega,\alpha,\lambda)$ and $H(\omega,\alpha,\lambda)$ of analytic functions with negative co-efficents defined in the unit disk. We estimate some properties of functions in these classes and obtain Co-efficient bounds, Neighbourhood and Integral means inequality for the function f(z) under these classes.

Keywords : *convex, coefficient estimate, neighbourhood and integral means inequality.*

GJSFR-F Classification : *30C45*

Strictly as per the compliance and regulations of :

Notes

Certain New Subclasses of Analytic Univalent Functions in the Unit Disk

B. Srutha Keerthi [α] & M. Revathi [σ]

Abstract - Introduction of new subclasses $K(\omega,\alpha,\lambda)$ and $H(\omega,\alpha,\lambda)$ of analytic functions with negative co-efficents defined in the unit disk. We estimate some properties of functions in these classes and obtain Co-efficient bounds, Neighbourhood and Integral means inequality for the function f(z) under these classes.

Keywords : *convex, coefficient estimate, neighbourhood and integral means inequality.*

I. Introduction and Definitions

Let A denote the class of functions $f(z)$ of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which are analytic in the unit disc $E = \{z : |z| < 1\}$.Let S denote the subclass of A consisting of univalent functions f(z) in E.

Let $A(\omega) \subset A$ denote the class of functions of the form

$$f(z) = (z-\omega) + \sum_{k=2}^{\infty} a_k (z-\omega)^k \tag{1.2}$$

which are analytic in the unit disk $E = \{z : |z| < 1\}$ and normalised with $f(\omega) = 0$ and $f'(\omega) - 1 = 0$ and ω is a fixed point in E.

Let $B(\omega,\alpha,\lambda)$ and $Q(\omega,\alpha,\lambda)$ denote the class of analytic function of order α in E defined as follows:

$S(\omega) = \{f \in A(\omega) : f \text{ is univalent in E}\}$

$$B(\omega,\alpha,\lambda) = \left\{ f \in S(\omega) : Re\left(\frac{\lambda(z-\omega)^2 f''(z) + (z-\omega)f'(z)}{(1-\lambda)f(z) + \lambda(z-\omega)f'(z)}\right) > \alpha \,, 0 \le \lambda \le 1, 0 \le \alpha < 1, z \in E \right\}$$

Author α : Department of Applied Mathematics Sri Venkateswara College of Engineering Sriperumbudur, Chennai - 602105, India. E-mail : sruthilaya06@yahoo.co.in
Author σ : Department of Mathematics Aalim Muhammed Salegh College of Engineering "Nizara Educational Campus" Muthapudupet, Avadi IAF, Chennai - 600 055. E-mail : revathi.joe@gmail.com

$$Q(\omega,\alpha,\lambda)=\left\{f\in S(\omega):Re\left(\frac{\lambda(z-\omega)^3f'''(z)+(1+2\lambda)(z-\omega)^2f''(z)+(z-\omega)f'(z)}{\lambda(z-\omega)^2f''(z)+(z-\omega)f'(z)}\right)>\alpha\ ,\right.$$

$$\left.0\le\lambda\le 1, 0\le\alpha<1, z\in E\right\}$$

Let $T(\omega)$ denote subclass of $S(\omega)$ whose elements can be expressed in the form

$$f(z)=(z-\omega)-\sum_{k=2}^{\infty}a_k(z-\omega)^k. \tag{1.3}$$

Notes

Here we denote by $H(\omega,\alpha,\lambda)$ and $K(\omega,\alpha,\lambda)$ respectively the subfamilies of $B(\omega,\alpha,\lambda)$ and $Q(\omega,\alpha,\lambda)$ obtained by taking intersection of $B(\omega,\alpha,\lambda)$ and $Q(\omega,\alpha,\lambda)$ with $T(\omega)$ that is,

$$H(\omega,\alpha,\lambda)=B(\omega,\alpha,\lambda)\cap T(\omega),$$

and

$$K(\omega,\alpha,\lambda)=Q(\omega,\alpha,\lambda)\cap T(\omega).$$

Also let $P(\omega)\subset P$(class of Cartheodory functions) denote the class of functions of the form

$$P_\omega(z)=1+\sum_{k=1}^{\infty}B_k(z-\omega)^k \tag{1.4}$$

that are regular in E and satisfy $P_\omega(\omega)=1$, $ReP_\omega(z)>0$ for $z\in E$ and ω is a fixed point in E and

$$|B_k|\le\frac{2}{(1+d)(1-d)^k}\ ,\quad k\ge 1,\quad d=|\omega|.$$

II. Co-efficient Estimates

For our main results we first derive the following:

Lemma2.1. A function $f(z)\in T(\omega)$ is in the class $H(\omega,\alpha,\lambda)$ if and only if

$$\sum_{k=2}^{\infty}(k-\alpha)(1-d)^{k-1}(1-\lambda+\lambda k)a_k\le 1-\alpha. \tag{2.1}$$

The result is sharp.

Proof. Assume that the inequality (2.1) holds and let $|z-\omega|=1-d<1$ then we have

$$\left|\frac{\lambda(z-\omega)^2f''(z)+(z-\omega)f'(z)}{(1-\lambda)f(z)+\lambda(z-\omega)f'(z)}-1\right|=\left|\frac{-\sum_{k=2}^{\infty}(k-1)a_k(z-\omega)^{k-1}(1-\lambda+\lambda k)}{1-\sum_{k=2}^{\infty}a_k(z-\omega)^{k-1}(1-\lambda+\lambda k)}\right|$$

$$\le\frac{\sum_{k=2}^{\infty}(k-1)a_k(1-d)^{k-1}(1-\lambda+\lambda k)}{1-\sum_{k=2}^{\infty}a_k(1-d)^{k-1}(1-\lambda+\lambda k)}$$

$$\le 1-\alpha.$$

Notes

This shows that the values of $\frac{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{(1-\lambda)f(z)+\lambda(z-\omega)f'(z)}$ lie in the circle centered at $\gamma = 1$ whose radius is $1-\alpha$. Hence $f(z)$ is in the class $H(\omega,\alpha,\lambda)$. Then

$$Re\left(\frac{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{(1-\lambda)f(z)+\lambda(z-\omega)f'(z)}\right) = Re\left(\frac{1-\sum_{k=2}^{\infty} k(1-\lambda+\lambda k)a_k(z-\omega)^{k-1}}{1-\sum_{k=2}^{\infty}(1-\lambda+\lambda k)a_k(z-\omega)^{k-1}}\right)$$
$$> \alpha. \tag{2.2}$$

for $z \in E$ and ω is a fixed point in E. Choose values of z on the real axis so that $\frac{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{(1-\lambda)f(z)+\lambda(z-\omega)f'(z)}$ is real.Upon clearing the denominator and letting $z \to 1^-$ through real values, we have

$$\alpha\left(1-\sum_{k=2}^{\infty} a_k(1-d)^{k-1}(1-\lambda+\lambda k)\right) \le 1-\sum_{k=2}^{\infty} k(1-\lambda+\lambda k)a_k(1-d)^{k-1} \tag{2.3}$$

which is the required result.

Finally, we note that the assertion (2.1) of lemma 2.1 is sharp, with the external function

$$f(z) = (z-\omega) - \frac{1-\alpha}{(k-\alpha)(1-\lambda+\lambda k)(1-d)^{k-1}}(z-\omega)^k. \tag{2.4}$$

Corollary 2.2. Let $f(z) \in T(\omega)$ be in the class $H(\omega,\alpha,\lambda)$. Then we have

$$a_k \le \frac{1-\alpha}{(k-\alpha)(1-\lambda+\lambda k)(1-d)^{k-1}}. \tag{2.5}$$

Equality in (2.5) holds true for the function $f(z)$ given by (2.4).

Lemma 2.3. A function $f(z) \in T(\omega)$ is in the class $K(\omega,\alpha,\lambda)$ if and only if

$$\sum_{k=2}^{\infty} k(k-\alpha)(1-d)^{k-1}(1-\lambda+\lambda k)a_k \le 1-\alpha. \tag{2.6}$$

The result is sharp.

Proof. Assume that the inequality (2.1) holds and let $|z-\omega| = 1-d < 1$ then we have

$$\left|\frac{\lambda(z-\omega)^3 f'''(z)+(1+2\lambda)(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)} - 1\right|$$

$$= \left| \frac{-\sum_{k=2}^{\infty} k(k-1)a_k(z-\omega)^{k-1}(1-\lambda+\lambda k)}{1-\sum_{k=2}^{\infty} ka_k(z-\omega)^{k-1}(1-\lambda+\lambda k)} \right|$$

$$\leq \frac{\sum_{k=2}^{\infty} k(k-1)a_k(1-d)^{k-1}(1-\lambda+\lambda k)}{1-\sum_{k=2}^{\infty} ka_k(1-d)^{k-1}(1-\lambda+\lambda k)} \leq 1-\alpha.$$

This shows that the values of $\frac{\lambda(z-\omega)^3 f'''(z)+(1+2\lambda)(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)}$ lie in the circle centered at $\gamma = 1$ whose radius is $1-\alpha$. Hence $f(z)$ is in the class $K(\omega, \alpha, \lambda)$. Then

$$Re\left(\frac{\lambda(z-\omega)^3 f'''(z)+(1+2\lambda)(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)}\right)$$

$$= Re\left(\frac{1-\sum_{k=2}^{\infty} k^2(1-\lambda+\lambda k)a_k(z-\omega)^{k-1}}{1-\sum_{k=2}^{\infty} k(1-\lambda+\lambda k)a_k(z-\omega)^{k-1}}\right) > \alpha \qquad (2.7)$$

for $z \in E$ and ω is a fixed point in E. Choose values of z on the real axis so that $\frac{\lambda(z-\omega)^3 f'''(z)+(1+2\lambda)(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)}$ is real.Upon clearing the denominator and letting $z \to 1^-$ through real values, we have

$$\alpha\left(1-\sum_{k=2}^{\infty} ka_k(1-d)^{k-1}(1-\lambda+\lambda k)\right) \leq 1-\sum_{k=2}^{\infty} k^2(1-\lambda+\lambda k)a_k(1-d)^{k-1} \qquad (2.8)$$

which is the required result.

Finally, we note that the assertion (2.6) of Lemma 2.3 is sharp, with the external function

$$f(z) = (z-\omega) - \frac{1-\alpha}{k(k-\alpha)(1-\lambda+\lambda k)(1-d)^{k-1}}(z-\omega)^k. \qquad (2.9)$$

Corollary 2.4. Let $f(z) \in T(\omega)$ be in the class $K(\omega, \alpha, \lambda)$. Then we have

$$a_k \leq \frac{1-\alpha}{k(k-\alpha)(1-\lambda+\lambda k)(1-d)^{k-1}}. \qquad (2.10)$$

Equality in (2.10) holds true for the function $f(z)$ given by (2.9).

Theorem 2.5. Let $f(z) \in H(\omega, \alpha, \lambda)$ and $f(z) = (z-\omega) - a_2(z-\omega)^2 - \cdots$ for $0 \leq \alpha < 1$, and ω is a fixed point in E. Then

$$|a_2| \leq \frac{-2(1-\alpha)}{(1+\lambda)(1-d^2)}, \qquad (2.11)$$

Notes

Global Journal of Science Frontier Research (F) Volume XIII Issue I Version I Year 2013

$$|a_3| \le \frac{-1}{1+2\lambda}\left[\frac{1-\alpha}{(1-d)(1-d^2)} + \frac{2(1-\alpha)^2}{(1-d^2)^2}\right],$$

$$|a_4| \le \frac{-1}{1+3\lambda}\left[\frac{2(1-\alpha)}{3(1+d)(1-d)^3} + \frac{2(1-\alpha)^2}{(1-d)(1-d^2)^2} + \frac{4(1-\alpha)^3}{3(1-d^2)^3}\right].$$

Notes

Proof. Let us define

$$\frac{\lambda(z-\omega)^2 f''(z) + (z-\omega)f'(z)}{(1-\lambda)f(z) + \lambda(z-\omega)f'(z)} = \alpha + (1-\alpha)P_\omega(z). \tag{2.12}$$

That is,

$$\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z) = [(1-\lambda)f(z)+\lambda(z-\omega)f'(z)][\alpha+(1-\alpha)\sum_{k=1}^{\infty} B_k(z-\omega)^k]. \tag{2.13}$$

On comparing the coefficient in(2.13)the result follow.

We define the δ-neighbourhood of the function $f(z) \in T(\omega)$ by

$$N_\delta = \{g \in T(\omega) : g(z) = (z-\omega) - \sum_{k=2}^{\infty} b_k(z-\omega)^k, \sum_{k=2}^{\infty} k(1-d)^{k-1}|b_k| \le \delta\}, \tag{2.14}$$

and in particular, for the identity function

$$e(z) = \left(1 - \frac{\omega}{z}\right) z, \tag{2.15}$$

we immediately have

$$N_\delta(e) = \{g \in T(\omega) : g(z) = (z-\omega) - \sum_{k=2}^{\infty} b_k(z-\omega)^k, \sum_{k=2}^{\infty} k(1-d)^{k-1}|b_k| \le \delta\}. \tag{2.16}$$

Theorem 2.6. Let $f(z) \in K(\omega, \alpha, \lambda)$ and $f(z) = (z-\omega) - a_2(z-\omega)^2 - \cdots$ for $0 \le \alpha < 1$, and ω is a fixed point in E. Then

$$|a_2| \le \frac{-(1-\alpha)}{(1+\lambda)(1-d^2)}, \tag{2.17}$$

$$|a_3| \le \frac{-1}{1+2\lambda}\left[\frac{1-\alpha}{3(1-d)(1-d^2)} + \frac{2(1-\alpha)^2}{3(1-d^2)^2}\right],$$

$$|a_4| \le \frac{-1}{1+3\lambda}\left[\frac{1-\alpha}{6(1+d)(1-d)^3} + \frac{(1-\alpha)^2}{2(1-d)(1-d^2)^2} + \frac{(1-\alpha)^3}{3(1-d^2)^3}\right].$$

Proof. Let us define

$$\frac{\lambda(z-\omega)^3 f'''(z)+(1+2\lambda)(z-\omega)^2 f''(z)+(z-\omega)f'(z)}{\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)}=\alpha+(1-\alpha)P_\omega(z). \tag{2.18}$$

That is,

$$\lambda(z-\omega)^3 f'''(z)+(1+2\lambda)(z-\omega)^2 f''(z)+(z-\omega)f'(z)=[\lambda(z-\omega)^2 f''(z)+(z-\omega)f'(z)]$$
$$[\alpha+(1-\alpha)\sum_{k=1}^{\infty}B_k(z-\omega)^k\,] \tag{2.19}$$

comparing the coefficients we get the required result.

From (2.14) and (2.15) we immediately have

$$N_\delta(e)=\{g\in T(\omega):g(z)=(z-\omega)-\sum_{k=2}^{\infty}b_k(z-\omega)^k,\sum_{k=2}^{\infty}k^2(1-d)^{k-1}|b_k|\le\delta\}. \tag{2.20}$$

Theorem 2.7. $H(\omega,\alpha,\lambda)\subset N_\delta(e)$ where $\delta=\frac{2(1-\alpha)}{(2-\alpha)(1-d)(1+\lambda)}$.

Proof. Let $f(z)\in H(\omega,\alpha,\lambda)$. Then in view of lemma 2.1, since $(k-\alpha)(1+\lambda)(1-d)^{k-1}$ is an increasing function of $k(k\ge 2)$, we have

$$(1-d)(1+\lambda)(2-\alpha)\sum_{k=2}^{\infty}a_k\le\sum_{k=2}^{\infty}(k-\alpha)(1-\lambda+\lambda k)(1-d)^{k-1}a_k\le 1-\alpha, \tag{2.21}$$

which immediately yields

$$\sum_{k=2}^{\infty}a_k\le\frac{1-\alpha}{(2-\alpha)(1-d)(1+\lambda)}. \tag{2.22}$$

On the other hand, we also find from(2.3) that

$$(1-d)\sum_{k=2}^{\infty}ka_k(1-\lambda+\lambda k)-\alpha(1-d)\sum_{k=2}^{\infty}a_k(1-\lambda+\lambda k)\le\sum_{k=2}^{\infty}(k-\alpha)a_k(1-d)^{k-1}(1-\lambda+\lambda k)$$
$$\le 1-\alpha. \tag{2.23}$$

From (2.22) and (2.23), we have

$$(1-d)(1+\lambda)\sum_{k=2}^{\infty}ka_k\le(1-\alpha)+\alpha(1-d)(1+\lambda)\sum_{k=2}^{\infty}a_k \tag{2.24}$$

Notes

Notes

$$\leq (1-\alpha) + \frac{\alpha(1-\alpha)}{(2-\alpha)}$$

$$\leq \frac{2(1-\alpha)}{2-\alpha}.$$

$$\sum_{k=2}^{\infty} k a_k \leq \frac{2(1-\alpha)}{(2-\alpha)(1-d)(1+\lambda)}. \tag{2.25}$$

which proved the theorem.

Theorem 2.8. $K(\omega, \alpha, \lambda) \subset N_\delta(e)$ where $\delta = \frac{1-\alpha}{(2-\alpha)(1-d)(1+\lambda)}$.

Proof. Let $f(z) \in K(\omega, \alpha, \lambda)$. Then in view of lemma 2.3, since $k(k-\alpha)(1-\lambda+\lambda k)(1-d)^{k-1}$ is an increasing function of $k (k \geq 2)$, we have

$$2(1-d)(1+\lambda)(2-\alpha)\sum_{k=2}^{\infty} a_k \leq \sum_{k=2}^{\infty} k(k-\alpha)(1-\lambda+\lambda k)(1-d)^{k-1} a_k \leq 1-\alpha, \tag{2.26}$$

which immediately yields

$$\sum_{k=2}^{\infty} a_k \leq \frac{1-\alpha}{2(2-\alpha)(1-d)(1+\lambda)}. \tag{2.27}$$

On the other hand, we also find from(2.8) that

$$2(1-d)(1+\lambda)\sum_{k=2}^{\infty} k a_k - 2\alpha(1-d)(1+\lambda)\sum_{k=2}^{\infty} a_k \leq \sum_{k=2}^{\infty} k(k-\alpha)(1-d)^{k-1} a_k (1-\lambda+\lambda k)$$

$$\leq 1-\alpha. \tag{2.28}$$

From (2.27) and (2.28), we have

$$2(1-d)(1+\lambda)\sum_{k=2}^{\infty} k a_k \leq (1-\alpha) + 2\alpha(1-d)(1+\lambda)\sum_{k=2}^{\infty} a_k \tag{2.29}$$

$$\leq (1-\alpha) + \frac{\alpha(1-\alpha)}{(2-\alpha)} = \frac{2(1-\alpha)}{2-\alpha}.$$

$$\sum_{k=2}^{\infty} k a_k \leq \frac{1-\alpha}{(2-\alpha)(1-d)(1+\lambda)} \tag{2.30}$$

which proved the theorem.

III. Integral Mean Inequality

Lemma3.1. If f and g are analytic in E with $f < g$ then

$$\int_0^{2\pi} |g(re^{i\theta})|^\delta d\theta \leq \int_0^{2\pi} |f(re^{i\theta})|^\delta d\theta \tag{3.1}$$

where $\delta > 0, z = re^{i\theta}, \omega = de^{i\theta}$ and $0 < r + d < 1$.

Applying Lemma 3.1 and(1.2) we prove the following.

Theorem3.2. Let $\delta > 0$. If $f(z) \in H(\omega, \alpha, \lambda)$, then $z = re^{i\theta}, \omega = de^{i\theta}$ and $0 \leq d < r < 1$, we have

$$\int_0^{2\pi} |f(re^{i\theta})|^\delta d\theta \leq \int_0^{2\pi} |f_2(re^{i\theta})|^\delta d\theta \tag{3.2}$$

where

$$f_2(z) = (z - \omega) - \frac{1 - \alpha}{(2 - \alpha)(1 - d)(1 + \lambda)} \ (z - \omega)^2. \tag{3.3}$$

Proof. Let $f(z)$ defined by (1.3) and $f_2(z)$ be given by (3.3) we must show that

$$\int_0^{2\pi} \left| 1 - \sum_{k=2}^{\infty} a_k (z - \omega)^{k-1} \right|^\delta d\theta \leq \int_0^{2\pi} \left| 1 - \frac{1 - \alpha}{(2 - \alpha)(1 - d)(1 + \lambda)} \ (z - \omega) \right|^\delta d\theta. \tag{3.4}$$

By Lemma 3.1, it suffices to show that

$$1 - \sum_{k=2}^{\infty} a_k (z - \omega)^{k-1} < 1 - \frac{1 - \alpha}{(2 - \alpha)(1 - d)(1 + \lambda)} (z - \omega), \tag{3.5}$$

setting

$$1 - \sum_{k=2}^{\infty} a_k (z - \omega)^{k-1} = 1 - \frac{1 - \alpha}{(2 - \alpha)(1 - d)(1 + \lambda)} h(z). \tag{3.6}$$

From(3.6) and (2.1) we obtain

$$|h(z)| = \left| \sum_{k=2}^{\infty} \frac{(2 - \alpha)(1 - d)(1 + \lambda)}{(1 - \alpha)} \ a_k (z - \omega)^{k-1} \right| \tag{3.7}$$

$$\leq |z - \omega| \sum_{k=2}^{\infty} \frac{(k - \alpha)(1 - d)^{k-1}(1 + \lambda)}{(1 - \alpha)} \ a_k \leq |z - \omega|.$$

This completes the proof of theorem 3.2.

Notes

Theorem3.3. Let $\delta > 0$. If $f(z) \in K(\omega, \alpha, \lambda)$, then $z = re^{i\theta}, \omega = de^{i\theta}$ and $0 \leq d < r < 1$, we have

$$\int_0^{2\pi} |f(re^{i\theta})|^\delta d\theta \leq \int_0^{2\pi} |f_2(re^{i\theta})|^\delta d\theta \tag{3.8}$$

Notes

where

$$f_2(z) = (z-\omega) - \frac{1-\alpha}{2(2-\alpha)(1-d)(1+\lambda)} \ (z-\omega)^2. \tag{3.9}$$

Proof. Let $f(z)$ defined by (1.3) and $f_2(z)$ be given by (3.9) we must show that

$$\int_0^{2\pi} \left|1 - \sum_{k=2}^{\infty} a_k (z-\omega)^{k-1}\right|^\delta d\theta \leq \int_0^{2\pi} \left|1 - \frac{1-\alpha}{2(2-\alpha)(1-d)(1+\lambda)} \ (z-\omega)\right|^\delta d\theta. \tag{3.10}$$

By lemma 3.1, it suffices to show that

$$1 - \sum_{k=2}^{\infty} a_k (z-\omega)^{k-1} < 1 - \frac{1-\alpha}{2(2-\alpha)(1-d)(1+\lambda)}(z-\omega), \tag{3.11}$$

setting

$$1 - \sum_{k=2}^{\infty} a_k (z-\omega)^{k-1} = 1 - \frac{1-\alpha}{2(2-\alpha)(1-d)(1+\lambda)} h(z). \tag{3.12}$$

From(3.12) and (2.6) we obtain

$$|h(z)| = \left|\sum_{k=2}^{\infty} \frac{2(2-\alpha)(1-d)(1+\lambda)}{(1-\alpha)} \ a_k (z-\omega)^{k-1}\right| \tag{3.13}$$

$$\leq |z-\omega| \sum_{k=2}^{\infty} \frac{2(k-\alpha)(1-d)^{k-1}(1+\lambda)}{(1-\alpha)} \ a_k$$

$$\leq |z-\omega|.$$

This completes the proof of theorem 3.3.

Acknowledgement

The first author thanks the support provided by Science and Engineering Research Board (DST), New Delhi. Project No: SR/S4/MS:716/10.

References Références Referencias

[1] Acu,Mugur and Shigeyoshi Owa(2005). On Some Subclasses of Univalent Functions, Journal of Inequalities in Pure and Applied Mathematics 6(3), 1-14.

[2] Goodman,A.W.(1957).Univalent Functions and analytic curves proc. Amer. Math. Soc. 8(3),598-601.

[3] Kanas, S and F.Ronning(1999). Uniformly Starlike and convex functions and other related classes of Univalent Functions. Anna Univ. Mariae curie - slowdowska Section A 53(53), 95-105.

[4] Oladipo, A.T.(2009)- On subclasses of analytic and univalent functions. Advances in Applied Mathematical Analysis 4(1), 87-93.

[5] Ruschewegh, Stephan(1981), Neighbourhoods of Univalent functions Proc. Amer. Math. Soc., 8(3),521-527.

[6] Wald, J.K(1978), On Starlike functions, Ph.D Thesis, University of Deleware, New Ark Deleware.

Notes