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Helical-One, Two, Three-Revolutional Cyclical Surfaces

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Abstract - This paper describes method for modelling of helical-*n*-revolutional cyclical surfaces. The axis of the cyclical surface Φ_1 is the helix s_1 created by revolving the point about n each other revolving axes o_n ($n = 1,2,3$), that move together with Frenet-Serret moving trihedron along the cylindrical helix s. Particular evolutions are determined by its angular velocity and orientation. The moving circle along the helix s or s_1, where its center lies on the helix and circle lies in the normal plane of the helix creates the cyclical surface.

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GJSFR-F Classification : *MSC 2010: 05B20*

HELICAL-ONE, TWO, THREE-REVOLUTIONAL CYCLICALSURFACES

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Abstract - This paper describes method for modelling of helical-*n*-revolutional cyclical surfaces. The axis of the cyclical surface Φ_1 is the helix s_1 created by revolving the point about *n* each other revolving axes o_n (n = 1,2,3), that move together with Frenet-Serret moving trihedron along the cylindrical helix *s*. Particular evolutions are determined by its angular velocity and orientation. The moving circle along the helix *s* or s_1, where its center lies on the helix and circle lies in the normal plane of the helix creates the cyclical surface.

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I. Introduction

Let three-dimensional Euclidean space E^3 is determined by Cartesian coordinate system $(0,x,y,z)$. In this space is given cylindrical helix s with axis identical with coordinate axis z determined by vector function (Fig.1)

$$\mathbf{r}(v) = (x_s, y_s, z_s, 1) = (a\cos mv, sg\, a\sin mv, bv, 1),\ v \in \langle 0, 2\pi \rangle, \tag{1}$$

where parameter a is radius of the helix, b is the reduced pitch, sg determined orientation of the helix, ($sg = +1$ for right-handed and $sg = -1$ for left-handed revolution), m is number of pitches. Let $(0', n, b, t)$ be Frenet-Serret moving trihedron of the cylindrical helix s represented by regular square matrix

$$\mathbf{M}(v) = \begin{pmatrix} n_x(v) & n_y(v) & n_z(v) & 0 \\ b_x(v) & b_y(v) & b_z(v) & 0 \\ t_x(v) & t_y(v) & t_z(v) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

where the matrix elements are the coordinates of unit vectors of the principle normal n, binormal b and tangent t of the helix s in the point $0' \in s$ in the coordinate system $(0,x,y,z)$

$$\mathbf{t}(v) = (t_x(v), t_y(v), t_z(v)) = \frac{\mathbf{r}'(v)}{|\mathbf{r}'(v)|}, \tag{3}$$

$$\mathbf{b}(v) = (b_x(v), b_y(v), b_z(v)) = \frac{\mathbf{r}'(v) \times \mathbf{r}''(v)}{|\mathbf{r}'(v) \times \mathbf{r}''(v)|}, \tag{4}$$

$$\mathbf{n}(v) = (n_x(v), n_y(v), n_z(v)) = \mathbf{b}(v) \times \mathbf{t}(v). \tag{5}$$

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Transformations of revolutions about coordinate axes x, y, z are represented by matices $\mathbf{T}_x(\varphi,\psi)$, $\mathbf{T}_y(\varphi,\psi)$, $\mathbf{T}_z(\varphi,\psi)$, where φ is angle and ψ is orientation of the revolution, transformation of translation is represented by matrix $\mathbf{T}(\pm d_x,\ \pm d_y, \pm d_z)$, where $(\pm d_x,\ \pm d_y, \pm d_z)$ is translation vector determined by its coordinates (6), (7):

$$\mathbf{T}_x(\varphi,\psi)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & \cos\varphi & \psi\sin\varphi & 0\\ 0 & -\psi\sin\varphi & \cos\varphi & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\ \mathbf{T}_y(\varphi,\psi)=\begin{pmatrix}\cos\varphi & 0 & \psi\sin\varphi & 0\\ 0 & 1 & 0 & 0\\ -\psi\sin\varphi & 0 & \cos\varphi & 0\\ 0 & 0 & 0 & 1\end{pmatrix}, \tag{6}$$

$$\mathbf{T}_z(\varphi,\psi)=\begin{pmatrix}\cos\varphi & \psi\sin\varphi & 0 & 0\\ -\psi\sin\varphi & \cos\varphi & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\ \mathbf{T}(\pm d_x,\ \pm d_y, \pm d_z)=\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ \pm d_x & \pm d_y & \pm d_z & 1\end{pmatrix}. \tag{7}$$

The moving circle $c=(0',r)$ along the helix s, where its center $0'$ lies in the normal plane determined by principal normal n and binormal b of the helix in the point $0'\in s$ creates the cyclical surface Φ. The vector function of this surface is

$$\mathbf{P}(u,v)=\mathbf{r}(v)+\mathbf{c}(u).\mathbf{M}(v),\ u\in\langle 0,2\pi\rangle,\ v\in\langle 0,2\pi\rangle, \tag{8}$$

where $\mathbf{r}(v)$ is vector function of the helix s expressed in equation (1), $\mathbf{M}(v)$ is transformation matrix of the coordinate system $(0',n,b,t)$ into coordinate system $(0,x,y,z)$ (2) and $\mathbf{c}(u)=(r\cos u,\ r\sin u,\ 0,1), u\in\langle 0,2\pi\rangle$ is vector function of the circle c determined by its center $0'$ and radius r (Fig.2). In Fig.3 there are displayed two screws of the right-handed cyclical surface Φ together with the cylindrical surface on which helix s is wound.

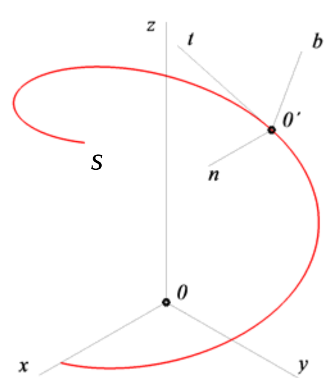

Fig. 1 : Helix s with Trihedron

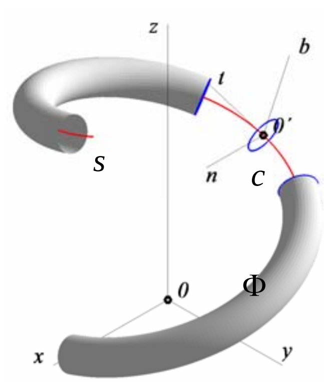

Fig. 2 : Cyclical Surface Φ

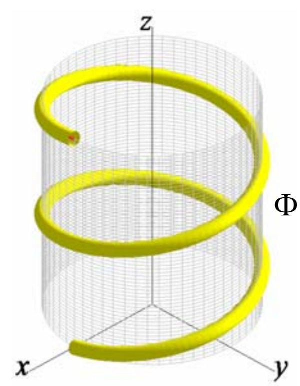

Fig. 3 : Surface Φ and Cylinder

II. Cyclical Helical Surface Created by One Revolution

The helix s_1 created by revolution of the point $P(x_0,y_0,z_0,1)$ about the axis o_1 connected to the moving trihedron of the helix s, is represented by vector function

$$\mathbf{r}_1(v)=\mathbf{r}(v)+(x_0,\ y_0,\ z_0,1).\ \mathbf{T}_1(m_1v,\ sg_1).\mathbf{M}(v) \tag{9}$$

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and cyclical surface Φ_1 created in a similar way as surface Φ by vector function

$$\mathbf{P}_1(u,v)=\mathbf{r}_1(v)+\mathbf{c}_1(u).\mathbf{M}_1(v),\ u\in\langle 0,2\pi\rangle,\ v\in\langle 0,2\pi\rangle, \tag{10}$$

where $\mathbf{r}_1(v)$ is vector function of the helix s_1 expressed in equation (9), $\mathbf{M}_1(v)$ is transformation matrix of the coordinate system $(0'',n',b',t')$ into coordinate system $(0,x,y,z)$ (11), $\mathbf{c}_1(u)=(r_1\cos u,\, r_1\sin u,0,1), u\in\langle 0,2\pi\rangle$ is vector function of the circle c_1 determined by center $0''\in s_1$ and radius r_1

$$\mathbf{M}_1(v)=\begin{pmatrix} n'_x(v) & n'_y(v) & n'_z(v) & 0\\ b'_x(v) & b'_y(v) & b'_z(v) & 0\\ t'_x(v) & t'_y(v) & t'_z(v) & 0\\ 0 & 0 & 0 & 1\end{pmatrix}. \tag{11}$$

Elements of this matrix are coordinates of unit vectors of the principle normal n', binormal b' and tangent t' of the helix s_1 in the point $0''\in s_1$ in the coordinate system $(0',n,b,t)$

$$\mathbf{t}'(v)=\left(t'_x(v),t'_y(v),t'_z(v)\right)=\frac{\mathbf{r}'_1(v)}{|\mathbf{r}'_1(v)|} \tag{12}$$

$$\mathbf{b}'(v)=\left(b'_x(v),b'_y(v),b'_z(v)\right)=\frac{\mathbf{r}'_1(v)\times\mathbf{r}''_1(v)}{|\mathbf{r}'_1(v)\times\mathbf{r}''_1(v)|}, \tag{13}$$

$$\mathbf{n}'(v)=\left(n'_x(v),n'_y(v),n'_z(v)\right)=\mathbf{b}'(v)\times\mathbf{t}'(v). \tag{14}$$

a) *Revolution about tangent t of the helix s*

The helix created by the revolution of the point P about the axis $o_1=t$ is expressed by vector function (9), in which matrix $\mathbf{T}_1(m_1 v, sg_1)=\mathbf{T}_z(m_1 v, sg_1)$. In Fig.4 is displayed cyclical surface Φ, whose axis is helix s with parameters $m=2, sg=+1$ and surface Φ_1, whose axis is helix s_1 created by revolution of the point $P=(d,0,0,1)$ about tangent t of the helix s with parameters $m_1=8m, sg_1=+1$.

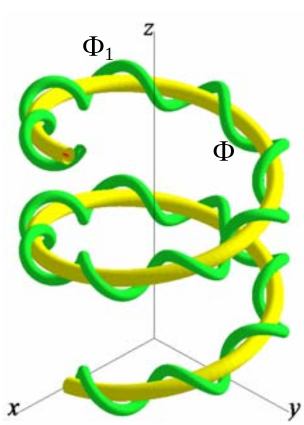

Fig. 4 : Cyclical Surfaces Φ, Φ_1

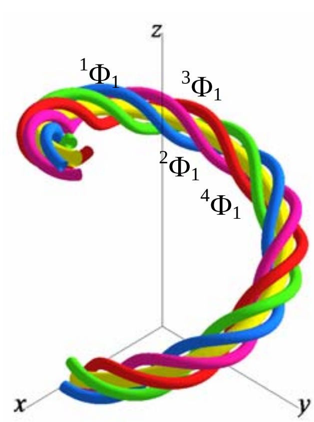

Fig. 5 : 4 Surfaces ${}^i\Phi_1$

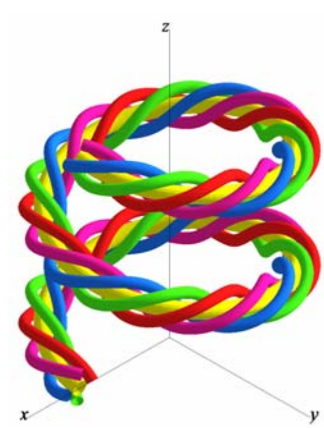

Fig. 6 : Left-handed Surfaces $\Phi, {}^i\Phi_1$

In Fig.5 are displayed $k=4$ surfaces ${}^{i}\Phi_1$, whose axes are helix ${}^{i}s_1$, $i=1,\ldots,k$ created by revolution of the points ${}^{i}P=(d\cos i\alpha, d\sin i\alpha, 0, 1)$, $\alpha=2\pi/k$ about tangent t of the helix s with parameters $m_1=4m, sg_1=+1$, in Fig.6 are displayed the same surfaces with altered orientation of the revolution $m=2, sg=-1$, $m_1=4m, sg_1=-1$.

b) Revolution about principal normal n of the helix s

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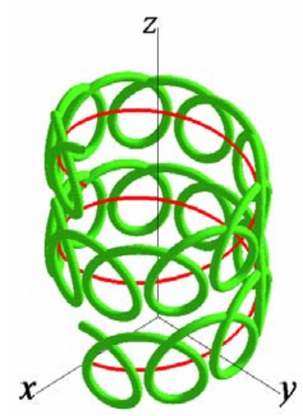

Fig. 7 : Normal Surface Φ_1

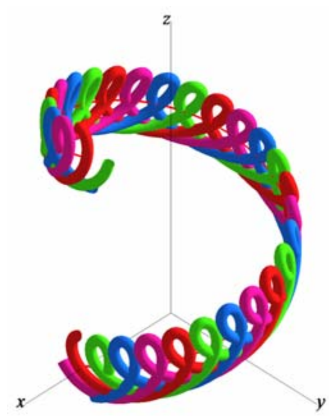

Fig. 8 : 4 Surfaces ${}^{i}\Phi_1$

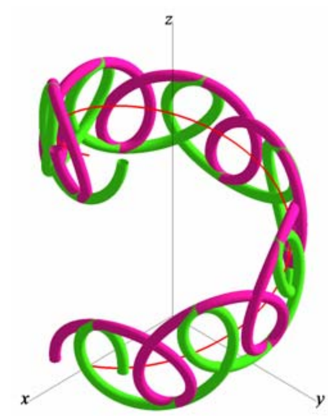

Fig. 9 : Normal Surfaces ${}^{1}\Phi_1$, ${}^{3}\Phi_1$

The helix s_1 created by the revolution of the point P about the axis $o_1=n$ is expressed by vector function (9), in which matrix $\mathbf{T}_1(m_1 v, sg_1)=\mathbf{T}_x(m_1 v, sg_1)$. In Fig.7 is displayed helix s with parameters $m=2, sg=+1$ and normal surface Φ_1, whose axis is helix s_1 created by revolution of the point $P=(0,d,0,1)$ about normal n of the helix s with parameters $m_1=10m, sg_1=+1$, in Fig.8 are displayed $k=4$ normal surfaces ${}^{i}\Phi_1$, whose axes are helix ${}^{i}s_1$, $i=1,\ldots,k$ created by revolution of the points ${}^{i}P=(0, d\cos i\alpha, d\sin i\alpha, 1)$, $\alpha=2\pi/k$ about normal n of the helix s with parameters $m_1=7m, sg_1=-1$, in Fig.9 are displayed surfaces ${}^{1}\Phi_1$, ${}^{3}\Phi_1$ with altered orientation of the revolution $sg_1=\pm1$.

c) Revolution about binormal b of the helix s

The helix created by the revolution of the point P about axis $o_1=b$ is expressed by vector function (9), in which matrix $\mathbf{T}_1(m_1 v, sg_1)=\mathbf{T}_y(m_1 v, sg_1)$. In Fig.10 is displayed helix s with parameters $m=2, sg=+1$ and binormal surface Φ_1, whose axis is helix s_1 created by revolution of the point $P=(d,0,0,1)$ about binormal b of the helix s with parameters $m_1=10m, sg_1=+1$, in Fig.11 is displayed binormal surface with parameters $m_1=8m, sg_1=-1$, in Fig.12 are surfaces ${}^{1}\Phi_1$, ${}^{3}\Phi_1$ created by revolution of the points ${}^{i}P=(d\cos i\alpha, d\sin i\alpha, 0, 1)$, $\alpha=2\pi/k$ about binormal b of the helix with altered orientation of the revolution $sg_1=\pm1$.

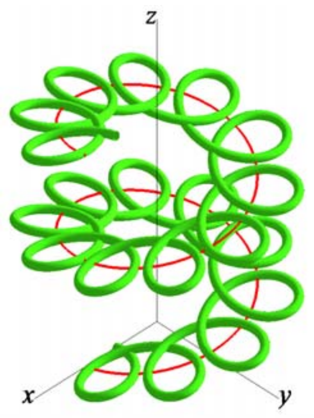

Fig. 10 : Binormal Surface Φ_1

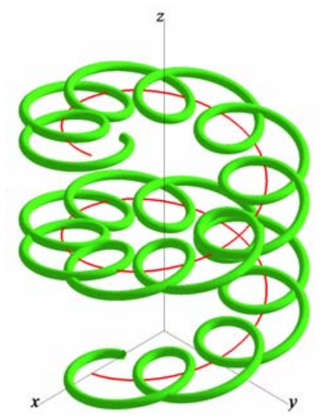

Fig. 11 : 4 Surfaces ${}^{i}\Phi_1$

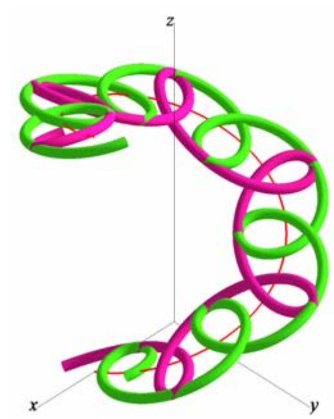

Fig. 12 : Binormal Surfaces ${}^{1}\Phi_1, {}^{3}\Phi_1$

III. Cyclical Helical Surface Created by Two Revolutions

The helix s_1 created by revolution of the point $P(x_0, y_0, z_0, 1)$ about axis o_2, which revolves about the axis o_1 identical with one edge of the moving trihedron of the helix s is represented by vector function

$$\mathbf{r}_1(v) = \mathbf{r}(v) + (x_0, y_0, z_0, 1).\mathbf{T}_2(m_2 v, sg_2).\mathbf{T}_1(m_1 v, sg_1).\mathbf{M}(v), \tag{15}$$

where matrix $\mathbf{T}_2(m_2 v, sg_2)$ represents revolution of the point P about the axis o_2 and matrix $\mathbf{T}_1(m_1 v, sg_1)$ represents revolution of the axis o_2 about the axis o_1.

a) Revolution about two parallel axes

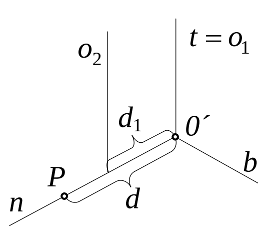

Fig. 13

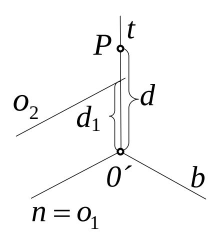

Fig. 14

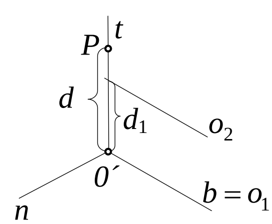

Fig. 15

If the helix s_1 is created by revolution of the point P about two parallel axes $o_2 \parallel o_1$ and $o_1 = t$, where $d_1 = |o_1 o_2|$ is the distance between them, then (Fig.13)

$$\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(-d_1, 0, 0).\mathbf{T}_z(m_2 v, sg_2).\mathbf{T}(+d_1, 0, 0), \ \mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_z(m_1 v, sg_1). \tag{16}$$

In Fig.16 is displayed this surface Φ_1 with parameters $m_1 = 8m, sg_1 = +1, m_2 = 4m_1, sg_2 = +1$.

If the helix s_1 is created by revolution of the point P about parallel axes $o_2 \parallel o_1$ and $o_1 = n$, where $d_1 = |o_1 o_2|$, then (Fig.14)

$$\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(0, 0, -d_1).\mathbf{T}_x(m_2 v, sg_2).\mathbf{T}(0, 0, +d_1), \ \mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_x(m_1 v, sg_1). \tag{17}$$

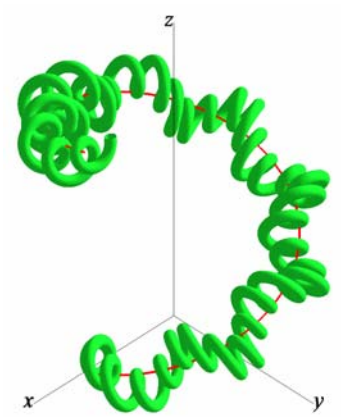

Fig. 16 : $o_2 \parallel o_1,\ o_1 = t$

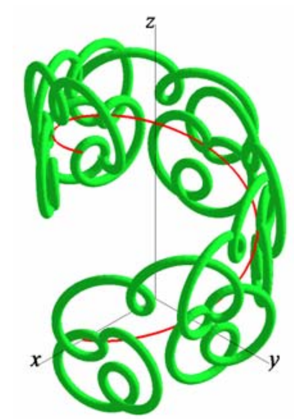

Fig. 17 : $o_2 \parallel o_1,\ o_1 = n$

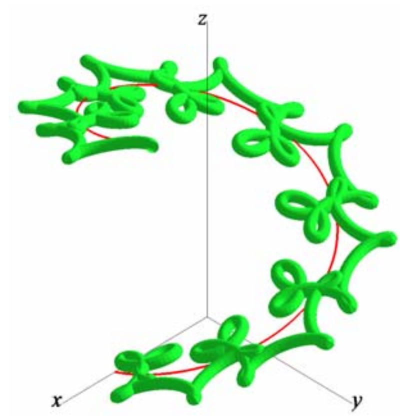

Fig. 18 : $o_2 \parallel o_1,\ o_1 = b$

In Fig.17 is displayed this surface Φ_1 with parameters $m_1 = 6m, sg_1 = +1, m_2 = 4m_1, sg_2 = +1$.

If the helix s_1 is created by revolution of the point $P = (0,0,d,1)$ about parallel axes $o_2 \parallel o_1$ and $o_1 = b$, where $d_1 = |o_1 o_2|$, then (Fig.15)

$$\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(0,0,-d_1).\mathbf{T}_y(m_2 v, sg_2).\mathbf{T}(0,0,+d_1),\ \mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_y(m_1 v, sg_1). \tag{18}$$

In Fig.18 is displayed this surface Φ_1 with parameters $m_1 = 8m, sg_1 = -1, m_2 = 5m_1, sg_2 = +1$.

b) Revolution about two intersecting axes

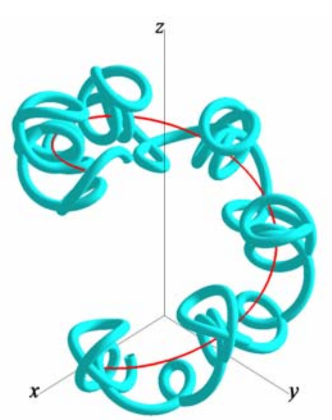

Fig. 19 : $(o_2 = t) \perp (o_1 = n)$

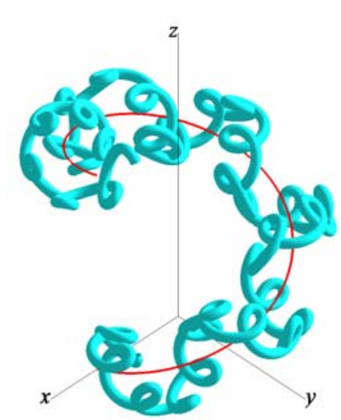

Fig. 20 : $(o_2 = n) \perp (o_1 = t)$

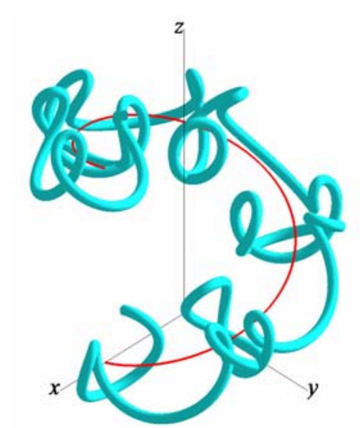

Fig. 21 : $(o_2 = n) \perp (o_1 = b)$

In Fig.19 is displayed surface created by revolution of the point $P = (2,2,0,1)$ about mutually perpendicular axes $(o_2 = t) \perp (o_1 = n)$ determined by the parameters $m_1 = 6m, sg_1 = -1$, $m_2 = 4m_1, sg_2 = -1$, where matrices $\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}_z(m_2 v, sg_2)$, $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_x(m_1 v, sg_1)$, in Fig.20 is displayed surface created by revolution of the point $P = (2.2,1.2,0,1)$ about mutually perpendicular axes $(o_2 = n) \perp (o_1 = t)$ determined by parameters $m_1 = 6m, sg_1 = +1$, $m_2 = 6m_1, sg_2 = +1$, and matrices $\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}_x(m_2 v, sg_2)$, $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_z(m_1 v, sg_1)$, here we see action of changing the order of the revolutions to form of the surfaces. In Fig.21 is displayed surface created by revolution of the point $P = (2.5,2.5,0,1)$ about mutually perpendicular axes $(o_2 = n) \perp (o_1 = b)$ determined by parameters

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$m_1 = 5m, sg_1 = -1$, $m_2 = 3m_1, sg_2 = +1$, $\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}_x(m_2 v, sg_2)$, $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_y(m_1 v, sg_1)$.In Figs.22,23 is displayed surface created by revolution of the point $P = (d,0,0,1)$ about intersecting axes $o_2 \times (o_1 = t)$ determined by the parameters $m_1 = 6m, sg_1 = +1$, $m_2 = 6m_1, sg_2 = -1$, $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_z(m_1 v, sg_1)$, $\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}_y(\alpha,+1).\mathbf{T}_x(m_2 v, sg_2).\mathbf{T}_y(\alpha,-1)$.

c) Revolution about two skew axes

In Figs.24,25 is displayed surface created by revolution of the point $P = (d,0,0,1)$ about mutually skew axes $(o_2 \parallel n) / (o_1 = t)$, determined by parameters $m_1 = 4m, sg_1 = +1$, $m_2 = 8m_1, sg_2 = +1$, where transformation matrices of two revolutions are

$$\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_z(m_1 v, sg_1),\ \ \mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(0,0,-d_1).\mathbf{T}_x(m_2 v, sg_2).\mathbf{T}(0,0,+d_1).$$

In Figs.26,27 is displayed surface created by revolution of the point $P = (d,0,0,1)$ about mutually skew axes $(o_2 \times t, o_2 \times n) / (o_1 = t)$ determined by parameters $m_1 = 6m, sg_1 = +1$, $m_2 = 4m_1, sg_2 = -1$, and transformation matrices $\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}_y(\alpha,+1).\mathbf{T}_x(m_2 v, sg_2).\mathbf{T}_y(\alpha,-1)$, $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_y(m_1 v, sg_1)$.

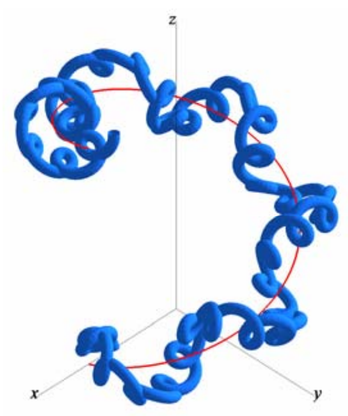

Fig. 22: $o_2 \times (o_1 = t)$

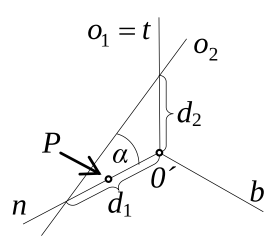

Fig. 23

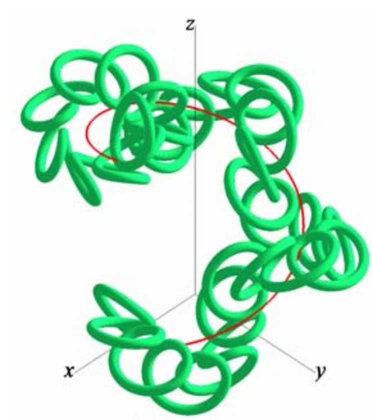

Fig. 24: $(o_2 \parallel n) / (o_1 = t)$

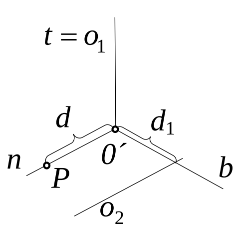

Fig. 25

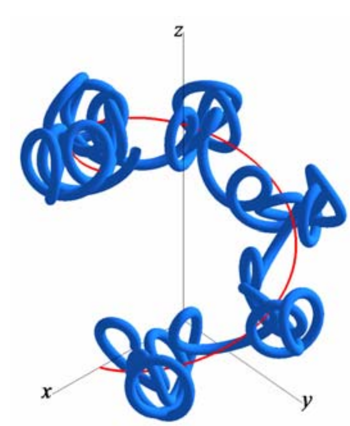

Fig. 26: $(o_2 \times t, o_2 \times n) / (o_1 = t)$

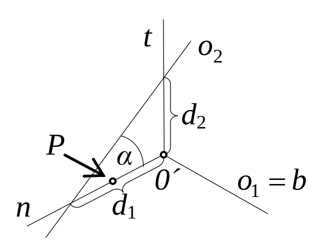

Fig. 27

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IV. Cyclical Helical Surface Created by Three Revolutions

The helix s_1 created by the revolution of the point $P=(x_0, y_0, z_0, 1)$ about the axis o_3, which revolves about the axis o_2 and this revolves about the axis o_1 identical with any edge of the moving trihedron of the helix *s* is represented by vector function

$$\mathbf{r}_1(v)=\mathbf{r}(v)+(x_0, y_0, z_0, 1)\,.\,\mathbf{T}_3(m_3 v, sg_3).\,\mathbf{T}_2(m_2 v, sg_2).\,\mathbf{T}_1(m_1 v, sg_1).\,\mathbf{M}(v), \qquad (19)$$

Notes

where matrix $\mathbf{T}_3(m_3 v, sg_3)$ represents revolution of the point *P* about the axis o_3, matrix $\mathbf{T}_2(m_2 v, sg_2)$ represents revolution of the axis o_3 about the axis o_2 and matrix $\mathbf{T}_1(m_1 v, sg_1)$ represents revolution of the axis o_2 about the axis o_1.

a) Revolution about three parallel axes

In Fig.28 is displayed surface created by revolution about three parallel axes $o_3 \parallel o_2 \parallel o_1 = t$ determined by parameters $m_1 = 4m, sg_1 = +1$, $m_2 = 4m_1, sg_2 = +1$, $m_3 = 3m_2, sg_1 = +1$, matrices $\mathbf{T}_3(m_3 v, sg_3) = \mathbf{T}(-d_2, 0, 0).\,\mathbf{T}_z(m_3 v, sg_3).\mathbf{T}(+d_2, 0, 0)$, $\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(-d_1, 0, 0).\,\mathbf{T}_z(m_2 v, sg_2).\mathbf{T}(+d_1, 0, 0)$, $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_z(m_1 v, sg_1)$. In Fig.29 is displayed surface created by revolution about three parallel axes $o_3 \parallel o_2 \parallel o_1 = n$ determined by parameters $m_1 = 4m, sg_1 = +1$, $m_2 = 4m_1, sg_2 = +1$, $m_3 = 4m_2, sg_1 = +1$ and by transformation matrices $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_x(m_1 v, sg_1)$
$\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(0,0,-d_1).\,\mathbf{T}_z(m_2 v, sg_2).\mathbf{T}(0,0,+d_1)$, $\mathbf{T}_3(m_3 v, sg_3) = \mathbf{T}(0,0,-d_2).\,\mathbf{T}_x(m_3 v, sg_3).\mathbf{T}(0,0,+d_2)$.

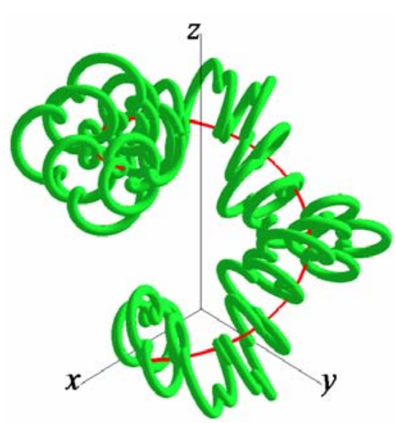

Fig. 28 : $o_3 \parallel o_2 \parallel o_1 = t$

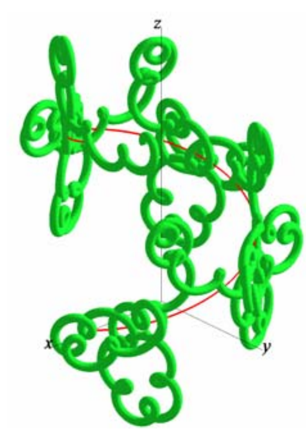

Fig. 29 : $o_3 \parallel o_2 \parallel o_1 = n$

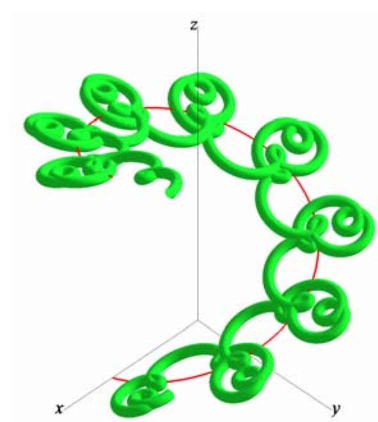

Fig. 30 : $o_3 \parallel o_2 \parallel o_1 = b$

In Fig.30 is displayed surface created by revolution about three parallel axes $o_3 \parallel o_2 \parallel o_1 = b$ determined by parameters $m_1 = 3m, sg_1 = +1$, $m_2 = 3m_1, sg_2 = +1$, $m_3 = 3m_2, sg_1 = +1$ and transformation matrices $\mathbf{T}_3(m_3 v, sg_3) = \mathbf{T}(0,0,-d_2).\,\mathbf{T}_y(m_3 v, sg_3).\mathbf{T}(0,0,+d_2)$,
$\mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(0,0,-d_1).\,\mathbf{T}_y(m_2 v, sg_2).\mathbf{T}(0,0,+d_1)$, $\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}_y(m_1 v, sg_1)$.

b) Revolution about three perpendicular axes

In Figs.31,32,33 are displayed surfaces created by revolution of the point $P = (d, d, 0, 1)$ about three perpendicular axes with common point $o_3 \perp o_2 \perp o_1$, which are identical with edges of the trihedron of the helix *s*, where parameters are the same $m_1 = 3m, sg_1 = +1$, $m_2 = 3m_1, sg_2 = +1$, $m_3 = 3m_2, sg_1 = +1$, but the order of the revolutions changes.

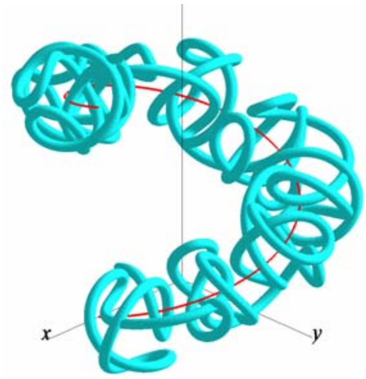

Fig. 31 : $o_3 = n, o_2 = b, o_1 = t$

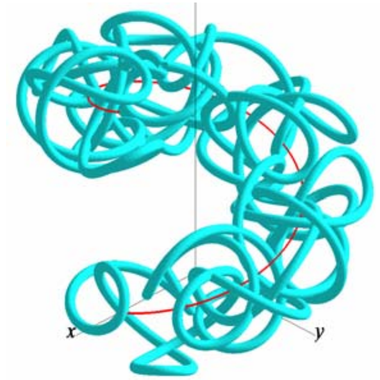

Fig. 32 : $o_3 = n, o_2 = t, o_1 = b$

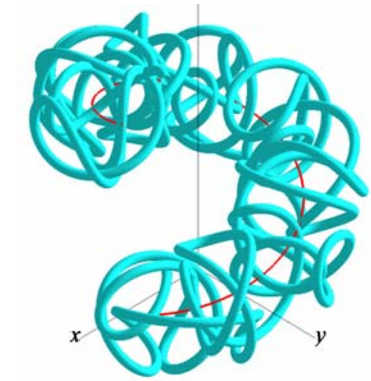

Fig. 33 : $o_3 = b, o_2 = n, o_1 = t$

c) *Revolution about three skew axes*

In Fig.34 are displayed surfaces created by revolution of the point $P = (0,0,0,1)$ about three skew axes $o_3 / o_2 / o_1$, which are parallel with edges of the trihedron of the helix s, $o_3 \parallel n$, $o_2 \parallel b$, $o_3 \parallel t$, where parameters are $m_1 = 4m, sg_1 = +1$, $m_2 = 2m_1, sg_2 = +1$, $m_3 = 6m_2, sg_1 = +1$, transformation matrices of three revolutions are

$$\mathbf{T}_3(m_3 v, sg_3) = \mathbf{T}(0,0,-d_3).\mathbf{T}_x(m_3 v, sg_3).\mathbf{T}(0,0,+d_3),\ \mathbf{T}_2(m_2 v, sg_2) = \mathbf{T}(-d_2,0,0).\mathbf{T}_y(m_2 v, sg_2).\mathbf{T}(+d_2,0,0),$$
$$\mathbf{T}_1(m_1 v, sg_1) = \mathbf{T}(0,-d_1,0).\mathbf{T}_z(m_1 v, sg_1).\mathbf{T}(0,+d_2,0).$$

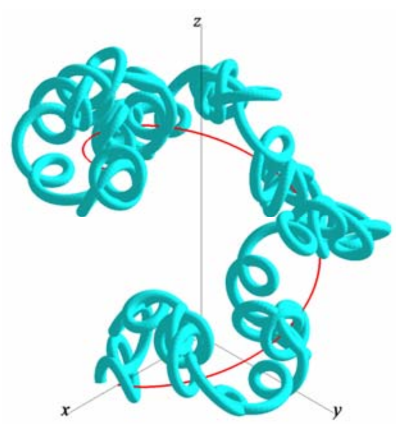

Fig. 34 : $o_3 \parallel n,\ o_2 \parallel b,\ o_3 \parallel t$

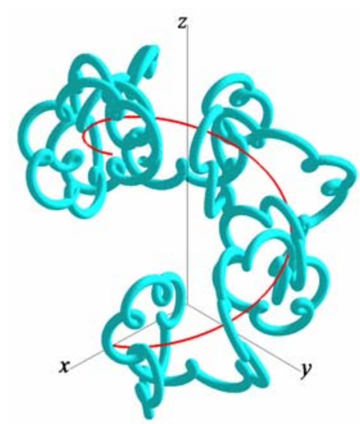

Fig. 35 : $o_3 \parallel n, o_2 = n, o_1 = b$

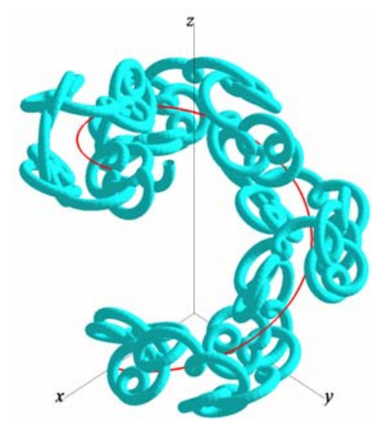

Fig. 36 : $o_3 \parallel n, o_2 = n, o_1 = t$

In Figs.35,36 are displayed surfaces created by revolution about the axes $o_3 \parallel n, o_2 = n, o_1 = b$ or axes $o_3 \parallel n, o_2 = n, o_1 = t$.

V. Conclusion

The described method of modeling of the helical-*n*-revolutional cyclical surfaces makes it possible to model different interest surfaces simply by changing the parameters.

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